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Abstract. In this study, hg-hypergroupoid and partial hg-hypergroupoid structures defined via neighborhood intersections are investigated within a parameter-dependent framework. Structural parameters and soft set parameters are treated separately, and the relationship between these two parameter sets is modeled through an appropriate mapping. This approach allows neighborhood-based hyperoperations to be examined as flexible structures that vary with respect to different parameter values. The partial nature arising from the possible existence of empty hyperproducts is analyzed in detail, and the conditions under which the structure becomes a total hg-hypergroupoid are explicitly characterized. Moreover, by means of examples defined on $\mathbf{Z_6}$ equipped with the modular distance, it is shown that the proposed theoretical framework can be reduced to a concrete and discrete special case.

2020 AMS Classification: 20N20, 54E35, 03E75

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1. Introduction

Hyperstructure theory was first introduced by Marty as a multivalued generalization of classical algebraic structures [1]. Within this theory, hypergroups and hypergroupoids have become effective tools for modeling problems involving uncertainty and multiple interactions. Later, hypergroupoid structures were systematically studied by Corsini and Leoreanu, providing the most general framework for hyperoperations [2].

In classical hypergroup definitions, the hyperoperation is required to be everywhere defined and nonempty. However, many natural constructions fail to satisfy this condition. This situation arises particularly in hyperoperations defined via neighborhood or distance-based approaches, where empty hyperproducts may occur. Such structures have been investigated in the literature under the name of partial hypergroupoids by various authors [3, 4, 5, 6, 7].

Neighborhood-intersection-based hyperoperations allow the construction of hg-hypergroupoid structures without assuming any additional algebraic structure. In this work, the abbreviation **"hg"** refers to a specific class of hyperstructures where the associative property is required to hold only when the involved hyperproducts are nonempty. Unlike classical hypergroups, hg-hypergroupoids do not necessitate the hyperoperation to be everywhere defined and non-empty, thus exhibiting a *partial* algebraic nature. This approach has been applied in several works on metric spaces and discrete structures [8, 9]. Nevertheless, in most of these studies, the neighborhood parameter is fixed, and the effect of parameter variation on the induced structure has not been systematically examined. [5, 6, 7]

On the other hand, soft set theory, introduced by Molodtsov, provides a flexible mathematical framework for modeling parameter-dependent uncertainty [9]. Soft sets were soon combined with algebraic structures, leading to various generalizations such as soft groups, soft rings, and soft hyperstructures in the literature [10, 11, 12, 13].Furthermore, the properties of soft relations, mappings, and topological structures have been extensively studied in recent years [14, 15, 16, 17, 18, 19].

In this study, neighborhood-based hg-hypergroupoid structures are considered in a parameter-dependent manner and are systematically investigated within the framework of soft set theory. A clear distinction is made between structural parameters and soft set parameters, and the relationship between these two parameter sets is modeled via an appropriate mapping. Consequently, it is shown that the classical definition of a soft hg-hypergroupoid appearing in the literature can be regarded as a special case of a more general parameter-dependent construction.

Terminology Remark.
In this work, the term *small* is used in accordance with the underlying soft structure. More precisely, for a soft partial hg-hypergroupoid (F, E), we say that the structure is *small* if the support of the soft structure does not cover the whole universe, that is,

$$F(e) \subsetneq H \quad \text{for some } e \in E.$$

In this case, the hyperoperation induced by $F(e)$ is defined only on a proper subset of H, and the resulting structure is referred to as a *soft small partial hg-hypergroupoid*. Thus, the notion of "small" reflects a restriction arising from the support of the soft set rather than from the cardinality of H.

2. Preliminaries

In this section, we recall the basic notions of hyperstructures, soft sets, and neighborhood-based hyperoperations that will be used throughout the paper.

Definition 2.1 ([2]). Let H be a nonempty set. A hyperoperation

$$\circ : H \times H \longrightarrow \mathcal{P}(H)$$

is given. For all $x, y \in H$, the hyperproduct

$$x \circ y \subseteq H$$

is interpreted in the set-theoretical sense as

$$x \circ y = \bigcup_{z \in x \circ y} \{z\}.$$

Then the pair $(H, \circ)$ is called a *hypergroupoid*.

Definition 2.2 ([2])**.** A hypergroupoid $(H, \circ)$ is called a *semihypergroup*, if the associative law

$$(x \circ y) \circ z = x \circ (y \circ z), \quad \forall x, y, z \in H,$$

is satisfied .

Definition 2.3 ([4])**.** A semihypergroup $(H, \circ)$ is said to be a *quasihypergroup*, if

$$x \in x \circ H \quad \text{and} \quad x \in H \circ x, \quad \forall x \in H,$$

hold .

Definition 2.4 ([1, 5])**.** According to Marty, a classical hypergroup $(H, \circ)$ satisfies the following axioms:

(H1) Totality:

$$x \circ y \neq \varnothing \quad \forall x, y \in H.$$

(H2) Associativity:

$$(x \circ y) \circ z = x \circ (y \circ z) \quad \forall x, y, z \in H.$$

(H3) Reproduction Axiom:

$$x \circ H = H \circ x = H \quad \forall x \in H.$$

Such structures are referred to as *total hypergroups* in the literature.

Definition 2.5 ([9])**.** Let U be an initial universe set and E be a set of parameters. Let $R \subseteq E$ and

$$F : R \longrightarrow \mathcal{P}(U)$$

be a mapping. Then the pair (F, R) is called a *soft set* over U.

Definition 2.6 ([20])**.** Let (X, d) be a metric space. For $r > 0$, the set

$$N_r(x) = \{y \in X \mid d(x, y) \leq r\}$$

is called the *r-neighborhood* of x.

Using these neighborhoods, the hyperoperation

$$x \oplus_r y = N_r(x) \cap N_r(y)$$

may be empty for some $x, y \in X$ [21].

Definition 2.7 ([4])**.** A hypergroupoid $(H, \circ)$ is called an *hg-hypergroupoid*, if the associative property holds whenever the involved hyperproducts are nonempty.

Definition 2.8 ([22, 23]). Let H be a nonempty set and E be a parameter set. Consider a soft structure

$$F : E \longrightarrow \mathcal{P}(\mathcal{P}(H)).$$

If for each $e \in E$, the set $F(e)$ forms an hg-hypergroupoid, then the pair (F, E) is called a *soft hg-hypergroupoid*.

2.1. **Notation Remark.** Throughout this paper, hyperoperations are interpreted in the set-theoretical (union) sense. Structural parameters such as neighborhood radii are clearly distinguished from soft set parameters, and this distinction plays a central role in our constructions.

Remark 2.9. Neighborhood structures have also been studied within the framework of cubic sets and related generalized set theories. In such approaches, uncertainty is typically modeled by interval–valued or pairwise membership functions. In contrast, the present work addresses uncertainty and proximity directly at the algebraic operational level by means of hyperoperations induced through neighborhood intersections. Therefore, although the underlying motivation is similar, the cubic set–based approaches and the proposed framework differ essentially in their mathematical mechanisms.

3. Main Results on Neighborhood–Based Soft hg–Hypergroupoids

In this section, we introduce and formalize the concept of neighborhood-based soft hg-hypergroupoids. The term hg-hypergroupoid is used here to denote a class of hypergroupoids in which the hyperoperation is defined via neighborhood intersections and satisfies associativity whenever the involved hyperproducts are nonempty. The main objective of this section is to establish a systematic connection between soft set theory and hyperstructure theory through neighborhood mappings. By defining hyperoperations as intersections of neighborhoods, we obtain a flexible algebraic framework that naturally incorporates proximity and uncertainty. This construction serves as a foundational step for the parameter-dependent soft hypergroupoid structures and the reduced small structures studied in the subsequent definitions and results.

Definition 3.1. Let H be a nonempty set, Λ a nonempty parameter set and let

$$N : \Lambda \times H \longrightarrow \mathcal{P}(H)$$

a neighborhood mapping which assigns to each pair $(\lambda, x) \in \Lambda \times H$ a subset $N(\lambda, x) \subseteq H$. For each $\lambda \in \Lambda$, define a hyperoperation $\oplus_\lambda$ on H by

$$x \oplus_\lambda y = N(\lambda, x) \cap N(\lambda, y) \quad \text{for all } x, y \in H.$$

Remark 3.2. The above definition is purely set-theoretical and does not require any additional algebraic structure on H. Specific algebraic properties can be obtained by imposing suitable conditions on the neighborhood mapping N.

Proposition 3.3. *Let H be a nonempty set, Λ a parameter set and $N : \Lambda \times H \to \mathcal{P}(H)$ be a neighborhood mapping. For a fixed $\lambda \in \Lambda$, define a hyperoperation on H by*

$$x \oplus_\lambda y = N(\lambda, x) \cap N(\lambda, y), \quad \textit{for all } x, y \in H.$$

Then $(H, \oplus_\lambda)$ is a hypergroupoid.

Proof. Since $N(\lambda, x) \subseteq H$ and $N(\lambda, y) \subseteq H$ for all $x, y \in H$, their intersection

$$x \oplus_\lambda y = N(\lambda, x) \cap N(\lambda, y)$$

is a subset of H. Then the mapping

$$\oplus_\lambda : H \times H \longrightarrow \mathcal{P}(H)$$

is well defined. Thus $(H, \oplus_\lambda)$ satisfies the defining condition of a hypergroupoid. □

Example 3.4. Let $H = \mathbb{R}$ be the set of real numbers equipped with the usual metric $d(x, y) = |x - y|$. Let the parameter set be $\Lambda = \mathbb{R}^+$.

For each $r \in \Lambda$ and $x \in H$, define the neighborhood mapping

$$N(r, x) = \{ y \in \mathbb{R} \mid |y - x| \leq r \}.$$

Using this neighborhood mapping, define a hyperoperation

$$x \oplus_r y = N(r, x) \cap N(r, y), \quad \text{for all } x, y \in H.$$

Since $N(r, x) = [x - r, x + r]$ and $N(r, y) = [y - r, y + r]$, we have

$$x \oplus_r y = [x - r, x + r] \cap [y - r, y + r].$$

In general, the hyperproduct $x \oplus_r y$ may be empty or a closed interval in $\mathbb{R}$. Then $(H, \oplus_r)$ forms an hg-hypergroupoid. Restricting to the nonempty hyperproducts yields a partial hg-hypergroupoid.

Remark 3.5. The metric space example above shows that the proposed hyperoperation is not restricted to modular structures. In fact, whenever a suitable distance function is defined, the neighborhood intersection naturally induces an hg-hypergroupoid structure. In particular, when the underlying set is $\mathbb{Z}_m$ equipped with the modular distance, the construction reduces to a finite and discrete special case. Hence, the hypergroupoid defined on $\mathbb{Z}_m$ can be regarded as a concrete illustration of the general framework introduced in this work.

Example 3.6. Let $m \geq 3$ be a fixed integer and let $H = \mathbb{Z}_m$. For a fixed parameter $r \in \mathbb{N}$, define the neighborhood mapping

$$N(r, x) = \{x + k \pmod{m} \mid |k| \leq r\} \quad x \in \mathbb{Z}_m.$$

The induced hyperoperation $\oplus_r$ on $\mathbb{Z}_m$ is given by

$$x \oplus_r y = N(r, x) \cap N(r, y) \quad \text{for all } x, y \in \mathbb{Z}_m.$$

Then $(\mathbb{Z}_m, \oplus_r)$ is a neighborhood–based parametric hypergroupoid.

For $m = 6, r = 1$

$\oplus_1$	0	1	2	3	4	5
0	$\{0,1,5\}$	$\{5,1\}$	$\{1\}$	$\varnothing$	$\{5\}$	$\{0,5\}$
1	$\{5,1\}$	$\{0,1,2\}$	$\{1,2\}$	$\{2\}$	$\{1\}$	$\{0\}$
2	$\varnothing$	$\{1,2\}$	$\{1,2,3\}$	$\{2,3\}$	$\{3\}$	$\varnothing$
3	$\varnothing$	$\{2\}$	$\{2,3\}$	$\{2,3,4\}$	$\{3,4\}$	$\{4\}$
4	$\{3\}$	$\varnothing$	$\{3\}$	$\{3,4\}$	$\{3,4,5\}$	$\{4,5\}$
5	$\{0,5\}$	$\{0\}$	$\varnothing$	$\{4\}$	$\{4,5\}$	$\{0,4,5\}$

For $m = 6, r = 2$

$\oplus_2$	0	1	2	3	4	5
0	$\{0,1,2,4,5\}$	$\{0,1,2,5\}$	$\{0,1,2,4\}$	$\{1,2,4,5\}$	$\{0,2,4,5\}$	$\{0,1,4,5\}$
1	$\{0,1,2,5\}$	$\{0,1,2,3,5\}$	$\{0,1,2,3\}$	$\{1,2,3,5\}$	$\{0,2,3,5\}$	$\{0,1,3,5\}$
2	$\{0,1,2,4\}$	$\{0,1,2,3\}$	$\{0,1,2,3,4\}$	$\{1,2,3,4\}$	$\{0,2,3,4\}$	$\{0,1,3,4\}$
3	$\{1,2,4,5\}$	$\{1,2,3,5\}$	$\{1,2,3,4\}$	$\{1,2,3,4,5\}$	$\{2,3,4,5\}$	$\{1,3,4,5\}$
4	$\{0,2,4,5\}$	$\{0,2,3,5\}$	$\{0,2,3,4\}$	$\{2,3,4,5\}$	$\{0,2,3,4,5\}$	$\{0,3,4,5\}$
5	$\{0,1,4,5\}$	$\{0,1,3,5\}$	$\{0,1,3,4\}$	$\{1,3,4,5\}$	$\{0,3,4,5\}$	$\{0,1,3,4,5\}$

Theorem 3.7. *Let $m \geq 3$, $H = \mathbb{Z}_m$, and $r \in \mathbb{N}$. For $x, y \in \mathbb{Z}_m$, the hyperoperation*

$$x \oplus_r y = N(r,x) \cap N(r,y)$$

is nonempty if and only if

$$d_m(x,y) \leq 2r.$$

Proof. ($\Rightarrow$) Suppose $x \oplus_r y \neq \varnothing$. Then there exists $z \in \mathbb{Z}_m$ such that $z \in N(r,x)$ and $z \in N(r,y)$. Thus

$$d_m(x,z) \leq r \quad \text{and} \quad d_m(y,z) \leq r.$$

By the triangle inequality on $\mathbb{Z}_m$,

$$d_m(x,y) \leq d_m(x,z) + d_m(z,y) \leq 2r.$$

($\Leftarrow$) Suppose $d_m(x,y) \leq 2r$. Then there exists $z \in \mathbb{Z}_m$ such that $d_m(x,z) \leq r$ and $d_m(y,z) \leq r$. Thus $z \in N(r,x) \cap N(r,y)$. So $x \oplus_r y \neq \varnothing$. □

Theorem 3.8. *Let $H = \mathbb{Z}_m$ with the neighborhood mapping defined above. Then for $x, y \in H$,*

$$x \oplus_r y \neq \varnothing \iff d_m(x,y) \leq 2r$$

and in this case

$$|x \oplus_r y| = 2r - d_m(x,y) + 1.$$

Proof. Let $x, y \in \mathbb{Z}_m$. By definition,

$$x \oplus_r y = N(r,x) \cap N(r,y).$$

Suppose $x \oplus_r y \neq \varnothing$. Then there exists $z \in \mathbb{Z}_m$ such that

$$z \in N(r,x) \cap N(r,y).$$

Thus

$$d_m(x,z) \leq r \quad \text{and} \quad d_m(y,z) \leq r.$$

Using the triangle inequality for the modular distance d_m, we obtain

$$d_m(x,y) \leq d_m(x,z) + d_m(z,y) \leq r + r = 2r.$$

Conversely, suppose $d_m(x,y) \leq 2r$. Since $N(r,x)$ and $N(r,y)$ are symmetric neighborhoods of radius r centered at x and y, respectively, this inequality implies that the two neighborhoods overlap. Then

$$N(r,x) \cap N(r,y) \neq \varnothing.$$

Thus $x \oplus_r y \neq \varnothing$.

Finally, when $d_m(x, y) \leq 2r$, the intersection of two intervals of radius r whose centers are $d_m(x, y)$ apart contains exactly

$$(2r + 1) - d_m(x, y)$$

elements. So

$$|x \oplus_r y| = 2r - d_m(x, y) + 1.$$

This completes the proof. □

Proposition 3.9. *Let $(H, \oplus_\lambda)$ be defined as above. Then $(H, \oplus_\lambda)$ is a partial hg-hypergroupoid; that is, the hyperoperation $\oplus_\lambda$ is well-defined and associative whenever the involved hyperproducts are nonempty.*

Proof. The hyperoperation is well-defined by construction as the intersection of subsets of H. Associativity follows from the associativity of set intersection whenever the corresponding hyperproducts are nonempty. □

Remark 3.10. The possible occurrence of empty set for some hyperproducts prevents the structure from being a hypergroup in the classical sense. However, the operation satisfies the hypergroupoid axioms on the domain where it is defined, which justifies the use of the term *partial hg-hypergroupoid* for the neighborhood-based structure under consideration.

Definition 3.11. Let H be a nonempty set, Λ a set of structural parameters, and E a set of soft parameters. Let

$$N : \Lambda \times H \longrightarrow \mathcal{P}(H)$$

be a neighborhood mapping. Assume that there exists a mapping

$$\phi : E \to \Lambda.$$

For each $e \in E$, define a hyperoperation $\oplus_e$ on H by

$$x \oplus_e y = N(\phi(e), x) \cap N(\phi(e), y), \quad \text{for all } x, y \in H.$$

Define a mapping

$$F : E \to \mathcal{P}(\mathcal{P}(H))$$

by

$$F(e) = \{x \oplus_e y \mid x, y \in H,\ x \oplus_e y \neq \varnothing\}.$$

Then the pair (F, E) is called a *soft partial hg-hypergroupoid* over H.

Proposition 3.12. *Let H be a nonempty set, Λ a set of structural parameters, and E a set of soft parameters. Assume that there exists a mapping $\phi : E \to \Lambda$. If $E = \Lambda$ and $\phi = \mathrm{id}_\Lambda$, then the soft partial hg-hypergroupoid defined in Definition 3.11 reduces to the soft hg-hypergroupoid given in Definition 3.13.*

Proof. Suppose $E = \Lambda$ and $\phi = \mathrm{id}_\Lambda$. Then for each $e \in E = \Lambda$, we have $\phi(e) = e$. Thus the hyperoperation

$$x \oplus_e y = N(\phi(e), x) \cap N(\phi(e), y)$$

coincides with

$$x \oplus_\lambda y = N(\lambda, x) \cap N(\lambda, y)$$

for $\lambda = e$. Moreover, the induced soft set mapping

$$F(e) = \{x \oplus_e y \mid x, y \in H,\ x \oplus_e y \neq \varnothing\}$$

agrees with the mapping

$$F(\lambda) = \{x \oplus_\lambda y \mid x, y \in H,\ x \oplus_\lambda y \neq \varnothing\}.$$

So the two constructions are identical and the former definition reduces to the latter one in this special case. □

Definition 3.13. Let H be a nonempty set and Λ a parameter set. Define a mapping

$$F : \Lambda \to \mathcal{P}(\mathcal{P}(H))$$

by

$$F(\lambda) = \{x \oplus_\lambda y \mid x, y \in H,\ x \oplus_\lambda y \neq \varnothing\},$$

where

$$x \oplus_\lambda y = N(\lambda, x) \cap N(\lambda, y).$$

Then (F, Λ) is called a *soft hg-hypergroupoid over H*.

Proposition 3.14. *Let (F, E) be a soft partial hg-hypergroupoid. Then the structure $(H, \oplus_e)$ is a* (total) hg-hypergroupoid *if and only if*

$$x \oplus_e y \neq \varnothing \quad \textit{for all } x, y \in H.$$

Otherwise, the structure remains a partial hg-hypergroupoid.

Proof. ($\Rightarrow$) Suppose $(H, \oplus_e)$ is a (total) hg-hypergroupoid. By definition, the hyperoperation $\oplus_e$ is defined on the whole set $H \times H$ and assigns a nonempty subset of H to each pair $(x, y) \in H \times H$. Then

$$x \oplus_e y \neq \varnothing \quad \text{for all } x, y \in H.$$

($\Leftarrow$) Conversely, suppose

$$x \oplus_e y \neq \varnothing \quad \text{for all } x, y \in H.$$

Since (F, E) is a soft partial hg-hypergroupoid, the hyperoperation $\oplus_e$ is associative whenever the involved hyperproducts are nonempty. The above hypothesis guarantees that all hyperproducts are nonempty. Then the associativity holds on the entire set $H \times H$. Thus the hyperoperation $\oplus_e$ is everywhere defined and associative, which shows that $(H, \oplus_e)$ is a (total) hg-hypergroupoid.

Otherwise, if there exist $x, y \in H$ such that $x \oplus_e y = \varnothing$, the hyperoperation is not everywhere defined and the structure remains a partial hg-hypergroupoid. □

Example 3.15. For A soft hg-Hypergroupoid on $\mathbb{Z}_6$ Example 3.6

Let $H = \mathbb{Z}_6$ and let the parameter set be

$$\Lambda = \{0, 1, 2\}.$$

Define a mapping

$$F : \Lambda \to \mathcal{P}(\mathcal{P}(H))$$

by

$$F(r) = \{x \oplus_r y \mid x, y \in H,\ x \oplus_r y \neq \varnothing\},$$

where

$$x \oplus_r y = N(r,x) \cap N(r,y), \quad N(r,x) = \{x+k \pmod 6 \mid |k| \le r\}.$$

The support of the soft set (F,Λ) is given by

$$\mathrm{Supp}(F) = \{r \in \Lambda \mid F(r) \neq \varnothing\}.$$

In this case, we have

$$\mathrm{Supp}(F) = \{0,1,2\}.$$

For each $r \in \mathrm{Supp}(F)$, the structure $(H,\oplus_r)$ forms an hg-hypergroupoid. Then (F,Λ) is a soft hg-hypergroupoid over H.

Definition 3.16. Let (H,F,Λ) be a soft partial hg-hypergroupoid. The support set of F is defined by

$$\mathrm{Supp}(F) = \{\lambda \in \Lambda \mid F(\lambda) \neq \varnothing\}.$$

The soft structure $(H,F^*,\mathrm{Supp}(F))$, where F^* is the restriction of F to $\mathrm{Supp}(F)$, is called the *reduced soft partial hg-hypergroupoid.*

Remark 3.17. The idea of disregarding parameters yielding empty values appears implicitly in the soft set literature. However, in the context of hg-hypergroupoids, this approach has not been explicitly and systematically formulated as a reduction process based on support sets. In this work, the notion of a reduced soft hg-hypergroupoid obtained via support sets is introduced to address this gap in the literature.

Example 3.18 (Non-trivial soft example on $\mathbb{Z}_6$ with reduction)**.** Let $H = \mathbb{Z}_6 = \{0,1,2,3,4,5\}$ and let the set of structural parameters be

$$\Lambda = \{\lambda_0, \lambda_1\}.$$

Fix the proper subset $M = \{0,1,2\} \subsetneq H$. Define a neighborhood mapping $N : \Lambda \times H \to \mathcal{P}(H)$ by

$$N(\lambda_0,x) = \varnothing \quad \text{for all } x \in H,$$

and

$$N(\lambda_1,x) = \begin{cases} \{x\} & x \in M \\ \varnothing & x \in H \setminus M. \end{cases}$$

For each $\lambda \in \Lambda$, define the neighborhood–intersection hyperoperation

$$x \oplus_\lambda y = N(\lambda,x) \cap N(\lambda,y) \qquad x,y \in H.$$

Next, take the set of soft parameters

$$E = \{e_0,e_1,e_2\}$$

and define a mapping $\phi : E \to \Lambda$ by

$$\phi(e_0) = \lambda_0, \qquad \phi(e_1) = \lambda_1, \qquad \phi(e_2) = \lambda_1.$$

For each $e \in E$, set

$$x \oplus_e y := N(\phi(e),x) \cap N(\phi(e),y)$$

and define

$$F(e) = \{x \oplus_e y \mid x,y \in H,\ x \oplus_e y \neq \varnothing\}.$$

Then $F(e_0) = \varnothing$, since $N(\lambda_0, x) = \varnothing$ for all x and hence $x \oplus_{e_0} y = \varnothing$ for all $x, y \in H$. Thus

$$\mathrm{Supp}(F) = \{e \in E \mid F(e) \neq \varnothing\} = \{e_1, e_2\}.$$

The reduced soft structure $(H, F^*, \mathrm{Supp}(F))$ is obtained by restricting F to $\mathrm{Supp}(F)$. Moreover, for $e \in \{e_1, e_2\}$, we have $\phi(e) = \lambda_1$. So for $x, y \in H$,

$$x \oplus_e y = \begin{cases} \{x\} & x = y \in M \\ \varnothing & \text{otherwise.} \end{cases}$$

In particular, when we restrict to M, every nonempty hyperproduct of elements of M is again a subset of M, and $M \subsetneq H$. So the reduction process is nontrivial (since e_0 is discarded) and the resulting reduced soft structure exhibits a genuine *small* behavior on the proper subset M. Since the hyperoperation is defined via neighborhood intersections and all nonempty hyperproducts are singletons, associativity holds whenever the hyperproducts are nonempty. Hence the resulting structure satisfies the hg–hypergroupoid condition.

Definition 3.19. Let (M, F, Λ) be a soft partial hg-subhypergroupoid and let

$$\mathrm{Supp}(F) = \{\lambda \in \Lambda \mid F(\lambda) \neq \varnothing\}.$$

Denote by F^* the restriction of F to $\mathrm{Supp}(F)$. The reduced soft partial hg-subhypergroupoid $(M, F^*, \mathrm{Supp}(F))$ is called a *reduced soft small partial hg-subhypergroupoid* of M, if $F^*(\lambda)$ is a small hg-subhypergroupoid of M for all $\lambda \in \mathrm{Supp}(F)$.

Example 3.20. Let $H = \mathbb{Z}_6$ be the set as in Example 3.6 and fix $r = 1$. Define the neighborhood mapping

$$N(1, x) = \{x - 1, x, x + 1\} \pmod 6.$$

The induced hyperoperation on H is defined by

$$x \oplus_1 y = N(1, x) \cap N(1, y).$$

Let $\Lambda = \{1\}$ and define a soft set

$$F : \Lambda \to \mathcal{P}(H), \qquad F(1) = \{0, 1, 2\}.$$

Then $(H, \oplus_1)$ is a partial hg-hypergroupoid. Moreover, $F(1)$ is closed under $\oplus_1$ and satisfies

$$F(1) \subsetneq H.$$

Thus $F(1)$ is a small hg-subhypergroupoid of H and (F, Λ) is a soft partial small hg-subhypergroupoid over H.

Example 3.21. Let $H = \mathbb{Z}_6$ be the set as in Example 3.6 and define the neighborhood mapping with $r = 3$ by

$$N(3, x) = \{x - 3, \ldots, x + 3\} \pmod 6.$$

Since $|H| = 6$, we have $N(3, x) = H$ for all $x \in H$. Hence, the induced hyperoperation satisfies

$$x \oplus_3 y = H \quad \text{for all } x, y \in H.$$

Let $\Lambda = \{3\}$ and define a soft set

$$F : \Lambda \to \mathcal{P}(H), \qquad F(3) = H.$$

Then (F, Λ) is a soft partial hg-hypergroupoid. However, since $F(3) = H$, the soft structure is not small. This example shows that an inappropriate choice of the parameter eliminates the possibility of obtaining a small soft substructure.

4. Conclusion

In this paper, neighborhood-intersection-based hg-hypergroupoid and partial hg-hypergroupoid structures have been investigated within a parameter-dependent framework. Structural parameters and soft set parameters have been clearly distinguished, and the relationship between these two classes of parameters has been modeled via an appropriate mapping. This approach demonstrates that neighborhood-based hyperoperations give rise to flexible hyperstructures whose behavior varies with respect to the chosen parameters.

Furthermore, the notion of *small* has been introduced in terms of the support of the underlying soft structure, highlighting the connection between partiality of the hyperoperation and the fact that the support does not cover the entire universe. It has been shown that, under suitable conditions eliminating empty hyperproducts, the resulting structure reduces to a total hg-hypergroupoid. Finally, examples defined on $\mathbb{Z}_m$ equipped with the modular distance illustrate that the proposed framework can be realized in a concrete and discrete setting.

As a direction for future research, the present framework may be extended by considering different types of neighborhood systems, alternative distance functions, and time– or scale–dependent parameters. In addition, the study of morphisms, soft continuity, and dynamic processes associated with neighborhood–based soft hg–hypergroupoids appears to be a natural continuation of the results presented in this work.

References

[1] F. Marty, Sur une generalization de la notion de groups, 8th Congress Math. Scandinaves, Stockholm (1934).

[2] P. Corsini and V. Leoreanu, Applications of Hyperstructure Theory, Vol. 5, Springer Science & Business Media, 2013.

[3] B. Davvaz, V. Leoreanu-Fotea and T. Vougiouklis, A survey on the theory of n-hypergroups, Mathematics 11 (3) (2023) 551.

[4] B. Davvaz and V. Leoreanu-Fotea, Hyperring Theory and Applications, Vol. 347, International Academic Press, USA, 2007.

[5] C. Massouros and G. Massouros, An overview of the foundations of the hypergroup theory, Mathematics 9 (9) (2021) 1014.

[6] I. Cristea and M. Stefanescu, Hypergroups and n-ary relations, European Journal of Combinatorics 31 (3) (2010) 780–789.

[7] M. Al Tahan, S. Hoskova-Mayerova and B. Davvaz, An overview of topological hypergroupoids, Journal of Intelligent and Fuzzy Systems 34 (3) (2018) 1907–1916.

[8] F. Harary, Graph Theory, Addison-Wesley, Reading, MA, USA, 1969.

[9] D. Molodtsov, Soft set theory—first results, Computers & Mathematics with Applications 37 (4-5) (1999) 19–31.

[10] U. Acar, F. Koyuncu and B. Tanay, Soft sets and soft rings, Computers & Mathematics with Applications 59 (11) (2010) 3458–3463.

[11] A. S. Sezer, A. O. Atagün and N. Çağman, A new view to N-group theory: soft N-groups, Fasciculi Mathematici 51 (2013) 123–140.

[12] S. Yamak, O. Kazancı and B. Davvaz, Soft hyperstructure, Computers & Mathematics with Applications 62 (2) (2011) 797–803.
[13] O. Kazancı, S. Hoskova-Mayerova and B. Davvaz, Algebraic hyperstructure of multi-fuzzy soft sets related to polygroups, Mathematics 10 (13) (2022) 2178.
[14] G. Şenel, J. G. Lee and K. Hur, Advanced soft relation and soft mapping, International Journal of Computational Intelligence Systems 14 (1) (2021) 461–470.
[15] G. Şenel, A new approach to Hausdorff space theory via the soft sets, Mathematical Problems in Engineering 2016 (2016) 1–6.
[16] G. Şenel and N. Çağman, Soft topological subspaces, Ann. Fuzzy Math. Inform. 10 (4) (2015) 525–535.
[17] G. Şenel and N. Çağman, Soft closed sets on soft bitopological space, Journal of New Results in Science 3 (5) (2014) 57–66.
[18] G. Şenel, J. G. Lee and K. Hur, Distance and similarity measures for octahedron sets and their application to MCGDM problems, Mathematics 8 (2020) 1690.
[19] J. G. Lee, G. Şenel, J. I. Baek, S. H. Han and K. Hur, Neighborhood structures and continuities via cubic sets, Axioms 11 (2022) 406.
[20] D. Burago, Y. Burago and S. Ivanov, A Course in Metric Geometry, Vol. 33, American Mathematical Society, Providence, RI, 2001.
[21] P. Corsini, Hypergraphs and hypergroups, Algebra Universalis 35 (4) (1996) 548–555.
[22] N. Çağman and S. Enginoğlu, Soft set theory and uni–int decision making, European Journal of Operational Research 207 (2) (2010) 848–855.
[23] M. A. Dehghanizadeh, Soft crossed hypermodules and soft HG-hypergroupoids, Journal of Mathematical Extension 18 (6) (2024) 1–18.

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