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Abstract. In this paper, the notion of fuzzy weakly F_σ-complemented space is introduced and studied. The conditions under which fuzzy F_σ-complemented spaces become fuzzy weakly F_σ-complemented spaces, are established. It is obtained that fuzzy weakly F_σ-complemented spaces are neither fuzzy almost P-spaces nor fuzzy quasi- F-spaces. The conditions under which fuzzy weakly F_σ-complemented spaces become fuzzy resolvable spaces are also obtained.

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Keywords: Fuzzy F_σ-set, Fuzzy dense set, Fuzzy nowhere dense set, Fuzzy nodef space, Fuzzy F' space, Fuzzy almost P-space, Fuzzy quasi-F-space, Fuzzy perfectly disconnected space.

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1. Introduction

The universe is a complex system filled with uncertainties. Problems of vagueness have probably always existed in human experience and vagueness is not regarded with suspicion, but is simply an acknowledged characteristic of the world around us. Mathematics equips us with the tools to quantify and manage these uncertainties. Human concepts have a graded structure in that whether or not a concept applies to a given object is a matter of degree, rather than a yes - or - no question and that people are capable of working with the degrees in a consistent way. In 1965, Zadeh [1] in his classic paper, called the concepts with a graded structure "*fuzzy concepts*" and proposed the notion of a "fuzzy set" that give birth to the field of fuzzy logic . The potential of fuzzy notion was realized by the researchers and has successfully been applied for new investigations in all the branches of science and technology for more than last five decades. In 1968, Chang [2] introduced the concept of fuzzy topological space. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

In the recent years, there has been a growing trend to introduce and study different forms of fuzzy topological spaces. In 2004, Henriksen and Woods [3] introduced the notion of cozero complemented space and several characterizations of these spaces are established. Levy and Shapiro [4] studied cozero complemented spaces under the name "z-good spaces". In [5], Azarpanah and Karavan studied cozero complemented spaces in the name "m-spaces". In 2009, Knox et al. [6] introduced the notion of weakly cozero complemented space. In essence, weakly cozero complemented spaces are a generalization of the idea of cozero complemented spaces, where the separation requirement is relaxed. They play a role in the study of functional spaces and other topological spaces, particularly in relation to minimal prime ideal spaces and related concepts.

In 2023, the notion of F_σ- complemented spaces in fuzzy setting was introduced and studied by Thangaraj and Vikraman in [7] . In this paper, the notion of fuzzy weakly F_σ-complemented space is introduced and studied. The conditions under which fuzzy F_σ- complemented spaces become fuzzy weakly F_σ-complemented spaces are explored. The conditions, under which fuzzy perfectly disconnected spaces become both fuzzy F_σ-complemented spaces and fuzzy weakly F_σ-complemented spaces, are identified. It is found that fuzzy weakly F_σ-complemented spaces are neither fuzzy almost P-spaces nor fuzzy quasi-F-spaces. The conditions, under which fuzzy weakly F_σ-complemented spaces become fuzzy resolvable spaces, are also obtained in this paper.

2. Preliminaries

Some basic notions and results used in the sequel, are given in order to make the exposition self-contained. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval [0,1]. A *fuzzy set* λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$ for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$ for all $x \in X$. For any fuzzy set λ in X and a family $(\lambda_i)_{i\in J}$ of fuzzy set in X, the *compliment* $\lambda^{'}$, the *union* $\bigvee_{i\in J}\lambda_i$ and *intersection* $\bigwedge_{i\in J}\lambda_i$ are defined respectively as follows: for each $x \in X$, $\lambda^{'}(x) = 1-\lambda(x)$, $(\bigvee_{i\in J}\lambda_i)(x) = sup_{i\in J}\lambda_i(x)$, $(\bigwedge_{i\in J}\lambda_i)(x) = inf_{i\in J}\lambda_i(x)$, where J is an index set.

Definition 2.1 ([2])**.** A *fuzzy topology* on a set X is a family T of fuzzy sets in X which satisfies the following conditions:

(i) $0_X \in T$ and $1_X \in T$,
(ii) if $A, B \in T$, then $A \wedge B \in T$,
(iii) if $A_i \in T$ for each $i \in J$, then $\bigvee_{i\epsilon J} A_i \in T$,

The pair (X, T) is called a *fuzzy topological space* (briefly, fts). Members of T are called *fuzzy open sets* in X and their complements are called *fuzzy closed sets* in X.

Definition 2.2 ([2])**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The *interior* and the *closure* of λ are defined respectively as follows:

(i) $int(\lambda) = \bigvee\{\mu/\mu \leq \lambda, \mu \in T\}$,

(ii) $cl(\lambda) = \bigwedge\{\mu^{'}/\lambda \leq \mu^{'}, \mu \in T\}$.

Lemma 2.3 ([8]). *Let λ be any fuzzy set in a fuzzy topological space (X, T). Then we have*

(1) $1 - cl(\lambda) = int(1 - \lambda)$,
(2) $1 - int(\lambda) = cl(1 - \lambda)$.

Definition 2.4. A fuzzy set λ in a fuzzy topological space (X, T) is called a

(i) *fuzzy regular open*, if $\lambda = intcl(\lambda)$ and *fuzzy regular closed* in X, if $clint(\lambda) = \lambda$ [8],
(ii) *fuzzy G_δ-set* in X, if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in T$ [9],
(iii) *fuzzy dense set* in X, if there exists no fuzzy closed set μ in X such that $\lambda < \mu < 1$, i.e., $cl(\lambda) = 1$ in X [10],
(iv) *fuzzy nowhere dense set* in X, if there exists no non-zero fuzzy open set μ in X such that $\mu < cl(\lambda)$, i.e., $intcl(\lambda) = 0$ in X [10],
(v) *fuzzy first category set* in X, if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where each λ_i is a fuzzy nowhere dense set in X. Any other fuzzy set in X is said to be *of fuzzy second category* [10],
(vi) *fuzzy somewhere dense set* in X, if there exists a non-zero fuzzy open set μ in X such that $\mu < cl(\lambda)$, i.e., $intcl(\lambda) \neq 0$ in X [11],
(vii) *fuzzy residual set* in X, if $1 - \lambda$ is a fuzzy first category set in X [12],
(viii) *fuzzy σ-nowhere dense set* in X, if λ is a fuzzy F_σ-set with $int(\lambda) = 0$ in X [13],
(ix) *fuzzy simply* open set* in X, if $\lambda = \mu \vee \delta$, where μ is a fuzzy open set and δ is a fuzzy nowhere dense set in X [14],
(x) *fuzzy σ-boundary set* in X, if $\lambda = \bigvee_{i=1}^{\infty}(\mu_i)$, where $\mu_i = cl(\lambda_i) \wedge (1 - \lambda_i)$ and each λ_i is a fuzzy regular open set in X [15],
(xi) *fuzzy pseudo-open set* in X, if $\lambda = \mu \vee \delta$, where μ is a non-zero fuzzy open set in X and δ is a fuzzy first category set in X [16],
(xii) *fuzzy Baire set* in X, if $\lambda = \mu \wedge \eta$, where μ is a fuzzy open set and η is a fuzzy residual set in X [16].

Definition 2.5. A fuzzy topological space (X, T) is called a

(i) *fuzzy regular space*, if each fuzzy open set A of X is a union of fuzzy open sets $(\lambda_i)'$s of X such that $cl(\lambda_i) \leq \lambda$ [8],
(ii) *fuzzy extremally disconnected space*, if the closure of every fuzzy open set of X is fuzzy open in X [17],
(iii) *fuzzy hyperconnected space*, if every non-null fuzzy open subset of X is fuzzy dense in X [18],
(iv) *fuzzy open hereditarily irresolvable space*, if for any non-zero fuzzy set λ in X, $intcl(\lambda) \neq 0$ imply that $int(\lambda) \neq 0$ in X [19],
(v) *fuzzy resolvable space*, if there exists a fuzzy dense set λ in X such that $cl(1 - \lambda) = 1$. Otherwise, (X, T) is called a *fuzzy irresolvable space* [19],
(vi) *fuzzy D-Baire space*, if every fuzzy first category set in X is a fuzzy nowhere dense set in X[20],
(vii) *fuzzy almost P-space*, if for each non- zero fuzzy G_δ-set λ in X, $int(\lambda) \neq 0$ in X [21],

(viii) *fuzzy F'-space*, if $\lambda \leq 1-\mu$ imply that $cl(\lambda) \leq 1 - cl(\mu)$ in X, where λ and μ are fuzzy F_σ-sets in X [22],
(ix) *fuzzy perfectly disconnected space*, if for any two non-zero fuzzy sets λ and μ defined on X such that $\lambda \leq 1-\mu$ in X, $cl(\lambda) \leq 1 - cl(\mu)$ in X [23],
(x) *fuzzy quasi-F space*, if $clint(\lambda \wedge \mu) = clint(\lambda) \wedge clint(\mu)$ for any two fuzzy G_δ-sets λ and μ in X [24],
(xi) *fuzzy nodef space* if each fuzzy nowhere dense set is a fuzzy F_σ-set in X [25],
(xii) *fuzzy DG_δ-space*, if each fuzzy dense (but not fuzzy open) set in X is a fuzzy G_δ-set in X [25],
(xiii) *fuzzy O_z-space*, if each fuzzy regular closed set is a fuzzy G_δ-set in X [26],
(xiv) *fuzzy F_σ-complemented space*, if for each fuzzy F_δ-set λ in X, there exist a fuzzy F_σ-set μ in X such that $\lambda \leq 1-\mu$ and $cl(\lambda \vee \mu) = 1$ [7],
(xv) *fuzzy fraction dense space*, if for each fuzzy open set λ in X, $cl(\lambda) = cl(\mu)$, where μ is a fuzzy F_σ-set in X [27],
(xvi) *fuzzy S*N-space*, if for each pair of fuzzy closed sets μ_1 and μ_2 in X with $\mu_1 \leq 1-\mu_2$, there exist fuzzy simply* open sets λ_1 and λ_2 in X such that $\mu_1 \leq \lambda_1$, $\mu_2 \leq \lambda_2$ and $\lambda_1 \leq 1-\lambda_2$ [28].

Theorem 2.6 ([8])**.** *In a fuzzy topological space,*

(1) *the closure of a fuzzy open set is a fuzzy regular closed set,*
(2) *the interior of a fuzzy closed set is a fuzzy regular open set.*

Theorem 2.7 ([19])**.** *If the fuzzy topological space (X,T) is a fuzzy open hereditarily irresolvable space, then for any non-zero fuzzy set λ in X, $cl(\lambda) = 1$ implies that $cl\ int(\lambda) = 1$ in X.*

Theorem 2.8 ([20])**.** *If a fuzzy topological space (X,T) has a fuzzy dense and fuzzy G_δ-set, then X is not a fuzzy D-Baire space.*

Theorem 2.9 ([21])**.** *A fuzzy topological space (X,T) is a fuzzy almost P-space if and only if the only fuzzy F_σ -set λ such that $cl(\lambda) = 1$ in X is 1_X.*

Theorem 2.10 ([13])**.** *If λ is a fuzzy σ-nowhere dense set in a fuzzy topological space (X,T), then $1-\lambda$ is a fuzzy residual set in X.*

Theorem 2.11 ([15])**.** *If λ is a fuzzy σ-boundary set in a fuzzy topological space (X,T), then λ is a fuzzy F_σ-set in X.*

Theorem 2.12 ([22])**.** *If a fuzzy topological space (X,T) is a fuzzy perfectly disconnected space, then X is a fuzzy F'-space.*

Theorem 2.13 ([23])**.** *If for any two fuzzy sets λ and μ defined on X in a fuzzy perfectly disconnected space (X,T), $\lambda \leq 1-\mu$, then there exists a fuzzy open set δ in X such that $intcl(\lambda) \leq \delta \leq 1 - cl[int(\mu)]$ and $int(\mu)$ is not a fuzzy dense set in X.*

Theorem 2.14 ([24])**.** *If for any two fuzzy F_σ-sets γ and δ in a fuzzy topological space (X,T), $intcl(\gamma \vee \delta) \leq intcl(\gamma) \vee intcl(\delta)$, then X is a fuzzy quasi-F-space.*

Theorem 2.15 ([25])**.** *If λ is a fuzzy nowhere dense (but not fuzzy closed) set in a fuzzy DG_δ- space (X,T), then λ is a fuzzy F_σ-set in X.*

Theorem 2.16 ([16]). *If λ is a fuzzy pseudo-open set in a fuzzy D-Baire space (X, T), then λ is a fuzzy simply* open set in X.*

Theorem 2.17 ([26]). *If λ is a fuzzy regular open set in a fuzzy O_z-space (X, T), then λ is a fuzzy F_σ-set in X.*

Theorem 2.18 ([26]). *If μ is a fuzzy regular open set in a fuzzy extremally disconnected space (X, T), then μ is a fuzzy closed F_σ-set in X.*

Theorem 2.19 ([7]). *If (X, T) is a topological space in which fuzzy F_σ-sets are fuzzy dense and fuzzy disjoint, then X is a fuzzy F_σ-complemented space.*

Theorem 2.20 ([29]). *If a fuzzy topological space (X, T) is a fuzzy regular space, then each fuzzy open set in X is a fuzzy F_σ-set in X.*

Theorem 2.21 ([27]). *If (X, T) is a fuzzy fraction dense and fuzzy DG_δ-space, then X is a fuzzy nodef space.*

Theorem 2.22 ([28]). *If a fuzzy set λ is a fuzzy simply* open set in a fuzzy topological space (X, T), then $cl(\lambda)$ is a fuzzy F_σ-set in X.*

3. Fuzzy weakly F_σ-complimented spaces

Motivated by the works of Knox et al. [6] on weakly cozero complemented spaces in classical topology, the notion of fuzzy weakly F_σ-complemented spaces is defined as follows.

Definition 3.1. A fuzzy topological space (X, T) is called a *fuzzy weakly F_σ-complemented space*, if for each pair of fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1 - \mu_2$ in X, there exist fuzzy F_σ-sets λ_1 and λ_2 with $\lambda_1 \leq 1 - \lambda_2$ in X such that $\mu_1 \leq \lambda_1$, $\mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$.

Proposition 3.2. *If a fuzzy topological space (X, T) is a fuzzy weakly F_σ-complemented space, then for each pair of fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \leq 1 - \mu_2$ in X, there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$.*

Proof. Let μ_1 and μ_2 be any two fuzzy F_σ-sets in X with $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy weakly F_σ-complemented space, there exist fuzzy F_σ-sets λ_1 and λ_2 with $\lambda_1 \leq 1 - \lambda_2$ in X such that $\mu_1 \leq \lambda_1$, $\mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. Now $\mu_1 \leq \lambda_1 \leq 1 - \lambda_2 \leq 1 - \mu_2$. Let $\delta = 1 - \lambda_2$. Then δ is a fuzzy G_δ-set in X. Thus for a pair of fuzzy F_σ-sets $\mu_1$and μ_2 with $\mu_1 \leq 1 - \mu_2$, there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$. □

Corollary 3.3. *If μ_1 and μ_2 are disjoint fuzzy F_σ-sets in a fuzzy weakly F_σ-complemented space (X, T), then there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta$ and $\mu_2 \leq 1 - \delta$.*

Proof. Let μ_1 and μ_2 be disjoint fuzzy F_σ-sets in X. Then $\mu_1 \wedge \mu_2 = 0$ implies that $\mu_1 \leq 1 - \mu_2$ in X and by Proposition 3.2, there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$. Now $\delta \leq 1 - \mu_2$ implies that $\mu_2 \leq 1 - \delta$. Thus there exists a fuzzy G_δ -set δ in X such that $\mu_1 \leq \delta$ and $\mu_2 \leq 1 - \delta$. □

Proposition 3.4. *If μ_1 and μ_2 are fuzzy F_σ-sets with $\mu_1 \le 1-\mu_2$ in a fuzzy weakly F_σ-complemented space (X,T), then there exist fuzzy somewhere dense sets λ_1 and λ_2 in X such that $\mu_1 \le \lambda_1$, $\mu_2 \le \lambda_2$ and $1 - cl(\lambda_1) \le cl(\lambda_2)$.*

Proof. Let μ_1 and μ_2 be fuzzy F_σ-sets in X with $\mu_1 \le 1-\mu_2$. Since X is a fuzzy weakly F_σ-complemented space, there exist fuzzy F_σ-sets λ_1 and λ_2 with $\lambda_1 \le 1-\lambda_2$ in X such that $\mu_1 \le \lambda_1$, $\mu_2 \le \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. Now $1-cl(\lambda_1 \vee \lambda_2) = 0$ implies that $1-[cl(\lambda_1) \vee cl(\lambda_2)] = 0$. Then $[1-cl(\lambda_1)] \wedge [1-cl(\lambda_2)] = 0$. This implies that $1-cl(\lambda_1) \le 1-[1-cl(\lambda_2)]$ and $1-cl(\lambda_1) \le cl(\lambda_2)$, in (X,T). Since $\lambda_1 \le 1-\lambda_2$, $\lambda_2 \le 1-\lambda_1$ and $cl(\lambda_2) \le cl(1-\lambda_1) = 1-int(\lambda_1)$ Thus $1-cl(\lambda_1) \le cl(\lambda_2) \le 1-int(\lambda_1)$. Now $1-cl(\lambda_1)$ is a fuzzy open set in X implies that $intcl(\lambda_2) \ne 0$. Then λ_2 is a fuzzy somewhere dense set in X. Also $1-cl(\lambda_1) \le cl(\lambda_2) \le 1-int(\lambda_1) \le 1$, implies that $1-cl(\lambda_1)$ is not a fuzzy dense set in X. Thus $cl[1-cl(\lambda_1)] \ne 1$ in X. Then by Lemma 2.3, $[1-intcl(\lambda_1)] \ne 1$ and $intcl(\lambda_1) \ne 0$. So λ_1 is a fuzzy somewhere dense set in X. Hence for the fuzzy F_σ-sets μ_1 and μ_2 with $\mu_1 \le 1-\mu_2$, there exist fuzzy somewhere dense sets λ_1 and λ_2 in X such that $\mu_1 \le \lambda_1, \mu_2 \le \lambda_2$ and $1-cl(\lambda_1) \le cl(\lambda_2)$. □

Corollary 3.5. *If μ_1 and μ_2 are disjoint fuzzy F_σ-sets in a fuzzy weakly F_σ-complemented space (X,T), then there exists a fuzzy F_σ-set λ_1 and λ_2 in X such that $\mu_1 \le \lambda_1$ and $\mu_2 \le \lambda_2$ and $1-cl(\lambda_1) \le (cl\lambda_2)$, where λ_1 and λ_2 are not fuzzy nowhere dense sets in X.*

Proposition 3.6. *If a fuzzy topological space (X,T) is a fuzzy weakly F_σ-complemented space, then for each pair of fuzzyF_σ-sets μ_1 and μ_2 with $\mu_1 \le 1-\mu_2$ in X, there exist fuzzy F_σ- sets λ_1 and λ_2 with $\lambda_1 \le 1-\lambda_2$, in X and $1-cl(\lambda_2) \le cl(\lambda_1)$.*

Proof. Let μ_1 and μ_2 be fuzzy F_σ-sets in X with $\mu_1 \le 1-\mu_2$. Since (X,T) is a fuzzy weakly F_σ-complemented space, there exist fuzzy F_σ-sets λ_1 and λ_2 with $\lambda_1 \le 1-\lambda_2$ in X such that $\mu_1 \le \lambda_1$, $\mu_2 \le \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. Then $\mu_1 \le \lambda_1 \le cl(\lambda_1)$, $\mu_2 \le \lambda_2 \le cl(\lambda_2)$ and $1-cl(\lambda_1 \vee \lambda_2) = 0$, in X. Now, by Lemma 2.3, $int(1-[\lambda_1 \vee \lambda_2]) = 1-cl(\lambda_1 \vee \lambda_2)$. Thus $int([1-\lambda_1] \wedge [1-\lambda_2]) = 0$. This implies that $int(1-\lambda_1) \wedge int(1-\lambda_2) = 0$ in X. So $int(1-\lambda_2) \le 1-int(1-\lambda_1)$ and $1-cl(\lambda_2) \le 1-[1-cl(\lambda_1)]$. Hence $1-cl(\lambda_2) \le cl(\lambda_1)$ in X. □

Remark 3.7. From proposition 3.6, it is understood that for the fuzzy F_σ-sets λ_1 and λ_2 with $\lambda_1 \le 1-\lambda_2$ in X, the relation $cl(\lambda_1) \le 1-cl(\lambda_2)$ need not hold in general and $cl(\lambda_1) \le 1-cl(\lambda_2)$ will hold only if the fuzzy topological space X is a fuzzy perfectly disconnected space.

Proposition 3.8. *If μ_1 and μ_2 are disjoint fuzzy σ-nowhere dense sets in a fuzzy weakly F_σ-complemented space (X,T), then there exist a fuzzy G_δ-set δ and a fuzzy residual set θ in X such that*

(i) $\mu_1 \le \delta \le 1-\mu_2$
(ii) $\mu_1 \le \delta \le \theta$.

Proof. Let μ_1 and μ_2 be disjoint fuzzy σ - nowhere dense sets in X. Then $\mu_1 \wedge \mu_2 = 0$. Thus $\mu_1 \le 1-\mu_2$.

(i) Since μ_1 and μ_2 are fuzzy σ-nowhere dense sets in X, μ_1 and μ_2 are fuzzy F_σ-sets in X with $int(\mu_1) = 0$ and $int(\mu_2) = 0$. Since X is a fuzzy weakly F_σ-complemented space, by Proposition 3.2, there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$.

(ii) By Theorem 2.10, for the fuzzy σ-nowhere dense set μ_2, $1 - \mu_2$ is a fuzzy residual set in X. Let $\theta = 1 - \mu_2$. Then there exist a fuzzy G_δ-set δ and a fuzzy residual set θ in X such that $\mu_1 \leq \delta \leq \theta$. □

Proposition 3.9. *Let (X, T) be a fuzzy topological space. If δ_1 and δ_2 are fuzzy G_δ-sets in X with $1 - \delta_1 \leq \delta_2$, then there exist fuzzy G_δ-sets η_1 and η_2 in X with $1 - \eta_1 \leq \eta_2$ such that $\eta_1 \leq \delta_1$, $\eta_2 \leq \delta_2$ and $int(\eta_1 \wedge \eta_2) = 0$.*

Proof. Let δ_1 and δ_2 be fuzzy G_δ-sets in X with $1 - \delta_1 \leq \delta_2$. Then $1 - \delta_1$ and $1 - \delta_2$ are fuzzy F_σ-sets in X with $1 - \delta_1 \leq 1 - (1 - \delta_2)$. Since X is a fuzzy weakly F_σ-complemented space, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1 - \lambda_2$ such that $1 - \delta_1 \leq \lambda_1$, $1 - \delta_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. This implies that $1 - \lambda_1 \leq \delta_1$, $1 - \lambda_2 \leq \delta_2$. Let $\eta_1 = 1 - \lambda_1$ and $\eta_2 = 1 - \lambda_2$. Thus η_1 and η_2 are fuzzy G_δ-sets in (X, T) and $\eta_1 \leq \delta_1$, $\eta_2 \leq \delta_2$. Now $cl(\lambda_1 \vee \lambda_2) = 1$, implies that $1 - cl(\lambda_1 \vee \lambda_2) = 0$. By Lemma 2.3, $int(1 - [\lambda_1 \vee \lambda_2]) = 0$ and $int([1 - \lambda_1] \wedge [1 - \lambda_2]) = 0$. So $int(\eta_1 \wedge \eta_2) = 0$. Also, $\lambda_1 \leq 1 - \lambda_2$ implies that $1 - (1 - \lambda_1) \leq 1 - \lambda_2$. Hence $1 - \eta_1 \leq \eta_2$. □

Proposition 3.10. *If μ_1 and μ_2 are disjoint fuzzy σ-boundary sets in a fuzzy weakly F_σ-complemented space (X, T), then there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$.*

Proof. Let μ_1 and μ_2 be disjoint fuzzy σ-boundary sets in X. Then $\mu_1 \wedge \mu_2 = 0$. This implies that $\mu_1 \leq 1 - \mu_2$. Since μ_1 and μ_2 are fuzzy σ-boundary sets , by Theorem 2.11 , μ_1 and μ_2 are fuzzy F_σ-sets in the fuzzy weakly F_σ-complemented space X. Thus by Proposition 3.2, there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$. □

4. Fuzzy weakly F_σ-complimented spaces and other fuzzy topological spaces

In this section, we relate fuzzy weakly F_σ-complemented spaces to some other well known fuzzy topological spaces.

The following proposition shows that fuzzy F_σ-complemented and fuzzy F' -spaces are fuzzy weakly F_σ-complemented spaces.

Proposition 4.1. *If (X, T) is a fuzzy F_σ-complemented and fuzzy F'-space, then X is a fuzzy weakly F_σ-complemented space.*

Proof. Let μ_1 be a non-zero fuzzy F_σ-set in X. Since X is a fuzzy F_σ-complemented space, there exists a fuzzy F_σ-set μ_2 in X such that $\mu_1 \leq 1 - \mu_2$ and $cl(\mu_1 \vee \mu_2) = 1$. Thus there exists a pair of fuzzy F_σ-sets μ_1 and μ_2 in X with $\mu_1 \leq 1 - \mu_2$. Now $cl(\mu_1)$ and $cl(\mu_2)$ are fuzzy F_σ-sets in X. Let $\lambda_1 = cl(\mu_1)$ and $\lambda_2 = cl(\mu_2)$. Now $cl[\lambda_1 \vee \lambda_2] = cl[cl(\mu_1) \vee cl(\mu_2)] = cl[cl(\mu_1 \vee \mu_2)] = cl(1) = 1$. Since X is a fuzzy F'-space, for the fuzzy F_σ-sets μ_1 and μ_2 in X with $\mu_1 \leq 1 - \mu_2$, $cl(\mu_1) \leq 1 - cl(\mu_2)$. Thus $\lambda_1 \leq 1 - \lambda_2$. So for a pair of fuzzy F_σ-sets μ_1 and μ_2 in X with $\mu_1 \leq 1 - \mu_2$, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1-\lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl[\lambda_1 \vee \lambda_2] = 1$. Hence X is a fuzzy weakly F_σ-complemented space. □

Proposition 4.2. *If (X, T) is a fuzzy F_σ-complemented and fuzzy perfectly disconnected space, then X is a fuzzy weakly F_σ-complemented space.*

Proof. Let μ_1 be a non-zero fuzzy F_σ-set in X. Since X is a fuzzy F_σ-complemented space, there exists a fuzzy F_σ-set μ_2 in X such that $\mu_1 \leq 1-\mu_2$ and $cl(\mu_1 \vee \mu_2) = 1$. Thus there exists a pair of fuzzy F_σ-sets μ_1 and μ_2 in X with $\mu_1 \leq 1-\mu_2$. Now $\mu_1 \leq cl(\mu_1),\ \mu_2 \leq cl(\mu_2)$ and $cl(\mu_1)$ and $cl(\mu_2)$ are fuzzy F_σ-sets in X. Since X is a fuzzy perfectly disconnected space, for the non-zero fuzzy sets μ_1 and μ_2 in X such that $\mu_1 \leq 1-\mu_2, cl(\mu_1) \leq 1-cl(\mu_2)$, in (X, T). Let $\lambda_1 = cl(\mu_1)$ and $\lambda_2 = cl(\mu_2)$. Now $cl[\lambda_1 \vee \lambda_2] = cl[cl(\mu_1) \vee cl(\mu_2)] = cl[cl(\mu_1 \vee \mu_2)] = cl(1) = 1$. So for a pair of fuzzy F_σ-sets μ_1 and μ_2 in X with $\mu_1 \leq 1-\mu_2$, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1-\lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl[\lambda_1 \vee \lambda_2] = 1$. Hence X is a fuzzy weakly F_σ-complemented space. □

The following proposition give conditions for fuzzy F'-spaces to become fuzzy weakly F_σ-complemented spaces

Proposition 4.3. *If $cl(\mu_1 \vee \mu_2) = 1$, for any two fuzzy F_σ-sets μ_1 and μ_2 in a fuzzy F'-space (X, T) with $\mu_1 \leq 1-\mu_2$, then X is a fuzzy weakly F_σ-complemented space.*

Proof. Let μ_1 and μ_2 be fuzzy F_σ-sets in X such that $\mu_1 \leq 1-\mu_2$ and $cl(\mu_1 \vee \mu_2) = 1$. Now $\mu_1 \leq cl(\mu_1), \mu_2 \leq cl(\mu_2)$ and $cl(\mu_1)$ and $cl(\mu_2)$ are fuzzy F_σ-sets in X. Since X is a fuzzy F'-space, for the non-zero fuzzy sets μ_1 and μ_2 in X with $\mu_1 \leq 1-\mu_2$, $cl(\mu_1) \leq 1-cl(\mu_2)$. Let $\lambda_1 = cl(\mu_1)$ and $\lambda_2 = cl(\mu_2)$. Now we have

$$cl[\lambda_1 \vee \lambda_2] = cl[cl(\mu_1) \vee cl(\mu_2)] = cl[cl(\mu_1 \vee \mu_2)] = cl(1) = 1.$$

Then for a pair of fuzzy $F_\sigma-$sets μ_1 and μ_2 in X with $\mu_1 \leq 1-\mu_2$, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1-\lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl[\lambda_1 \vee \lambda_2] = 1$. Thus X is a fuzzy weakly F_σ-complemented space. □

The following proposition give conditions for fuzzy perfectly disconnected spaces to become fuzzy weakly F_σ-complemented spaces.

Proposition 4.4. *If $cl(\mu_1 \vee \mu_2) = 1$, for any two fuzzy F_σ-sets μ_1 and μ_2 in a fuzzy perfectly disconnected space (X, T) with $\mu_1 \leq 1-\mu_2$, then X is a fuzzy weakly F_σ-complemented space.*

Proof. The proof follows from Theorem 2.12 and Proposition 4.3. □

Corollary 4.5. *If fuzzy F_σ-sets are fuzzy dense and fuzzy disjoint in a fuzzy perfectly disconnected space (X, T), then X is a fuzzy weakly F_σ-complemented space.*

Remark 4.6. In view of theorem 2.19 and corollary 4.5, one will have the following result:

"Fuzzy perfectly disconnected spaces, in which fuzzy F_σ-sets are fuzzy dense and fuzzy disjoint, are fuzzy F_σ-complemented spaces as well as fuzzy weakly F_σ-complemented spaces ".

Proposition 4.7. *If μ_1 and μ_2 are disjoint fuzzy regular open sets in a fuzzy O_z and fuzzy weakly F_σ-complemented space (X, T), then there exist a fuzzy G_δ-set δ in X such that*

(i) $\mu_1 \leq \delta \leq 1 - \mu_2$,
(ii) $int(\mu_1) \leq clint(\delta) \leq 1 - \mu_2$,
(iii) $\mu_1 \leq intcl(\delta) \leq 1 - int(\mu_2)$.

Proof. Suppose that μ_1 and μ_2 are disjoint fuzzy regular open sets in X. Then $\mu_1 \wedge \mu_2 = 0$. This implies that $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy O_z-space, by Theorem 2.17, μ_1 and μ_2 are fuzzy F_σ-sets in X. Thus μ_1 and μ_2 are disjoint fuzzy F_σ-sets in the fuzzy weakly F_σ-complemented space X.

(i) By Corollary 3.3, there exists a fuzzy G_δ-set δ in (X, T) such that $\mu_1 \leq \delta \leq 1 - \mu_2$.

(ii) Now $\mu_1 \leq \delta \leq 1 - \mu_2$ implies that $clint(\mu_1) \leq clint(\delta) \leq clint(1 - \mu_2)$. Then $int(\mu_1) \leq clint(\mu_1) \leq clint(\delta) \leq 1 - intcl(\mu_2) = 1 - \mu_2$. Thus it follows that $int(\mu_1) \leq clint(\delta) \leq 1 - \mu_2$.

(iii) Now $\mu_1 \leq \delta \leq 1 - \mu_2$ implies that $intcl(\mu_1) \leq intcl(\delta) \leq intcl(1 - \mu_2)$. Then $\mu_1 = intcl(\mu_1) \leq intcl(\delta) \leq 1 - clint(\mu_2) \leq 1 - int(\mu_2)$. Thus it follows that $\mu_1 \leq intcl(\delta) \leq 1 - int(\mu_2)$. □

Proposition 4.8. *If μ_1 and μ_2 are disjoint fuzzy nowhere dense sets in a fuzzy nodef and fuzzy weakly F_σ-complemented space (X, T), then there exists a fuzzy G_δ-set δ in (X, T) such that $\mu_1 \leq \delta \leq 1 - \mu_2$.*

Proof. Suppose that μ_1 and μ_2 are disjoint fuzzy nowhere dense sets in X. Then $\mu_1 \wedge \mu_2 = 0$ implies that $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy nodef space, the fuzzy nowhere dense sets μ_1 and μ_2 are fuzzy F_σ-sets in X. Thus μ_1 and μ_2 are disjoint fuzzy F_σ-sets in the fuzzy weakly F_σ-complemented space X. By Corollary 3.3, there exists a fuzzy G_δ-set δ in (X, T) such that $\mu_1 \leq \delta \leq 1 - \mu_2$. □

Corollary 4.9. *If μ_1 and μ_2 are disjoint fuzzy nowhere dense sets in a fuzzy nodef and fuzzy weakly F_σ-complemented space (X, T), then there exists a fuzzy somewhere dense set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$.*

Proof. Suppose that μ_1 and μ_2 are disjoint fuzzy nowhere dense sets in a fuzzy nodef and fuzzy weakly F_σ-complemented space X. By Proposition 4.8, there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$. This implies that $intcl(\mu_1) \leq intcl(\delta) \leq intcl(1 - \mu_2)$ Then $0 \leq intcl(\delta) \leq 1 - clint(\mu_2)$. Since μ_2 is a fuzzy nowhere dense set in X, $intcl(\mu_2) = 0$ and $int(\mu_2) \leq intcl(\mu_2)$ implies that $int(\mu_2) = 0$. Thus $clint(\mu_2) = 0$, in (X, T). So $0 \leq intcl(\delta) \leq 1$ and then $intcl(\delta) \neq 0$. Hence δ is a fuzzy somewhere dense set in X. □

Proposition 4.10. *If (X, T) is a fuzzy weakly F_σ-complemented space, then X is not a fuzzy almost P-space.*

Proof. Suppose that μ_1 and μ_2 are fuzzy F_σ-sets in X with $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy weakly F_σ-complemented space, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1 - \lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. Now, $\lambda_1 \vee \lambda_2$ is a fuzzy F_σ-set in X such that $cl(\lambda_1 \vee \lambda_2) = 1$. Then there exists a fuzzy F_σ-set

$\lambda_1 \vee \lambda_2$ in X such that $cl(\lambda_1 \vee \lambda_2) = 1$. Thus by Theorem 2.9, X is not a fuzzy almost P-space. □

Proposition 4.11. *If μ_1 and μ_2 are disjoint fuzzy nowhere dense sets in a fuzzy nodef, fuzzy F_σ-complemented and fuzzy $F^{'}$-space, then there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$.*

Proof. The proof follows from 4.1 and Proposition 4.8. □

Proposition 4.12. *If μ_1 and μ_2 are disjoint fuzzy nowhere dense sets in a fuzzy weakly F_σ-complemented, fuzzy fraction dense and fuzzy DG_δ-space (X, T), then there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$.*

Proof. The proof follows from Theorem 2.21 and Proposition 4.8. □

Proposition 4.13. *If μ_1 and μ_2 are disjoint fuzzy nowhere dense (but not fuzzy closed) sets in a fuzzy weakly F_σ-complemented and fuzzy DG_δ-space (X, T), then there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1-\lambda_2$ such that $\mu_1 \leq \lambda_1,\ \mu_2 \leq \lambda_2$ and $int(\lambda_1) \leq 1 - int(\lambda_2)$.*

Proof. Suppose that μ_1 and μ_2 are disjoint fuzzy nowhere dense (but not fuzzy closed) sets in X. Then $\mu_1 \wedge \mu_2 = 0$ implies that $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy DG_δ-space, by Theorem 2.15, μ_1 and μ_2 are fuzzy F_σ-sets in X and X being a fuzzy weakly F_σ-complemented space, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1-\lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. Now $\mu_1 \leq \lambda_1 \leq 1-\lambda_2 \leq 1-\mu_2$. This implies that $intcl(\mu_1) \leq intcl(\lambda_1) \leq intcl(1-\lambda_2) \leq intcl(1-\mu_2)$. Thus $intcl(\mu_1) \leq intcl(\lambda_1) \leq 1 - clint(\lambda_2) \leq 1 - clint(\mu_2)$. Since μ_1 and μ_2 are fuzzy nowhere dense sets in X, $intcl(\mu_1) = 0$ and $intcl(\mu_2) = 0$ and $int(\mu_2) \leq intcl(\mu_2)$ implies that $int(\mu_2) = 0$. So $0 \leq intcl(\lambda_1) \leq 1 - clint(\lambda_2) \leq 1 - cl(0) = 1$ and $0 \leq intcl(\lambda_1) \leq 1-clint(\lambda_2) \leq 1$. Now $int(\lambda_1) \leq intcl(\lambda_1) \leq 1-clint(\lambda_2) \leq 1-int(\lambda_2)$. Hence $int(\lambda_1) \leq 1 - int(\lambda_2)$. □

Proposition 4.14. *If μ_1 and μ_2 are disjoint fuzzy regular open sets in a fuzzy extremally disconnected and fuzzy weakly F_σ-complemented space (X, T), then there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq cl(\delta) \leq 1 - int(\mu_2)$.*

Proof. Suppose that μ_1 and μ_2 are disjoint fuzzy regular open sets in X. Then $\mu_1 \wedge \mu_2 = 0$. This implies that $\mu_1 \leq 1-\mu_2$. Since X is a fuzzy extremally disconnected space, by Theorem 2.18, μ_1 and μ_2 are fuzzy closed F_σ-sets in X. Thus μ_1 and μ_2 are disjoint fuzzy F_σ-sets in the fuzzy weakly F_σ-complemented space X. So by Corollary 3.3, there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$. This implies that $cl(\mu_1) \leq cl(\delta) \leq cl(1 - \mu_2)$. Hence $\mu_1 \leq cl(\delta) \leq 1 - int(\mu_2)$. □

Proposition 4.15. *If (X, T) is a fuzzy weakly F_σ-complemented space, then X is not a fuzzy quasi-F-space.*

Proof. Suppose that μ_1 and μ_2 are fuzzy F_σ-sets in X with $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy weakly F_σ-complemented space, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1 - \lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. Now $cl(\lambda_1 \vee \lambda_2) = cl(\lambda_1) \vee cl(\lambda_2)$. Then $intcl(\lambda_1 \vee \lambda_2) = int[cl(\lambda_1) \vee cl(\lambda_2)] \geq intcl(\lambda_1) \vee intcl(\lambda_2)$ and $intcl(\lambda_1) \vee intcl(\lambda_2) \leq 1$. Thus it follows that $intcl(\lambda_1 \vee \lambda_2) \nleq intcl(\lambda_1) \vee intcl(\lambda_2)$

for the fuzzy F_σ-sets λ_1 and λ_2 in X. So by Theorem 2.14, X is not a fuzzy quasi-F-space. □

Proposition 4.16. *If μ_1 and μ_2 are disjoint fuzzy open sets in a fuzzy weakly F_σ-complemented and fuzzy regular space (X, T), then there exist fuzzy F_σ-sets λ_1 and λ_2 with $\lambda_1 \leq 1 - \lambda_2$ in X such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$.*

Proof. Suppose that μ_1 and μ_2 are disjoint fuzzy open sets in X. Then $\mu_1 \wedge \mu_2 = 0$ implies that $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy regular space, by Theorem 2.20, the open sets μ_1 and μ_2 are fuzzy F_σ-sets in X. Thus μ_1 and μ_2 are fuzzy F_σ-sets such that $\mu_1 \leq 1 - \mu_2$ in the fuzzy weakly F_σ-complemented space (X, T). So there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1 - \lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. □

Proposition 4.17. *If there exists a pair of disjoint fuzzy open sets in a fuzzy weakly F_σ-complemented and fuzzy regular space (X, T), then X is not a fuzzy hyperconnected space.*

Proof. Suppose that μ_1 and μ_2 are a pair of disjoint fuzzy open sets in X. Since X is a fuzzy weakly F_σ-complemented and fuzzy regular space, by Proposition 4.16, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1-\lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. Now $cl(\mu_1) \leq cl(\lambda_1) \leq cl(\lambda_1 \vee \lambda_2)$ and $cl(\mu_1) \leq cl(\lambda_1) \leq 1$ implies that $cl(\mu_1) \neq 1$. Then for the fuzzy open set μ_1, $cl(\mu_1) \neq 1$ implies that X is not a fuzzy hyperconnected space. □

Proposition 4.18. *If μ_1 and μ_2 are disjoint fuzzy simply* open sets in a fuzzy weakly F_σ-complemented and fuzzy perfectly disconnected space (X, T), then there exist fuzzy F_σ- sets λ_1 and λ_2 in X with $\lambda_1 \leq 1 - \lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$.*

Proof. Suppose that μ_1 and μ_2 are disjoint fuzzy simply* open sets in X. Then $\mu_1 \wedge \mu_2 = 0$ implies that $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy perfectly disconnected space, for the fuzzy sets μ_1 and μ_2 in X with $\mu_1 \leq 1 - \mu_2$, $cl(\mu_1) \leq 1 - cl(\mu_2)$. By Theorem 2.22, for the fuzzy simply* open sets μ_1 and μ_2, in X $cl(\mu_1)$ and $cl(\mu_2)$ are fuzzy F_σ-sets in X. Since X is fuzzy weakly F_σ-complemented, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1 - \lambda_2$ such that $cl(\mu_1) \leq \lambda_1, cl(\mu_2) \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. Now $\mu_1 \leq cl(\mu_1) \leq \lambda_1$ and $\mu_2 \leq cl(\mu_2) \leq \lambda_2$. Thus for the disjoint fuzzy simply* open sets μ_1 and μ_2 in X, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1 - \lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. □

Proposition 4.19. *If μ_1 and μ_2 are disjoint fuzzy simply* open sets in a fuzzy weakly F_σ-complemented and fuzzy perfectly disconnected space (X, T), then there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$.*

Proof. Suppose that μ_1 and μ_2 are disjoint fuzzy simply* open sets in X. Then $\mu_1 \wedge \mu_2 = 0$ implies that $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy perfectly disconnected space, for the fuzzy sets μ_1 and μ_2 in X with $\mu_1 \leq 1 - \mu_2, cl(\mu_1) \leq 1 - cl(\mu_2)$. By Theorem 2.22, for the fuzzy simply* open sets μ_1 and $\mu_2, cl(\mu_1)$ and $cl(\mu_2)$ are fuzzy F_σ-sets in X. Since X is a fuzzy weakly F_σ-complemented space, by Proposition

3.2, there exists a fuzzy G_δ-set δ in X such that $cl(\mu_1) \leq \delta \leq 1 - cl(\mu_2)$. Thus $\mu_1 \leq cl(\mu_1) \leq \delta \leq 1 - cl(\mu_2) \leq 1 - \mu_2$. So $\mu_1 \leq \delta \leq 1 - \mu_2$. □

Proposition 4.20. *If μ_1 and μ_2 are any two disjoint fuzzy dense sets such that $\mu_1 \vee \mu_2 = 1$, in a fuzzy open hereditarily irresolvable, fuzzy nodef and fuzzy weakly F_σ-complemented space (X, T), then there exists a fuzzy G_δ-set δ in X such that $1 - \mu_1 \leq \delta \leq \mu_2$.*

Proof. Suppose that μ_1 and μ_2 are any two fuzzy dense sets such that $\mu_1 \vee \mu_2 = 1$. Now $1 - (\mu_1 \vee \mu_2) = 0$ implies that $(1 - \mu_1) \wedge (1 - \mu_2) = 0$. Thus $(1 - \mu_1)$ and $(1 - \mu_2)$ are disjoint fuzzy sets in X. Since X is a fuzzy open hereditarily irresolvable space, for the fuzzy dense sets μ_1 and μ_2 in X by Theorem 2.7, $clint(\mu_1) = 1$ and $clint(\mu_2) = 1$. So $1 - clint(\mu_1) = 0$ and $1 - clint(\mu_2) = 0$. By Lemma 2.3, $intcl(1 - \mu_1) = 1 - clint(\mu_1) = 0$ and $intcl(1 - \mu_2) = 1 - clint(\mu_2) = 0$. So $1 - \mu_1$ and $1 - \mu_2$ are fuzzy nowhere dense sets in X. Hence $1 - \mu_1$ and $1 - \mu_2$ are disjoint fuzzy nowhere dense sets in X. Since X is a fuzzy nodef and fuzzy weakly F_σ-complemented space, by Proposition 4.8, for the disjoint fuzzy nowhere dense sets $1 - \mu_1$ and $1 - \mu_2$, there exists a fuzzy G_δ-set δ in X such that $1 - \mu_1 \leq \delta \leq 1 - (1 - \mu_2)$, i.e., $1 - \mu_1 \leq \delta \leq \mu_2$. □

Proposition 4.21. *If μ_1 and μ_2 are disjoint fuzzy pseudo-open sets in a fuzzy weakly F_σ-complemented, fuzzy D-Baire and fuzzy perfectly disconnected space (X, T), then there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$.*

Proof. Suppose that μ_1 and μ_2 are disjoint fuzzy pseudo-open sets in X. Since X is a fuzzy D-Baire space, by Theorem 2.16, the fuzzy pseudo-open sets μ_1 and μ_2 are fuzzy simply* open sets in X. Since X is a fuzzy weakly F_σ-complemented and fuzzy perfectly disconnected space, for the disjoint fuzzy simply* open sets μ_1 and μ_2 in X by Proposition 4.19, there exists a fuzzy G_δ-set δ in X such that $\mu_1 \leq \delta \leq 1 - \mu_2$. □

Proposition 4.22. *If μ_1 and μ_2 are disjoint fuzzy closed sets in a fuzzy weakly F_σ-complemented, fuzzy perfectly disconnected and fuzzy S*N-space (X, T), then there exist fuzzy F_σ-sets δ_1 and δ_2 in X with $\delta_1 \leq 1 - \delta_2$ such that $\mu_1 \leq \delta_1, \mu_2 \leq \delta_2$ and $cl(\delta_1 \vee \delta_2) = 1$.*

Proof. Suppose that μ_1 and μ_2 are disjoint fuzzy closed sets in X. Then $\mu_1 \wedge \mu_2 = 0$ implies that $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy S*N-space, for the pair of fuzzy closed sets μ_1 and μ_2 in X with $\mu_1 \leq 1 - \mu_2$, there exist fuzzy simply* open sets λ_1 and λ_2 in X such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $\lambda_1 \leq 1 - \lambda_2$. Since X is a fuzzy perfectly disconnected space, for the fuzzy sets λ_1 and λ_2 in X with $\lambda_1 \leq 1 - \lambda_2, cl(\lambda_1) \leq 1 - cl(\lambda_2)$. By Theorem 2.22, for the fuzzy simply* open sets λ_1 and λ_2 in X, $cl(\lambda_1)$ and $cl(\lambda_2)$ are fuzzy F_σ-sets in X. Thus $cl(\lambda_1)$ and $cl(\lambda_2)$ are fuzzy F_σ-sets in X with $cl(\lambda_1) \leq 1 - cl(\lambda_2)$. Since X is a fuzzy weakly F_σ-complemented space, there exist fuzzy F_σ-sets δ_1 and δ_2 in X with $\delta_1 \leq 1 - \delta_2$ such that $cl(\lambda_1) \leq \delta_1, cl(\lambda_2) \leq \delta_2$ and $cl(\delta_1 \vee \delta_2) = 1$. Now $\mu_1 \leq \lambda_1 \leq cl(\lambda_1) \leq \delta_1$ and $\mu_2 \leq \lambda_2 \leq cl(\lambda_2) \leq \lambda_1$. So for the disjoint fuzzy closed sets μ_1 and μ_2 in X, there exist fuzzy F_σ-sets δ_1 and δ_2 in X with $\delta_1 \leq 1 - \delta_2$ such that $\mu_1 \leq \delta_1, \mu_2 \leq \delta_2$ and $cl(\delta_1 \vee \delta_2) = 1$. □

Corollary 4.23. *If μ_1 and μ_2 are disjoint fuzzy closed sets in a fuzzy weakly F_σ-complemented, fuzzy perfectly disconnected and fuzzy S*N-space (X, T), then there exist fuzzy simply* open sets λ_1 and λ_2 and fuzzy F_σ-sets δ_1 and δ_2 in X such that $\mu_1 \le \lambda_1 \le \delta_1 \le 1 - \delta_2 \le 1 - \lambda_2 \le 1 - \mu_2$.*

Proposition 4.24. *If $\mu_1 \le 1 - \mu_2$, for each pair of fuzzy F_σ-sets μ_1 and μ_2 with $cl(\mu_1) = 1$, in a fuzzy weakly F_σ-complemented space (X, T), then X is not a fuzzy D-Baire space.*

Proof. Suppose that μ_1 and μ_2 are fuzzy F_σ-sets in X with $\mu_1 \le 1 - \mu_2$. Since X is a fuzzy weakly F_σ-complemented space, by Proposition3.2, there exists a fuzzy G_δ-set δ in X such that $\mu_1 \le \delta \le 1 - \mu_2$. By hypothesis, $cl(\mu_1) = 1$. Now $\mu_1 \le \delta$, implies that $cl(\mu_1) \le cl(\delta)$. Then $cl(\delta) = 1$. Thus X has a fuzzy dense and fuzzy G_δ-set δ in X. So by Theorem 2.8, X is not a fuzzy D-Baire space. □

The following proposition give conditions for fuzzy weakly F_σ-complemented spaces to become fuzzy resolvable spaces.

Proposition 4.25. *If there exists a pair of disjoint fuzzy F_σ-sets μ_1 and μ_2 which are fuzzy dense in a fuzzy weakly F_σ-complemented space (X, T), then X is a fuzzy resolvable space.*

Proof. Suppose that μ_1 and μ_2 are fuzzy F_σ-sets in X with $\mu_1 \le 1 - \mu_2$. Since X is a fuzzy weakly F_σ-complemented space, by corollary 3.3, there exists a fuzzy G_δ-set δ in X such that $\mu_1 \le \delta$ and $\mu_2 \le 1 - \delta$. Then $cl(\mu_1) \le cl(\delta)$ and $cl(\mu_2) \le cl(1 - \delta)$. By hypothesis, $cl(\mu_1) = 1$ and $cl(\mu_2) = 1$. Thus $1 \le cl(\delta)$ and $1 \le cl(1 - \delta)$, i.e., $cl(\delta) = 1$ and $cl(1 - \delta) = 1$. So there exists a fuzzy dense set δ in X such that $cl(1 - \delta) = 1$. Hence X is a fuzzy resolvable space. □

Proposition 4.26. *If μ_1 and μ_2 are disjoint fuzzy F_σ-sets in a fuzzy weakly F_σ-complemented and fuzzy perfectly disconnected space (X, T), then there exist fuzzy sets δ_1 and δ_2 which are both fuzzy F_σ-sets and fuzzy G_δ-sets in X such that $\mu_1 \le \delta_1 and \mu_2 \le \delta_2$.*

Proof. Let μ_1 and μ_2 be fuzzy F_σ-sets in X with $\mu_1 \le 1 - \mu_2$. Since X is a fuzzy weakly F_σ-complemented space, by Proposition 3.6, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \le 1 - \lambda_2$ and $1 - cl(\lambda_2) \le cl(\lambda_1)$. Let $cl(\lambda_1) = \delta_1$ and $cl(\lambda_2) = \delta_2$. Then δ_1 and δ_2 are fuzzy F_σ-sets in X such that

$$\mu_1 \le \delta_1, \ \mu_2 \le \delta_2 \text{ and } 1 - \delta_2 \le \delta_1. \tag{4.1}$$

Since X is a fuzzy perfectly disconnected space, for the fuzzy sets λ_1 and λ_2 with $\lambda_1 \le 1 - \lambda_2, cl(\lambda_1) \le 1 - cl(\lambda_2)$, we have

$$\delta_1 \le 1 - \delta_2. \tag{4.2}$$

From (4.1) and (4.2), $\delta_1 = 1 - \delta_2$. Since the fuzzy set δ_2 is a fuzzy F_σ-set in X, $1 - \delta_2$ is a fuzzy G_δ-set in X. Thus δ_1 is a fuzzy G_δ-set in X. So δ_1 is both fuzzy F_σ-set and fuzzy G_δ-set in X. Similarly, δ_2 is both fuzzy F_σ-set and fuzzy G_δ-set in X. Hence for the fuzzy F_σ-sets μ_1 and μ_2 in X, there exist fuzzy sets δ_1 and δ_2 in X which are both fuzzy F_σ-sets and fuzzy G_δ-sets in X such that $\mu_1 \le \delta_1$ and $\mu_2 \le \delta_2$. □

Proposition 4.27. *If μ_1 and μ_2 are disjoint fuzzy F_σ-sets in a fuzzy weakly F_σ-complemented and fuzzy perfectly disconnected space (X,T), then there exists a fuzzy open set δ in X such that $int(\mu_1) \leq \delta \leq 1 - int(\mu_2)$.*

Proof. Suppose μ_1 and μ_2 are disjoint fuzzy F_σ-sets in X. Then $\mu_1 \wedge \mu_2 = 0$ implies that $\mu_1 \leq 1 - \mu_2$. Since X is a fuzzy weakly F_σ-complemented space, there exist fuzzy F_σ-sets λ_1 and λ_2 in X with $\lambda_1 \leq 1 - \lambda_2$ such that $\mu_1 \leq \lambda_1, \mu_2 \leq \lambda_2$ and $cl(\lambda_1 \vee \lambda_2) = 1$. Since (X,T) is a fuzzy perfectly disconnected space, for the fuzzy sets λ_1 and λ_2 in X with $\lambda_1 \leq 1-\lambda_2$, by Theorem 2.13, there exists a fuzzy open set δ in X such that $intcl(\lambda_1) \leq \delta \leq 1 - cl[int(\lambda_2)]$ and $int(\lambda_2)$ is not a fuzzy dense set in X. Then $int(\mu_1) \leq int(\lambda_1) \leq intcl(\lambda_1) \leq \delta \leq 1 - cl[int(\lambda_2)] \leq 1 - int(\lambda_2) \leq 1 - int(\mu_2)$. Thus $int(\mu_1) \leq \delta \leq 1 - int(\mu_2)$. □

5. Conclusion

In this paper, the notion of fuzzy weakly F_σ-complemented space is introduced by means of fuzzy F_σ-sets. Several characterizations of fuzzy weakly F_σ-complemented spaces are established. It is established that fuzzy F_σ-complemented, fuzzy F'-spaces and fuzzy F_σ-complemented, fuzzy perfectly disconnected spaces are fuzzy weakly F_σ-complemented spaces. The conditions, under which fuzzy F'-spaces become fuzzy weakly F_σ-complemented spaces, are also obtained. It is obtained that fuzzy perfectly disconnected spaces, in which fuzzy F_σ-sets are fuzzy dense and fuzzy disjoint, are fuzzy F_σ-complemented spaces as well as fuzzy weakly F_σ-complemented spaces. It is found that fuzzy weakly F_σ-complemented spaces are neither fuzzy almost P-spaces nor fuzzy quasi-F-spaces. It is obtained that those fuzzy weakly F_σ-complemented spaces which contain a pair of disjoint fuzzy F_σ-sets which are fuzzy dense, are fuzzy resolvable spaces . It is established that the existence of a pair of disjoint fuzzy open sets in a fuzzy weakly F_σ-complemented and fuzzy regular space makes them as non-fuzzy hyperconnected spaces.

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