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G. Thangaraj, D. Vijayan

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Abstract. In this paper, a new type of resolvablity of fuzzy topological spaces, namely fuzzy extraresolvability is introduced by means of fuzzy nowhere denseness and fuzzy denseness of fuzzy sets. It is obtained that the existence of fuzzy σ-nowhere dense sets, fuzzy dense and fuzzy G_δ-sets in fuzzy extraresolvable spaces ensures their fuzzy resolvability. It is obtained that fuzzy Baire spaces, fuzzy DG_δ-spaces, fuzzy weakly Baire spaces become fuzzy resolvable spaces when they possesses fuzzy extraresolvability.

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Keywords: Fuzzy first category set, Fuzzy residual set, Fuzzy G_δ-set, Fuzzy resolvable space, Fuzzy Baire space, Fuzzy Brown space, Fuzzy DG_δ-space, Fuzzy hyperconnected space, Fuzzy almost P-space.

Corresponding Author: G. Thangaraj (g.thangaraj@rediffmail.com)

1. Introduction

The application of mathematical concepts in the real world drives our modern world and is closely intertwined with various fields from engineering and finance to astronomy and medicine. From the elegant dance of celestial bodies to the meticulous precision of medical treatments, mathematics offers a universal language for deciphering complex relationships and opening up new possibilities. In 1965, the concept of fuzzy set as an innovative approach for modelling uncertainties was introduced by Zadeh [1]. The term “fuzzy” often refers to things that are not clear-cut or well-defined. In various contexts it might describe something that is ambiguous or has blurred boundaries. For example, in fuzzy logic, it represents reasoning that is approximate rather than fixed and exact. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. In 1968, Chang [2] introduced the concept of fuzzy topological space. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Recently, a considerable amount of research has been done on various types of fuzzy topological spaces.

At the beginning of twentieth century, the problem of resolvability of topological spaces became a matter of intense research. Research in this area stems from the papers of Hewitt [3] and Katetov [4]. In 1969, El'Kin [5] introduced open hereditarily irresolvable spaces in classical topology. Motivated by their works on resolvability, the concepts of resolvability, irresolvability and open hereditarily irresolvability of fuzzy topological spaces were introduced and studied by Thangaraj and Balasubramanian [6]. In classical topology, the concept of extraresolvability of topological spaces was introduced by Malykhin [7] as a generalization of ω-resolvability. "Extraresolvability"is a less common term, but it suggests something that can be resolved or clarified beyond the usual level of resolution. It might imply a deeper understanding or additional layers of detail that can be uncovered. The term "space"can refer to physical dimensions, abstract concepts, or contexts in which things exist. In physics, it's the boundless three-dimensional continuum in which objects and events occur. In a more abstract sense, it might refer to conceptual or informational spaces. Ferreira et.al [8, 9], carried out a detailed study on extraresolvable spaces. Motivated on these lines, the notion of extraresolvability of fuzzy topological spaces is introduced in this paper. In Section 3, several characterizations of fuzzy extraresolvable spaces are established. In Section 4, inter-relations between fuzzy extraresolvable spaces and other fuzzy topological spaces such as fuzzy D-Baire spaces, fuzzy P-spaces and fuzzy hyperconnected spaces are investigated in this paper. The conditions under which fuzzy extraresolvable spaces become fuzzy resolvable spaces are also established in this section.

In recent years, the topological space theory has been embedding in the soft set theory to obtain some interesting applications [10, 11, 12]. Many authors redefined the classical topological concepts via soft topological structure. Recently, Şenel et al. [13] applied the concept of octahedron sets proposed by Lee et al. [14] to multi-criteria group decision making problems. Also, Şenel et al. [15] provided an insight into a cubic crisp sets and their applications to topology. On these lines, there is a need and scope of investigation considering different types of fuzzy resolvability, for applying some fuzzy topological concepts to information science and decision-making problems.

2. Preliminaries

Some basic notions and results used in the sequel, are given in order to make the exposition self-contained. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval $[0, 1]$. A *fuzzy set* λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$ for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$ for all $x \in X$. For any fuzzy set λ in X and a family $(\lambda_i)_{i\in J}$ of fuzzy sets in X, the *complement* $\lambda^{'}$, the *union* $\bigvee_{i\in J} \lambda_i$ and the *intersection* $\bigwedge_{i\in J} \lambda_i$ are defined respectively as follows: for each $x \in X$,

$$\lambda^{'}(x) = 1 - \lambda(x),\ (\bigvee_{i\in J} \lambda_i)(x) = sup_{i\in J}\lambda_i(x),\ (\bigwedge_{i\in J} \lambda_i)(x) = inf_{i\in J}\lambda_i(x),$$

where J is an index set.

Definition 2.1 ([2])**.** A *fuzzy topology* on a set X is a family T of fuzzy sets in X which satisfies the following conditions :

(i) $0_X \in T$ and $1_X \in T$,
ii) if $A,\ B \in T$, then $A \wedge B \in T$,
(iii) if $A_i \in T$ for each $i \in J$, then $\bigvee_{i \in J} A_i \in T$.

The pair (X, T) is called a *fuzzy topological space*. Members of T are called *fuzzy open sets* in X and their complements are called *fuzzy closed sets* in X.

Definition 2.2 ([2])**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The *interior* and the *closure* of λ are defined respectively as follows:

(i) $int(\lambda) = \bigvee\{\mu \in T : \mu \leq \lambda\}$,
(ii) $cl(\lambda) = \bigwedge\{\mu^{'} : \lambda \leq \mu^{'},\ \mu \in T\}$.

Definition 2.3. A fuzzy set λ in a fuzzy topological space (X, T) is called a

(i) *fuzzy regular-open* [resp. *closed*] *set* in X, if $\lambda = intcl(\lambda)$ [resp. $\lambda = clint(\lambda)$] [16],
(ii) *fuzzy* G_δ [resp. F_σ]*-set* in X, if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$ [resp. $\lambda = \bigvee_{i=1}^{\infty} \lambda_i^{'}$], where $\lambda_i \in T$ for $i \in J$ [17],
(iii) *fuzzy dense set* in X, if there exists no fuzzy closed set μ in X such that $\lambda < \mu < 1$, i.e., $cl(\lambda) = 1$ [18],
(iv) *fuzzy nowhere dense set* in X, if there exists no non-zero fuzzy open set μ in X such that $\mu < cl(\lambda)$, i.e., $intcl(\lambda) = 0$ [18],
(v) *fuzzy first category set* in X, if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$, where each λ_i is a fuzzy nowhere dense set in X. Any other fuzzy set in X is said to be of *fuzzy second category* [18],
(vi) *fuzzy residual set* in X, if $\lambda^{'}$ is a fuzzy first category set in X [19],
(vii) *fuzzy somewhere dense set* in X, if there exists a non-zero fuzzy open set μ in X such that $\mu < cl(\lambda)$, i.e., $intcl(\lambda) \neq 0$ [20],
(viii) *fuzzy Baire set* in X, if $\lambda = \mu \wedge \eta$, where $\mu \in T$ and η is a fuzzy residual set in X [21],
(ix) *fuzzy σ-nowhere dense set* in X, if λ is a fuzzy F_σ-set with $int(\lambda) = 0$ [22],
(x) *fuzzy regular σ-nowhere dense set* in X, if $\lambda = \bigvee_{i=1}^{\infty} cl(\lambda_i)$, where $int(\lambda_i) = 0$ [23],
(xi) *fuzzy σ-boundary set* in X if $\mu = \bigvee_{i=1}^{\infty} \mu_i$, where $\mu_i = cl(\lambda_i) \wedge \lambda_i^{'}$ and each λ_i is a fuzzy regular open set in X [24].

Definition 2.4. A fuzzy topological space (X, T) is called a

(i) *fuzzy submaximal space*, if for each fuzzy set λ in X such that $cl(\lambda) = 1,\ \lambda \in T$ [17],
(ii) *fuzzy hyperconnected space*, if every non null fuzzy open subset of (X, T) is fuzzy dense in X [25],
(iii) *fuzzy open hereditarily irresolvable space*, if $intcl(\lambda) \neq 0$ for any non-zero fuzzy set λ in X implies $int(\lambda) \neq 0$ [6],
(iv) *fuzzy P-space*, if each fuzzy G_δ-set in X is fuzzy open in X [26],

(v) *fuzzy almost P-space* , if for each non- zero fuzzy G_δ-set λ in X, $int(\lambda) \neq 0$ [27],
(vi) *fuzzy Baire space*, if $int(\bigvee_{i=1}^{\infty} \lambda_i) = 0$, where each λ_i is a fuzzy nowhere dense set in X [19],
(vii) *fuzzy D-Baire space*, if every fuzzy first category set in X is a fuzzy nowhere dense set in X [28],
(viii) *fuzzy DG_δ-space*, if each fuzzy dense (but not fuzzy open) set in X is a fuzzy G_δ-set in X [29],
(ix) *fuzzy nodef space*, if each fuzzy nowhere dense set is a fuzzy F_σ-set in X [29],
(x) *fuzzy resolvable space*, if there exists a fuzzy dense set λ in X such that $cl(1-\lambda) = 1$. Otherwise, (X, T) is called a *fuzzy irresolvable space* [6],
(xi) *fuzzy Baire-dominated space*, if for each collection $\lambda_i (i = 1\ to\ \infty)$ of fuzzy closed sets with $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 ...$ and $\bigwedge_{i=1}^{\infty} \lambda_i = 0$, there exists a collection $\lambda_i (i = 1\ to\ \infty)$ of fuzzy Baire sets in X with $\lambda_i \leq \mu_i$ for each i and $\bigwedge_{i=1}^{\infty} \mu_i = 0$ [30],
(xii) *fuzzy weakly Baire space*, if $int(\bigvee_{i=1}^{\infty} \lambda_i) = 0$, where $\mu_i = cl(\lambda_i) \wedge \lambda_i^{'}$ and each λ_i is a fuzzy regular open set in X [24].

Lemma 2.5 ([16]). *For a fuzzy set λ of a fuzzy topological space X,*
(1) $(int(\lambda))^{'} = cl(\lambda^{'})$,
(2) $(cl(\lambda))^{'} = int(\lambda^{'})$.

Theorem 2.6 ([16]). *In a fuzzy topological space,*
(1) *the closure of a fuzzy open set is a fuzzy regular closed set,*
(2) *the interior of a fuzzy closed set is a fuzzy regular open set.*

Theorem 2.7 ([6]). *If a fuzzy topological space (X, T) is a fuzzy open hereditarily irresolvable space, then $cl(\lambda) = 1$ for any non-zero fuzzy set λ in X implies that $clint(\lambda) = 1$.*

Theorem 2.8 ([19]). *Let (X, T) be a fuzzy topological space. Then the following are equivalent:*
(1) *X is a fuzzy Baire space,*
(2) *$Int(\lambda) = 0$ for every fuzzy first category set λ in X,*
(3) *$Cl(\mu) = 1$ for every fuzzy residual set μ in X.*

Theorem 2.9 ([22]). *If λ is a fuzzy σ-nowhere dense set in a fuzzy topological space X, then $\lambda^{'}$ is a fuzzy residual set in X.*

Theorem 2.10 ([29]). *If λ is a fuzzy dense and fuzzy open set in a fuzzy nodef space X, then λ is a fuzzy residual set in X.*

Theorem 2.11 ([31]). *If a fuzzy topological space X is a fuzzy Brown space, then X is a fuzzy open hereditarily irresolvable space.*

Theorem 2.12 ([22]). *If λ is a fuzzy σ-nowhere dense set in a fuzzy topological space X, then λ is a fuzzy first category set in X.*

Theorem 2.13 ([23]). *If $int(\lambda) = 0$, where λ is a fuzzy regular σ-nowhere dense set in a fuzzy topological space X, then λ is a fuzzy first category set in X.*

Theorem 2.14 ([24])**.** *If λ is a fuzzy σ-boundary set in a fuzzy weakly Baire space X, then λ is a fuzzy σ-nowhere dense set in X.*

Theorem 2.15 ([24])**.** *Let (X,T) be a fuzzy topological space. Then the following are equivalent:*

(1) *X is a fuzzy weakly Baire space,*
(2) *$int(\lambda)=0$ for each fuzzy σ-boundary set λ in X,*
(3) *$cl(\gamma)=1$ for each fuzzy co-σ-boundary set γ in X.*

3. Fuzzy extraresolvable spaces

Motivated by the works of Malykhin [7] on extraresolvability of topological spaces in classical topology, the concept of extraresolvability of fuzzy topological spaces is introduced as follows :

Definition 3.1. A fuzzy topological space (X,T) is called a *fuzzy extraresolvable space*, if there exists a family $(\lambda_i)_{i\in J}$ of fuzzy dense sets in X such that $\lambda_i\wedge\lambda_j$ is a fuzzy nowhere dense set in (X,T), whenever $\lambda_i\neq\lambda_j$ for any $i,\ j\in J$.

Example 3.2. Let $X=\{a,b,c\}$. Consider the fuzzy sets $\alpha,\ \beta,\ \gamma$ and δ in X defined as follows:

$$\alpha(a)=0.8,\ \alpha(b)=0.2,\ \alpha(c)=0.4,\ \beta(a)=0.3,\ \beta(b)=0.5,\ \beta(c)=0.4,$$

$$\gamma(a)=0,\ \gamma(b)=0.9,\ \gamma(c)=0.5,\ \delta(a)=0.2,\ \delta(b)=0.5,\ \delta(c)=0.7.$$

Then $T=\{0,\alpha,\beta,\alpha\vee\beta,\alpha\wedge\beta,1\}$ is a fuzzy topology on X. By computation, we obtain the followings:

$cl(\alpha)=1,\ cl(\beta)=1-\beta,\ cl(\alpha\vee\beta)=1,\ cl(\alpha\wedge\beta)=1-\beta,$
$cl(\gamma)=1,\ cl(1-\gamma)=1,\ cl(\delta)=1,\ cl(1-\delta)=1$

and

$int(1-\alpha)=0,\ int(1-\beta)=\beta,\ int(1-[\alpha\vee\beta])=0,\ int(1-[\alpha\wedge\beta])=\beta.$

Now $\mathscr{D}=\{\alpha,\beta,\gamma\}$ is a family of fuzzy dense sets in X. Also, by computation, one can find that

$$intcl(\alpha\wedge\beta)=int(1-[\alpha\vee\beta])=0,$$
$$intcl(\alpha\wedge\gamma)=int(1-[\alpha\vee\beta])=0.$$
$$intcl(\beta\wedge\gamma)=int(1-[\alpha\vee\beta])=0.$$

Then for $\alpha,\beta,\gamma\in\mathscr{D},\alpha\wedge\beta,\alpha\wedge\gamma$ and $\beta\wedge\gamma$ are fuzzy nowhere dense sets in X. Thus (X,T) is a fuzzy extraresolvable space.

Example 3.3. Let $X=\{a,b,c\}$. Consider the fuzzy sets $\alpha,\ \beta,\ \gamma,\ \delta$ and η in X defined as follows:

$$\alpha(a)=0.8,\ \alpha(b)=0.6,\ \alpha(c)=0.7,\ \beta(a)=0.6,\ \beta(b)=0.9,\ \beta(c)=0.8,$$

$$\gamma(a)=0.7,\ \gamma(b)=0.5,\ \gamma(c)=0.9,\ \delta(a)=0.3,\ \delta(b)=0.4,\ \delta(c)=0.5,$$

$$\eta(a)=0.2,\ \eta(b)=0.5,\ \eta(c)=0.4.$$

Then $T=\{0,\alpha,\beta,\gamma,\alpha\vee\beta,\alpha\vee\gamma,\beta\vee\gamma,\alpha\wedge\beta,\alpha\wedge\gamma,\beta\wedge\gamma,\alpha\vee[\beta\wedge\gamma],\beta\vee[\alpha\wedge\gamma],$
$\gamma\vee[\alpha\wedge\beta],\alpha\wedge[\beta\vee\gamma],\beta\wedge[\alpha\vee\gamma],\gamma\wedge[\alpha\vee\beta],\alpha\vee\beta\vee\gamma,\alpha\wedge\beta\wedge\gamma,1\}$

is a fuzzy topology on X. By computation, one can find that all the fuzzy open sets are fuzzy dense in X and $cl(\delta)=1,\ cl(1-\delta)=1,\ cl(\eta)=1,\ cl(1-\eta)=1.$

Now $\mathscr{D} = \{\alpha, \delta, \eta\}$ constitutes a family of fuzzy dense sets in X and for $\alpha,\ \delta \in \mathscr{D}$, $intcl(\alpha \wedge \delta) = intcl(\delta) = int(1) = 1 \neq 0$. Then $\alpha \wedge \delta$ is not a fuzzy nowhere dense set in X. Also, $\mathscr{D}_1 = \{\alpha, 1-\delta, 1-\eta\}$ constitutes another family of fuzzy dense sets in X and for $\alpha,\ 1-\delta \in \mathscr{D}$, $intcl(\alpha \wedge [1-\delta]) = intcl(1-\delta) = 1 \neq 0$. Thus $\alpha \wedge [1-\delta]$ is not a fuzzy nowhere dense set in X. On the other hand, $\mathscr{D}_2 = \{\alpha, \delta, 1-\eta\}$ constitutes another family of fuzzy dense sets in X and for $\alpha,\ 1-\eta \in \mathscr{D}$, $intcl(\alpha \wedge [1-\eta]) = intcl(1-\eta) = 1 \neq 0$. So $\alpha \wedge [1-\eta]$ is not a fuzzy nowhere dense set in X. Also, $\mathscr{D}_3 = \{1-\delta, 1-\eta\}$ constitutes another family of fuzzy dense sets in X and for $1-\delta,\ 1-\eta \in \mathscr{D}$, $intcl((1-\delta) \wedge [1-\eta]) = int(1) = 1 \neq 0$. Hence $(1-\delta) \wedge (1-\eta)$ is not a fuzzy nowhere dense set in X. Therefore (X, T) is not a fuzzy extraresolvable space.

Proposition 3.4. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then $int(\lambda_i) \leq cl(1-\lambda_j)$, whenever $\lambda_i \neq \lambda_j$ for any $i,\ j \in J$, where $\mathscr{D} = (\lambda_i)_{i \in J}$ is a family of fuzzy dense sets in X.*

Proof. Suppose X is a fuzzy extraresolvable space. Then there exists a family $\mathscr{D} = (\lambda_i)_{i \in J}$ of fuzzy dense sets in X such that $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in X, where $\lambda_i \neq \lambda_j$. Thus $intcl(\lambda_i \wedge \lambda_j) = 0$. Since $int(\lambda_i \wedge \lambda_j) \leq intcl(\lambda_i \wedge \lambda_j)$, $int(\lambda_i \wedge \lambda_j) = 0$. So $int(\lambda_i) \wedge int(\lambda_j) = 0$ and $int(\lambda_i) \leq 1 - int(\lambda_j)$. By Lemma 2.5, $1 - int(\lambda_j) = cl(1-\lambda_j)$. Hence $int(\lambda_i) \leq cl(1-\lambda_j)$, where $\lambda_i,\ \lambda_j \in \mathscr{D}$. □

Proposition 3.5. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then there exists a family $\mathscr{D} = (\lambda_i)_{i \in J}$ of fuzzy dense sets in X in which only one fuzzy dense set is fuzzy open in X.*

Proof. Suppose X is a fuzzy extraresolvable space. Then there exists a family $\mathscr{D} = (\lambda_i)_{i \in J}$ of fuzzy dense sets in X such that $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in X, whenever $\lambda_i \neq \lambda_j$ for any $\lambda_i,\ \lambda_j \in \mathscr{D}$. Assume that $\lambda_i,\ \lambda_j \in \mathscr{D}$ are fuzzy open sets in X. Then $\lambda_i \wedge \lambda_j \in T$. Thus $\lambda_i \wedge \lambda_j \leq intcl(\lambda_i \wedge \lambda_j)$. So $intcl(\lambda_i \wedge \lambda_j) \neq 0$. Since $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in X, $intcl(\lambda_i \wedge \lambda_j) = 0$. This is a contradiction. So it follows that there exists only one fuzzy dense set in $\mathscr{D}$ which is fuzzy open in X. □

Proposition 3.6. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then there exists a family $\mathscr{D} = (\lambda_i)_{i \in J}$ of fuzzy dense sets in (X, T) and $clint(\lambda_j) \neq 1$, whenever $\lambda_j \notin T$ for each $j \in J$.*

Proof. Suppose X) is a fuzzy extraresolvable space. Then, there exists a family $\mathscr{D} = (\lambda_i)_{i \in J}$ of fuzzy dense sets in X such that $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in X, whenever $\lambda_i \neq \lambda_j$ for any $i,\ j \in J$. By Proposition 3.4, $int(\lambda_i) \leq cl(1-\lambda_j)$. Thus $int[int(\lambda_i)] \leq int[cl(1-\lambda_j)]$. So $int(\lambda_i) \leq intcl(1-\lambda_j)$ and $intcl(1-\lambda_j) \neq 0$. By Lemma 2.5, it follows that $1 - clint(\lambda_j) \neq 0$, i.e., $clint(\lambda_j) \neq 1$. Assume that $\lambda_j \in T$. Then $cl(\lambda_j) = clint(\lambda_j) \neq 1$, in (X, T) which is a contradiction, since $\lambda_j \in \mathscr{D}$. Thus $\lambda_j \notin T$. □

Proposition 3.7. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then there exists a family $\mathscr{D} = (\lambda_i)_{i \in J}$ of fuzzy dense sets in X and whenever $\lambda_j \in \mathscr{D}$ and $\lambda_j \notin T$, there exists a fuzzy regular closed set δ in X such that $\delta \leq cl(1-\lambda_j)$.*

Proof. Suppose X is a fuzzy extraresolvable space. Then there exists a family $\mathscr{D} = (\lambda_i)_{i\in J}$ of fuzzy dense sets in X such that $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in X, whenever $\lambda_i \neq \lambda_j$for any $i,\ j \in J$. By Proposition 3.4, $int(\lambda_i) \leq cl(1-\lambda_j)$. Thus $clint(\lambda_i) \leq clcl(1-\lambda_j)$, i.e., $clint(\lambda_i) \leq cl(1-\lambda_j)$. So by Theorem 2.6, $clint(\lambda_i)$ is a fuzzy regular closed set in X, say $\delta = clint(\lambda_i)$. Moreover, $\delta \leq cl(1-\lambda_j)$, whenever $\lambda_j \in \mathscr{D}$ and $\lambda_j \notin T$. □

Remark 3.8. If the fuzzy dense set λ_j is a fuzzy open set in Proposition 3.7, then $1-\lambda_j$ is a fuzzy closed set in X, i.e., $cl(1-\lambda_j) = 1-\lambda_j$. Thus $clint(\lambda_i) \leq cl(1-\lambda_j) = 1-\lambda_j$. So $\lambda_j \leq 1 - clint(\lambda_i)$. Hence $1 = cl(\lambda_j) \leq cl[1-clint(\lambda_i)]$, i.e., $cl[1-clint(\lambda_i)] = 1$ and $intclint(\lambda_i) = 0$. Now $int(\lambda_i) = intint(\lambda_i) \leq intclint(\lambda_i)$ implies that $int(\lambda_i) = 0$ and $\delta = clint(\lambda_i) = 0$.

Proposition 3.9. *If a fuzzy topological space (X,T) is a fuzzy extraresolvable space, then there exists a family $\mathscr{D} = (\lambda_i)_{i\in J}$ of fuzzy dense sets X, whenever $\lambda_j \in \mathscr{D}$ and $\lambda_j \notin T$, $1-\lambda_j$ is a fuzzy somewhere dense set in X.*

Proof. Suppose (X,T) is a fuzzy extraresolvable space. Then there exists a family $\mathscr{D} = (\lambda_i)_{i\in J}$ of fuzzy dense sets in X such that $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in X, whenever $\lambda_i \neq \lambda_j$ for any $i,\ j \in J$. Let $\lambda_j \in \mathscr{D}$ and $\lambda_j \notin T$. Thus by Proposition 3.6, $clint(\lambda_j) \neq 1$, i.e., $1-clint(\lambda_j) \neq 0$. By Lemma 2.5, $intcl(1-\lambda_j) \neq 0$. So $1-\lambda_j$ is a fuzzy somewhere dense set in X. □

Proposition 3.10. *If a fuzzy topological space (X,T) is a fuzzy extraresolvable space, then there exists a family $\mathscr{D} = (\lambda_i)_{i\in J}$ of fuzzy dense sets in X and whenever $\lambda_i,\ \lambda_j \in \mathscr{D}$ $(\lambda_i \neq \lambda_j)$, $[(1-\lambda_i)\vee(1-\lambda_j)]$ is a fuzzy dense set in X, where $int(1-\lambda_i) = 0$ and $int(1-\lambda_j) = 0$.*

Proof. Suppose X is a fuzzy extraresolvable space. Then there exists a family $\mathscr{D} = (\lambda_i)_{i\in J}$ of fuzzy dense sets in X such that $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in X, whenever $\lambda_i \neq \lambda_j$ for any $i,\ j \in J$. Thus $intcl(\lambda_i \wedge \lambda_j) = 0$. Since $int(\lambda_i \wedge \lambda_j) \leq intcl(\lambda_i \wedge \lambda_j)$, $int(\lambda_i \wedge \lambda_j) = 0$, i.e., $1 - int(\lambda_i \wedge \lambda_j) = 1$. By Lemma 2.5, $cl[1-(\lambda_i \wedge \lambda_j)] = 1$, i.e., $cl[(1-\lambda_i)\vee(1-\lambda_j)] = 1$. So $[(1-\lambda_i)\vee(1-\lambda_j)]$ is a fuzzy dense set in X. Since $\lambda_i,\ \lambda_j \in \mathscr{D}$, $cl(\lambda_i) = 1,\ cl(\lambda_j) = 1$. Hence we have

$$int(1-\lambda_i) = 1 - cl(\lambda_i) = 1-1 = 0,\ int(1-\lambda_j) = 1-cl(\lambda_j) = 1-1 = 0.$$

□

Proposition 3.11. *If each λ_β $(\beta = 1\ \ to\ \ \infty)$ is a fuzzy dense set in a fuzzy extraresolvable space (X,T), then $cl[\bigvee_{\beta=1}^{\infty}(1-\lambda_\beta)] = 1$.*

Proof. Let X be a fuzzy extraresolvable space and suppose each λ_β $(\beta = 1\ \ to\ \ \infty)$ is a fuzzy dense set in X. Then there exists a family $\mathscr{D} = (\lambda_i)_{i\in J}$ of fuzzy dense sets in X such that $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in X, whenever $\lambda_i \neq \lambda_j$ for any $i,\ j \in J$. By Proposition 3.10, $[(1-\lambda_i)\vee(1-\lambda_j)]$ is a fuzzy dense set in X, where $int(1-\lambda_i) = 0$ and $int(1-\lambda_j) = 0$. Let $\{\lambda_\beta\}$ $(\beta = 1\ \ to\ \ \infty)$ be a family of fuzzy dense sets in X including $\mathscr{D}$. Then $cl(\lambda_\beta) = 1$, i.e., $1 - cl(\lambda_\beta) = 0$. By Lemma 2.5, $int(1-\lambda_\beta) = 0$. On the other hand, $[(1-\lambda_i)\vee(1-\lambda_j)] \leq \bigvee_{\beta=1}^{\infty}(1-\lambda_\beta)$. Thus $cl[(1-\lambda_i)\vee(1-\lambda_j)] \leq cl[\bigvee_{\beta=1}^{\infty}(1-\lambda_\beta)]$. So $1 \leq cl[\bigvee_{\beta=1}^{\infty}(1-\lambda_\beta)]$, i.e., $cl[\bigvee_{\beta=1}^{\infty}(1-\lambda_\beta)] = 1$. □

Corollary 3.12. *If each λ_β ($\beta = 1$ to ∞) is a fuzzy dense set in a fuzzy extraresolvable space (X,T), then $int(\bigwedge_{\beta=1}^{\infty} \lambda_\beta) = 0$.*

Proposition 3.13. *If each μ_β ($\beta = 1$ to ∞) is a fuzzy nowhere dense sets in a fuzzy extraresolvable space (X,T), then $cl(\bigvee_{\beta=1}^{\infty} \mu_\beta) = 1$.*

Proof. Let X be fuzzy extraresolvable space and suppose each μ_β ($\beta = 1$ to ∞) is a fuzzy nowhere dense sets in X. Then $intcl(\mu_\beta) = 0$ and $int(\mu_\beta) \leq intcl(\mu_\beta)$. Thus $int(\mu_\beta) = 0$. So $cl(1-\mu_\beta) = 1 - int(\mu_\beta) = 1 - 0 = 1$. Hence $1 - \mu_\beta$ is a fuzzy dense set in X. By Proposition 3.11, $cl[\bigvee_{\beta=1}^{\infty}(1 - \{1-\mu_\beta\})] = 1$, i.e., $cl(\bigvee_{\beta=1}^{\infty} \mu_\beta) = 1$. □

If (X,T) is a fuzzy extraresolvable space, then there exists a family $\mathscr{D} = (\lambda_i)_{i\in J}$ of fuzzy dense sets in X such that $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in X, whenever $\lambda_i \neq \lambda_j$ for any $i,\ j \in J$. Let $\delta_{ij} = \lambda_i \wedge \lambda_j$. Then $\{\delta_{ij}\}$ $(i \neq j)$ is a family of fuzzy nowhere dense sets in X and $\delta = \bigvee_{i,j\in J, i\neq j} \delta_{ij}$ is a fuzzy first category set in X. This establishes the existence of fuzzy first category sets in fuzzy extraresolvable spaces.

The following proposition shows that fuzzy first category sets in fuzzy extraresolvable spaces are fuzzy dense sets.

Proposition 3.14. *If λ is a fuzzy first category set in a fuzzy extraresolvable space (X,T), then λ is a fuzzy dense set in X.*

Proof. Suppose λ is a fuzzy first category set in X. Then $\lambda = \bigvee_{k=1}^{\infty} \lambda_k$, where each λ_k is a fuzzy nowhere dense set in X. Since $X$is a fuzzy extraresolvable space, by Proposition 3.13, $cl(\bigvee_{k=1}^{\infty} \lambda_k) = 1$. Thus $cl(\lambda) = 1$. Soλ is a fuzzy dense set in X. □

The following proposition shows that fuzzy residual sets are having zero interior in fuzzy extraresolvable spaces.

Proposition 3.15. *If λ is a fuzzy residual set in a fuzzy extraresolvable space (X,T), then $int(\lambda) = 0$.*

Proof. Suppose λ is a fuzzy residual set in X. Then, $1-\lambda$ is a fuzzy first category set in X. Since X is a fuzzy extraresolvable space, by Proposition 3.14, $1 - \lambda$ is a fuzzy dense set in (X,T) and $cl(1-\lambda) = 1$. By Lemma 2.5, $1 - int(\lambda) = cl(1-\lambda) = 1$. So $int(\lambda) = 0$. □

Proposition 3.16. *If λ is a fuzzy first category set in a fuzzy extraresolvable space (X,T), then there exists a fuzzy G_δ-set μ in X such that $\mu \leq 1-\lambda$ and $int(\mu) = 0$.*

Proof. Suppose λ is a fuzzy first category set in X. Then $\lambda = \bigvee_{k=1}^{\infty} \lambda_k$, where each λ_k is a fuzzy nowhere dense sets in X. Thus $intcl(\lambda_k) = 0$ for each k. So $cl(\lambda_k) \neq 1$. [For, if $cl(\lambda_k) = 1$, then $intcl(\lambda_k) = 1$, a contradiction]. Hence $1 - cl(\lambda_k)$ is a non-zero fuzzy open set in X. Let $\mu = \bigwedge_{k=1}^{\infty}(1 - cl(\lambda_k))$. Then μ is a fuzzy G_δ-set in X and $\mu \leq \bigwedge_{k=1}^{\infty}(1-\lambda_k) = 1 - \bigvee_{k=1}^{\infty}(\lambda_k) = 1 - \lambda$. Thus $\mu \leq 1 - \lambda$. Thus by Proposition 3.14, $cl(\lambda) = 1$ and $int(\mu) \leq int(1-\lambda) = 1 - cl(\lambda) = 1 - 1 = 0$. So $int(\mu) = 0$. □

Proposition 3.17. *If λ is a fuzzy first category set in a fuzzy extraresolvable space (X,T), then there exists a fuzzy F_σ-set γ such that $\lambda \leq \gamma$ and $cl(\gamma) = 1$.*

Proof. Suppose λ is a fuzzy first category set in X. Since X is a fuzzy extraresolvable space, by Proposition 3.16, there exists a fuzzy G_δ-set μ in X such that $\mu \leq 1 - \lambda$ and $int(\mu) = 0$. Thus $\lambda \leq 1 - \mu$. Let $\gamma = 1 - \mu$. Then clearly, γ is a fuzzy F_σ-set in X such that $\lambda \leq \gamma$. Thus $cl(\lambda) \leq cl(\gamma)$. By Proposition 3.14, $cl(\lambda) = 1$. So $1 \leq cl(\gamma)$, i.e., $cl(\gamma) = 1$. Hence there exists a fuzzy F_σ-set γ such that $\lambda \leq \gamma$ and $cl(\gamma) = 1$. □

Proposition 3.18. *If α is a fuzzy residual set in a fuzzy extraresolvable space (X, T), then there exists a fuzzy G_δ-set η such that $\eta \leq \alpha$ and $int(\eta) = 0$.*

Proof. Suppose α is a fuzzy residual set in X. Then $1 - \alpha$ is a fuzzy first category set in X. Since X is a fuzzy extraresolvable space, by Proposition 3.17, there exists a fuzzy F_σ-set γ such that $1 - \alpha \leq \gamma$ and $cl(\gamma) = 1$. Thus $1 - \gamma \leq \alpha$ and $int(1 - \gamma) = 1 - cl(\gamma) = 1 - 1 = 0$, say $\eta = 1 - \gamma$. So there exists a fuzzy G_δ-set η such that $\eta \leq \alpha$ and $int(\eta) = 0$. □

The following proposition shows that fuzzy σ-nowhere dense sets in fuzzy extraresolvable spaces are fuzzy dense sets.

Proposition 3.19. *If λ is a fuzzy σ-nowhere dense set in a fuzzy extraresolvable space (X, T), then λ is a fuzzy dense set in X.*

Proof. Suppose λ is a fuzzy σ-nowhere dense set in X). Then by Theorem 2.12, λ is a fuzzy first category set in X. Since X is a fuzzy extraresolvable space, by Proposition 3.13, λ is a fuzzy dense set in X. □

Proposition 3.20. *If $int(\lambda) = 0$, where λ is a fuzzy regular σ-nowhere dense set in a fuzzy extraresolvable space (X, T), then λ is a fuzzy dense and fuzzy F_σ-set in X.*

Proof. Suppose λ is a fuzzy regular σ-nowhere dense set in X. Then $\lambda = \bigvee_{i=1}^{\infty} cl(\lambda_i)$, where $int(\lambda_i) = 0$. Thus λ is a fuzzy F_σ-set in X. By hypothesis, $int(\lambda) = 0$. So by Theorem 2.13, λ is a fuzzy first category set in X. Since X is a fuzzy extraresolvable space, by Proposition 3.14, λ is a fuzzy dense set in X. Hence λ is a fuzzy dense and fuzzy F_σ-set in X. □

4. Fuzzy extraresolvable spaces and other fuzzy topological spaces

Proposition 4.1. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then X is not a fuzzy submaximal space.*

Proof. Suppose X be a fuzzy extraresolvable space. Then there exists a family $\mathscr{D} = (\lambda_\alpha)_{i \in J}$ of fuzzy dense sets in X such that $\lambda_i \wedge \lambda_j$ is a fuzzy nowhere dense set in X for $\lambda_i \neq \lambda_j$. By Proposition 3.5, there exists only one fuzzy open set which is fuzzy dense in X, i.e., if $\lambda_i,\ \lambda_j \in \mathscr{D}$ and $\lambda_i \neq \lambda_j$, then either λ_i is a fuzzy open set or λ_j is a fuzzy open set in X. Thus all the fuzzy dense sets are not fuzzy open sets in X. So X is not a fuzzy submaximal space. □

Proposition 4.2. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then X is not a fuzzy almost P-space.*

Proof. Suppose X is a fuzzy extraresolvable space and let λ be a fuzzy first category set in X. Then by the hypothesis and by Proposition 3.16, there exists a fuzzy G_δ-set μ such that $\mu \leq 1 - \lambda$ and $int(\mu) = 0$. Thus $int(\mu) = 0$. So X is not a fuzzy almost P-space. □

Corollary 4.3. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then X is not a fuzzy P-space. For, in the proof of Proposition 4.2, $int(\mu) = 0 \neq \mu$, for the fuzzy G_δ-set μ in (X, T).*

Proof. In the proof of Proposition 4.2, $int(\mu) = 0 \neq \mu$, for the fuzzy G_δ-set μ in X. □

Proposition 4.4. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then X is not a fuzzy D-Baire space.*

Proof. Suppose X is a fuzzy extraresolvable space and let λ be a fuzzy first category set in X. Then by the hypothesis and by Proposition 3.14, λ is a fuzzy dense set in X. Thus $cl(\lambda) = 1$. So $intcl(\lambda) = int(1) = 1 \neq 0$. Hence λ is not a fuzzy nowhere dense set in X. Therefore X is not a fuzzy D-Baire space. □

Proposition 4.5. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then X is not a fuzzy open hereditarily irresolvable space.*

Proof. Suppose X is a fuzzy extraresolvable space and let X be a fuzzy extraresolvable space. Then by Proposition 3.6, $clint(\lambda_j) \neq 1$ for each $\lambda_j \in \mathscr{D}$, where $\mathscr{D}$ is a family of fuzzy dense sets in X and $\lambda_j \in T$. Thus for any non-zero fuzzy set λ_j in X with $cl(\lambda_j) = 1$, $clint(\lambda_j) \neq 1$. So by Theorem 2.7, X is not a fuzzy open hereditarily irresolvable space. □

Remark 4.6. It is to be noted that fuzzy resolvable spaces need not be fuzzy extraresolvable spaces. For, in example 3.3, (X, T) is not a fuzzy extraresolvable space and $cl(\gamma) = 1$, $cl(1 - \gamma) = 1$, $cl(\delta) = 1$, $cl(1 - \delta) = 1$, shows that (X, T) is a fuzzy resolvable space.

The following example shows that there are fuzzy topological spaces which are both fuzzy resolvable and fuzzy extraresolvable.

Example 4.7. Let $X = \{a, b, c\}$. Consider the fuzzy sets α and β in X defined as follows:

$$\alpha(a) = 0.5,\ \alpha(b) = 0.5,\ \alpha(c) = 0.6,\ \beta(a) = 0.4,\ \beta(b) = 0.8,\ \beta(c) = 0.2.$$

Then $T = \{0, \alpha, 1\}$ is a fuzzy topology on X. Furthermore, β is a fuzzy dense set in (X, T). By computation, one can find that

$$cl(\alpha) = 1,\ cl(\beta) = 1,\ cl(1 - \beta) = 1.$$

Thus by Definition 2.4(x), (X, T) is a fuzzy resolvable space. Now let $\mathscr{D} = \{\beta, 1-\beta\}$ be the family of fuzzy dense sets in (X, T). Then $intcl(\beta \wedge [1 - \beta]) = int(1 - \alpha) = 0 \neq 0$. Thus $\beta \wedge [1 - \beta]$ is a fuzzy nowhere dense set in (X, T). So (X, T) is a fuzzy extraresolvable space.

Remark 4.8. It should be noted in example 4.7 that β is a fuzzy dense set in (X, T) but not a fuzzy open set in (X, T).

The following proposition shows that the existence of fuzzy σ-nowhere dense sets in a fuzzy extraresolvable space implies its fuzzy resolvability.

Proposition 4.9. *If there exists a fuzzy σ-nowhere dense set λ in a fuzzy extraresolvable space (X, T), then X is a fuzzy resolvable space.*

Proof. Suppose λ is a fuzzy σ-nowhere dense set in X. Then by Theorem 2.9, $1 - \lambda$ is a fuzzy residual set in X. Since X is a fuzzy extraresolvable space, by Proposition 3.15, $int(1 - \lambda) = 0$. By Lemma 2.5, $1 - cl(\lambda) = int(1 - \lambda) = 0$, i.e., $cl(\lambda) = 1$. Since λ is a fuzzy σ-nowhere dense set in X, λ is a fuzzy F_σ-set with $int(\lambda) = 0$, i.e., $1 - int(\lambda) = 1$. Thus $cl(1 - \lambda) = 1$. So there exists a fuzzy set λ in X such that $cl(\lambda) = 1$ and $cl(1 - \lambda) = 1$. Hence X is a fuzzy resolvable space. □

The following proposition shows that the existence of fuzzy dense and fuzzy G_δ-sets in a fuzzy extraresolvable space implies its resolvability.

Proposition 4.10. *If there exists a fuzzy dense and fuzzy G_δ-set in a fuzzy extraresolvable space (X, T), then X is a fuzzy resolvable space.*

Proof. Suppose λ is a fuzzy dense and fuzzy G_δ-set in X. Then $1 - \lambda$ is a fuzzy F_σ-set in X. Thus $int(1 - \lambda) = 1 - cl(\lambda) = 1 - 1 = 0$. So $1 - \lambda$ is a fuzzy σ-nowhere dense set in X. Hence by Proposition 4.9, X is a fuzzy resolvable space. □

The following propositions give conditions for fuzzy extraresolvable spaces to become fuzzy resolvable spaces.

Proposition 4.11. *If a fuzzy topological space (X, T) is a fuzzy Baire and fuzzy extraresolvable space, then X is a fuzzy resolvable space.*

Proof. Suppose X is a fuzzy Baire and fuzzy extraresolvable space, and let λ be a fuzzy first category set in X. Since X is a fuzzy extraresolvable space, by Proposition 3.14, $cl(\lambda) = 1$. Since (X, T) is a fuzzy Baire space, by Theorem 2.8, $int(\lambda) = 0$ for the fuzzy first category set λ in X. Then $cl(1 - \lambda) = 1 - int(\lambda) = 1$. Thus there exists a fuzzy set λ in X such that $cl(\lambda) = 1$ and $cl(1 - \lambda) = 1$. So X is a fuzzy resolvable space. □

Proposition 4.12. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable and fuzzy DG_δ-space, then X is a fuzzy resolvable space.*

Proof. Suppose X is a fuzzy extraresolvable and fuzzy DG_δ-space and let λ be a fuzzy dense (but not fuzzy open) set in X. Since X is a fuzzy DG_δ-space, λ is a fuzzy G_δ-set in X. Thus λ is a fuzzy dense and fuzzy G_δ-set in the fuzzy extraresolvable space X. So by Proposition 4.10, X is a fuzzy resolvable space. □

Proposition 4.13. *If there exists a fuzzy regular σ-nowhere dense set in a fuzzy extraresolvable space (X, T) such that $int(\lambda) = 0$, then X is a fuzzy resolvable space.*

Proof. Suppose λ is a fuzzy regular σ-nowhere dense set in X such that $int(\lambda) = 0$. Since (X, T) is a fuzzy extraresolvable space, by Proposition 3.20, λ is a fuzzy dense and fuzzy F_σ-set in X. Since $int(\lambda) = 0$, $1 - int(\lambda) = 1 - 0 = 1$ and $cl(1 - \lambda) = 1 - int(\lambda) = 1$. Then there exists a fuzzy set λ in X such that $cl(\lambda) = 1$ and $cl(1 - \lambda) = 1$. Thus X is a fuzzy resolvable space. □

Proposition 4.14. *If a fuzzy topological space (X, T) is a fuzzy weakly Baire and fuzzy extraresolvable space, then X is a fuzzy resolvable space.*

Proof. Suppose X is a fuzzy weakly Baire and fuzzy extraresolvable space and let λ be a fuzzy σ-boundary set in X. Since X is a fuzzy weakly Baire space, by Theorem 2.15, λ is a fuzzy σ-nowhere dense set in X. Since X is a fuzzy extraresolvable space, by Proposition 3.19, λ is a fuzzy dense set in X. Then $cl(\lambda) = 1$. Since X is a fuzzy weakly Baire space, by Theorem 2.14, $int(\lambda) = 0$. Thus $cl(1 - \lambda) = 1 - int(\lambda) = 1$. So there exists a fuzzy set λ in X such that $cl(\lambda) = 1$ and $cl(1 - \lambda) = 1$. Hence X is a fuzzy resolvable space. □

The following proposition shows that fuzzy nodef spaces are not fuzzy extraresolvable spaces.

Proposition 4.15. *If a fuzzy topological space (X, T) is a fuzzy nodef space, then X is not a fuzzy extraresolvable space.*

Proof. Suppose X is a fuzzy nodef space and let λ be a fuzzy dense and fuzzy open set in X. Since X is a fuzzy nodef space, by Theorem 2.10, λ is a fuzzy residual set in X. Since $\lambda \in T$, $int(\lambda) = \lambda \neq 0$. Then by Proposition 3.15, X is not a fuzzy extraresolvable space. [For, if X is a fuzzy extraresolvable space, then for the residual set λ, it should be that $int(\lambda) = 0$]. □

The following proposition shows that fuzzy extraresolvable spaces are not fuzzy Brown spaces.

Proposition 4.16. *If a fuzzy topological space (X, T) is a fuzzy extraresolvable space, then X is not a fuzzy Brown space.*

Proof. Suppose X is a fuzzy extraresolvable space and assume that X is a fuzzy Brown space. Then by Theorem 2.11, X is a fuzzy open hereditarily irresolvable space. This is a contradiction by Proposition 4.5. Thus X is not a fuzzy Brown space. □

Remark 4.17. It should be noted in example 3.2 that β is a fuzzy open set in (X, T), but not a fuzzy dense set in X and this implies that fuzzy extraresolvable spaces need not be fuzzy hyperconnected spaces.

Remark 4.18. It is established in [30] that fuzzy Baire-dominated and fuzzy extraresolvable spaces are fuzzy hyperconnected spaces.

5. Conclusions

In this paper, the notion of fuzzy extraresolvability of fuzzy topological spaces was introduced by means of fuzzy nowhere denseness and fuzzy denseness of fuzzy sets. It was obtained that fuzzy first category sets, fuzzy σ-nowhere dense sets in fuzzy extraresolvable spaces are fuzzy dense sets and fuzzy residual sets are having zero interior in fuzzy extraresolvable spaces. It was obtained that fuzzy extraresolvable spaces are neither fuzzy almost P-spaces, nor fuzzy D-Baire spaces. It was also obtained that fuzzy extraresolvable spaces are neither fuzzy open hereditarily irresolvable spaces nor fuzzy Brown spaces. It was obtained that fuzzy nodef spaces are not fuzzy extraresolvable spaces. It was established that the existence of fuzzy

σ-nowhere dense sets, fuzzy dense and fuzzy G_δ -sets, in fuzzy extraresolvable spaces ensures its fuzzy resolvability. It was obtained that fuzzy Baire and fuzzy extraresolvable spaces, fuzzy DG_δ and fuzzy extraresolvable spaces, fuzzy weakly Baire and fuzzy extraresolvable spaces, are fuzzy resolvable spaces.

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G. Thangaraj (g.thangaraj@rediffmail.com)
Department of Mathematics, Thiruvalluvar University, Serkkadu, Vellore-632 115. Tamilnadu, India

D.Vijayan (jyoshijeev@gmail.com)
Department of Mathematics, Muthurangam Government Arts College(Autonomous), Vellore - 632 001. Tamilnadu, India