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# Some corrections to primals and their topological structures

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Abstract. Al-Saadi and Al-Malki [1] corrected the primals on some examples given in there article [2]. In this note, we mention and correct as possible some examples of primals in the articles cited in [3, 4, 5, 6, 7] by giving remarks and correct examples.

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## 1. Introduction

For the duration of this document $\mathcal{P}$ denotes the primal on any set $X$. The power set of a set $X$ will be symbolized by $2^X$.

**Definition 1.1** ([8])**.** Let $X$ be a non-empty set. Then the *primal* $\mathcal{P} \subseteq 2^X$ satisfies the following conditions:
(i) $X \notin \mathcal{P}$,
(ii) if $A \in \mathcal{P}$ and $B \subseteq A$, then $B \in \mathcal{P}$,
(iii) if $A \cap B \in \mathcal{P}$, then $A \in \mathcal{P}$ or $B \in \mathcal{P}$.

The remarks and counter-examples bellow give the correct statements for the corresponding statements on the cited articles.

**Corollary 1.2** ([8])**.** *Let $X$ be a non-empty set. Then $\mathcal{P} \subseteq 2^X$ is a primal on $X$ if and only if it satisfies the following conditions:*
(1) $X \notin \mathcal{P}$,
(2) *if $B \notin \mathcal{P}$ and $B \subseteq A$, then $A \notin \mathcal{P}$,*
(3) *if $A \notin \mathcal{P}$ and $B \notin \mathcal{P}$, then $A \cap B \notin \mathcal{P}$.*

**Definition 1.3** ([8])**.** A topological space $(X, \tau)$ with a primal $\mathcal{P}$ on $X$ is called a *primal topological space* and denoted by $(X, \tau, \mathcal{P})$.

**Definition 1.4** ([8])**.** Let $(X, \tau, \mathcal{P})$ be a primal topological space. The family of all open neighborhoods of a point $x$ of $X$ will be denoted by $O(X, x)$. We consider a map $(\cdot)^\diamond : 2^X \to 2^X$ as $A^\diamond(X, \tau, \mathcal{P}) = \{x \in X | (\forall U \in O(X, x))(A^c \cup U^c \in \mathcal{P})\}$ for any subset $A$ of $X$. We can also write $A^\diamond$ as $A^\diamond(X, \tau, \mathcal{P})$ to specify the primal as per our requirements.

**Definition 1.5** ([2])**.** The symbol $(X, \mathfrak{g}, \mathcal{P})$ references a *generalized primal topological space* (GPT space), which is a generalized topological space $(X, \mathfrak{g})$ together with a primal set $\mathcal{P}$ over $X$. The members of $(X, \mathfrak{g}, \mathcal{P})$ are called $(\mathfrak{g}, \mathcal{P})$-*open sets* in $X$ and their complements are called $(\mathfrak{g}, \mathcal{P})$-*closed sets* in $X$. The entire set of $(\mathfrak{g}, \mathcal{P})$-closed symbols is referred to as $C_{(\mathfrak{g},\mathcal{P})}(X)$. In addition, $\mathrm{cl}_{(\mathfrak{g},\mathcal{P})}(E)$ denotes the closure of $E \subseteq X$.

**Definition 1.6** ([6])**.** Let $(X, \mathfrak{g}, \mathcal{P})$ be a GPT space and $E \subseteq X$. Then $E$ is called a:

(i) $(\mathfrak{g}, \mathcal{P})$-*semi-open set*, if $E \subseteq \mathrm{cl}^\diamond\left(i_\mathfrak{g}(E)\right)$,
(ii) $(\mathfrak{g}, \mathcal{P})$-*pre-open set*, if $E \subseteq i_\mathfrak{g}\left(cl^\diamond(E)\right)$,
(iii) $a(\mathfrak{g}, \mathcal{P})$-*regular open set*, if $E = i_\mathfrak{g}\left(\mathrm{cl}^\diamond(E)\right)$,
(iv) $(\mathfrak{g}, \mathcal{P})$ - $\beta$-*open set*, if $E \subseteq c_\mathfrak{g}\left(i_\mathfrak{g}\left(cl^\triangleright(E)\right)\right)$,
(v) $(\mathfrak{g}, \mathcal{P})$-$\alpha$-*open set*, if $E \subseteq i_\mathfrak{g}\left(cl^\triangleright\left(i_\mathfrak{g}(E)\right)\right)$.

The whole set of $(\mathfrak{g}, \mathcal{P})$-semi-open sets is symbolized by $\sigma$, while the whole set of $(\mathfrak{g}, \mathcal{P})$-pre-open sets is symbolized by $\pi$. Moreover, the whole set of $(\mathfrak{g}, \mathcal{P})$ - $\alpha$-open sets is symbolized by $\alpha$, while $\beta$ is the symbolization of all $(\mathfrak{g}, \mathcal{P})$-$\beta$-open sets. In addition, $c_\mathfrak{g}(E)$ and $i_\mathfrak{g}(E)$ symbolize the closure and interior of E, respectively, which are described as in the general situation.

## 2. The corrections

**Remark 2.1.** In Example 2 in [3], the primal is defined as $\mathcal{P}_f =$ all finite subsets of the real line $\mathbb{R}$ whose complement is not finite. Clearly, $\mathcal{P}_f$ is not a primal, since for example $\{1\} \in \mathcal{P}_f$ and $\{1\} = (-\infty, 1] \cap [1, \infty)$ but neither $(-\infty, 1]$ nor $[1, \infty) \in \mathcal{P}_f$.

**Remark 2.2.** Example 3.1 in [4] is not correct. Now it is corrected in the following example.

**Example 2.3.** Let $T = \{1, 2, 3\}$, $\sigma = \{\phi, T, \{1\}, \{2\}, \{1, 2\}\}$ and $\mathcal{P} = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$. Let $S = \{1, 3\}$. Then we have $S^\diamond = \gamma(S) = \{3\}$ and $\prod(S) = \{1, 2, 3\}$. Thus $\prod(S) \nsubseteq \gamma(S)$ and $\prod(S) \nsubseteq S^\diamond$.

**Remark 2.4.** In Example 3.1 in [5], let $\mathcal{A} = \{x, y, z, r\}$ be a universe with parameter $\rho = \{\alpha\}$ and consider the following soft sets over $\mathcal{A}$ defined as follows:

$$\Phi_\rho(1) = (\Phi(1), \rho) = \{(\alpha, \{z\})\},\ \Phi_\rho(2) = (\Phi(2), \rho) = \{(\alpha, \{r\})\},$$
$$\Phi_\rho(6) = (\Phi(6), \rho) = \{(\alpha, \{z, r\})\},\ \phi_\rho = (\Phi(1), \rho) = \{(\alpha, \phi)\}.$$

The primal is defined as $\mathcal{P} = \{\phi_\rho, \Phi_\rho(1), \Phi_\rho(2), \Phi_\rho(6)\}$. Clearly, this primal is incorrect. For example, $\Phi_\rho(1) = (\Phi(1), \rho) = \{(\alpha, \{z\})\} = \{(\alpha, \{x, z\})\} \sqcap \{(\alpha, \{y, z\})\} \in \mathcal{P}$ but neither $\{(\alpha, \{x, z\})\}$ nor $\{(\alpha, \{y, z\})\} \in \mathcal{P}$. Next, we suggest a possible correction.

**Example 2.5.** Let $\mathcal{A} = \{x, y, z,\}$ with parameter $\rho = \{\alpha\}$. Consider the following soft sets: $\Phi_\rho(1) = (\Phi(1), \rho) = \{(\alpha, \{x\})\}$, $\Phi_\rho(2) = (\Phi(2), \rho) = \{(\alpha, \{y\})\}$, $\Phi_\rho(3) = (\Phi(3), \rho) = \{(\alpha, \{z\})\}$, $\Phi_\rho(4) = (\Phi(4), \rho) = \{(\alpha, \{x, y\})\}$, $\Phi_\rho(5) = (\Phi(5), \rho) = \{(\alpha, \{x, z\})\}$, $\Phi_\rho(6) = (\Phi(6), \rho) = \{(\alpha, \{y, z\})\}$, $\Phi_\rho(7) = (\Phi(7), \rho) = \{(\alpha, \{x, y, z\})\}$ and $\phi_\rho = (\Phi(1), \rho) = \{(\alpha, \phi)\}$. Then $\Delta_s = \{\phi_\rho, \Phi_\rho(1), \Phi_\rho(2), \Phi_\rho(4)\}$ is a soft primal topology and $\mathcal{P} = \{\phi_\rho, \Phi_\rho(1), \Phi_\rho(2), \Phi_\rho(4)\}$ is a soft primal on $\mathcal{A}$ with parameters $\rho$. Thus we have $\Lambda(\Phi_\rho(5)) = \Phi_\rho(7)$ and $\Phi^\circ_\rho(5) = \Phi_\rho(3)$. It is clear that $\Lambda(\Phi_\rho) \nsubseteq \Phi^\circ_\rho$.

**Remark 2.6.** In Example 1 in [6], the primal is defined on a universe $X = \{x_1, x_2, x_3, x_4\}$ as $\mathcal{P} = \{\phi, \{x_3\}, \{x_4\}\}$. This primal is incorrect, since for example $\{x_3\} = \{x_1, x_3\} \cap \{x_2, x_3\} \in \mathcal{P}$ but neither $\{x_1, x_3\}$ nor $\{x_2, x_3\} \in \mathcal{P}$. Now, it is corrected as below:

**Example 2.7.** Suppose that $(X, \mathfrak{g}, \mathcal{P})$ is a GPT space, where $X = \{x_1, x_2, x_3, x_4\}$, $\mathfrak{g} = \{\phi, \{x_1\}, \{x_2\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}\{x_1, x_3\}, \{x_1, x_2, x_3\}, X\}$ and $\mathcal{P} = 2^X \backslash \{X\}$. Consider $E = \{x_1, x_2, x_4\}$. Then $E$ is $(\mathfrak{g}, \mathcal{P})$-$\beta$-open.

**Remark 2.8.** In Example 2 in [6], the primal is defined on a universe $X = \{x_1, x_2, x_3, x_4\}$ as $\mathcal{P} = \{\phi, \{x_2\}, \{x_4\}\}$. This primal is incorrect, since for example $\{x_2\} = \{x_1, x_2\} \cap \{x_2, x_3\} \in \mathcal{P}$ but neither $\{x_1, x_2\}$ nor $\{x_2, x_3\} \in \mathcal{P}$. Now, it is corrected as below:

**Example 2.9.** Suppose that $(X, \mathfrak{g}, \mathcal{P})$ is a GPT space, where $X = \{x_1, x_2, x_3, x_4\}$, $\mathfrak{g} = \{\phi, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}, X\}$ and $\mathcal{P} = 2^X \backslash \{X\}$. Consider $A = \{x_1, x_3, x_4\}$. Then $A$ is $(\mathfrak{g}, \mathcal{P})$-semi-open.

**Remark 2.10.** In Example 3 in [6], the primal is defined on a universe $X = \{x_1, x_2, x_3, x_4\}$ as $\mathcal{P} = \{\phi, \{x_3\}, \{x_4\}\}$. This primal is incorrect, since for example $\{x_4\} = \{x_1, x_4\} \cap \{x_2, x_4\} \in \mathcal{P}$ but neither $\{x_1, x_4\}$ nor $\{x_2, x_4\} \in \mathcal{P}$. Now, it is corrected as below:

**Example 2.11.** Suppose that $(X, \mathfrak{g}, \mathcal{P})$ is a GPT space, where $X = \{x_1, x_2, x_3, x_4\}$, $\mathfrak{g} = \{\phi, \{x_1, x_2, x_3\}\}$ and $\mathcal{P} = 2^X \backslash \{X\}$. Consider $E = \{x_1, x_2\}$. Then $E$ is $(\mathfrak{g}, \mathcal{P})$-pre-open.

**Remark 2.12.** In the same way, the defined primals in Example 7 in [7] and Examples 4, 5, 6, 7, 8, 10, 12, 13 and 14 in [6] are not correct. In all these examples, the defined primal on the given universe doesn't meet the axiom (iii) mentioned in definition 1.1.

**For more explanation:**

1. In Example 7 in [7], the soft primal on the set of real numbers $\mathbb{R}$ is defined as $\mathcal{P}_f$ = all countable soft subsets of the real line $\mathbb{R}$ with a set of parameters $\Delta$. Clearly, $\mathcal{P}_f$ is not a soft primal, since for example $(\{5\}, \Delta) \in \mathcal{P}_f$ (countable soft subset) and $(\{5\}, \Delta) = [(-\infty, 5], \Delta) \cap ([5, \infty), \Delta)]$ but neither $(-\infty, 5]$ nor $[5, \infty) \in \mathcal{P}_f$, because $(-\infty, 5]$ and $[5, \infty)$ are uncountable soft subsets.
2. In [6], Examples 4, 5, 6 depend on Examples 1, 2, 3, respectively, which are incorrect as shown in Remarks 2.6, 2.8, 2.10.
3. In Example 7 in [6], the primal is defined on a universe $X = \{a, b, c, d\}$ as $\mathcal{P} = \{\phi, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$. This primal is incorrect, since for example $\phi = \{b\} \cap \{a, c, d\} \in \mathcal{P}$ but neither $\{b\}$ nor $\{a, c, d\} \in \mathcal{P}$.

(4) In Example 8 in [6], the primal is defined on a universe $X = \{a, b, c, d\}$ as $\mathcal{P} = \{\phi, \{b\}, \{b, d\}\}$. This primal is incorrect, since for example $\{b\} = \{a, b\} \cap \{b, c\} \in \mathcal{P}$ but neither $\{a, b\}$ nor $\{b, c\} \in \mathcal{P}$.
(5) In Example 10 in [6], the primal is defined on a universe $X = \{a, b, c, d\}$ as $\mathcal{P} = \{\phi, \{b\}\}$. This primal is incorrect, since for example $\{b\} = \{b, c\} \cap \{b, d\} \in \mathcal{P}$ but neither $\{b, c\}$ nor $\{b, d\} \in \mathcal{P}$.
(6) In Examples 12 in [6], the primal is defined on a universe $X = \{a, b, c, d\}$ as $\mathcal{P} = \{\phi, \{b, d\}\}$. This primal is incorrect, since for example $\{b, d\} = \{b, c, d\} \cap \{a, b, d\} \in \mathcal{P}$ but neither $\{b, c, d\}$ nor $\{a, b, d\} \in \mathcal{P}$.
(7) In Examples 13 in [6], the primal is defined on a universe $X = \{a, b, c, d\}$ as $\mathcal{P} = \{\phi, \{b, d\}\}$. This primal is incorrect, since for example $\phi = \{a\} \cap \{b\} \in \mathcal{P}$ but neither $\{a\}$ nor $\{b\} \in \mathcal{P}$.
(8) In Examples 14 in [6], the primal is defined on a universe $X = \{a, b, c, d\}$ as $\mathcal{P} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. This primal is incorrect, since for example $\{c, d\} = \{b, c, d\} \cap \{a, c, d\} \in \mathcal{P}$ but neither $\{b, c, d\}$ nor $\{a, c, d\} \in \mathcal{P}$.

Anyone can add correct primals to modify these examples and obtain the required aim of them. Also, it should be noted that there may be other incorrect examples of primal structure that were not included in this paper.

**Here** we propose extra explanations related to all possible primals formed in some finite universes:

All primals on $X = \phi$:
$\mathcal{P}_1 = \phi = 2^X \backslash \{X\}$.

All primals on $X = \{a\}$:
$\mathcal{P}_1 = \phi$,
$\mathcal{P}_2 = \{\phi\} = 2^X \backslash \{X\}$.

All primals on $X = \{a, b\}$:
$\mathcal{P}_1 = \phi$,
$\mathcal{P}_2 = \{\phi, \{a\}\}$,
$\mathcal{P}_3 = \{\phi, \{b\}\}$,
$\mathcal{P}_4 = \{\phi, \{a\}, \{b\}\} = 2^X \backslash \{X\}$.

All primals on $X = \{a, b, c\}$:
$\mathcal{P}_1 = \phi$,
$\mathcal{P}_2 = \{\phi, \{a\}, \{b\}, \{a, b\}\}$,
$\mathcal{P}_3 = \{\phi, \{a\}, \{c\}, \{a, c\}\}$,
$\mathcal{P}_4 = \{\phi, \{b\}, \{c\}, \{b, c\}\}$,
$\mathcal{P}_5 = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$,
$\mathcal{P}_6 = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$,
$\mathcal{P}_7 = \{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$,
$\mathcal{P}_8 = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\} = 2^X \backslash \{X\}$.

All primals on $X = \{a, b, c, d\}$:
$\mathcal{P}_1 = \phi$,
$\mathcal{P}_2 = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$,

$\mathcal{P}_3 = \{\phi, \{a\}, \{b\}, \{d\}, \{a,b\}, \{a,d\}, \{b,d\}, \{a,b,d\}\}$,
$\mathcal{P}_4 = \{\phi, \{a\}, \{c\}, \{d\}, \{a,c\}, \{a,d\}, \{c,d\}, \{a,c,d\}\}$,
$\mathcal{P}_5 = \{\phi, \{b\}, \{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{b,c,d\}\}$,
$\mathcal{P}_6 = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{b,c,d\}\}$,
$\mathcal{P}_7 = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,d\}, \{b,c,d\}\}$,
$\mathcal{P}_8 = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}\}$,
$\mathcal{P}_9 = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}\}$,
$\mathcal{P}_{10} = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}\}$,
$\mathcal{P}_{11} = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$,
$\mathcal{P}_{12} = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$,
$\mathcal{P}_{13} = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}\}$,
$\mathcal{P}_{14} = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$,
$\mathcal{P}_{15} = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$,
$\mathcal{P}_{16} = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$
$= 2^X \setminus \{X\}$.

## 3. Conclusion

In this note, we have mentioned some alleged examples of primals in primal topological spaces given [3, 4, 5, 6, 7]. In addition, we suggested certain changes to them. Furthermore, we explained why theese primals are incorrect. Finally, we provided all possible primals that can be created in some finite universes.

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## References

[1] H. Al-Saadi and H. Al-Malki, Correction: Generalized primal topological spaces, AIMS Mathematics 9 (7) (2024) 19068–19069.
[2] H. Al-Saadi and H. Al-Malki, Generalized primal topological spaces, AIMS Mathematics 8 (10) (2023) 24162–24175.
[3] Al-Omari and M. H. Alqahtani, Primal structure with closure operators and their applications, Mathematics 11 (24) (2023) 13 pages.
[4] O. Alghamdi, Al-Omari and M. H. Alqahtani, Novel operators in the frame of primal topological spaces, AIMS Mathematics 9 (9) (2024) 25792–25808.
[5] Al-Omari and M. H. Alqahtani, Some operators in soft primal spaces, AIMS Mathematics 9 (5) (2024) 10756–10774.
[6] H. Al-Saadi and H. Al-Malki, Categories of open sets in generalized primal topological spaces, Mathematics 12 (2) (2024) 14 pages.
[7] T. M. Al-shami, Z. A. Ameen, R. Abu-Gdairi and A. Mhemdi, On primal soft topology, Mathematics 11 (10) (2023) 15 pages.
[8] S. Acharjee, M. Özkoç and F. Y. Issaka, Primal topological spaces arXiv (2022) 9 pages.

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