Annals of Fuzzy Mathematics and Informatics
Volume 9, No. 2, (February 2015), pp. 279–292
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Soft interval-valued intuitionistic fuzzy rough sets

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Received 17 April 2014; Revised 24 July 2014; Accepted 22 August 2014

Abstract. Soft set theory, fuzzy set theory and rough set theory are all mathematical tools for dealing with uncertainties and are closely related. Feng et al. introduced the notions of rough soft set, soft rough set and soft rough fuzzy set by combining fuzzy set, rough set and soft set all together. This paper is devoted to the discussions of the combinations of interval-valued intuitionistic fuzzy set, rough set and soft set. A new model, namely soft interval-valued intuitionistic fuzzy rough set is proposed and it's properties are derived. Also a soft interval-valued intuitionistic fuzzy rough set based multi criteria group decision making scheme is presented. The proposed scheme is illustrated by an example regarding the car selection problem.

2010 AMS Classification: 03E72, 90B50, 54A40, 06D72

Keywords: Soft set, Rough set, Soft rough set, Soft fuzzy rough set, Soft interval valued intuitionistic fuzzy rough set, Decision making.

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1. Introduction

The soft set theory, initiated by Molodtsov [19] in 1999, is a completely generic mathematical tool for modeling vague concepts. In soft set theory there is no limited condition to the description of objects; so researchers can choose the form of parameters they need, which greatly simplifies the decision making process and make the process more efficient in the absence of partial information. Although many mathematical tools are available for modeling uncertainties such as probability theory, fuzzy set theory, rough set theory, interval valued mathematics etc, but there are inherent difficulties associated with each of these techniques. Moreover all these techniques lack in the parameterization of the tools and hence they could not be applied successfully in tackling problems especially in areas like economic, environmental and social problems domains. Soft set theory is standing in a unique way in the sense that it is free from the above difficulties and it has a unique scope for many applications in a multi-dimentional way. Soft set theory has a rich potential for application in many directions, some of which are reported by Molodtsov [19] in his work. He successfully applied soft set theory in areas such as the smoothness of functions, game theory, operations research, Riemann integration and so on. Later on Maji et al.[15] presented some new definitions on soft sets such as subset, union, intersection and complements of soft sets and discussed in detail the application of soft set in decision making problem. Based on the analysis of several operations on soft sets introduced in [15], Ali et al. [2] presented some new algebraic operations for soft sets and proved that certain De Morgan's law holds in soft set theory with respect to these new definitions. Chen et al. [6] presented a new definition of soft set parameterization reduction and compared this definition with the related concept of knowledge reduction in the rough set theory. Kong et al. [14] introduced the definition of normal parameter reduction into soft sets and then presented a heuristic algorithm to compute normal parameter reduction of soft sets. By amalgamating the soft sets and algebra, Aktas and Cagman [1] introduced the basic properties of soft sets, compared soft sets to the related concepts of fuzzy sets [28] and rough sets [20], pointed out that every fuzzy set and every rough set may be considered as a soft set. Jun [13] applied soft sets to the theory of BCK/BCI-algebra and introduced the concept of soft BCK/BCI-algebras. Feng et al. [9] defined soft semi rings and several related notions to establish a connection between the soft sets and semi rings. Sun et al. [23] presented the definition of soft modules and constructed some basic properties using modules and Molodtsov's definition of soft sets. Maji et al. [16] presented the concept of the fuzzy soft set which is based on a combination of the fuzzy set and soft set models. Roy and Maji [21] presented a fuzzy soft set theoretic approach towards a decision making problem. Yang et al. [26] defined the operations on fuzzy soft sets, which are based on three fuzzy logic operations: negation, triangular norm and triangular co-norm. Xiao et al. [24] proposed a combined forecasting approach based on fuzzy soft set theory. Yang et al. [25] introduced the concept of interval valued fuzzy soft set and a decision making problem was analyzed by the interval valued fuzzy soft set. Feng et al. [10] presented an adjustable approach to fuzzy soft set based decision making and give some illustrative examples. The notion of intuitionistic fuzzy set was initiated by Atanassov [3] as a generalization of fuzzy set. Combining soft sets with intuitionistic fuzzy sets, Maji et al. [17] introduced intuitionistic fuzzy soft sets, which are rich potentials for solving decision making problems. The notion of the interval-valued intuitionistic fuzzy set was introduced by Atanassov and Gargov [4]. The distance between intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets was proposed by A.K.Shyamal and M.Pal[22] in 2007. In 2009 M.Bhowmik and M. Pal [5]introduced the concept of partition of generalized interval-valued intuitionistic fuzzy sets. In 2010, Jiang et al. [12] introduced the concept of interval valued intuitionistic fuzzy soft sets. Over the years, the theories of fuzzy sets and rough sets have become much closer to each other for practical needs to use both of these two theories complementarily for managing uncertainty that arises from inexact, noisy or incomplete information. Hybrid models combining fuzzy set with rough sets have arisen in various guises in different settings. For instance, based on the equivalence relation on the universe of discourse, Dubois and Prade [7] introduced the lower and upper approximation of fuzzy sets in a Pawlak's approximation space [20] and obtained a new notion called rough fuzzy sets. Alternatively, a fuzzy similarity relation can be used to replace an equivalence relation, and the resulting notion is called fuzzy rough sets [7]. In general, a rough fuzzy set is the approximation of a fuzzy set in a crisp approximation space, whereas a fuzzy rough set is the approximation of a crisp set or fuzzy set in a fuzzy approximation space. Feng el al. [11] provided a framework to combine rough sets and soft sets all together, which gives rise to several interesting new concepts such as soft rough sets and rough soft sets. A rough soft set is the approximation of a soft set in a Pawlak approximation space, where as a soft rough set is based on soft rough approximations in a soft approximation space. Feng [8] presented a soft rough set based multi-criteria group decision making scheme. Motivated by Dubois and Prade's original idea about rough fuzzy set, Feng et al. [11] introduced lower and upper soft rough approximations of fuzzy sets in a soft approximation space and obtained a new hybrid model called soft rough fuzzy set. By employing a fuzzy soft set to granulate the universe of discourse, Meng et al. [18] introduced a more general model called soft fuzzy rough set. The aim of this paper is to introduce the concept of soft interval valued intuitionistic fuzzy rough sets. Also some properties based on soft interval valued intuitionistic fuzzy rough sets are presented. Finally a soft interval valued intuitionistic fuzzy rough set based multi criteria group decision making scheme is presented. The proposed scheme is illustrated by an example regarding the car selection problem. Actually the structure of soft fuzzy rough set introduced by Meng et al. is of complex form and will be more complicated whenever we extend it to soft intuitionistic fuzzy rough set and soft interval valued intuitionistic fuzzy rough set. But we have defined soft interval valued intuitionistic fuzzy rough sets in such a manner that calculations portion in solving decision making problems can be simplified. .

2. Preliminaries

This section presents a review of some fundamental notions of fuzzy sets, soft sets and their combinations and generalizations. We refer to [1], [2], [3], [4], [15], [16], [17], [19], [28] for details. The theory of fuzzy sets initiated by Zadeh provides an appropriate framework for representing and processing vague concepts by allowing partial memberships. Since establishment, this theory has been actively studied by both mathematicians and computer scientists. Many applications of fuzzy set theory have arisen over the years, for instance, fuzzy logic, fuzzy neural networks, fuzzy automata, fuzzy control systems and so on.

Definition 2.1 ([28])**.** Let X be a non empty set. Then a fuzzy set A on X is a set having the form A=$\{$(x, μ_A(x)): x $\in$ X$\}$, where the function μ_A:X$\rightarrow$[0,1] is called the membership function and μ_A(x) represents the degree of membership of each element x$\in$ X.

Definition 2.2 ([19])**.** Let U be an initial universe and E be a set of parameters. Let P(U) denotes the power set of U and A$\subseteq$ E. Then the pair (F, A) is called a soft set over U, where F is a mapping given by F: A$\rightarrow$ P(U).

In other words, the soft set is not a kind of set, but a parameterized family of subsets of U. For e∈A, F(e)⊆U may be considered as the set of e-approximate elements of the soft set (F, A).

Maji et al. [16] initiated the study on hybrid structures involving both fuzzy sets and soft sets. They introduced the notion of fuzzy soft sets, which can be seen as a fuzzy generalization of (crisp) soft sets.

Definition 2.3 ([16]). Let U be an initial universe and E be a set of parameters. Let I^U be the set of all fuzzy subsets of U and A⊆E. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A → I^U.

It is easy to see that every (classical) soft set may be considered as a fuzzy soft set [29]. For e∈A, F(e) is a fuzzy subset of U and is called the fuzzy value set of the parameter e. Let us denote $\mu_{F(e)}$(x) by the membership degree that object x holds parameter e, where e∈A and x∈U. Then F(e) can be written as a fuzzy set such that F(e)={(x, $\mu_{F(e)}$(x)): x∈U}.

Before introduce the notion of the intuitionistic fuzzy soft sets, let us give the concept of intuitionistic fuzzy set [3].

Definition 2.4 ([3]). Let X be a non empty set. Then an intuitionistic fuzzy set (IFS for short) A is an object having the form A={(x, μ_A(x), γ_A(x)): x ∈ X}where the function μ_A:X→[0,1] and γ_A: X→[0,1] represents the degree of membership and the degree of non-membership respectively of each element x∈X and 0≤μ_A(x)+ γ_A(x)≤1 for each x∈X.

By introducing the concept of intuitionistic fuzzy sets into the theory of soft sets, Maji et al. [17] proposed the concept of the intuitionistic fuzzy soft sets as follows:

Definition 2.5 ([17]). Let U be an universe set and E be a set of parameters. Let IF^U be the set of all intuitionistic fuzzy subsets of U and A⊆E. Then the pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by F: A → IF^U.

For e∈ A, F(e) is an intuitionistic fuzzy subset of U and is called the intuitionistic fuzzy value set of the parameter e. Let us denote $\mu_{F(e)}$(x) by the membership degree that object x holds parameter e and $\gamma_{F(e)}$(x) by the membership degree that object x doesn't hold parameter e where e ∈A and x ∈U. Then F(e) can be written as an intuitionistic fuzzy set such that F(e)={(x, $\mu_{F(e)}$(x), $\gamma_{F(e)}$(x)): x∈ U}. If ∀x ∈U, $\gamma_{F(e)}$(x)=1- $\mu_{F(e)}$(x), then F(e) will generated to be a standard fuzzy set and then (F, A) will be generated to be a traditional fuzzy soft set.

Now before introduce the notion of the interval valued intuitionistic fuzzy soft sets, let us give the concept of interval-valued intuitionistic fuzzy set which was first introduced by Atanassov and Gargov [4]. Actually an interval- valued intuitionistic fuzzy set is characterized by an interval-valued membership degree and an interval-valued non-membership degree.

Definition 2.6 ([4]). An interval-valued intuitionistic fuzzy set (IVIFS for short) A on an universe set U is defined as the object of the form

A={<x, μ_A(x), γ_A(x)>: x ∈U}, where μ_A: U →Int ([0, 1]) and γ_A: U →Int ([0, 1]) are functions such that the condition: x∈ U, sup μ_A(x)+supγ_A(x)≤ 1 is satisfied (where Int[0,1] is the set of all closed intervals of [0,1]).

We denote the class of all interval-valued intuitionistic fuzzy sets on U by IVIFS^U. The union and intersection of the interval valued intuitionistic fuzzy sets are defined as follows:

Let A, B $\in\mathrm{IVIFS}^U$. Then the union of A and B is denoted by A∪B where

A∪B={(x,[max(infμ_A(x), infμ_B(x)), max(sup μ_A(x), supμ_B(x))], [min(infγ_A(x), infγ_B(x)), min(supγ_A(x), supγ_B(x))]): x ∈U}

the intersection of A and B is denoted by A∩B where

A∩B={(x,[min(infμ_A(x), infμ_B(x)), min(sup μ_A(x), supμ_B(x))], [max(infγ_A(x), infγ_B(x)), max(supγ_A(x), supγ_B(x))]): x ∈U}

Atanassov and Gargov shows in [4] that A∪B and A∩B are again IVIFSs.

Definition 2.7 ([12])**.** Let U be an universe set and E be a set of parameters. Let IVIFS^U be the set of all interval-valued intuitionistic fuzzy sets on U and A⊆E. Then the pair (F, A) is called an interval-valued intuitionistic fuzzy soft set (IVIFSS for short) over U, where F is a mapping given by F: A → IVIFS^U. In other words, an interval-valued intuitionistic fuzzy soft set is a parameterized family of interval valued intuitionistic fuzzy subsets of U. For any parameter e ∈A, F(e) can be written as an interval valued intuitionistic fuzzy set such that F(e)={(x, $\mu_{F(e)}$(x), $\gamma_{F(e)}$(x)): x ∈U} where $\mu_{F(e)}$(x) is the interval-valued fuzzy membership degree that object x holds parameter e and $\gamma_{F(e)}$(x) is the interval-valued fuzzy membership degree that object x doesn't hold parameter e.

3. Rough sets, rough fuzzy sets, soft rough sets and rough soft sets

The rough set theory provides a systematic method for dealing with vague concepts caused by isdiscernability in situation with incomplete information or a lack of knowledge. The rough set philosophy is founded on the assumption that with every object in the universe, we associate some information (data\knowledge). In general, a fuzzy set may be viewed as a class with unsharp boundaries, whereas a rough set is a coarsely described crisp set [27].

Definition 3.1 ([20])**.** Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R\mu_{F(e)}$. Now for X ⊆U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R_*X and R^*X and are defined by

R_*X={x ∈U: $[x]_R$ ⊆X},

R^*X={x ∈U: $[x]_R$ ∩X≠ϕ }.

Now if $R_*X=R^*X$, then X is called definable; otherwise X is called a rough set.

Based on the equivalence relation on the universe of discourse, Dubois and Prade [3] introduced the lower and upper approximation of fuzzy sets in a Pawlak's approximation space [20] and obtained a new notion called rough fuzzy sets.

Definition 3.2 ([7])**.** Let (U, R) be a Pawlak approximation space and $\mu\in I^U$. Then the lower and upper rough approximations of μ in (U, R) are denoted by $\underline{R}(\mu)$and $\overline{R}(\mu)$, respectively, which are fuzzy subsets in U defined by

$\underline{R}(\mu)(x)= \wedge\{\mu(y)\colon y\in [x]_R\}$ and

$\overline{R}(\mu)(x)= \vee\{ \mu(y)\colon y \in[x]_R\}$, for all x∈ U.

The operators $\underline{R}$ and $\overline{R}$ are called the lower and upper rough approximation operators on fuzzy sets. μ is said to be definable in U if $\underline{R}(\mu)=\overline{R}(\mu)$; otherwise μ is called a rough fuzzy set.

Feng el al. [11] provided a framework to combine if rough sets and soft sets all together, which gives rise to several interesting new concepts such as soft rough sets and rough soft sets.

Definition 3.3 ([11])**.** Let Θ =(f, A) be a soft set over U. The pair S=(U, Θ) is called a soft approximation space. Based on S, the operators $\overline{apr}_S$ and $\underline{apr}_S$ are defined as:

$\underline{apr}_S$(X)={u$\in$ U: $\exists$a $\in$ A (u $\in$f(a)$\subseteq$ X)},

$\overline{apr}_S$(X)={u $\in$U: $\exists$a $\in$ A (u$\in$ f(a), f(a)$\cap$ X $\neq\phi$)} for every X $\subseteq$U.

The two sets $\underline{apr}_S$(X) and $\overline{apr}_S$(X) are called the lower and upper soft rough approximations of X in S respectively. If $\underline{apr}_S(X) = \overline{apr}_S$ (X), then X is said to be soft definable; otherwise X is called a soft rough set.

Definition 3.4 ([11])**.** Let (U, R) be a Pawlak approximation space and Θ=(f, A) be a soft set over U. Then the lower and upper rough approximations of Θ in (U, R) are denoted by $R_*(\Theta)=(F_*,A)$ and $R^*(\Theta)=(F^*,A)$, respectively, which are soft sets over U defined by:

$F_*(x)=\underline{R}(F(x))=\{y\in U: [y]_R\subseteq F(x)\}$, and

$F^*(x)= \overline{R}(F(x))=\{y \in U: [y]_R\cap F(x)\neq\phi\}$ for all x $\in$U.

The operators R_* and R^* are called the lower and upper rough approximation operators on soft sets. If $R_*(\Theta) = R^*(\Theta)$, the soft set Θ is said to be definable; otherwise Θ is called a rough soft set.

4. Soft rough fuzzy sets and soft fuzzy rough soft sets

Motivated by Dubois and Prade's original idea about rough fuzzy set, Feng et al. [11] introduced lower and upper soft rough approximations of fuzzy sets in a soft approximation space and obtained a new hybrid model called soft rough fuzzy set.

Definition 4.1 ([11])**.** Let Θ=(f, A) be a full soft set over U i.e; $\cup_{a\in A}$f(a)=U and the pair S=(U, Θ) be the soft approximation space. Then for a fuzzy set $\lambda\in I^U$, the lower and upper soft rough approximations of λ with respect to S are denoted by $\underline{sap}_S(\lambda)$ and $\overline{sap}_S(\lambda)$ respectively, which are fuzzy sets in U given by:

$\underline{sap}_S(\lambda)=\{(x, \underline{sap}_S(\lambda)(x)): x\in U\}$,

$\overline{sap}_S(\lambda)=\{(x, \overline{sap}_S(\lambda)(x)): x \in U\}$,

where $\underline{sap}_S(\lambda)(x)= \wedge\{ \mu_\lambda(y): \exists a\in A (\{x, y\}\subseteq f(a))\}$ and

$\overline{sap}_S(\lambda)(x)= \vee\{ \mu_\lambda(y): \exists a\in A (\{x, y\}\subseteq f(a))\}$ for every x$\in$ U.

The operators $\underline{sap}_S$ and $\overline{sap}_S$ are called the lower and upper soft rough approximation operators on fuzzy sets. If $\underline{sap}_S(\lambda) =\overline{sap}_S(\lambda)$, then λ is said to be fuzzy soft definable; otherwise λ is called a soft rough fuzzy set.

Meng et al. [18] introduced the lower and upper soft fuzzy rough approximations of a fuzzy set by granulating the universe of discourse with the help of a fuzzy soft set and obtained a new model called soft fuzzy rough set.

Definition 4.2 ([18])**.** Let Θ=(f, A) be a fuzzy soft set over U. Then the pair SF=(U, Θ) is called a soft fuzzy approximation space. Then for a fuzzy set $\lambda\in I^U$, the lower and upper soft fuzzy rough approximations of λ with respect to SF are denoted by $\underline{Apr}_{SF}(\lambda)$ and $\overline{Apr}_{SF}(\lambda)$ respectively, which are fuzzy sets in U given by:

$\underline{Apr}_{SF}(\lambda)$={(x, $\underline{Apr}_{SF}(\lambda)$(x)): x∈ U},

$\overline{Apr}_{SF}(\lambda)$={(x, $\overline{Apr}_{SF}(\lambda)$(x)): x ∈U} where

$\underline{Apr}_{SF}(\lambda)$(x)= $\wedge_{a\in A}$((1-f(a)(x))∨ ($\wedge_{y\in U}$((1-f(a)(y))∨ μ_λ(y))) and

$\overline{Apr}_{SF}(\lambda)$(x)= $\vee_{a\in A}$((f(a)(x))∧ ($\vee_{y\in U}$(f(a)(y)∧ μ_λ(y))) for every x ∈U and μ_λ(y) is the degree of membership of y∈ U.

The operators $\underline{Apr}_{SF}$ and $\overline{Apr}_{SF}$ are called the lower and upper soft fuzzy rough approximation operators on fuzzy sets. If $\underline{Apr}_{SF}(\lambda)$= $\overline{Apr}_{SF}(\lambda)$, then λ is said to be soft fuzzy definable; otherwise λ is called a soft fuzzy rough set.

5. Soft interval-valued intuitionistic fuzzy rough sets

In this section we use an interval-valued intuitionistic fuzzy soft set to granulate the universe of discourse and obtain a new hybrid model called soft interval-valued intuitionistic fuzzy rough set.

Definition 5.1. Let us consider an interval-valued intuitionistic fuzzy set τ defined by τ={<x, μ_τ(x), γ_τ(x)>: x ∈ U} where μ_τ(x), γ_τ(x)∈Int([0, 1]) for each x ∈U and 0≤ supμ_τ(x)+ supγ_τ(x)≤ 1.

Now let Θ=(f, A) be an interval-valued intuitionistic fuzzy soft set over U and SIVIF=(U, Θ) be the soft interval-valued intuitionistic fuzzy approximation space.

Let f: A → IVIFS^U be defined by f(a)={<x, $\mu_{f(a)}$(x), $\gamma_{f(a)}$(x)>: x ∈ U}, for a∈ A .

Then the lower and upper soft interval-valued intuitionistic fuzzy rough approximations of τ with respect to SIVIF are denoted by ↓Apr$_{SIVIF}(\tau)$ and ↑Apr$_{SIVIF}(\tau)$ respectively, which are interval-valued intuitionistic fuzzy sets in U given by:

↓Apr$_{SIVIF}(\tau)$={<x, [$\wedge_{a\in A}$(inf$\mu_{f(a)}$(x)∧infμ_τ(x)), $\wedge_{a\in A}$(sup$\mu_{f(a)}$(x)∧supμ_τ(x))], [$\wedge_{a\in A}$(inf$\gamma_{f(a)}$(x)∨infγ_τ(x)), $\wedge_{a\in A}$(sup$\gamma_{f(a)}$(x)∨supγ_τ(x))]>: x∈U},

↑Apr$_{SIVIF}(\tau)$={<x, [$\wedge_{a\in A}$(inf$\mu_{f(a)}$(x)∨infμ_τ(x)), $\wedge_{a\in A}$(sup$\mu_{f(a)}$(x)∨supμ_τ(x))], [$\wedge_{a\in A}$(inf$\gamma_{f(a)}$(x)∧infγ_τ(x)), $\wedge_{a\in A}$(sup$\gamma_{f(a)}$(x)∧supγ_τ(x))]>: x∈U}.

The operators ↓Apr$_{SIVIF}$ and ↑Apr$_{SIVIF}$ are called the lower and upper soft interval valued intuitionistic fuzzy rough approximation operators on interval-valued intuitionistic fuzzy sets. If ↓Apr$_{SIVIF}(\tau)$= ↑Apr$_{SIVIF}(\tau)$, then τ is said to be soft interval-valued intuitionistic fuzzy definable; otherwise τ is called a soft interval-valued intuitionistic fuzzy rough set. It is to be noted that if μ_τ(x), γ_τ(x)∈[0,1] for each x ∈U and 0≤ μ_τ(x)+ γ_τ(x)≤ 1, then soft interval-valued intuitionistic fuzzy rough set becomes soft intuitionistic fuzzy rough set and if μ_τ(x)∈[0,1] with γ_τ(x)=1-μ_τ(x) then soft intuitionistic fuzzy rough set becomes soft fuzzy rough set.

Example 5.2. Let U={x, y} and A={a, b}. Let (f, A) be an interval-valued intuitionistic fuzzy soft set over U where f: A → IVIFS^U be defined by

f(a)={⟨x, [0.2, 0.5], [0.3, 0.4]⟩, ⟨y, [0.6, 0.7], [0.1, 0.2]⟩},

f(b)={⟨x, [0.1, 0.3], [0.4, 0.5]⟩, ⟨y, [0.5, 0.8], [0.1, 0.2]⟩}.

Let τ={⟨x, [0.3, 0.4], [0.3, 0.4]⟩, ⟨y, [0.2, 0.4], [0.4, 0.5]⟩}. Then

↓Apr$_{SIVIF}(\tau$)={⟨x, [0.1, 0.3], [0.3, 0.4]⟩, ⟨y, [0.2, 0.4], [0.4, 0.5]⟩},

$\uparrow$Apr$_{SIVIF}(\tau)$={⟨x, [0.3, 0.4], [0.3, 0.4]⟩, ⟨y, [0.5, 0.7], [0.1, 0.2]⟩}. Then τ is a soft interval-valued intuitionistic fuzzy rough set.

Theorem 5.3. *let* Θ *=(f, A) be an interval-valued intuitionistic fuzzy soft set over U and SIVIF=(U,* Θ *) be the soft interval-valued intuitionistic fuzzy approximation space. Then*
(i) $\downarrow Apr_{SIVIF}(\phi)=\phi$
(ii) $\uparrow Apr_{SIVIF}(U)=U$
(iii) $\downarrow Apr_{SIVIF}(\tau)\subseteq\tau\subseteq \uparrow Apr_{SIVIF}(\tau)$ *for* $\tau\in IVIFS^U$.

Proof. Straight forward. □

Theorem 5.4. *let* Θ *=(f, A) be an interval-valued intuitionistic fuzzy soft set over U and SIVIF=(U,* Θ *) be the soft interval-valued intuitionistic fuzzy approximation space. Then for* τ, $\delta \in IVIFS^U$*, we have*
(i) $\tau\subseteq \delta\Rightarrow \uparrow Apr_{SIVIF}(\tau)\subseteq \uparrow Apr_{SIVIF}(\delta)$
(ii) $\tau\subseteq \delta\Rightarrow\downarrow Apr_{SIVIF}(\tau)\subseteq \downarrow Apr_{SIVIF}(\delta)$
(iii) $\uparrow Apr_{SIVIF}(\tau\cap\delta)\subseteq \uparrow Apr_{SIVIF}(\tau)\cap\uparrow Apr_{SIVIF}(\delta)$
(iv) $\downarrow Apr_{SIVIF}(\tau\cap\delta)\subseteq \downarrow Apr_{SIVIF}(\tau)\cap\downarrow Apr_{SIVIF}(\delta)$
(v)$\uparrow Apr_{SIVIF}(\tau)\cup\uparrow Apr_{SIVIF}(\delta)\subseteq\uparrow Apr_{SIVIF}(\tau\cup\delta)$
(vi) $\downarrow Apr_{SIVIF}(\tau)\cup\downarrow Apr_{SIVIF}(\delta)\subseteq\downarrow Apr_{SIVIF}(\tau\cup\delta)$

Proof. (i)-(ii) are straight forward.
(iii) We have $\uparrow$Apr$_{SIVIF}(\tau\cap\delta)$
$=\{<$x, $[\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee \inf\mu_{\tau\cap\delta}(x)), \wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee \sup\mu_{\tau\cap\delta}(x))]$,
$[\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\wedge \inf\gamma_{\tau\cap\delta}(x)), \wedge_{a\in A}(\sup\gamma_{f(a)}(x)\wedge \sup\gamma_{\tau\cap\delta}(x))]>$: x$\in$U}

$=\{<$x, $[\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee \min(\inf\mu_{\tau}(x),\inf\mu_{\delta}(x))), \wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee \min(\sup\mu_{\tau}(x),\sup\mu_{\delta}(x)))]$,
$[\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\wedge \max(\inf\gamma_{\tau}(x),\inf\gamma_{\delta}(x))), \wedge_{a\in A}(\sup\gamma_{f(a)}(x)\wedge \max(\sup\gamma_{\tau}(x),\sup\gamma_{\delta}(x)))]>$: x$\in$U}

Now $\uparrow$Apr$_{SIVIF}(\tau)\cap\uparrow$Apr$_{SIVIF}(\delta)$
$=\{<$x, $[\min(\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee \inf\mu_{\tau}(x)),\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf\mu_{\delta}(x)))$,
$\min(\wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\sup\mu_{\tau}(x)), \wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\sup\mu_{\delta}(x)))]$,
$[\max(\bigwedge_{a\in A}(\inf\gamma_{f(a)}(x)\wedge\inf\gamma_{\tau}(x)), \bigwedge_{a\in A}(\inf\gamma_{f(a)}(x)\wedge\inf\gamma_{\delta}(x)))$,
$\max(\wedge_{a\in A}(\sup\gamma_{f(a)}(x)\wedge\sup\gamma_{\tau}(x)), \wedge_{a\in A}(\sup\gamma_{f(a)}(x)\wedge\sup\gamma_{\delta}(x)))]>$: x$\in$U}

Since $\min(\inf\mu_{\tau}(x), \inf \mu_{\delta}(x))\leq \inf \mu_{\tau}(x)$ and $\min(\inf\mu_{\tau}(x), \inf\mu_{\delta}(x))\leq \inf\mu_{\delta}(x)$, so $\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\min(\inf\mu_{\tau}(x), \inf \mu_{\delta}(x)))\leq \wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf \mu_{\tau}(x))$ and $\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\min(\inf\mu_{\tau}(x), \inf\mu_{\delta}(x)))\leq \wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf\mu_{\delta}(x))$.

Hence $\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\min(\inf\mu_{\tau}(x), \inf \mu_{\delta}(x)))\leq\min(\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf \mu_{\tau}(x)), \wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf\mu_{\delta}(x)))$.

Similarly, $\wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\min(\sup\mu_{\tau}(x), \sup\mu_{\delta}(x)))\leq\min(\wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee \sup \mu_{\tau}(x)), \wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\sup\mu_{\delta}(x)))$.

Now as $\max(\inf\gamma_{\tau}(x), \inf\gamma_{\delta}(x))\geq \inf\gamma_{\tau}(x)$ and $\max(\inf\gamma_{\tau}(x), \inf\gamma_{\delta}(x))\geq \inf\gamma_{\delta}(x)$, we have $\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\max(\inf\gamma_{\tau}(x), \inf \gamma_{\delta}(x)))\geq \wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\inf \gamma_{\tau}(x))$ and $\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\max(\inf\gamma_{\tau}(x), \inf\gamma_{\delta}(x)))\geq \wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\inf\gamma_{\delta}(x))$.

Therefore, $\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\max(\inf\gamma_\tau(x), \inf\gamma_\delta(x)))\geq\max(\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\inf\gamma_\tau(x)), \wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\inf\gamma_\delta(x)))$.

Similarly, $\wedge_{a\in A}(\sup\gamma_{f(a)}(x)\vee\max(\sup\gamma_\tau(x), \sup\gamma_\delta(x)))\geq\max(\wedge_{a\in A}(\sup\gamma_{f(a)}(x)\vee\sup\gamma_\tau(x)), \wedge_{a\in A}(\sup\gamma_{f(a)}(x)\vee\sup\gamma_\delta(x)))$.

Consequently, $\uparrow\mathrm{Apr}_{SIVIF}(\tau\cap\delta)\subseteq\uparrow\mathrm{Apr}_{SIVIF}(\tau)\cap\uparrow\mathrm{Apr}_{SIVIF}(\delta)$.

(iv) Proof is similar to (iii).

(v) We have $\uparrow\mathrm{Apr}_{SIVIF}(\tau\cup\delta)$

$=\{<x, [\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf\mu_{\tau\cup\delta}(x)), \wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\sup\mu_{\tau\cup\delta}(x))],$

$[\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\wedge\inf\gamma_{\tau\cup\delta}(x)), \wedge_{a\in A}(\sup\gamma_{f(a)}(x)\wedge\sup\gamma_{\tau\cup\delta}(x))]>: x\in U\}$

$=\{<x, [\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\max(\inf\mu_\tau(x),\inf\mu_\delta(x))), \wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\max(\sup\mu_\tau(x),\sup\mu_\delta(x)))],$

$[\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\wedge\min(\inf\gamma_\tau(x),\inf\gamma_\delta(x))), \wedge_{a\in A}(\sup\gamma_{f(a)}(x)\wedge\min(\sup\gamma_\tau(x),\sup\gamma_\delta(x)))]>: x\in U\}$

Now $\uparrow\mathrm{Apr}_{SIVIF}(\tau)\cup\uparrow\mathrm{Apr}_{SIVIF}(\delta)$

$=\{<x, [\max(\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf\mu_\tau(x)),\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf\mu_\delta(x))),$

$\max(\wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\sup\mu_\tau(x)), \wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\sup\mu_\delta(x)))],$

$[\min(\vee_{a\in A}(\inf\gamma_{f(a)}(x)\wedge\inf\gamma_\tau(x)), \vee_{a\in A}(\inf\gamma_{f(a)}(x)\wedge\inf\gamma_\delta(x))),$

$\min(\wedge_{a\in A}(\sup\gamma_{f(a)}(x)\wedge\sup\gamma_\tau(x)), \wedge_{a\in A}(\sup\gamma_{f(a)}(x)\wedge\sup\gamma_\delta(x)))]>: x\in U\}$

Since $\max(\inf\mu_\tau(x), \inf\mu_\delta(x))\geq\inf\mu_\tau(x)$ and $\max(\inf\mu_\tau(x), \inf\mu_\delta(x))\leq\inf\mu_\delta(x)$, so $\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\max(\inf\mu_\tau(x), \inf\mu_\delta(x)))\geq\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf\mu_\tau(x))$ and $\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\max(\inf\mu_\tau(x), \inf\mu_\delta(x)))\geq\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf\mu_\delta(x))$.

Hence $\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\max(\inf\mu_\tau(x), \inf\mu_\delta(x)))\geq\max(\wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf\mu_\tau(x)), \wedge_{a\in A}(\inf\mu_{f(a)}(x)\vee\inf\mu_\delta(x)))$.

Similarly, $\wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\max(\sup\mu_\tau(x), \sup\mu_\delta(x)))\leq\max(\wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\sup\mu_\tau(x)), \wedge_{a\in A}(\sup\mu_{f(a)}(x)\vee\sup\mu_\delta(x)))$.

Now as $\min(\inf\gamma_\tau(x), \inf\gamma_\delta(x))\leq\inf\gamma_\tau(x)$ and $\min(\inf\gamma_\tau(x), \inf\gamma_\delta(x))\leq\inf\gamma_\delta(x)$, we have $\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\min(\inf\gamma_\tau(x), \inf\gamma_\delta(x)))\leq\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\inf\gamma_\tau(x))$ and $\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\min(\inf\gamma_\tau(x), \inf\gamma_\delta(x)))\leq\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\inf\gamma_\delta(x))$.

Therefore, $\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\min(\inf\gamma_\tau(x), \inf\gamma_\delta(x)))\leq\min(\wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\inf\gamma_\tau(x)), \wedge_{a\in A}(\inf\gamma_{f(a)}(x)\vee\inf\gamma_\delta(x)))$.

Similarly, $\wedge_{a\in A}(\sup\gamma_{f(a)}(x)\vee\min(\sup\gamma_\tau(x), \sup\gamma_\delta(x)))\leq\min(\wedge_{a\in A}(\sup\gamma_{f(a)}(x)\vee\sup\gamma_\tau(x)), \wedge_{a\in A}(\sup\gamma_{f(a)}(x)\vee\sup\gamma_\delta(x)))$.

Consequently, $\uparrow\mathrm{Apr}_{SIVIF}(\tau)\cup\uparrow\mathrm{Apr}_{SIVIF}(\delta)\subseteq\uparrow\mathrm{Apr}_{SIVIF}(\tau\cup\delta)$.

(vi) Proof is similar to (v). □

Theorem 5.5. *Every soft interval-valued intuitionistic fuzzy rough set is an interval-valued intuitionistic fuzzy soft set.*

Proof. let $\Theta=(f, A)$ be an interval-valued intuitionistic fuzzy soft set over U and SIVIF=(U, Θ) be the soft interval-valued intuitionistic fuzzy approximation space.Let τ be a soft interval- valued intuitionistic fuzzy rough set. Let us define an interval-valued intuitionistic fuzzy set ψ by:

$$\psi=\{(x,[\frac{\wedge_{a\in A}(inf\mu_{f(a)}(x)\wedge inf\mu_\tau(x))}{\wedge_{a\in A}(inf\mu_{f(a)}(x)\bigvee inf\mu_\tau(x))},\frac{\wedge_{a\in A}(sup\mu_{f(a)}(x)\wedge sup\mu_\tau(x))}{\wedge_{a\in A}(sup\mu_{f(a)}(x)\bigvee sup\mu_\tau(x))}]$$
$$,[\frac{\wedge_{a\in A}(inf\gamma_{f(a)}(x)\wedge inf\gamma_\tau(x))}{\wedge_{a\in A}(inf\gamma_{f(a)}(x)\bigvee inf\gamma_\tau(x))},\frac{\wedge_{a\in A}(sup\gamma_{f(a)}(x)\wedge sup\gamma_\tau(x))}{\wedge_{a\in A}(sup\gamma_{f(a)}(x)\bigvee sup\gamma_\tau(x))}]):x\in U\}$$

Now for $\alpha \in [0,1]$, we consider the following four sets:
$F_1(\alpha) = \{x \in U : \frac{\wedge_{a\in A}(inf\mu_{f(a)}(x)\wedge inf\mu_{\tau}(x))}{\wedge_{a\in A}(inf\mu_{f(a)}(x)\bigvee inf\mu_{\tau}(x))} \geq \alpha\}$
$F_2(\alpha) = \{x \in U : \frac{\wedge_{a\in A}(sup\mu_{f(a)}(x)\wedge sup\mu_{\tau}(x))}{\wedge_{a\in A}(sup\mu_{f(a)}(x)\bigvee sup\mu_{\tau}(x))} \geq \alpha\}$
$F_3(\alpha) = \{x \in U : \frac{\wedge_{a\in A}(inf\gamma_{f(a)}(x)\wedge inf\gamma_{\tau}(x))}{\wedge_{a\in A}(inf\gamma_{f(a)}(x)\bigvee inf\gamma_{\tau}(x))} \geq \alpha\}$
$F_4(\alpha) = \{x \in U : \frac{\wedge_{a\in A}(sup\gamma_{f(a)}(x)\wedge sup\gamma_{\tau}(x))}{\wedge_{a\in A}(sup\gamma_{f(a)}(x)\bigvee sup\gamma_{\tau}(x))} \geq \alpha\}$
Then $\vartheta(\alpha) = \{(x, [inf\{\alpha : x \in F_1(\alpha)\}, inf\{\alpha : x \in F_2(\alpha)\}], [inf\{\alpha : x \in F_3(\alpha)\}$
$, inf\{\alpha : x \in F_4(\alpha)\}]) : x \in U\}$
is an interval-valued intuitionistic fuzzy set over U for each $\alpha \in [0,1]$.
Consequently (ϑ, α) is an interval-valued intuitionistic fuzzy soft set over U. □

6. A multicriteria group decision making problem

Let U=$\{$o$_1$, o$_2$, o$_3$,.....,o$_r\}$ be a set of objects and E be a set of parameters and A=$\{$e$_1$, e$_2$, e$_3$,.....,e$_m\}$ $\subseteq$E and S=(F, A) be an interval-valued intuitionistic fuzzy soft set over U. Let us assume that we have an expert group G=$\{$T$_1$, T$_2$, ...,T$_n\}$ consisting of n specialists to evaluate the objects in U. Each specialist will examine all the objects in U and will point out his/her evaluation result. Let X$_i$ denote the primary evaluation result of the specialist T$_i$. It is easy to see that the primary evaluation result of the whole expert group G can be represented as an interval-valued intuitionistic fuzzy evaluation soft set S*=(F*, G) over U, where F*: G IVIFS^U is given by F*(T$_i$)=X$_i$, for i=1, 2,..., n.

Now we consider the soft interval-valued intuitionistic fuzzy rough approximations of the specialist T$_i$'s primary evaluation result X$_i$ w.r.t the soft interval-valued intuitionistic fuzzy approximation space SIVIF=(U, S). Then we obtain two other interval-valued intuitionistic fuzzy soft sets $\downarrow$S*=($\downarrow$F*, G) and $\uparrow$S*=($\uparrow$F*, G) over U, where $\downarrow$F*: G $\rightarrow$ IVIFS^U is given by $\downarrow$F*(T$_i$)= $\downarrow$Apr$_{SIVIF}$(X$_i$) and $\uparrow$F*: G$\rightarrow$IVIFS^U is given by $\uparrow$F*(T$_i$)= $\uparrow$Apr$_{SIVIF}$(X$_i$), for i=1, 2,..., n.

Here $\downarrow$S* can be considered as the evaluation result for the whole expert group G with 'low confidence' , $\uparrow$S* can be considered as the evaluation result for the whole expert group G with 'high confidence' and S* can be considered as the evaluation result for the whole expert group G with 'middle confidence'.

Let us define two interval-valued intuitionistic fuzzy sets IVIFS$\downarrow_{S^*}$ and IVIFS$\uparrow_{S^*}$ by

IVIFS$\downarrow_{S^*}$ $=\{<$o$_k$, [$\frac{1}{n}\sum_{j=1}^{n}inf\mu\downarrow_{F^*(T_j)}$(o$_k$), $\frac{1}{n}\sum_{j=1}^{n}sup\mu\downarrow_{F^*(T_j)}$(o$_k$)]
, [$\frac{1}{n}\sum_{j=1}^{n}inf\gamma\downarrow_{F^*(T_j)}$(o$_k$), $\frac{1}{n}\sum_{j=1}^{n}sup\gamma\downarrow_{F^*(T_j)}$(o$_k$)]$>$: k=1, 2,...., r$\}$ and
IVIFS$\uparrow_{S^*}$ $=\{<$o$_k$, [$\frac{1}{n}\sum_{j=1}^{n}inf\mu\uparrow_{F^*(T_j)}$(o$_k$), $\frac{1}{n}\sum_{j=1}^{n}sup\mu\uparrow_{F^*(T_j)}$(o$_k$)]
, [$\frac{1}{n}\sum_{j=1}^{n}inf\gamma\uparrow_{F^*(T_j)}$(o$_k$), $\frac{1}{n}\sum_{j=1}^{n}sup\gamma\uparrow_{F^*(T_j)}$(o$_k$)]$>$: k=1, 2,...., r$\}$.

Now we define another interval-valued intuitionistic fuzzy set IVIFS$_{S^*}$ by
IVIFS$_{S^*}$ $=\{<$o$_k$, [$\frac{1}{n}\sum_{j=1}^{n}inf\mu_{F^*(T_j)}$(o$_k$), $\frac{1}{n}\sum_{j=1}^{n}sup\mu_{F^*(T_j)}$(o$_k$)]
, [$\frac{1}{n}\sum_{j=1}^{n}inf\gamma_{F^*(T_j)}$(o$_k$), $\frac{1}{n}\sum_{j=1}^{n}sup\gamma_{F^*(T_j)}$(o$_k$)]$>$: k=1, 2,...., r$\}$.
Then clearly, IVIFS$\downarrow_{S^*}$ $\subseteq$ IVIFS$_{S^*}$ $\subseteq$ IVIFS$\uparrow_{S^*}$.

Let C=$\{$L(low confidence), M(middle confidence), H(high confidence)$\}$ be a set of parameters. Let us consider the interval valued intuitionistic fuzzy soft set S**=(f,

C) over U, where f: C$\rightarrow$IVIFS^U is given by f(H)=IVIFS$\uparrow_{S^*}$, f(M)=IVIFS$_{S^*}$, f(L)= IVIFS$\downarrow_{S^*}$.

Now given a weighting vector W=(w_L, w_M, w_H) such that w_L, w_M, w_H $\in$Int([0, 1]), we define α: U $\rightarrow$R$^+$ by

$\alpha(o_k))=\sup w_L * \sup\mu_{f(L)}(o_k) + \sup w_M * \sup\mu_{f(M)}(o_k) + \sup w_H * \sup\mu_{f(H)}(o_k)$

, $o_k \in$U (* represents ordinary multiplication).

Here $\alpha(o_k)$ is called the weighted evaluation value of the alternative $o_k\in$U. Finally, we can select the object o_p such that $\alpha(o_p)$=max{$\alpha(o_k)$: k=1, 2,...., r} as the most preferred alternative.

Algorithm:

(1) Input the original description interval-valued intuitionistic fuzzy soft set (F, A).

(2) Construct the interval-valued intuitionistic fuzzy evaluation soft set S*=(F*, G).

(3) Compute the soft interval-valued intuitionistic fuzzy rough approximations and then construct the interval-valued intuitionistic fuzzy soft sets $\downarrow_{S^*}$ and $\uparrow_{S^*}$.

(4) Construct the interval-valued intuitionistic fuzzy sets IVIFS$\uparrow_{S^*}$, IVIFS$_{S^*}$, IVIFS$\downarrow_{S^*}$.

(5) Construct the interval-valued intuitionistic fuzzy soft set S**.

(6) Input the weighting vector W and compute the weighted evaluation values $\alpha(o_k)$ of each alternative o_k $\in$U.

(7) Select the object o_p such that $\alpha(o_p)$=max{$\alpha(o_k)$: k=1, 2,...., r} as the most preferred alternative.

An illustrative example:

Let us consider a car selection problem to buy a car for the family of Mr. X. Let U={c_1, c_2, c_3, c_4, c_5} is the universe set consisting of five cars. Let us consider the soft set S=(F, A), which describes the "quality of the car", where A={e_1(expensive), e_2(fuel efficient), e_3(attractive), e_4(challenging internal structure with maximum seat capacity)}.

Let the tabular representation of the interval-valued intuitionistic fuzzy soft set (F, A) be:

Table-1: Representation of the interval- valued intuitionistic fuzzy soft set (F, A)

	c_1	c_2	c_3	c_4	c_5
e_1	([.2,.3],[.4,.5])	([.5,.7],[.1,.3])	([.4,.5],[.2,.4])	([.1,.2],[.1,.3])	([.3,.5],[.3,.4])
e_2	([.3,.6],[.1,.2])	([.1,.3],[.2,.3])	([.3,.6],[.2,.4])	([.5,.6],[.2,.3])	([.1,.3],[.3,.6])
e_3	([.4,.5],[.2,.3])	([.2,.4],[.2,.5])	([.1,.3],[.4,.6])	([.3,.4],[.3,.4])	([.4,.6],[.1,.3])
e_4	([.2,.4],[.2,.4])	([.6,.7],[.1,.2])	([.3,.4],[.3,.4])	([.2,.4],[.4,.6])	([.5,.7],[.1,.2])

Let G={T_1, T_2, T_3, T_4, T_5} be the set of members of the family of Mr. X to judge the quality of the car in U. Now if X_i denote the primary evaluation result of the member T_i (for i=1, 2, 3, 4, 5), then the primary evaluation result of the whole expert group G can be represented as an interval-valued intuitionistic fuzzy evaluation soft set S*=(F*, G) over U, where F*: G $\rightarrow$IVIFS^U is given by F*(T_i)=X_i for i=1, 2, 3, 4, 5.

Let the tabular representation of S* be given as;

Table-2: Representation of the interval-valued intuitionistic fuzzy soft set S*

	c_1	c_2	c_3	c_4	c_5
T_1	([.4,.6],[.1,.2])	([.3,.4],[.3,.4])	([.2,.3],[.2,.3])	([.6,.8],[.1,.2])	([.1,.4],[.2,.4])
T_2	([.3,.5],[.2,.4])	([.5,.7],[.1,.3])	([.4,.6],[.1,.3])	([.3,.5],[.1,.3])	([.4,.5],[.2,.3])
T_3	([.1,.3],[.5,.6])	([.2,.3],[.4,.5])	([.1,.4],[.2,.4])	([.2,.3],[.5,.6])	([.3,.6],[.2,.3])
T_4	([.2,.3],[.3,.4])	([.4,.7],[.1,.2])	([.3,.5],[.4,.5])	([.4,.5],[.2,.4])	([.5,.7],[.1,.2])
T_5	([.6,.7],[.1,.2])	([.3,.5],[.2,.5])	([.5,.6],[.3,.4])	([.1,.3],[.3,.6])	([.1,.2],[.6,.8])

Let us choose P=(U, S) as the soft interval-valued intuitionistic fuzzy approximation space. Let us consider the interval valued intuitionistic fuzzy evaluation soft sets ↓S*=(↓F*, G) and ↑S*=(↑F*, G) over U.

Then after calculation we get the tabular representation of these sets as:

Table-3: Representation of the interval-valued intuitionistic fuzzy soft set ↓S*

	c_1	c_2	c_3	c_4	c_5
T_1	([.2,.3],[.1,.2])	([.1,.3],[.3,.4])	([.1,.3],[.2,.4])	([.1,.2],[.1,.3])	([.1,.3],[.2,.4])
T_2	([.2,.3],[.2,.4])	([.1,.3],[.1,.3])	([.1,.3],[.2,.4])	([.1,.2],[.1,.3])	([.1,.3],[.2,.3])
T_3	([.1,.3],[.5,.6])	([.1,.3],[.4,.5])	([.1,.3],[.2,.4])	([.1,.2],[.5,.6])	([.1,.3],[.2,.3])
T_4	([.2,.3],[.3,.4])	([.1,.3],[.1,.2])	([.1,.3],[.4,.5])	([.1,.2],[.2,.4])	([.1,.3],[.1,.2])
T_5	([.2,.3],[.1,.2])	([.1,.3],[.2,.5])	([.1,.3],[.3,.4])	([.1,.2],[.3,.6])	([.1,.2],[.6,.8])

Table-4: Representation of the interval-valued intuitionistic fuzzy soft set ↑S*

	c_1	c_2	c_3	c_4	c_5
T_1	([.4,.6],[.1,.2])	([.3,.4],[.1,.2])	([.2,.3],[.2,.3])	([.6,.8],[.1,.2])	([.1,.4],[.1,.2])
T_2	([.3,.5],[.1,.2])	([.5,.7],[.1,.2])	([.4,.6],[.1,.3])	([.3,.5],[.1,.3])	([.4,.5],[.1,.2])
T_3	([.2,.3],[.1,.2])	([.2,.3],[.1,.2])	([.1,.4],[.2,.4])	([.2,.3],[.1,.3])	([.3,.6],[.1,.2])
T_4	([.2,.3],[.1,.2])	([.4,.7],[.1,.2])	([.3,.5],[.2,.4])	([.4,.5],[.1,.3])	([.5,.7],[.1,.2])
T_5	([.6,.7],[.1,.2])	([.3,.5],[.1,.2])	([.5,.6],[.2,.4])	([.1,.3],[.1,.3])	([.1,.3],[.1,.2])

Here, ↓S*⊆S* ⊆ ↑S*.

Then we have IVIFS$\uparrow_{S^*}$= $\{< c_1, [0.34, 0.48], [0.10, 0.20] >, < c_2, [0.34, 0.52]$
$, [0.10, 0.20] >, < c_3, [0.30, 0.48], [0.18, 0.36] >, < c_4, [0.32, 0.48], [0.10, 0.28] >$
$, < c_5, [0.28, 0.50], [0.10, 0.20] >\}$,

IVIFS$\downarrow_{S^*}$= $\{< c_1, [0.18, 0.30], [0.24, 0.36] >, < c_2, [0.10, 0.30], [0.22, 0.38] >, < c_3$
$, [0.10, 0.30], [0.26, 0.42] >, < c_4, [0.10, 0.20], [0.24, 0.44] >, < c_5$
$, [0.10, 0.28], [0.26, 0.40] >\}$,

IVIFS$_{S^*}$ = $\{< c_1, [0.32, 0.48], [0.24, 0.36] >, < c_2, [0.34, 0.52], [0.22, 0.38] >, < c_3$
$, [0.30, 0.48], [0.24, 0.38] >, < c_4, [0.32, 0.48], [0.24, 0.42] >, < c_5$
$, [0.28, 0.48], [0.26, 0.40] >\}$.

Thus, IVIFS$\downarrow_{S^*}$ ⊆ IVIFS$_{S^*}$ ⊆ IVIFS$\uparrow_{S^*}$.

Let C={L(low confidence), M(middle confidence), H(high confidence)} be a set of parameters. Let us consider the interval- valued intuitionistic fuzzy soft set S**=(f, C) over U, where f: C→ IVIFS^U is given by

f(H)=IVIFS$\uparrow_{S^*}$, f(M)=IVIFS$_{S^*}$, f(L)= IVIFS$\downarrow_{S^*}$.

Now assuming the weighting vector W=(w_L, w_M, w_H) such that w_L=[0.5, 0.6], w_M=[0.4, 0.5], w_H=[0.4, 0.7], we have,

$\alpha(c_1)$= 0.6*0.30+ 0.5*0.48+ 0.7*0.48=0.756,

$\alpha(c_2)$= 0.6*0.30+ 0.5*0.52+ 0.7*0.52=0.804,

$\alpha(c_3)$= 0.6*0.30+ 0.5*0.48+ 0.7*0.48=0.756,

$\alpha(c_4)$= 0.6*0.20+ 0.5*0.48+ 0.7*0.48=0.696,
$\alpha(c_5)$= 0.6*0.28+ 0.5*0.48+ 0.7*0.50=0.758.
Since max{ $\alpha(c_1)$, $\alpha(c_2)$, $\alpha(c_3)$, $\alpha(c_4)$, $\alpha(c_5)$}= 0.804, so the car c_2 will be selected as the most preferred alternative.

7. Conclusions

In this paper we first defined soft interval-valued intuitionistic fuzzy rough set which are the extension of soft intuitioistic fuzzy rough set and soft fuzzy rough set . We also investigated some basic properties of soft interval-valued intuitionistic fuzzy rough set. Finally we have proposed a soft interval-valued intuitionistic fuzzy rough set based multicriteria group decision making scheme and presented an example regarding the car selection problem for a family to buy a car to show that this scheme successfully works. It is to be noted that we defined soft interval-valued intuitionistic fuzzy rough set in such a way so that complicated calculations in decision making problems will be avoided.

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