Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 30, No. 3, (December 2025) pp. 309–318
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr
https://doi.org/10.30948/afmi.2025.30.3.309

@FMI

Strong inverse domination in fuzzy graphs

O. T. Manjusha

Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 30, No. 3, December 2025

Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 30, No. 3, (December 2025) pp. 309–318
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr
https://doi.org/10.30948/afmi.2025.30.3.309

@FMI

Strong inverse domination in fuzzy graphs

O. T. Manjusha

Received 1 May 2025; Revised 13 July 2025; Accepted 12 September 2025

Abstract. The concept of domination in graph theory has been widely studied due to its applications in network theory, social sciences, and optimization problems. In recent years, this concept has been extended to fuzzy graphs, where uncertainty and partial membership are incorporated into the structure of vertices and edges. In this paper, the inverse domination in fuzzy graphs using strong arcs is introduced. The strong inverse domination number of classes of fuzzy graphs, such as complete fuzzy graphs, complete bipartite fuzzy graphs, and fuzzy cycles, is determined. A relation is established between the strong inverse domination number and the strong independence number. Finally, the practical importance of the new concept of inverse domination in fuzzy graphs is nicely presented through a real-life example.

2020 AMS Classification: 05C69

Keywords: Fuzzy graph, Strong arcs, Connectedness, Fuzzy trees, Fuzzy cycles, Inverse domination.

Corresponding Author: O. T. Manjusha (manjushaot@gmail.com)

1. Introduction

Graph theory provides powerful tools for modeling relationships and interactions in a wide range of real-world systems, including communication networks, biological systems, and social structures. Among its many concepts, domination plays a central role, particularly in applications like resource allocation, monitoring, and control systems. Traditionally, a dominating set ensures that every vertex in the graph is either in the set or adjacent to a vertex in the set. This has led to the development of several extensions—one of which is the concept of inverse domination, where the focus is on finding largest minimal dominating sets.

Parallelly, fuzzy graph theory, introduced by Rosenfeld (1975), enhances classical graph models by incorporating uncertainty and partial relationships. In a fuzzy graph, both vertices and edges are associated with membership values between 0 and 1, allowing for a more realistic representation of systems where relationships are not strictly binary. This modeling approach is especially valuable in uncertain or imprecise environments, such as social networks or decision-making systems.

Despite the growing body of work on domination in crisp and fuzzy settings, the inverse domination problem in fuzzy graphs remains largely unexplored. Most existing studies focus on classical (crisp) graphs or consider only standard domination in fuzzy graphs, without addressing inverse domination or incorporating structural measures like strong arcs, which quantify directional influence or strength more meaningfully.

This paper aims to introduce and investigate inverse domination in fuzzy graphs using strong arcs—a refined concept that allows better modeling of influence strength between vertices. We define new terms, establish key properties, and provide structural results that extend existing domination theories into the fuzzy domain.

The novelty of this work lies in combining the concept of inverse domination with strong arc-based fuzzy graphs, creating a new framework that captures both domination structure and the graded strength of influence. This opens up new possibilities for applications in uncertain systems, including fault-tolerant networks, ambiguous decision-making environments, and influence modeling in social media platforms. Several researchers have investigated domination-related concepts in fuzzy graphs. Among them, Muhammad Akram and his research group have made significant contributions to the development of fuzzy domination theory. Their work includes the study of fuzzy dominating sets, independent dominating sets, and domination parameters in intuitionistic and bipolar fuzzy graphs. In particular, Akram et al. have extensively explored structural properties, algorithmic strategies, and applications of these domination parameters in uncertain environments [1, 2]. However, while their studies lay a strong foundation for fuzzy domination, the concept of inverse domination—especially using strong arcs—has not been explored in their framework. This paper builds on the foundational principles of fuzzy graph theory established by Akram and his collaborators but introduces a novel approach by defining and analyzing inverse domination in terms of strong arcs, thereby extending existing domination frameworks in a new direction.

2. Preliminaries

We summarize briefly some basic definitions in fuzzy graphs which are presented in [3, 4, 5, 6, 7, 8, 9, 10].

A *fuzzy graph* is denoted by $G : (V, \sigma, \mu)$, where V is a node set, σ is a fuzzy subset of V and μ is a fuzzy relation on σ. i.e., $\mu(x, y) \leq \sigma(x) \bigwedge \sigma(y)$ for all $x, y \in V$. We call σ the fuzzy node set of G and μ the fuzzy arc set of G, respectively. We consider fuzzy graph G with no loops and assume that V is finite and nonempty, μ is reflexive (i.e.,$\mu(x, x) = \sigma(x)$, for all x) and symmetric (i.e., $\mu(x, y) = \mu(y, x)$, for all (x, y)). In all the examples σ is chosen suitably. Also, we denote the underlying crisp graph by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V : \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V : \mu(u, v) > 0\}$. Throughout we assume that $\sigma^* = V$. The fuzzy graph $H : (\tau, \nu)$ is said to be a *partial fuzzy subgraph* of $G : (\sigma, \mu)$, if $\nu \subseteq \mu$ and $\tau \subseteq \sigma$. In particular, we call $H : (\tau, \nu)$, a *fuzzy subgraph* of $G : (\sigma, \mu)$, if $\tau(u) = \sigma(u)$ for all $u \in \tau^*$ and $\nu(u, v) = \mu(u, v)$

for all $(u,v) \in \nu^*$. We say that *a fuzzy subgraph* $H:(\tau,\nu)$ *spans the fuzzy graph* $G:(V,\sigma,\mu)$, if $\tau=\sigma$. The fuzzy graph $H:(P,\tau,\nu)$ is called an *induced fuzzy subgraph* of $G:(V,\sigma,\mu)$ induced by P, if $P\subseteq V$ and $\tau(u)=\sigma(u)$ for all $u\in P$ and $\nu(u,v)=\mu(u,v)$ for all $u,v\in P$. We shall use the notation $< P >$ to denote the fuzzy subgraph induced by P. $G:(V,\sigma,\mu)$ is called *trivial*, if $|\sigma^*|=1$.

In a fuzzy graph $G:(V,\sigma,\mu)$, a *path* P of length n is a sequence of distinct nodes $u_0,u_1,\cdots,u_n$ such that $\mu(u_{i-1},u_i)>0, i=1,2,\cdots,n$ and the degree of membership of a weakest arc is defined as its strength. If $u_0=u_n$ and $n\geq 3$ then P is called a cycle and P is called a fuzzy cycle, if it contains more than one weakest arc. The strength of a cycle is the strength of the weakest arc in it. The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x,y)$.

A fuzzy graph $G:(\sigma,\mu)$ is *connected,* if for every x,y in $\sigma^*, CONN_G(x,y)>0$.

An arc (u,v) of a fuzzy graph is called an *effective arc* (M-strong arc), if $\mu(u,v)=\sigma(u)\bigwedge\sigma(v)$. Then u and v are called effective neighbors. The set of all effective neighbors of u is called effective neighborhood of u and is denoted by $EN(u)$.

A fuzzy graph G is said to be *complete*, if $\mu(u,v)=\sigma(u)\bigwedge\sigma(v)$, for all $u,v\in\sigma^*$ and is denoted by K_σ.

The *order* p and *size* q of a fuzzy graph $G:(\sigma,\mu)$ are defined to be:

$$p=\sum_{x\in V}\sigma(x) \text{ and } q=\sum_{(x,y)\in V\times V}\mu(x,y).$$

Let $G:(V,\sigma,\mu)$ be a fuzzy graph and $S\subseteq V$. Then the *scalar cardinality* of S is defined to be $\sum_{v\in S}\sigma(v)$ and it is denoted by $|S|$. Let p denotes the scalar cardinality of V, also called the order of G.

An arc of a fuzzy graph is called *strong,* if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted. Depending on $CONN_G(x,y)$ of an arc (x,y) in a fuzzy graph G, Mathew and Sunitha [10] defined three different types of arcs. Note that $CONN_{G-(x,y)}(x,y)$ is the the strength of connectedness between x and y in the fuzzy graph obtained from G by deleting the arc (x,y). An arc (x,y) in G is *α-strong,* if $\mu(x,y)>CONN_{G-(x,y)}(x,y)$. An arc (x,y) in G is *β-strong*, if $\mu(x,y)=CONN_{G-(x,y)}(x,y)$. An arc (x,y) in G is *δ-arc*, if $\mu(x,y)<CONN_{G-(x,y)}(x,y)$. Thus an arc $(x,y)$is a *strong arc*, if it is either α-strong or β-strong. A path P is called a *strong path*, if P contains only strong arcs.

A fuzzy graph G is said to be *bipartite* [9] if the node set V can be partitioned into two non empty sets V_1 and V_2 such that $\mu(v_1,v_2)=0$ if $v_1,v_2\in V_1$ or $v_1,v_2\in V_2$. Further if $\mu(u,v)=\sigma(u)\bigwedge\sigma(v)$ for all $u\in V_1$ and $v\in V_2$, then G is called a *complete bipartite graph* and is denoted by K_{σ_1,σ_2}, where σ_1 and σ_2 are respectively the restrictions of σ to V_1 and V_2.

A connected fuzzy graph $G=(V,\sigma,\mu)$ is called a *fuzzy tree*, if it has a fuzzy spanning subgraph $F:(\sigma,\nu)$, which is a tree [spanning tree], where for all arcs (x,y) not in F there exists a path from x to y in F whose strength is more than $\mu(x,y)$ [8]. Note that here F is a tree which contains all nodes of G and hence is a spanning tree of G.

A *maximum spanning tree* of a connected fuzzy graph $G : (V, \sigma, \mu)$ is a fuzzy spanning subgraph $T : (\sigma, \nu)$ such that T is a tree, and for which $\sum_{u \neq v} \nu(u, v)$ is maximum. A node which is not an endnode of T is called an *internal node* of T [6]. A node u is said to be *isolated*, if $\mu(u, v) = 0$ for all $v \neq u$.

3. Strong inverse domination in fuzzy graphs

Graph theory is a powerful tool for modeling complex systems where entities and their interactions are represented as vertices and edges. One of the fundamental notions in this field is that of domination, which deals with identifying a subset of vertices that can "control" or influence the entire graph. Specifically, a dominating set is a set of vertices such that every vertex in the graph is either in this set or adjacent to a vertex in it. This concept has numerous applications in areas like network security, communication systems, and social networks.

As real-world systems often involve uncertainty and partial relationships, classical graph models may not fully capture the nuances of such scenarios. This has led to the development of fuzzy graph theory, introduced by Rosenfeld [8] in 1975, which extends traditional graphs by allowing vertices and edges to have degrees of membership in the interval $[0, 1]$. These fuzzy graphs better model situations where interactions are not purely binary, such as trust levels in social networks or connection strength in sensor grids.

Within this fuzzy framework, the concept of domination has also been extended, considering the degree of dominance based on fuzzy membership values. However, while much work has been done on standard domination in fuzzy graphs, the concept of inverse domination—which focuses on the maximum size of minimal dominating sets—has received limited attention.

A minimal dominating set in a fuzzy graph is one where no proper subset is itself dominating, and the inverse domination number represents the largest size among all such sets. This idea is particularly relevant in scenarios where distributed control, redundancy, or influence maximization is important, but minimality (i.e., no unnecessary elements) must still be preserved.

This study aims to formalize and explore the notion of inverse domination in fuzzy graphs, providing definitions, properties, and examples that bridge the gap between classical domination theory and fuzzy systems. The insights gained from this work may contribute to enhanced models for decision-making, information dissemination, and system resilience in uncertain environments. The concept of domination in graphs was introduced by Ore and Berge in 1962. The domination number and independent domination number are introduced by Haynes and Hedetniemi [11].Inverse domination in graphs was discussed by Kulli and Sigarkanti [12]. For the terminology of domination and inverse domination in crisp graphs we refer to [11, 12]. Nagoorgani and Chandrasekaran [7] introduced the concept of domination using strong arcs in fuzzy graphs. In this paper, the concept of domination in fuzzy graphs using strong arcs is taken from the paper [7], which is given as follows.

Definition 3.1 ([7])**.** A node v in a fuzzy graph G is said to be *strongly dominate itself*, if each of its strong neighbors, i.e., v strongly dominates the nodes in $N_s[v]$.

A set D of nodes of G is a *strong dominating set* of G, if every node of $V(G) - D$ is a strong neighbor of some node in D.

Manjusha and Sunitha [13] defined strong domination number using membership values (weights) of arcs in fuzzy graphs as follows.

Definition 3.2 ([13])**.** The *weight* of a strong dominating set D is defined as $W(D) = \sum_{u \in D} \mu(u,v)$, where $\mu(u,v)$ is the minimum of the membership values(weights) of strong arcs incident on u. The *strong domination number* of a fuzzy graph G is defined as the minimum weight of strong dominating sets of G and it is denoted by $\gamma_s(G)$ or simply γ_s. A*minimum strong dominating set* in a fuzzy graph G is a strong dominating set of minimum weight.

Let $\gamma_s(\overline{G})$ or $\overline{\gamma_s}$ denote the strong domination number of the complement of a fuzzy graph G.

Now, we define inverse domination in fuzzy graphs using strong arcs as follows.

Definition 3.3. Let D ba a minimum strong dominating set of a fuzzy graph $G : (V, \sigma, \mu)$. If $V(G) - D$ contains a strong dominating set $D*$ of G. then $D*$ is called a *strong inverse dominating set of* G *with respect to* D.

Definition 3.4. The *weight* of a strong inverse dominating set D is defined as $W(D) = \sum_{u \in D*} \mu(u,v)$, where $\mu(u,v)$ is the minimum of the membership values(weights) of strong arcs incident on u. The *strong inverse domination number* of a fuzzy graph G is defined as the minimum weight of strong inverse dominating sets of G and it is denoted by $\gamma_{si}(G)$ or simply γ_{si}. A *minimum strong inverse dominating set* in a fuzzy graph G is a strong inverse dominating set of minimum weight.

Let $\gamma_{si}(\overline{G})$ or $\overline{\gamma_{si}}$ denote the strong inverse domination number of the complement of a fuzzy graph G.

Example 3.5. Consider the fuzzy graph in Figure 1. In this fuzzy graph, the minimum strong dominating sets are $D_1 = \{v, w\}$ and $D_2 = \{w, x\}$ and the corresponding strong inverse dominating sets are$D_1^* = \{u, x\}$ and $D_2^* = \{u, v\}$. Both are minimum strong inverse dominating sets since $W(D_1^*) = 0.4 + 0.2 = 0.6$ and $W(D_2^*) = 0.4{+}0.2 = 0.6$ Then the strong inverse domination number is $\gamma_{si}(G) = 0.6$.

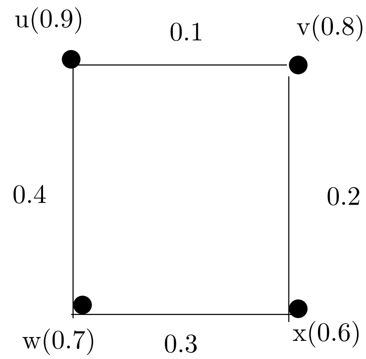

Figure 1: Example of a strong inverse dominating set

Proposition 3.6. *The necessary and sufficient condition for the existence of at least one strong inverse dominating set of G is that G contains no isolated nodes.*

4. Strong inverse domination number for classes of fuzzy graphs

In this section, we have determined the strong inverse domination number of complete fuzzy graphs, complete bipartite fuzzy graphs, fuzzy cycles, and the join of a fuzzy graph with a complete fuzzy graph.

Proposition 4.1. *If $G : (V, \sigma, \mu)$ is a complete fuzzy graph, then*

$$\gamma_{si}(G) = \gamma_s(G) = \wedge\{\mu(u, v) : u, v \in \sigma^*\}.$$

Proof. Since G is a complete fuzzy graph, all arcs are strong [14] and each node is adjacent to all other nodes. Hence, for any minimum strong dominating set $D = \{u\}$, its neighbor set $N(u)$ is a complete fuzzy graph, and hence the result follows. □

Theorem 4.2. *Let D be a minimum strong dominating set of G. If for every node $v \in D$, the induced fuzzy subgraph $< N_s(v) >$ is a complete fuzzy graph of order at least 2, then $\gamma_{si}(G) = \gamma_s(G)$.*

Proof. Let $D = u_1, u_2, \cdots, u_n$ be a minimum strong dominating set of G. Let $v_1, v_2, \cdots, v_n$ be the nodes strongly adjacent to $u_1, u_2, \cdots, u_n$ respectively. By the assumption, for each node $u_i \in D$, the graph $< N_s(u_i) >$ is complete. Then $< N_s(u_i) > \subset < N_s(v_i) >$. Thus $V(G) = N_s(u_1) \cup N_s(u_2) \cup \cdots \cup N_s(u_n) \subset N_s(v_1) \cup N_s(v_2) \cup \cdots \cup N_s(v_n) = V(G)$. So $v_1, v_2, \cdots, v_n$ is a minimum strong inverse dominating set of G. Hence $\gamma_{si}(G) = \gamma_s(G)$. □

Proposition 4.3.

$$\gamma_{si}(K_{\sigma_1,\sigma_2}) = \begin{cases} |V_2|\, \mu(u, v) & \text{if } |V_1| = 1 \\ |V_1|\, \mu(u, v) & \text{if } |V_2| = 1 \\ 2\mu(u, v) & \text{if } |V_1| \geq 2 \text{ and } |V_2| \geq 2, \end{cases}$$

where $\mu(u, v)$ is the weight of a weakest arc in K_{σ_1,σ_2}.

Proof. In K_{σ_1,σ_2}, all arcs are strong. Also each node in V_1 is adjacent with all nodes in V_2. Then in K_{σ_1,σ_2}, the strong dominating sets are V_1, V_2 and any set containing at least 2 nodes, one in V_1 and other in V_2.

If V_1 is a strong dominating set, then V_2 is a strong inverse dominating set and vice versa. Thus strong dominating sets and strong inverse dominating sets act complementary. So $\gamma_{si}(K_{\sigma_1,\sigma_2}) = |V_1|\,\mu(u,v)$ if $|V_2| = 1$ or $|V_2|\,\mu(u,v)$ if $|V_1| = 1$, where $\mu(u,v)$ is the minimum weight of arcs incident on u.

If both V_1 and V_2 contain more than one element, then the set $\{u,v\}$ of nodes of any weakest arc (u,v) in K_{σ_1,σ_2} forms a strong inverse dominating set. Thus $\gamma_{si}(K_{\sigma_1,\sigma_2}) = \mu(u,v) + \mu(u,v) = 2\mu(u,v)$. So the result holds. □

Theorem 4.4. *Let $G : (V,\sigma,\mu)$ be a fuzzy cycle where G^* is a cycle. Then* $\gamma_{si}(G) = \bigwedge \left\{W(D) : D \text{ is a strong inverse dominating set in } G \text{ with} |D| \geq \lceil \frac{n}{3} \rceil \right\}$, *where n is the number of nodes in G.*

Proof. In a fuzzy cycle, every arc is strong. Also, the number of nodes in a strong inverse dominating set of G and G^* is the same because each arc in both graphs is strong. In graph G^*, the strong inverse domination number of G^* is obtained as $\lceil \frac{n}{3} \rceil$ [12]. Then the minimum number of nodes in a strong inverse dominating set of G is $\lceil \frac{n}{3} \rceil$. Thus the result follows. □

Example 4.5. Consider the fuzzy cycle in Figure 2. In this fuzzy cycle, every arc is strong. Hence any set containing any two nodes is a strong dominating set. Then any set containing any two nodes is also a strong inverse dominating set. Thus the result follows.

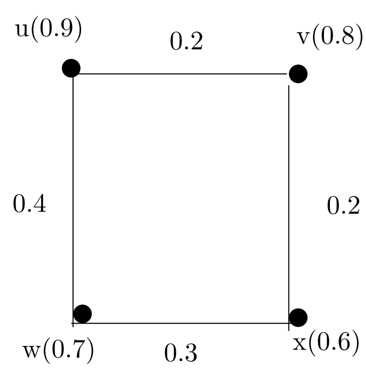

Figure 2: Example of a strong inverse dominating set in a fuzzy cycle

Definition 4.6 ([7])**.** Two nodes in a fuzzy graph $G : (V,\sigma,\mu)$ are said to be *strongly independent*, if there is no strong arc between them. A set of nodes in G is strongly independent if two nodes in the set are strongly independent.

Definition 4.7 ([15]). The *fuzzy weight* of a strong independent set D in a fuzzy graph $G : (V, \sigma, \mu)$ is defined as $W(D) = \sum_{u \in D} \mu(u, v)$, where $\mu(u, v)$ is the minimum of the membership values(weights) of strong arcs incident on u. The *strong independence number* of a fuzzy graph G is defined as the maximum fuzzy weight of strong independent sets of nodes in G and it is denoted by $\beta_{s_o}(G)$ or simply β_{s_o}. A *maximum strong independent set* in a fuzzy graph G is a strong independent set of maximum fuzzy weight.

Theorem 4.8. *For any fuzzy graph $G : (V, \sigma, \mu)$ without isolated nodes, $\gamma_{si}(G) \leq \beta_{s_o}(G)$. Equality holds if $G = K_\sigma$.*

Proof. Let D be a minimum strong dominating set of G. Let S be a maximal strong independent set in $V - D$. Here, we consider two cases.

Case 1: Suppose $V - D - S = \phi$. Then $V - D = S$ is a strong independent inverse-dominating set of G. Thus $\gamma_{si}(G) \leq |V - D|\, \mu(u, v) = |S|\, \mu(u, v) \leq \beta_{s_o}(G)$, where $\mu(u, v)$ is the weight of the weakest arc incident with $u \in V - D$.

Case 2: Suppose $V - D - S \neq \phi$. Then every node in $V - D - S$ is strongly adjacent to at least one node in S. If every node in D is strongly adjacent to at least one node in S, then S is a strong inverse dominating set of G. Otherwise, let $D^l \subset D$ be a set of nodes in D such that no node of D^l is strongly adjacent to the nodes of S. Since D is a minimum strong dominating set, every node in D^1 must be strongly adjacent to at least one node in V–D–S. Let $S^l \subset V - D - S$, be such that every node of D^1 is strongly adjacent to at least one node in S^1. Clearly $\left|S^l\right| \subset \left|D^l\right|$ and $S \cup S^l$ is strong inverse dominating set. Thus we have

$$\gamma_{si}(G) \leq \left|S \cup S^l\right| \mu(u, v) \leq \left|S \cup D^l\right| \mu(u, v) \leq \beta_{s_o}(G).$$

Clearly, equality holds if $G = K_\sigma$. □

5. Practical application

Greenhouse with Fuzzy Sensor Network: Imagine that you have a greenhouse with 5 sensors placed in different places. Each sensor monitors temperature and humidity and communicates with others. However, the signal strength (or influence) between sensors is not binary; it varies depending on obstacles, distance, or interference. We represent this as a fuzzy graph:

Nodes = Sensors $(S_1, S_2, S_3, S_4, S_5)$

Arcs = Degree of influence or connectivity between sensors, ranging from 0 to 1. Let us define the fuzzy adjacency matrix F where $F[i][j]$ = influence of node i on node j:

	S_1	S_2	S_3	S_4	S_5
S_1	1	.7	.3	0	0
S_2	.7	1	.5	.2	0
S_3	.3	.5	1	.6	0
S_4	0	.2	.6	1	.8
S_5	0	0	0	.8	1

This shows, for instance, that S_1 influences S_2 at 0.7 and S_3 at 0.3. Let's say we define a dominating threshold of 0.5. That is: A node is dominated if it is connected

to at least one other node with influence ≥ 0.5. We want to find the set of least dominated nodes — these are the "inverse dominated" nodes.
Let us check each sensor:

S_1: Connected to S_2 (0.7) $\rightarrow$ strongly dominated,
S_2 : Connected to S_1 (0.7), S_3 (0.5) $\rightarrow$ strongly dominated,
S_3 : Connected to S_2 (0.5),S_4 (0.6) $\rightarrow$ strongly dominated,
S_4 : Connected to S_3 (0.6),S_5 (0.8) $\rightarrow$ strongly dominated,
S_5 : Only connected to S_4 (0.8) $\rightarrow$ strongly dominated.

That is, all nodes are strongly dominated in this case. Let's change the threshold to 0.6 to simulate weaker connections being insufficient. Now $S_2 - S_3 = 0.5\rightarrow$ Not enough to strongly dominate:

$S_3 - S_2 = 0.5 \rightarrow$ Not enough,
$S_3 - S_4 = 0.6 \rightarrow$ Barely dominated,
$S_1 - S_2 = 0.7\rightarrow$ Still good,
$S_5 - S_4 = 0.8 \rightarrow$ Good.

Then we get:

S_1: strongly dominated (via S_2),
S_2 : strongly dominated (via S_1),
S_3 : Not dominated,
S_4 : strongly dominated (via S_5),
S_5 : strongly dominated (via S_4).

This means sensor S_3 is strongly inverse dominated — no strong enough connections (≥ 0.6) to other sensors. In practice, it may be poorly connected, at risk of data loss, or a low-priority node. In this case, S_3 could be Boosted with a signal repeater. Removed or ignored if budget/power is limited. Flagged for manual inspection.

6. Conclusion

In this paper, we have introduced and explored the concept of inverse domination in fuzzy graphs using strong arcs, extending classical domination theory to accommodate the graded relationships characteristic of fuzzy systems. By incorporating strong arcs, we provide a more nuanced view of influence and connectivity, which is essential for modeling real-world systems with uncertainty.

We have determined the strong inverse domination number for several classes of fuzzy graphs, including complete fuzzy graphs, complete bipartite fuzzy graphs, and fuzzy cycles. These results establish foundational benchmarks for further studies and applications in fuzzy graph theory. Additionally, we have derived a significant relation between the strong inverse domination number and the strong independence number, highlighting structural connections that may inform both theoretical analysis and algorithmic design.

Our methodological approach, based on adapting classical domination concepts to the fuzzy setting with strong arcs, provides a flexible and powerful framework for analyzing domination-related parameters in uncertain environments. By generalizing existing graph invariants, we offer a new lens through which to understand domination in systems that are not strictly binary or deterministic.

To emphasize the practical significance of our findings, we presented a real-world example illustrating how inverse domination in fuzzy graphs can be applied to influence modeling in uncertain decision-making or social systems. This example demonstrates the relevance of our theoretical work to domains such as network resilience, social influence analysis, and fuzzy control systems.

Overall, the results of this study open several promising directions for future research, including the development of efficient algorithms for computing strong inverse domination numbers in large-scale fuzzy networks, and extending the concept to dynamic or weighted fuzzy graphs..

References

[1] M. Akram and W. A. Dudek, Domination in fuzzy graphs, Journal of Intelligent & Fuzzy Systems 28 (5) (2015) 2183–2192.

[2] M. Akram and S. Sarwar, Independent domination in fuzzy graphs and its applications, Journal of Applied Mathematics and Computing, 55 (1–2) (2017) 493–510.

[3] K. R. Bhutani, On automorphisms of fuzzy graphs, Pattern Recognition Letters 9 (1989) 159–162.

[4] K. R. Bhutani and A. Rosenfeld, Strong arcs in fuzzy graphs, Inform. Sci. 152 (2003) 319–322.

[5] K. R. Bhutani and Abdullah Batton, On M-strong fuzzy graphs, Inform. Sci. 1559 (2003) 103–109.

[6] J. N. Mordeson and P. S. Nair, Fuzzy graphs and fuzzy hypergraphs, Physica - Verlag 2000.

[7] A. Nagoorgani and V. T. Chandrasekaran, Domination in fuzzy graph, Adv. in Fuzzy Sets and Systems I (1) (2006) 17–26.

[8] A. Rosenfeld, Fuzzy graphs, in fuzzy sets and their application to cognitive and decision processes Academic Press (1975) 77–95.

[9] A. Somasundaram and S. Somasundaram, Domination in fuzzy graphs-I, Pattern Recognition Letters 19 (1998) 787–791.

[10] Sunil Mathew and M. S. Sunitha, Types of arcs in a fuzzy graph, Inform. Sci. 179 (2009) 1760–1768.

[11] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of domination in graphs, Marcel Dekker, Inc., New York 1998.

[12] V. R. Kulli and S. C. Sigarkanti, Inverse domination in graphs, Nat. Acad. Sci. Letters 14 (1991) 473–475.

[13] O. T. Manjusha and M. S. Sunitha, Strong domination in fuzzy graphs, Fuzzy Inf. Eng. 7 (2015) 369–377.

[14] Sunil Mathew and M. S. Sunitha, Node connectivity and arc connectivity of a fuzzy graph, Inform. Sci. 180 (2010) 519–531.

[15] O. T. Manjusha and M. S. Sunitha, Coverings, matchings and paired domination in fuzzy graphs using strong arcs, Iranian Journal of Fuzzy Systems 16 (1) (2019) 145–157.

O. T. Manjusha (manjushaot@gmail.com)
PG and Research Department of Mathematics, Govt.Arts and Science College,Kondotty, Malappuram, postal code 673641, Kerala, India