Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 29, No. 2, (April 2025) pp. 177–190
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr
https://doi.org/10.30948/afmi.2025.29.2.177

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Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 29, No. 2, April 2025

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Generalized version of α^θ-continuity and its applications in fuzzy setting

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Received 30 November 2024; Revised 21 December 2024; Accepted 4 January 2025

Abstract. In this paper different types of generalized version of fuzzy continuity are discussed. Also new types of fuzzy separation axioms and fuzzy compactness are introduced and studied. Some applications of these functions on the spaces defined here are established.

2020 AMS Classification: 54A40, 03E72

Keywords: Fuzzy regular open set, Fuzzy semiopen set, Fuzzy α-open set, $f\alpha^\theta g$-open set, $f\alpha^\theta g$-continuity, $f\alpha^\theta g$-irresoluteness, Strongly $f\alpha^\theta g$-continuity, Weakly $f\alpha^\theta g$-continuity, $f\alpha^\theta g$-regular space, $f\alpha^\theta g$-normal space.

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1. Introduction

After fuzzy sets was introduced in [1], Chang [2] introduced fuzzy topology and fuzzy continuity. In [3], fuzzy regular open and fuzzy semiopen sets are introduced whereas in [4] fuzzy α-open set is defined. In [5, 6], fuzzy generalized version of closed set is introduced. Using fuzzy α-open set as a basic tool, in [7] $f\alpha^\theta g$-closed set is introduced and studied. However, fg-continuity [6], fmg-continuity [8], fwg-continuity [9], $frwg$-continuity [10], $fswg$-continuity [11] are introduced and studied. In this paper taking $f\alpha^\theta g$-closed set as a basic tool, we introduce first $f\alpha^\theta g$-continuity, the class of which is strictly larger than that of fuzzy continuity, fmg-continuity and $fswg$-continuity, but smaller than fwg-continuity and $frwg$-continuity. Also fg-continuity and $f\alpha^\theta g$-continuity are independent concepts. Again we introduce $f\alpha^\theta g$-irresolute function, the class of which is strictly smaller than that of $f\alpha^\theta g$-continuity. Here we introduce $f\alpha^\theta g$-regular, $f\alpha^\theta g$-normal and $f\alpha^\theta g$-compact spaces, the classes of which are strictly weaker than that of fuzzy regular [12], fuzzy normal [13] and fuzzy compact [2] spaces respectively.

Recently, new types of fuzzy sets, viz., fuzzy soft set and fuzzy octahedron set are introduced and studied. A new branch in fuzzy system is developed using these types of fuzzy sets. In this context we have to mention [14, 15, 16, 17, 18].

2. Preliminaries

Throughout this paper, by (X, τ) or simply by X we shall mean a fuzzy topological space (fts, for short) in the sense of Chang [2]. Zadeh [1] introduced the concept of fuzzy sets as follows: A *fuzzy set* A in a non-empty set X is a function from X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The *support* [1] of a fuzzy set A, denoted by $suppA$ and is defined by $suppA = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t $(0 < t \leq 1)$ will be denoted by x_t. 0_X and 1_X are the constant fuzzy sets taking values 0 and 1, respectively in X. The *complement* of a fuzzy set A in X is denoted by $1_X \setminus A$ and is defined by $(1_X \setminus A)(x) = 1 - A(x)$ for each $x \in X$. For any two fuzzy sets A, B in X, $A \leq B$ means $A(x) \leq B(x)$ for all $x \in X$ while AqB means A is *quasi-coincident* (q-coincident, for short) with B, if there exists $x \in X$ such that $A(x) + B(x) > 1$ [19]. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not{q} B$ respectively. For a fuzzy point x_t and a fuzzy set A, $x_t \in A$ means $A(x) \geq t$, i.e., $x_t \leq A$. For a fuzzy set A, clA and $intA$ will stand for the fuzzy closure and the fuzzy interior [2], respectively. A fuzzy set A is called a *fuzzy neighbourhood* (fuzzy nbd, for short) of a fuzzy point x_α, if there exists a fuzzy open set U in X such that $x_\alpha \in U \leq A$ [19]. If, in addition, A is fuzzy open, then A is called a *fuzzy open nbd* of x_α [19]. A fuzzy set A is called a *fuzzy quasi neighbourhood* (fuzzy q-nbd, for short) [19] of a fuzzy point x_α in a fts X, if there is a fuzzy open set U in X such that $x_\alpha qU \leq A$. If, in addition, A is fuzzy open, then A is called a *fuzzy open q-nbd* [19] of x_α. A fuzzy set A in X is called *fuzzy regular open* [3] (resp., *fuzzy semiopen* [3], *fuzzy α-open* [4]), if $A = int(clA)$ (resp., $A \leq cl(intA)$, $A \leq intclintA$). The complement of a fuzzy regular open (resp., fuzzy α-open) set is called *fuzzy regular closed* [3] (resp., *fuzzy α-closed* [4]). The union (resp., intersection) of all fuzzy α-open (resp., fuzzy α-closed) sets contained in (resp., containing) a fuzzy set A is called the *fuzzy α-interior* [4] (resp., the *fuzzy α-closure* [4]) of A, to be denoted by $\alpha intA$ (resp., αclA). The collection of all fuzzy open (resp., fuzzy regular open, fuzzy semiopen, fuzzy α-open) sets in a fts (X, τ) is denoted by τ (resp., $FRO(X)$, $FSO(X)$, $F\alpha O(X)$). The collection of all fuzzy closed (resp., fuzzy regular closed, fuzzy α-closed) sets in an fts X is denoted by τ^c (resp., $FRC(X)$, $F\alpha C(X)$).

3. $f\alpha^\theta g$-closed Set: Some well-known properties

In [7], we have introduced and studied $f\alpha^\theta g$-closed set. Now we recall some properties of it which will be used in this paper.

Definition 3.1 ([7])**.** Let (X, τ) be an fts and $A \in I^X$. Then A is called an *$f\alpha^\theta g$-closed set* in X, if $clintA \leq U$, whenever $A \leq U \in F\alpha O(X)$.

The complement of $f\alpha^\theta g$-closed set is called an *$f\alpha^\theta g$-open set* in X.
The collection of all $f\alpha^\theta g$-closed (resp., $f\alpha^\theta g$-open) sets in an fts X is denoted by $F\alpha^\theta GC(X)$ (resp., $F\alpha^\theta GO(X)$).

Remark 3.2 ([7])**.** Union and intersection of two $f\alpha^\theta g$-closed sets may not be so.

Definition 3.3 ([7]). Let (X,τ) be an fts and $A \in I^X$. Then the *$f\alpha^\theta g$-closure* and the *$f\alpha^\theta g$-interior* of A, denoted by $f\alpha^\theta gcl(A)$ and $f\alpha^\theta gint(A)$, are defined as follows:

$$f\alpha^\theta gcl(A) = \bigwedge\{F : A \leq F, F \ is \ an \ f\alpha^\theta g - closed \ set \ in \ X\},$$

$$f\alpha^\theta gint(A) = \bigvee\{G : G \leq A, G \ is \ an \ f\alpha^\theta g - open \ set \ in \ X\}.$$

Definition 3.4 ([7]). A fts (X,τ) is called an *$fT_{\alpha^\theta g}$-space*, if every $f\alpha^\theta g$-closed set in X is a fuzzy closed set in X.

Definition 3.5. [7]Let (X,τ) be a fts and x_t an fuzzy point in X. A fuzzy set A is called an *$f\alpha^\theta g$-neighbourhood* ($f\alpha^\theta g$-nbd, for short) of x_t, if there exists a $f\alpha^\theta g$-open set U in X such that $x_t \in U \leq A$. If, in addition, A is $f\alpha^\theta g$-open set in X, then A is called an *$f\alpha^\theta g$-open nbd* of x_t.

Definition 3.6 ([7]). Let (X,τ) be an fts and x_t a fuzzy point in X. A fuzzy set A is called an *$f\alpha^\theta g$-quasi neighbourhood* ($f\alpha^\theta g$-q-nbd, for short) of x_t, if there is an $f\alpha^\theta g$-open set U in X such that $x_t qU \leq A$. If, in addition, A is $f\alpha^\theta g$-open set in X, then A is called an *$f\alpha^\theta g$-open q-nbd* of x_t.

Definition 3.7 ([7]). An fts (X,τ) is called an *$f\alpha^\theta g$-T_2-space*, if for any two distinct fuzzy points x_t and y_s in X,

when $x \neq y$, there exist $f\alpha^\theta g$-open sets U, V in X such that $x_t qU$, $y_s qV$ and $U \not{q} V$,

when $x = y$ and $t < s$ (say), x_t has an $f\alpha^\theta g$-open nbd U and y_s has an $f\alpha^\theta g$-open q-nbd V such that $U \not{q} V$.

Theorem 3.8 ([20]). *An fts (X,τ) is a fuzzy T_2-space if and only if for any two distinct fuzzy points x_α and y_β in X,*

when $x \neq y$, there exist fuzzy open sets U, V in X such that $x_\alpha qU$, $y_\beta qV$ and $U \not{q} V$,

when $x = y$ and $\alpha < \beta$ (say), x_α has a fuzzy open nbd U and y_β has a fuzzy open q-nbd V such that $U \not{q} V$.

Now we recall the following definitions from [5, 6, 21] for ready references.

Definition 3.9. Let (X,τ) be an fts and $A \in I^X$. Then A is called an:

(i) *fg-closed set* [5, 6], if $clA \leq U$, whenever $A \leq U \in \tau$, the complement of an fg-closed set is called an *fg-open set*,

(ii) *fmg-closed set* [21], if $clintA \leq U$, whenever $A \leq U$, U is an fg-open set in X,

(iii) *fwg-closed set* [21], if $clintA \leq U$, whenever $A \leq U \in \tau$,

(iv) *$frwg$-closed set* [21], if $clintA \leq U$, whenever $A \leq U \in FRO(X)$,

(v) *$fswg$-closed set* [21], if $clintA \leq U$, whenever $A \leq U \in FSO(X)$.

Definition 3.10 ([22]). A function $f : X \to Y$ is called a *fuzzy open function*, if $f(U)$ is fuzzy open set in Y for every fuzzy open set U in X.

Definition 3.11. A function $h : X \to Y$ is called:

(i) a *fuzzy continuous function* [2], if $h^{-1}(U)$ is a fuzzy closed set in X for all fuzzy closed set U in Y,

(ii) an *fg-continuous function* [6], if $h^{-1}(U)$ is an fg-closed set in X for all fuzzy closed set U in Y,

(iii) an *fmg-continuous function* [8], if $h^{-1}(U)$ is an fmg-closed set in X for all fuzzy closed set U in Y,

(iv) an *fwg-continuous function* [9], if $h^{-1}(U)$ is an fwg-closed set in X for all fuzzy closed set U in Y,

(v) an *$frwg$-continuous function* [10], if $h^{-1}(U)$ is an $frwg$-closed set in X for all fuzzy closed set U in Y,

(vi) an *$fswg$-continuous function* [11]. if $h^{-1}(U)$ is an $fswg$-closed set in X for all fuzzy closed set U in Y.

4. $f\alpha^\theta g$-CONTINUOUS FUNCTION

In this section $f\alpha^\theta g$-continuous function is introduced and characterized, the class of which is strictly larger than that of fuzzy continuity, fmg-continuity and $frwg$-continuity, but weaker than that of fwg-continuity and $fswg$-continuity. Afterwards, we introduce $f\alpha^\theta g$-irresolute function which implies $f\alpha^\theta g$-continuous function, but independent of fuzzy continuous function. Lastly, we introduce strongly $f\alpha^\theta g$-continuous function which implies fuzzy continuous function, $f\alpha^\theta g$-continuous function and $f\alpha^\theta g$-irresolute function.

Definition 4.1. A function $h : X \to Y$ is said to be an *$f\alpha^\theta g$-continuous function*, if $h^{-1}(V)$ is an $f\alpha^\theta g$-closed (resp., $f\alpha^\theta g$-open) set in X for every fuzzy closed (resp., fuzzy open) set V in Y.

Theorem 4.2. *Let $h : (X, \tau) \to (Y, \sigma)$ be a function. Then the following statements are equivalent:*

(1) *h is an $f\alpha^\theta g$-continuous function,*

(2) *for each fuzzy point x_t in X and each fuzzy open nbd V of $h(x_t)$ in Y , there exists an $f\alpha^\theta g$-open nbd U of x_t in X such that $h(U) \leq V$,*

(3) *$h(f\alpha^\theta gcl(A)) \leq cl(h(A))$ for all $A \in I^X$,*

(4) *$f\alpha^\theta gcl(h^{-1}(B)) \leq h^{-1}(clB)$ for all $B \in I^Y$.*

Proof. (1) ⇒ (2) Suppose the condition (1) holds and let x_t be a fuzzy point in X and V any fuzzy open nbd of $h(x_t)$ in Y. Then $h^{-1}(V)$ is an $f\alpha^\theta g$-open set in X and $x_t \in h^{-1}(V)$. Let $U = h^{-1}(V)$. Then $h(U) = h(h^{-1}(V)) \leq V$.

(2) ⇒ (1) Suppose the condition (2) holds and let A be any fuzzy open set in Y and x_t. a fuzzy point in X such that $x_t \in h^{-1}(A)$. Then $h(x_t) \in A$, where A is a fuzzy open nbd of $h(x_t)$ in Y. By (2), there exists an $f\alpha^\theta g$-open nbd U of x_t in X such that $h(U) \leq A$. Thus $x_t \in U \leq h^{-1}(A)$. So $x_t \in U = f\alpha^\theta gint(U) \leq f\alpha^\theta gint(h^{-1}(A))$. Since x_t is taken arbitrarily and $h^{-1}(A)$ is the union of all fuzzy points in $h^{-1}(A)$, $h^{-1}(A) \leq f\alpha^\theta gint(h^{-1}(A))$. Hence $h^{-1}(A)$ is an $f\alpha^\theta g$-open set in X. Therefore h is an $f\alpha^\theta g$-continuous function.

(1) ⇒ (3) Suppose the condition (1) holds and let $A \in I^X$. Then $cl(h(A))$ is a fuzzy closed set in Y. By (1), $h^{-1}(cl(h(A)))$ is $f\alpha^\theta g$-closed set in X. On the other hand, $A \leq h^{-1}(h(A)) \leq h^{-1}(cl(h(A)))$. Thus we have

$$f\alpha^\theta gcl(A) \leq f\alpha^\theta gcl(h^{-1}(cl(h(A)))) = h^{-1}(cl(h(A))).$$

So $h(f\alpha^\theta gcl(A)) \leq cl(h(A))$.

(3) ⇒ (1) Suppose the condition (3) holds and let V be a fuzzy closed set in Y. Put $U = h^{-1}(V)$. Then $U \in I^X$. By (3), we get

$$h(f\alpha^\theta gcl(U)) \leq cl(h(U)) = cl(h(h^{-1}(V))) \leq clV = V.$$

Thus $f\alpha^\theta gcl(U) \leq h^{-1}(V) = U$. So U is an $f\alpha^\theta g$-closed set in X. Hence h is an $f\alpha^\theta g$-continuous function.

(3) ⇒ (4) Suppose the condition (3) holds and let $B \in I^Y$ and $A = h^{-1}(B)$. Then $A \in I^X$. By (3), $h(f\alpha^\theta gcl(A)) \leq cl(h(A))$. Thus we have

$$h(f\alpha^\theta gcl(h^{-1}(B))) \leq cl(h(h^{-1}(B))) \leq clB.$$

So $f\alpha^\theta gcl(h^{-1}(B)) \leq h^{-1}(clB)$.

(4) ⇒ (3) Suppose the condition (4) holds and let $A \in I^X$. Then $h(A) \in I^Y$. By (4), $f\alpha^\theta gcl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A)))$. Thus we get

$$f\alpha^\theta gcl(A) \leq f\alpha^\theta gcl(h^{-1}(h(A))) \leq h^{-1}(cl(h(A))).$$

So $h(f\alpha^\theta gcl(A)) \leq cl(h(A))$. □

Remark 4.3. Composition of two $f\alpha^\theta g$-continuous functions need not be so, as it is seen from the following example.

Example 4.4. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = B(b) = 0.4$. Then (X, τ_1), (X, τ_2) and (X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Clearly, i_1 and i_2 are $f\alpha^\theta g$-continuous functions. Let $i_3 = i_2 \circ i_1$. Then $i_3 : (X, \tau_1) \to (X, \tau_3)$. We claim that i_3 is not an $f\alpha^\theta g$-continuous function. Now $1_X \setminus B \in \tau_3^c$. Then $i_3^{-1}(1_X \setminus B) = 1_X \setminus B \leq 1_X \setminus B \in F\alpha O(X, \tau_1)$. But $cl_{\tau_1} int_{\tau_1}(1_X \setminus B) = 1_X \not\leq 1_X \setminus B$. Thus $1_X \setminus B \notin F\alpha^\theta GC(X, \tau_1)$. So i_3 is not an $f\alpha^\theta g$-continuous function.

Remark 4.5. It is clear from definitions that

(1) fuzzy continuity, fmg-continuity and $fswg$-continuity imply $f\alpha^\theta g$-continuity. Indeed, a fuzzy open set, an fmg-open set, an $fswg$-open set are $f\alpha^\theta g$-open sets [7]. But the reverse implications may not be true, in general, follow from the next examples,

(2) $f\alpha^\theta g$-continuity implies fwg-continuity and $frwg$-continuity. Indeed, an $f\alpha^\theta g$-open set is an fwg-open set, an $frwg$-open set [7]. But the reverse implications are not necessarily true, in general, follow from the following examples,

(3) fg-continuity and $f\alpha^\theta g$-continuity are independent concepts follow from the next examples.

Example 4.6. $f\alpha^\theta g$-continuity $\not\Rightarrow$ fuzzy continuity, $fswg$-continuity, fmg-continuity.

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.4, B(a) = B(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly, i is not a fuzzy continuous function. Now $FSO(X, \tau_1) = \{0_X, 1_X, U\}$, where $A \leq U \leq 1_X \setminus A$, $F\alpha O(X, \tau_1) = \tau_1$ and the collection of fg-open sets in (X, τ_1) is $\{0_X, 1_X, V\}$, where $V \not\geq 1_X \setminus A$. Now $1_X \setminus B = B \in \tau_2^c$, $i^{-1}(B) = B < 1_X \in F\alpha O(X, \tau_1)$ only and then $cl_{\tau_1} int_{\tau_1} B = 1_X \setminus A < 1_X$. Thus $B \in F\alpha^\theta GC(X, \tau_1)$. So i is an $f\alpha^\theta g$-continuous function.

On the other hand, $B \leq B \in FSO(X, \tau_1)$ as well as B is an fg-open set in (X, τ_1). Then $cl_{\tau_1} int_{\tau_1} B = 1_X \setminus A \not\leq B$. Thus B is not $fswg$-closed as well as an fmg-closed set in (X, τ_1). So i is not an $fswg$-continuous function as well as an fmg-continuous function.

Example 4.7. $f\alpha^\theta g$-continuity $\not\Rightarrow$ fg-continuity.

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$ where $A(a) = 0.5, A(b) = 0.6, B(a) = B(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $F\alpha O(X, \tau_1) = \{0_X, 1_X, U\}$, where $U \geq A$. Now $1_X \setminus B = B \in \tau_2^c, i^{-1}(B) = B \leq A \in F\alpha O(X, \tau_1)$ and $cl_{\tau_1} int_{\tau_1} B = 0_X < A$. Then $B \in F\alpha^\theta GC(X, \tau_1)$. Thus i is an $f\alpha^\theta g$-continuous function. But $B < A \in \tau_1$ and $cl_{\tau_1} B = 1_X \not\leq A$. So B is not an fg-closed set in (X, τ_1). Hence i is not an fg-continuous function.

Example 4.8. $frwg$-continuity $\not\Rightarrow$ $f\alpha^\theta g$-continuity.

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$, where $A(a) = 0.5, A(b) = 0.6, B(a) = B(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $FRO(X, \tau_1) = \{0_X, 1_X\}$, $F\alpha O(X, \tau_1) = \{0_X, 1_X, U\}$, where $U \geq A$. Now $1_X \setminus B \in \tau_2^c$, $i^{-1}(1_X \setminus B) = 1_X \setminus B < 1_X \in FRO(X, \tau_1)$ only and then $cl_{\tau_1} int_{\tau_1} (1_X \setminus B) = 1_X \leq 1_X$. Thus $1_X \setminus B$ is an $frwg$-closed set in (X, τ_1). So i is an $frwg$-continuous function. But $1_X \setminus B \leq 1_X \setminus B \in F\alpha O(X, \tau_1)$ and $cl_{\tau_1} int_{\tau_1} (1_X \setminus B) = 1_X \not\leq 1_X \setminus B$. Hence $1_X \setminus B$ is not an $f\alpha^\theta g$-closed set in (X, τ_1). Therefore i is not an $f\alpha^\theta g$-continuous function.

Example 4.9. fwg-continuity, fg-continuity $\not\Rightarrow$ $f\alpha^\theta g$-continuity.

Let $X = \{a, b\}, \tau_1 = \{0_X, 1_X, A, B\}, \tau_2 = \{0_X, 1_X, C\}$, where $A(a) = 0.45, A(b) = 0.55, B(a) = 0.4, B(b) = 0.5, C(a) = C(b) = 0.5$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Now $F\alpha O(X, \tau_1) = \{0_X, 1_X, B, U\}$, where $U \geq A$. Now $C \in \tau_2^c$, $i^{-1}(C) = C < 1_X \in \tau_1$ only and $cl_{\tau_1} C = 1_X \setminus B < 1_X$ and $cl_{\tau_1} int_{\tau_1} C = 1_X \setminus B < 1_X$. Thus C is an fg-closed as well as an fwg-closed set in (X, τ_1). So i is an fg-continuous as well as an fwg-continuous function.

On the other hand, $C < D \in F\alpha O(X, \tau_1)$, where $D(a) = 0.5, D(b) = 0.55$. Then $cl_{\tau_1} int_{\tau_1} C = 1_X \setminus B \not\leq D$. Thus C is not an $f\alpha^\theta g$-closed set in (X, τ_1). So i is not an $f\alpha^\theta g$-continuous function.

Definition 4.10. A function $h : X \to Y$ is called an *$f\alpha^\theta g$-irresolute function*, if $h^{-1}(U)$ is an $f\alpha^\theta g$-closed set in X for every $f\alpha^\theta g$-closed set U in Y.

Theorem 4.11. *A function $h : X \to Y$ is an $f\alpha^\theta g$-irresolute function if and only if for each fuzzy point x_t in X and each $f\alpha^\theta g$-open nbd V in Y of $h(x_t)$, there exists an $f\alpha^\theta g$-open nbd U in X of x_t such that $h(U) \leq V$.*

Proof. The proof is same as that of Theorem 4.2 (1)$\Leftrightarrow$(2). □

Definition 4.12. A function $h : X \to Y$ is called a *strongly $f\alpha^\theta g$-continuous function*, if $h^{-1}(U)$ is a fuzzy closed set in X for all $f\alpha^\theta g$-closed set U in Y.

Theorem 4.13. *A function $h : X \to Y$ is a strongly $f\alpha^\theta g$-continuous function if and only if for each fuzzy point x_t in X and each $f\alpha^\theta g$-open nbd V in Y of $h(x_t)$, there exists a fuzzy open nbd U in X of x_t such that $h(U) \leq V$.*

Proof. The proof is same as that of Theorem 4.2 (1)⇔(2). □

Definition 4.14. A function $h : X \to Y$ is called a *weakly $f\alpha^\theta g$-continuous function*, if $h^{-1}(U)$ is $f\alpha^\theta g$-closed set in X for all fuzzy regular closed set U in Y.

Theorem 4.15. *A function $h : X \to Y$ is a weakly $f\alpha^\theta g$-continuous function if and only if for each fuzzy point x_t in X and each $V \in FRO(Y)$ with $h(x_t) \in V$, there exists an $f\alpha^\theta g$-open nbd U in X of x_t such that $h(U) \leq V$.*

Proof. The proof is same as that of Theorem 4.2 (1)⇔(2). □

Remark 4.16. It is clear from definitions that

(1) as every fuzzy closed set is $f\alpha^\theta g$-closed set, so strongly $f\alpha^\theta g$-continuity implies fuzzy continuity, $f\alpha^\theta g$-continuity and $f\alpha^\theta g$-irresoluteness and $f\alpha^\theta g$-irresoluteness implies $f\alpha^\theta g$-continuity which implies weakly $f\alpha^\theta g$-continuity but the reverse implications are not necessarily true, follow from the following examples,

(2) fuzzy continuity and $f\alpha^\theta g$-irresoluteness are independent concepts follow from the following examples,

(3) the composition of two $f\alpha^\theta g$-irresolute (resp., strongly $f\alpha^\theta g$-continuous) functions is also so. But the composition of two weakly $f\alpha^\theta g$-continuous functions may not be so, as it is seen from the following example.

Example 4.17. Fuzzy continuity, $f\alpha^\theta g$-continuity $\not\Rightarrow$ $f\alpha^\theta g$-irresoluteness, strongly $f\alpha^\theta g$-continuity.

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, where $A(a) = 0.5, A(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly, i is a fuzzy continuous function as well as an $f\alpha^\theta g$-continuous function, since every fuzzy set in (X, τ_2) is $f\alpha^\theta g$-closed in (X, τ_2). Now $F\alpha O(X, \tau_1) = \{0_X, 1_X, U\}$, where $U \geq A$. Consider a fuzzy set C defined by $C(a) = C(b) = 0.6$. Then $C \in F\alpha^\theta GC(X, \tau_2)$. Now $i^{-1}(C) = C \leq C \in F\alpha O(X, \tau_1)$. But $cl_{\tau_1} int_{\tau_1} C = 1_X \not\leq C$. Thus $C \notin F\alpha^\theta GC(X, \tau_1)$. So i is not a $f\alpha^\theta g$-irresolute function. Again, $C \notin \tau_1^c$. Hence i is not a strongly $f\alpha^\theta g$-continuous function.

Example 4.18. $f\alpha^\theta g$-irresoluteness $\not\Rightarrow$ fuzzy continuity, strongly $f\alpha^\theta g$-continuity.

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X\}$, $\tau_2 = \{0_X, 1_X, A\}$, where $A(a) = A(b) = 0.4$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Clearly, i is not a fuzzy continuous function. Since every fuzzy set in (X, τ_1) is an $f\alpha^\theta g$-closed set in (X, τ_1), i is clearly an $f\alpha^\theta g$-irresolute function. Here $1_X \setminus A$ being a fuzzy closed set in (X, τ_2) is an $f\alpha^\theta g$-closed set in (X, τ_2). Now $i^{-1}(1_X \setminus A) = 1_X \setminus A \notin \tau_1^c$. Thus i is not a strongly $f\alpha^\theta g$-continuous function.

Example 4.19. Weakly $f\alpha^\theta g$-continuity $\not\Rightarrow$ $f\alpha^\theta g$-continuity.

Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X, B\}$, where $A(a) = 0.5, A(b) = 0.4, B(a) = 0.5, B(b) = 0.6$. Then (X, τ_1) and (X, τ_2) are fts's. Consider the identity function $i : (X, \tau_1) \to (X, \tau_2)$. Since $FRC(X, \tau_2) = \{0_X, 1_X\}$, i is a weakly $f\alpha^\theta g$-continuous function. Now $1_X \setminus B \in \tau_2^c$, $i^{-1}(1_X \setminus B) = 1_X \setminus B \leq A \in F\alpha O(X, \tau_1)$. But $cl_{\tau_1} int_{\tau_1}(1_X \setminus B) = 1_X \setminus A \not\leq A$. Thus $1_X \setminus B$ is not an $f\alpha^\theta g$-closed set in (X, τ_2). So i is not an $f\alpha^\theta g$-continuous function.

Example 4.20. Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A\}$, $\tau_2 = \{0_X, 1_X\}$, $\tau_3 = \{0_X, 1_X, B\}$, where $A(a) = 0.5, A(b) = 0.6, B(a) = 0.5, B(b) = 0.4$. Then $(X, \tau_1), (X, \tau_2)$ and

(X, τ_3) are fts's. Consider two identity functions $i_1 : (X, \tau_1) \to (X, \tau_2)$ and $i_2 : (X, \tau_2) \to (X, \tau_3)$. Clearly, i_1 and i_2 are weakly $f\alpha^\theta g$-continuous functions. Let $i_3 = i_2 \circ i_1$. Then $1_X \setminus B \in \tau_3^c$, $i_3^{-1}(1_X \setminus B) = 1_X \setminus B \leq A \in F\alpha O(X, \tau_1)$. But $cl_{\tau_1} int_{\tau_1}(1_X \setminus B) = 1_X \not\leq A$. Thus $1_X \setminus B$ is not an $f\alpha^\theta g$-closed set in (X, τ_1). So i_3 is not a weakly $f\alpha^\theta g$-continuous function.

Theorem 4.21. *If $h_1 : X \to Y$ is a strongly $f\alpha^\theta g$-continuous function and $h_2 : Y \to Z$ is an $f\alpha^\theta g$-continuous function, then $h_2 \circ h_1 : X \to Z$ is a fuzzy continuous function.*

Proof. Obvious. □

Note 4.22. Let $h : X \to Y$ be an $f\alpha^\theta g$-continuous function from a $fT_{\alpha^\theta g}$-space X onto a fts Y. Then h is fuzzy continuous, fg-continuous, fmg-continuous, fwg-continuous, $frwg$-continuous and $fswg$-continuous.

5. $f\alpha^\theta g$-REGULAR, $f\alpha^\theta g$-NORMAL AND $f\alpha^\theta g$-COMPACT SPACES

In this section, two new types of separation axioms are introduced and studied. Also a new type of compactness is introduced. Finally the mutual relationships of these spaces with the spaces defined in [2, 12, 13, 23, 24] are established.

Definition 5.1. An fts (X, τ) is said to be an $f\alpha^\theta g$-*regular space*, if for any fuzzy point x_t in X and each $f\alpha^\theta g$-closed set F in X with $x_t \notin F$, there exist $U, V \in \tau$ such that $x_t \in U, F \leq V$ and $U \not{q} V$.

Theorem 5.2. *In an fts (X, τ), the following statements are equivalent:*
(1) *X is $f\alpha^\theta g$-regular,*
(2) *for each fuzzy point x_t in X and any $f\alpha^\theta g$-open q-nbd U of x_t, there exists $V \in \tau$ such that $x_t \in V$ and $clV \leq U$,*
(3) *for each fuzzy point x_t in X and each $f\alpha^\theta g$-closed set A of X with $x_t \notin A$, there exists $U \in \tau$ with $x_t \in U$ such that $clU \not{q} A$.*

Proof. (1) ⇒ (2) Suppose the condition (1) holds and let x_t be a fuzzy point in X and U any $f\alpha^\theta g$-open q-nbd of x_t. Then $x_t q U$. Thus $U(x)+t > 1$. So $x_t \notin 1_X \setminus U$ which is an $f\alpha^\theta g$-closed set in X. By (1), there exist $V, W \in \tau$ such that $x_t \in V, 1_X \setminus U \leq W$ and $V \not{q} W$. Hence $V \leq 1_X \setminus W$. Therefore $clV \leq cl(1_X \setminus W) = 1_X \setminus W \leq U$.

(2) ⇒ (3) Suppose the condition (2) holds and let x_t be a fuzzy point in X and A an $f\alpha^\theta g$-closed set in X with $x_t \notin A$. Then $A(x) < t$. Thus $x_t q(1_X \setminus A)$ which being an $f\alpha^\theta g$-open set in X is an $f\alpha^\theta g$-open q-nbd of x_t. So by (2), there exists $V \in \tau$ such that $x_t \in V$ and $clV \leq 1_X \setminus A$. Hence $clV \not{q} A$.

(3) ⇒ (1) Suppose the condition (3) holds and let x_t be a fuzzy point in X and F be any $f\alpha^\theta g$-closed set in X with $x_t \notin F$. Then by (3), there exists $U \in \tau$ such that $x_t \in U$ and $clU \not{q} F$. Thus $F \leq 1_X \setminus clU$ (=V, say). So $V \in \tau$ and $V \not{q} U$ as $U \not{q} (1_X \setminus clU)$. Hence X is an $f\alpha^\theta g$-regular space. □

Definition 5.3. An fts (X, τ) is called an $f\alpha^\theta g$-*normal space*, if for each pair of $f\alpha^\theta g$-closed sets A, B in X with $A \not{q} B$, there exist $U, V \in \tau$ such that $A \leq U, B \leq V$ and $U \not{q} V$.

Theorem 5.4. *An fts (X, τ) is an $f\alpha^\theta g$-normal space if and only if for every $f\alpha^\theta g$-closed set F and $f\alpha^\theta g$-open set G in X with $F \leq G$, there exists $H \in \tau$ such that $F \leq H \leq clH \leq G$.*

Proof. Suppose X is an $f\alpha^\theta g$-normal space and let F be an $f\alpha^\theta g$-closed set and G an $f\alpha^\theta g$-open set in X with $F \leq G$. Then $F \not{q}(1_X \setminus G)$, where $1_X \setminus G$ is an $f\alpha^\theta g$-closed set in X. By the hypothesis, there exist $H, T \in \tau$ such that $F \leq H, 1_X \setminus G \leq T$ and $H \not{q} T$. Thus $H \leq 1_X \setminus T \leq G$. So $F \leq H \leq clH \leq cl(1_X \setminus T) = 1_X \setminus T \leq G$.

Conversely, suppose the necessary condition holds and let A, B be two $f\alpha^\theta g$-closed sets in X with $A \not{q} B$. Then $A \leq 1_X \setminus B$. By the hypothesis, there exists $H \in \tau$ such that $A \leq H \leq clH \leq 1_X \setminus B$. Thus $A \leq H, B \leq 1_X \setminus clH$ (=V, say). So $V \in \tau$. Hence $B \leq V$. Also as $H \not{q}(1_X \setminus clH)$, $H \not{q} V$. Therefore X is an $f\alpha^\theta g$-normal space. □

Let us now recall the following definitions from [2, 25] for ready references.

Definition 5.5. Let (X, τ) be an fts and $A \in I^X$. A collection $\mathcal{U}$ of fuzzy sets in X is called a *fuzzy cover* of A, if $\bigcup \mathcal{U} \geq A$ [25]. If each member of $\mathcal{U}$ is fuzzy open (resp., fuzzy regular open, $f\alpha^\theta g$-open) in X, then $\mathcal{U}$ is called a *fuzzy open* [25] (resp., *fuzzy regular open* [3], *$f\alpha^\theta g$-open*) cover of A. If, in particular, $A = 1_X$, we get the definition of fuzzy cover of X as $\bigcup \mathcal{U} = 1_X$ [2].

Definition 5.6. Let (X, τ) be an fts and $A \in I^X$. Then a fuzzy cover $\mathcal{U}$ of A (resp., of X) is said to *have a finite subcover* $\mathcal{U}_0$, if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigcup \mathcal{U}_0 \geq A$ [25]. If, in particular, $A = 1_X$, we get $\bigcup \mathcal{U}_0 = 1_X$ [2].

Definition 5.7. Let (X, τ) be a fts and $A \in I^X$. Then A is called a *fuzzy compact* [2] (resp., *fuzzy almost compact* [23], *fuzzy nearly compact* [20]) set, if every fuzzy open (resp., fuzzy open, fuzzy regular open) cover $\mathcal{U}$ of A has a finite subcollection $\mathcal{U}_0$ such that $\bigcup \mathcal{U}_0 \geq A$ (resp., $\bigcup_{U \in \mathcal{U}_0} clU \geq A$, $\bigcup \mathcal{U}_0 \geq A$). If, in particular, $A = 1_X$, we get the definition of fuzzy compact [2] (resp., fuzzy almost compact [23], fuzzy nearly compact [24]) space as $\bigcup \mathcal{U}_0 = 1_X$ (resp., $\bigcup_{U \in \mathcal{U}_0} clU = 1_X$, $\bigcup \mathcal{U}_0 = 1_X$).

Let us now introduce the following concept.

Definition 5.8. Let (X, τ) be an fts and $A \in I^X$. Then A is said to be *$f\alpha^\theta g$-compact*, if every fuzzy cover $\mathcal{U}$ of A by $f\alpha^\theta g$-open sets of X has a finite subcover. If, in particular, $A = 1_X$, we get the definition of $f\alpha^\theta g$-compact space X.

Theorem 5.9. *Every $f\alpha^\theta g$-closed set in an $f\alpha^\theta g$-compact space X is $f\alpha^\theta g$-compact.*

Proof. Let $A (\in I^X)$ be an $f\alpha^\theta g$-closed set in an $f\alpha^\theta g$-compact space X. Let $\mathcal{U}$ be a fuzzy cover of A by $f\alpha^\theta g$-open sets of X. Then $\mathcal{V} = \mathcal{U} \bigcup (1_X \setminus A)$ is a fuzzy cover of X by $f\alpha^\theta g$-open sets of X. As X is an $f\alpha^\theta g$-compact space, $\mathcal{V}$ has a finite subcollection $\mathcal{V}_0$ which also covers X. If $\mathcal{V}_0$ contains $1_X \setminus A$, we omit it and get a finite subcover of A. Thus A is a $f\alpha^\theta g$-compact set. □

Next we recall the following two definitions from [12, 13] for ready references.

Definition 5.10 ([12])**.** An fts (X, τ) is called a *fuzzy regular spac,e* if for each fuzzy point x_t in X and each fuzzy closed set F in X with $x_t \notin F$, there exist $U, V \in \tau$ such that $x_t \in U$, $F \leq V$ and $U \not{q} V$.

Definition 5.11 ([13])**.** An fts (X, τ) is called a *fuzzy normal space*, if for each pair of fuzzy closed sets A, B of X with $A \not{q} B$, there exist $U, V \in \tau$ such that $A \leq U, B \leq V$ and $U \not{q} V$.

Remark 5.12. It is clear from above discussion that

(1) an $f\alpha^\theta g$-regular (resp., $f\alpha^\theta g$-normal, $f\alpha^\theta g$-compact) space is a fuzzy regular (resp., fuzzy normal, fuzzy compact) space, but the converses are not true, in general, follow from the following example,

(2) in an $fT_{\alpha^\theta g}$-space, fuzzy regularity (resp., fuzzy normality, fuzzy compactness) implies $f\alpha^\theta g$-regularity (resp., $f\alpha^\theta g$-normality, $f\alpha^\theta g$-compactness).

Example 5.13. Let $X = \{a\}$, $\tau = \{0_X, 1_X\}$. Then (X, τ) is a fts. Clearly, (X, τ) is a fuzzy regular space, a fuzzy normal space and a fuzzy compact space. Here every fuzzy set is an $f\alpha^\theta g$-open set as well as an $f\alpha^\theta g$-closed set in (X, τ). Consider the fuzzy point $a_{0.4}$ and the fuzzy set A defined by $A(a) = 0.1$. Then $a_{0.4} \notin A$ which is an $f\alpha^\theta g$-closed set in X. But there does not exist $U, V \in \tau$ such that $a_{0.4} \in U, A \leq V$ and $U \not{q} V$. Thus (X, τ) is not an $f\alpha^\theta g$-regular space. Similarly, considering two fuzzy sets A, B defined by $A(a) = 0.2, B(a) = 0.1$. Then A and B are $f\alpha^\theta g$-closed sets in X with $A \not{q} B$. But there does not exist $U, V \in \tau$ such that $A \leq U$, $B \leq V$ and $U \not{q} V$. So (X, τ) is not an $f\alpha^\theta g$-normal space. Again let $\mathcal{U} = \{U_n(a) : n \in N\}$, where $U_n(a) = \frac{n}{n+1}$ for all $n \in N$ of X. Then $\mathcal{U}$ is an $f\alpha^\theta g$-open covering of X which has no finite subcovering. Hence (X, τ) is not an $f\alpha^\theta g$-compact space.

6. APPLICATIONS

In this section, several applications of the functions defined in this paper are discussed.

Theorem 6.1. *If a bijective function $h : X \to Y$ is an $f\alpha^\theta g$-continuous, fuzzy open function from an $f\alpha^\theta g$-regular space X onto a fts Y, then Y is a fuzzy regular space.*

Proof. Let y_t be a fuzzy point in Y and F, a fuzzy closed set in Y with $y_t \notin F$. As h is bijective, there exists unique $x \in X$ such that $h(x) = y$. Then $h(x_t) \notin F$. Thus $x_t \notin h^{-1}(F)$, where $h^{-1}(F)$ is an $f\alpha^\theta g$-closed set in X (as h is an $f\alpha^\theta g$-continuous function). As X is an $f\alpha^\theta g$-regular space, there exist fuzzy open sets U, V in X such that $x_t \in U$, $h^{-1}(F) \leq V$ and $U \not{q} V$. So $h(x_t) \in h(U)$, $F = h(h^{-1}(F))$ (as h is bijective) $\leq h(V)$ and $h(U) \not{q} h(V)$, where $h(U)$ and $h(V)$ are fuzzy open sets in Y. (Indeed, $h(U)qh(V) \Rightarrow$ there exists $z \in Y$ such that $[h(U)](z) + [h(V)](z) > 1 \Rightarrow U(h^{-1}(z)) + V(h^{-1}(z)) > 1$ as h is bijective $\Rightarrow UqV$, a contradiction). Hence Y is a fuzzy regular space. □

In a similar manner, we can state the following theorems easily the proofs of which are same as that of Theorem 6.1.

Theorem 6.2. *If a bijective function $h : X \to Y$ is an $f\alpha^\theta g$-continuous, fuzzy open function from an $f\alpha^\theta g$-normal space X onto an fts Y, then Y is a fuzzy normal space.*

Theorem 6.3. *If a bijective function $h : X \to Y$ is an $f\alpha^\theta g$-continuous, fuzzy open function from a fuzzy regular* (resp., *fuzzy normal*), *an $fT_{\alpha^\theta g}$-space X onto an fts Y, then Y is fuzzy regular* (resp., *fuzzy normal*) *space.*

Theorem 6.4. *If a bijective function $h : X \to Y$ is an $f\alpha^\theta g$-irresolute, fuzzy open function from an $f\alpha^\theta g$-regular* (resp., *$f\alpha^\theta g$-normal*) *space X onto an fts Y, then Y is an $f\alpha^\theta g$-regular* (resp., *$f\alpha^\theta g$-normal*) *space.*

Theorem 6.5. *If a bijective function $h : X \to Y$ is an $f\alpha^\theta g$-irresolute, fuzzy open function from an $f\alpha^\theta g$-regular* (resp., *$f\alpha^\theta g$-normal*) *space X onto an fts Y, then Y is a fuzzy regular* (resp., *fuzzy normal*) *space.*

Theorem 6.6. *If a bijective function $h : X \to Y$ is an $f\alpha^\theta g$-irresolute, fuzzy open function from a fuzzy regular* (resp., *fuzzy normal*), *an $fT_{\alpha^\theta g}$-space X onto an fts Y, then Y is a fuzzy regular* (resp., *fuzzy normal*) *space.*

Theorem 6.7. *If a bijective function $h : X \to Y$ is a strongly $f\alpha^\theta g$-continuous, fuzzy open function from a fuzzy regular* (resp., *fuzzy normal*) *space X onto an fts Y, then Y is an $f\alpha^\theta g$-regular* (resp., *$f\alpha^\theta g$-normal*) *space.*

Theorem 6.8. *If a bijective function $h : X \to Y$ is a strongly $f\alpha^\theta g$-continuous, fuzzy open function from a fuzzy regular* (resp., *fuzzy normal*) *space X onto an fts Y, then Y is a fuzzy regular* (resp., *fuzzy normal*) *space.*

Theorem 6.9. *Let $h : X \to Y$ be an $f\alpha^\theta g$-continuous function from X onto an fts Y and $A(\in I^X)$ an $f\alpha^\theta g$-compact set in X. Then $h(A)$ is a fuzzy compact* (resp., *fuzzy almost compact*, *fuzzy nearly compact*) *set in Y.*

Proof. Let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy cover of $h(A)$ by fuzzy open (resp., fuzzy open, fuzzy regular open) sets of Y. Then $h(A) \leq \bigcup\limits_{\alpha\in\Lambda} U_\alpha$. Thus $A \leq h^{-1}(\bigcup\limits_{\alpha\in\Lambda} U_\alpha) = \bigcup\limits_{\alpha\in\Lambda} h^{-1}(U_\alpha)$. So $\mathcal{V} = \{h^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a fuzzy cover of A by $f\alpha^\theta g$-open sets of X, as h is an $f\alpha^\theta g$-continuous function. As A is an $f\alpha^\theta g$-compact set in X, there exists a finite subcollection Λ_0 of Λ such that $A \leq \bigcup\limits_{\alpha\in\Lambda_0} h^{-1}(U_\alpha)$. Hence $h(A) \leq h(\bigcup\limits_{\alpha\in\Lambda_0} h^{-1}(U_\alpha)) \leq \bigcup\limits_{\alpha\in\Lambda_0} U_\alpha$. Therefore $h(A)$ is fuzzy compact (resp., fuzzy almost compact, fuzzy nearly compact) set in Y. □

Since fuzzy open set is $f\alpha^\theta g$-open, we can state the following theorems easily the proofs of which are same as that of Theorem 6.9.

Theorem 6.10. *Let $h : X \to Y$ be an $f\alpha^\theta g$-continuous function from an $f\alpha^\theta g$-compact space X onto an fts Y. Then Y is a fuzzy compact* (resp., *fuzzy almost compact*, *fuzzy nearly compact*) *space.*

Theorem 6.11. *Let $h : X \to Y$ be an $f\alpha^\theta g$-irresolute function from X onto an fts Y and $A(\in I^X)$ an $f\alpha^\theta g$-compact set in X. Then $h(A)$ is an $f\alpha^\theta g$-compact* (resp., *fuzzy compact*, *fuzzy almost compact*, *fuzzy nearly compact*) *set in Y.*

Theorem 6.12. *Let $h : X \to Y$ be an $f\alpha^\theta g$-irresolute function from an $f\alpha^\theta g$-compact space X onto an fts Y. Then Y is an $f\alpha^\theta g$-compact* (resp., *fuzzy compact, fuzzy almost compact, fuzzy nearly compac*) *set in Y.*

Theorem 6.13. *Let $h : X \to Y$ be an $f\alpha^\theta g$-continuous function from a fuzzy compact, $fT_{\alpha^\theta g}$-space X onto an fts Y. Then Y is a fuzzy compact* (resp., *fuzzy almost compact, fuzzy nearly compact*) *space.*

Theorem 6.14. *Let $h : X \to Y$ be an $f\alpha^\theta g$-irresolute function from a fuzzy compact, $fT_{\alpha^\theta g}$-space X onto an fts Y. Then Y is an $f\alpha^\theta g$-compact* (resp., *fuzzy compact, fuzzy almost compact, fuzzy nearly compact*) *space.*

Theorem 6.15. *Let $h : X \to Y$ be a strongly $f\alpha^\theta g$-continuous function from X onto an fts Y and $A(\in I^X)$ be an $f\alpha^\theta g$-compact set in X. Then $h(A)$ is a fuzzy compact* (resp., *fuzzy almost compact, fuzzy nearly compact, $f\alpha^\theta g$-compact*) *set in Y.*

Theorem 6.16. *Let $h : X \to Y$ be a strongly $f\alpha^\theta g$-continuous function from a fuzzy compact space X onto an fts Y. Then Y is an $f\alpha^\theta g$-compact* (resp., *fuzzy compact, fuzzy almost compact, fuzzy nearly compact*) *space.*

Theorem 6.17. *Let $h : X \to Y$ be a weakly $f\alpha^\theta g$-continuous function from an fts X onto an fts Y and $A(\in I^X)$ be an $f\alpha^\theta g$-compact set in X. Then $h(A)$ is a fuzzy nearly compact set in Y.*

Theorem 6.18. *Let $h : X \to Y$ be a weakly $f\alpha^\theta g$-continuous function from an $f\alpha^\theta g$-compact space X onto an fts Y. Then Y is a fuzzy nearly compact space.*

Theorem 6.19. *Let $h : X \to Y$ be a weakly $f\alpha^\theta g$-continuous function from an $fT_{\alpha^\theta g}$-space X onto an fts Y and $A(\in I^X)$ be a fuzzy compact set in X. Then $h(A)$ is a fuzzy nearly compact set in Y.*

Theorem 6.20. *Let $h : X \to Y$ be a weakly $f\alpha^\theta g$-continuous function from a fuzzy compact, $fT_{\alpha^\theta g}$-space X onto an fts Y. Then Y is a fuzzy nearly compact space.*

Theorem 6.21. *If an injective function $h : X \to Y$ is an $f\alpha^\theta g$-continuous function from an fts X onto a fuzzy T_2-space Y, then X is an $f\alpha^\theta g$-T_2-space.*

Proof. Let x_t and y_s be two distinct fuzzy points in X. Then $h(x_t)$ $(= z_t$, say) and $h(y_s)$ $(= w_s$, say) are two distinct fuzzy points in Y.

Case I. Suppose $x \neq y$. Then $z \neq w$. Since Y is a fuzzy T_2-space, there exist fuzzy open sets U, V in Y such that $z_t qU, w_s qV$ and $U \not{q} V$. As h is an $f\alpha^\theta g$-continuous function, $h^{-1}(U)$ and $h^{-1}(V)$ are $f\alpha^\theta g$-open sets in X with $x_t qh^{-1}(U), y_s qh^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$ [Indeed, $z_t qU \Rightarrow U(z) + t > 1 \Rightarrow U(h(x)) + t > 1 \Rightarrow [h^{-1}(U)](x) + t > 1 \Rightarrow x_t qh^{-1}(U)$. Again, $h^{-1}(U)qh^{-1}(V) \Rightarrow$ there exists $p \in X$ such that $[h^{-1}(U)](p) + [h^{-1}(V)](p) > 1 \Rightarrow U(h(p)) + V(h(p)) > 1 \Rightarrow UqV$, a contradiction].

Case II. Suppose $x = y$ and $t < s$ (say). Then $z = w$ and $t < s$. Since Y is a fuzzy T_2-space, there exist a fuzzy open nbd U of x_t and a fuzzy open q-nbd V of w_s such that $U \not{q} V$. Thus $U(z) \geq t \Rightarrow [h^{-1}(U)](x) \geq t \Rightarrow x_t \in h^{-1}(U), y_s qh^{-1}(V)$ and $h^{-1}(U) \not{q} h^{-1}(V)$, where $h^{-1}(U)$ and $h^{-1}(V)$ are $f\alpha^\theta g$-open sets in X as h is an $f\alpha^\theta g$-continuous function. Consequently, X is an $f\alpha^\theta g$-T_2-space. □

Similarly, we can state the following theorems easily the proofs of which are similar to that of Theorem 6.21.

Theorem 6.22. *If a bijective function $h : X \to Y$ is an $f\alpha^\theta g$-irresolute function from an fts X onto an $f\alpha^\theta g$-T_2-space* (resp., *fuzzy T_2-space*) *Y, then X is an $f\alpha^\theta g$-T_2-space.*

Theorem 6.23. *If a bijective function $h : X \to Y$ is an $f\alpha^\theta g$-continuous function from an $fT_{\alpha^\theta g}$-space X onto a fuzzy T_2-space Y, then X is a fuzzy T_2-space.*

Theorem 6.24. *If a bijective function $h : X \to Y$ is an $f\alpha^\theta g$-irresolute function from an $fT_{\alpha^\theta g}$-space X onto an $f\alpha^\theta g$-T_2-space* (resp., *fuzzy T_2-space*) *Y, then X is a fuzzy T_2-space.*

Theorem 6.25. *If a bijective function $h : X \to Y$ is a strongly $f\alpha^\theta g$-continuous function from an fts X onto an $f\alpha^\theta g$-T_2-space* (resp., *fuzzy T_2-space*) *Y, then X is a fuzzy T_2-space.*

7. Conclusions

Using the concept of $f\alpha^\theta g$-closed set here we introduce and study three different types of fuzzy continuous-like functions. Several applications of these functions on fuzzy regular, fuzzy normal, fuzzy compact and fuzzy T_2-spaces are shown here

Acknowledgements. I express my sincere gratitude to the referees for their valuable remark.

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