Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 28, No. 1, (August 2024) pp. 75–83
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr
https://doi.org/10.30948/afmi.2024.28.1.75

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Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 28, No. 1, August 2024

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Received 18 January 2024; Revised 18 February 2024; Accepted 12 April 2024

Abstract. A generalization of classical metric, namely parametric metric was introduced by Hussain et al. in [1]. This construction has a twofold significance: it deals with metric conditions and allows for the possibility of performing parametric soft S-metric space, specifically for the training of fixed point type theorems. This paper presents new insights into this construction and establishes various fixed point theorems in this space.

2020 AMS Classification: 54H25, 47H10

Keywords: Parametric S-metric, Parametric soft S-metric space, Soft contractive mapping, Fixed point theorem.

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1. Introduction and preliminaries

Metric spaces have an important role in topology and other applied sciences. Nowadays, the extensions of metric spaces are the most well studied structures. Thus there are different species of these extensions in literature [2, 3] such as parametric metric space [1], bipolar metric space [4], parametric S-metric space [5], parametric b-metric spaces [6] etc. Recently, many researchers have studied generalized metric space by changing the triangle inequality of metric conditions. Soft set theory was introduced by Molodtsov [7] as a new mathematical structure. Since applications of soft set theory in other disciplines and real life problems was progressing rapidly [8, 9, 10, 11] , the study of soft metric space which is based on soft point of soft sets was initiated by Das and Samanta [12]. Yazar et al. [13] examined some important properties of soft metric spaces and soft continuous mappings. They also proved some fixed point theorems of soft contractive mappings on soft metric spaces. A number of authors introduced contractive type mapping on a complete metric space which are generalizations of Banach contraction which has the property that each such mapping has a unique fixed point in [14, 15]. By using contractive type mapping, some authors have studied the fixed point theory for soft functions on different soft metric spaces which are generalizations of metric spaces such as soft S-metric space [16], bipolar soft metric space [17] etc.

A generalization of classical metric, namely parametric metric was introduced by Hussain et al. in [1]. The significance of investigate this construction firstly take care of metric conditions and secondly, chance of performing some kind of metric space namely parametric soft S-metric space to training of fixed point type theorems. In this paper we present new impression in this construction and set up varied fixed point theorem in parametric soft S-metric space. Initially, we acquaint the notions of parametric metric, parametric S-metric and soft S-metric. Later we successfully give parametric soft S-metric space using the definitions of parametric metric and soft S-metric. It is known that contractive mappings has a major area in the fixed point theory. In the classical Banach theorem, fixed point theorems are proven using contractive mapping. This is very important as the fixed point theorem is proven under different conditions in this paper.

Throughout this paper, X denotes initial universe, E denotes the set of all parameters, $P(X)$ denotes the power set of X.

Definition 1.1 ([7]). A pair F_E is called a *soft set* over X, where F is a mapping given by $F : E \to P(X)$.

Definition 1.2 ([18]). If $F(a) = \varnothing$ for all $a \in E$, then F_E is said to be a *null soft set* and denoted by Φ. If $F(a) = X$ for all $a \in E$, then F_E is said to be an *absolute soft set* and denoted by $\widetilde{X}$.

Definition 1.3 ([12, 19]). Let F_E be a soft set over X. Then F_E is called a *soft point with the value* $x \in X$ *and the support* $a \in E$ in X, denoted by x_a, if $F(a) = \{x\}$ and $F(a') = \varnothing$ for all $a' \in E - \{a\}$.

It is obvious that each soft set can be expressed as a union of soft points. For this reason, to give the family of all soft sets on X it is sufficient to give only soft points on X.

Definition 1.4 ([19]). Let F_E be a soft set over X. Then a soft point x_a is said to *belong to* F_E, denoted by $x_a \widetilde{\in} F_E$, if $x_a(a) \in F(a)$, i.e., $\{x\} \subseteq F(a)$.

Definition 1.5 ([12]). Let $\mathbb{R}$ be the set of all real numbers, $B(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$ and E be taken as a set of parameters. Then a mapping $F : E \to B(\mathbb{R})$ is called a *soft real set* in $\mathbb{R}$. A soft real set is called a *singleton set* in $\mathbb{R}$, if $F(a) = r$ for each $a \in E$. In this case, it will be called a *soft real number* of $\mathbb{R}$ and denoted by $\widetilde{r}, \widetilde{s}, \widetilde{t}$, etc. It is clear that $\widetilde{0}(a) = 0$, $\widetilde{1}(a) = 1$ for all $a \in E$, respectively.

Definition 1.6 ([12]). The collection of nonnegative soft real numbers is denoted by $\mathbb{R}(E)^*$ and the pair $\left(\mathbb{R}(E)^*, \widetilde{\leq}\right)$ is a partially ordered set, such that for $\widetilde{r}, \widetilde{s}$ as two soft real numbers,

(i) $\widetilde{r} \widetilde{\leq} \widetilde{s}$, if $\widetilde{r}(a) \leq \widetilde{s}(a)$ for all $a \in E$,
(ii) $\widetilde{r} \widetilde{\geq} \widetilde{s}$, if $\widetilde{r}(a) \geq \widetilde{s}(a)$ for all $a \in E$,
(iii) $\widetilde{r} \widetilde{<} \widetilde{s}$, if $\widetilde{r}(a) < \widetilde{s}(a)$ for all $a \in E$,
(iv) $\widetilde{r} \widetilde{>} \widetilde{s}$ if $\widetilde{r}(a) > \widetilde{s}(a)$ for all $a \in E$.

Definition 1.7 ([1])**.** Let X be a nonempty set and $P : X \times X \times (0, \infty) \to [0, \infty)$ be a function satisfing the following conditions: for all $x, y, a \in X$ and $t > 0$,
(i) $P(x, y, t) = 0$ if and only if $x = y$,
(ii) $P(x, y, t) = P(y, x, t)$,
(iii) $P(x, y, t) \leq P(x, a, t) + P(a, y, t)$.
Then P is called a *parametric metric* on X and the pair (X, P) is called a *parametric metric space*.

Definition 1.8 ([5])**.** Let X be a nonempty set and $S : X^3 \times (0, \infty) \to [0, \infty)$ be a function satisfing the following conditions: for all $x, y, z, a \in X$ and $t > 0$,
(i) $S(x, y, z, t) = 0$ if and only if $x = y = z$,
(ii) $S(x, y, z, t) \leq S(x, x, a, t) + S(y, y, a, t) + S(z, z, a, t)$.
Then S is called a *parametric S-metric* on X and the pair (X, S) is called a *parametric S-metric space*. The family of all soft points of the set $\widetilde{X}$ is denoted by $SP(\widetilde{X})$.

Definition 1.9 ([16])**.** A *soft S-metric* on X is a mapping $S : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \to \mathbb{R}(E)^*$ that satisfies the following conditions: for any $x_a, y_b, z_c, u_d \in SP(\widetilde{X})$,
(S1) $S(x_a, y_b, z_c) \widetilde{\geq} \widetilde{0}$,
(S2) $S(x_a, y_b, z_c) = \widetilde{0}$ if and only if $x_a = y_b = z_c$,
(S3) $S(x_a, y_b, z_c) \widetilde{\leq} S(x_a, x_a, u_d) + S(y_b, y_b, u_d) + S(z_c, z_c, u_d)$.
Then the soft set $\widetilde{X}$ with a soft S-metric S is called a *soft S-metric space* and denoted by $\left(\widetilde{X}, S, E\right)$.

2. Parametric soft S-metric spaces

In next section, we mention the concept of parametric soft S-metric space and give some results in complete parametric soft S-metric space. Let $\widetilde{X}$ be the absolute soft set, E be a nonempty set of parameters.

Definition 2.1. A *parametric soft S-metric* on $\widetilde{X}$ is a mapping $d_S : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ that satisfies the following conditions: for any $x_a, y_b, z_c, u_d \in SP(\widetilde{X})$ and $\widetilde{t} > \widetilde{0}$,
(PS1) $d_S\left(x_a, y_b, z_c, \widetilde{t}\right) \widetilde{\geq} \widetilde{0}$,
(PS2) $d_S\left(x_a, y_b, z_c, \widetilde{t}\right) = \widetilde{0}$ if and only if $x_a = y_b = z_c$,
(PS3) $d_S\left(x_a, y_b, z_c, \widetilde{t}\right) \widetilde{\leq} d_S\left(x_a, x_a, u_d, \widetilde{t}\right) + d_S\left(y_b, y_b, u_d, \widetilde{t}\right) + d_S\left(z_c, z_c, u_d, \widetilde{t}\right)$.
Then the soft set $\widetilde{X}$ with a parametric soft S-metric d_S is called a *parametric soft S-metric space* and denoted by $\left(\widetilde{X}, d_S, E\right)$.

Example 2.2. Let $E = \mathbb{R}$ be a parameter set and $X = \mathbb{R}^2$,$\widetilde{t} > \widetilde{0}$. Consider usual metric on $X = \mathbb{R}^2$ with norm. Then

$$d_S : SP(\widetilde{X}) \times SP(\widetilde{X}) \times SP(\widetilde{X}) \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$$

is defined by: for any $x_a, y_b, z_c \in SP(\widetilde{X})$ and $\widetilde{t} > \widetilde{0}$,

$$d_S\left(x_a, y_b, z_c, \widetilde{t}\right) = \widetilde{t}\left(|b + a - 2c| + |b - a| + \|y + x - 2z\| + \|y - x\|\right).$$

It is verified that d_S is a parametric soft S-metric space on $SP(\widetilde{X})$.

Definition 2.3. Let $\left(\widetilde{X}, d_S, E\right)$ be a parametric soft S-metric space and $\left\{x_{a_n}^n\right\}$ be a sequence of soft points in $\widetilde{X}$.

(i) A sequence $\left\{x_{a_n}^n\right\}$ is said to *be convergent*, if there exists a soft point x_a in $\widetilde{X}$ such that $\lim\limits_{n\to\infty} d_S\left(x_{a_n}^n, x_{a_n}^n, x_a, \widetilde{t}\right) = \widetilde{0}$ for all $\widetilde{t}\widetilde{>}\widetilde{0}$ and denoted by $\lim\limits_{n\to\infty} x_{a_n}^n = x_a$.

(ii) A sequence $\left\{x_{a_n}^n\right\}$ is called a *Cauchy sequence*, if $\lim\limits_{n,m\to\infty} d_S\left(x_{a_n}^n, x_{a_n}^n, x_{a_m}^m, \widetilde{t}\right) = \widetilde{0}$ for all $\widetilde{t}\widetilde{>}\widetilde{0}$.

Definition 2.4. If every Cauchy sequence is convergent in parametric soft S-metric space $\left(\widetilde{X}, d_S, E\right)$, then $\left(\widetilde{X}, d_S, E\right)$ is called a *complete parametric soft S-metric space*.

A soft mapping on $\left(\widetilde{X}, d_S, E\right)$ consists of the mappings $f : X \to X$ and $\varphi : E \to E$ and denoted by f_φ. If f and φ are surjective (injective), then f_φ is called *surjective (injective) soft mapping*.

Definition 2.5. Let $\left(\widetilde{X}, d_S, E\right)$ be a parametric soft S-metric space. Then a soft mapping $f_\varphi : \left(\widetilde{X}, d_S, E\right) \to \left(\widetilde{X}, d_S, E\right)$ is said to be a *soft contraction mapping* in parametric soft S-metric space, if there exists a soft real number $\widetilde{q} \in \mathbb{R}(E), \widetilde{0}\widetilde{\leq}\widetilde{q}\widetilde{<}\widetilde{1}$ such that

$$d_S\left(f_\varphi\left(x_a\right), f_\varphi\left(x_a\right), f_\varphi\left(y_b\right), \widetilde{t}\right) \widetilde{\leq} \widetilde{q} d_S\left(x_a, x_a, y_b, \widetilde{t}\right)$$

for all $x_a, y_b \in SP(\widetilde{X})$ and $\widetilde{t} > \widetilde{0}$.

Remark 2.6. A soft contraction mapping in a parametric soft S-metric space is a soft continuous mapping because if $\lim\limits_{n\to\infty} x_{a_n}^n = x_a$ in the above condition, we get $\lim\limits_{n\to\infty} f_\varphi\left(x_{a_n}^n\right) = f_\varphi\left(x_a\right)$.

Remark 2.7. In this study, we take $f_\varphi^n\left(x_a\right), x_a \in SP(\widetilde{X})$ and $n \in \{0, 1, 2, ...\}$, inductively by $f_\varphi^0\left(x_a\right) = x_a$ and $f_\varphi^{n+1}\left(x_a\right) = f_\varphi\left(f_\varphi^n\left(x_a\right)\right)$.

Lemma 2.8. *Let $\left(\widetilde{X}, d_S, E\right)$ be a parametric soft S-metric space. Then*

$$d_S\left(x_a, x_a, y_b, \widetilde{t}\right) = d_S\left(y_b, y_b, x_a, \widetilde{t}\right)$$

for all $x_a, y_b \in SP(\widetilde{X})$ and $\widetilde{t} > \widetilde{0}$.

Proof. Let $x_a,\ y_b \in SP(\widetilde{X})$ and $\widetilde{t} > \widetilde{0}$. Then by using condition of parametric soft $S-$metric, we have

$$\begin{aligned} d_S\left(x_a, x_a, y_b, \widetilde{t}\right) &\leq 2d_S\left(x_a, x_a, x_a, \widetilde{t}\right) + d_S\left(y_b, y_b, x_a, \widetilde{t}\right) \\ &= d_S\left(y_b, y_b, x_a, \widetilde{t}\right), \qquad (1) \\ d_S\left(y_b, y_b, x_a, \widetilde{t}\right) &\leq 2d_S\left(y_b, y_b, y_b, \widetilde{t}\right) + d_S\left(x_a, x_a, y_b, \widetilde{t}\right) \\ &= d_S\left(x_a, x_a, y_b, \widetilde{t}\right). \qquad (2) \end{aligned}$$

From the inequalities (1) and (2), $d_S\left(x_a, x_a, y_b, \widetilde{t}\right) = d_S\left(y_b, y_b, x_a, \widetilde{t}\right)$ is satisfied. □

Definition 2.9. Let $\left(\widetilde{X}, d_S, E\right)$ be a parametric soft S-metric space. Then a soft mapping $f_\varphi : \left(\widetilde{X}, d_S, E\right) \to \left(\widetilde{X}, d_S, E\right)$ is called a *soft self-mapping.*

Theorem 2.10. *Let $\left(\widetilde{X}, d_S, E\right)$ be a complete parametric soft S-metric space and $f_\varphi : \left(\widetilde{X}, d_S, E\right) \to \left(\widetilde{X}, d_S, E\right)$ be a surjective soft self-mapping. If there is soft real numbers $\widetilde{r}, \widetilde{s} \geq \widetilde{0}$ and $\widetilde{k} > \widetilde{1}$ such that*

$$\begin{aligned} d_S\left(f_\varphi(x_a), f_\varphi(x_a), f_\varphi(y_b), \widetilde{t}\right) &\geq \widetilde{k} d_S\left(x_a, x_a, y_b, \widetilde{t}\right) + \widetilde{r} d_S\left(f_\varphi(x_a), f_\varphi(x_a), x_a, \widetilde{t}\right) \\ &+ \widetilde{s} d_S\left(f_\varphi(y_b), f_\varphi(y_b), y_b, \widetilde{t}\right) \end{aligned}$$

for any $x_a,\ y_b \in SP(\widetilde{X})$ and $\widetilde{t} > \widetilde{0}$, then f_φ has a unique soft point in $SP(\widetilde{X})$.

Proof. f_φ is an injective soft mapping. Actually, if we get $f_\varphi(x_a) = f_\varphi(y_b)$ and using condition the assumption, for all $\widetilde{t} > \widetilde{0}$, we obtain

$$\begin{aligned} \widetilde{0} &= d_S\left(f_\varphi(x_a), f_\varphi(x_a), f_\varphi(x_a), \widetilde{t}\right) \\ &\geq \widetilde{k} d_S\left(x_a, x_a, y_b, \widetilde{t}\right) + \widetilde{r} d_S\left(f_\varphi(x_a), f_\varphi(x_a), x_a, \widetilde{t}\right) + \widetilde{s} d_S\left(f_\varphi(x_a), f_\varphi(x_a), y_b, \widetilde{t}\right). \end{aligned}$$

Then $d_S\left(x_a, x_a, y_b, \widetilde{t}\right) = \widetilde{0}$. Since $\widetilde{k} > \widetilde{1}$, $x_a = y_b$ is devised. Now we sign inverse mapping of f_φ as γ_ψ. Let x_a^0 be an arbitrary soft point in $SP(\widetilde{X})$. We define a soft sequence as follows:

$$\begin{aligned} x_{a_1}^1 &= \gamma_\psi\left(x_a^0\right), \\ x_{a_2}^2 &= \gamma_\psi\left(x_{a_1}^1\right) = \gamma_\psi^2\left(x_a^0\right), \\ &\dots \\ x_{a_{n+1}}^{n+1} &= \gamma_\psi^n\left(x_{a_n}^n\right) = \gamma_\psi^{n+1}\left(x_a^0\right), \dots \end{aligned}$$

Let us consider $x_{a_{n+1}}^{n+1} \neq x_{a_n}^n$ for each $n \in \mathbb{N}$. From the condition of theorem,

$$\begin{aligned} d_S\left(x_{a_{n-1}}^{n-1}, x_{a_{n-1}}^{n-1}, x_{a_n}^n, \widetilde{t}\right) &= d_S\left(f_\varphi\left(f_\varphi^{-1}\left(x_{a_{n-1}}^{n-1}\right)\right), f_\varphi\left(f_\varphi^{-1}\left(x_{a_{n-1}}^{n-1}\right)\right), f_\varphi\left(f_\varphi^{-1}\left(x_{a_n}^n\right)\right), \widetilde{t}\right) \\ &\geq \widetilde{k} d_S\left(f_\varphi^{-1}\left(x_{a_{n-1}}^{n-1}\right), f_\varphi^{-1}\left(x_{a_{n-1}}^{n-1}\right), f_\varphi^{-1}\left(x_{a_n}^n\right), \widetilde{t}\right) \\ &+ \widetilde{r} d_S\left(f_\varphi\left(f_\varphi^{-1}\left(x_{a_{n-1}}^{n-1}\right)\right), f_\varphi\left(f_\varphi^{-1}\left(x_{a_{n-1}}^{n-1}\right)\right), f_\varphi^{-1}\left(x_{a_{n-1}}^{n-1}\right), \widetilde{t}\right) \\ &+ \widetilde{s} d_S\left(f_\varphi\left(f_\varphi^{-1}\left(x_{a_n}^n\right)\right), f_\varphi\left(f_\varphi^{-1}\left(x_{a_n}^n\right)\right), f_\varphi^{-1}\left(x_{a_n}^n\right), \widetilde{t}\right) \\ &= \widetilde{k} d_S\left(\gamma_\psi\left(x_{a_{n-1}}^{n-1}\right), \gamma_\psi\left(x_{a_{n-1}}^{n-1}\right), \gamma_\psi\left(x_{a_n}^n\right), \widetilde{t}\right) \\ &+ \widetilde{r} d_S\left(x_{a_{n-1}}^{n-1}, x_{a_{n-1}}^{n-1}, \gamma_\psi\left(x_{a_{n-1}}^{n-1}\right), \widetilde{t}\right) \\ &+ \widetilde{s} d_S\left(x_{a_n}^n, x_{a_n}^n, \gamma_\psi\left(x_{a_n}^n\right), \widetilde{t}\right) \\ &= \widetilde{k} d_S\left(x_{a_n}^n, x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) + \widetilde{r} d_S\left(x_{a_{n-1}}^{n-1}, x_{a_{n-1}}^{n-1}, x_{a_n}^n, \widetilde{t}\right) \\ &+ \widetilde{s} d_S\left(x_{a_n}^n, x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) \\ &= \left(\widetilde{k} + \widetilde{s}\right) d_S\left(x_{a_n}^n, x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) + \widetilde{r} d_S\left(x_{a_{n-1}}^{n-1}, x_{a_{n-1}}^{n-1}, x_{a_n}^n, \widetilde{t}\right). \end{aligned}$$

We get

$$\left(\widetilde{1}-\widetilde{r}\right) d_S\left(x_{a_{n-1}}^{n-1}, x_{a_{n-1}}^{n-1}, x_{a_n}^n, \widetilde{t}\right) \geq\left(\widetilde{k}+\widetilde{s}\right) d_S\left(x_{a_n}^n, x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right).$$

It is clear that $\widetilde{k}+\widetilde{s} \neq \widetilde{0}$ and $\widetilde{1}-\widetilde{r}>\widetilde{0}$. Thus

$$d_S\left(x_{a_n}^n, x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) \leq \frac{\widetilde{1}-\widetilde{r}}{\widetilde{k}+\widetilde{s}} d_S\left(x_{a_{n-1}}^{n-1}, x_{a_{n-1}}^{n-1}, x_{a_n}^n, \widetilde{t}\right). \qquad (3)$$

Here $\widetilde{h}=\frac{\widetilde{1}-\widetilde{r}}{\widetilde{k}+\widetilde{s}}<\widetilde{1}$. By repeating this process in (3), we have

$$d_S\left(x_{a_n}^n, x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right) \leq \widetilde{h}^n d_S\left(x_a^0, x_a^0, x_{a_1}^1, \widetilde{t}\right) \qquad (4)$$

for all $\widetilde{t}>\widetilde{0}$. Let $m, n \in \mathbb{N}$ as $m>n \geq 1$. By using (4) and condition of parametric soft metric space,

$$d_S\left(x_{a_n}^n, x_{a_n}^n, x_{a_m}^m, \widetilde{t}\right) \leq \frac{2 \widetilde{h}^n}{\widetilde{1}-\widetilde{h}} d_S\left(x_a^0, x_a^0, x_{a_1}^1, \widetilde{t}\right).$$

This implies $d_S\left(x_{a_n}^n, x_{a_n}^n, x_{a_m}^m, \widetilde{t}\right) \rightarrow \widetilde{0}$ as $m, n \rightarrow \infty$. So $\left\{x_{a_n}^n\right\}$ is a soft Cauchy sequence, by the completeness of $\widetilde{X}$, $\left\{x_{a_n}^n\right\}$ converges to a soft point $x_c^* \in SP(\widetilde{X})$. Since f_{φ} is a surjective soft self-mapping, $f_{\varphi}\left(u_d\right)=x_c^*$ for some $u_d \in SP(\widetilde{X})$. From the condition of theorem, we obtain

$$\begin{aligned}
& d_S\left(x_{a_n}^n, x_{a_n}^n, x_c^*, \widetilde{t}\right) \\
= & d_S\left(f_{\varphi}\left(x_{a_{n+1}}^{n+1}\right), f_{\varphi}\left(x_{a_{n+1}}^{n+1}\right), f_{\varphi}\left(u_d\right), \widetilde{t}\right) \\
\geq & \widetilde{k} d_S\left(x_{a_{n+1}}^{n+1}, x_{a_{n+1}}^{n+1}, u_d, \widetilde{t}\right)+\widetilde{r} d_S\left(x_{a_n}^n, x_{a_n}^n, x_{a_{n+1}}^{n+1}, \widetilde{t}\right)+\widetilde{s} d_S\left(x_c^*, x_c^*, u_d, \widetilde{t}\right).
\end{aligned}$$

For $n \rightarrow \infty$, this inequality

$$\widetilde{0} \geq\left(\widetilde{k}+\widetilde{s}\right) d_S\left(x_c^*, x_c^*, u_d, \widetilde{t}\right)$$

is obtained, i.e. $x_c^*=u_d$. Hence we have $f_{\varphi}\left(x_c^*\right)=x_c^*$, i.e., f_{φ} has a fixed soft point.

For the uniqueness, we consider the converse which means there could be two different soft points x_c^*, z_a^* of f_{φ}. Then

$$\begin{aligned}
d_S\left(x_c^*, x_c^*, z_a^*, \widetilde{t}\right) & = d_S\left(f_{\varphi}\left(x_c^*\right), f_{\varphi}\left(x_c^*\right), f_{\varphi}\left(z_a^*\right), \widetilde{t}\right) \\
& \geq \widetilde{k} d_S\left(x_c^*, x_c^*, z_a^*, \widetilde{t}\right)+\widetilde{r} d_S\left(x_c^*, x_c^*, x_c^*, \widetilde{t}\right)+\widetilde{s} d_S\left(z_a^*, z_a^*, z_a^*, \widetilde{t}\right) \\
& = \widetilde{k} d_S\left(x_c^*, x_c^*, z_a^*, \widetilde{t}\right).
\end{aligned}$$

Since $\widetilde{k}>\widetilde{1}$, $x_c^*=z_a^*$. Therefore there exists a unique soft point of f_{φ}. □

Remark 2.11. If we take $\widetilde{r}=\widetilde{s}$ in Theorem 2.10, then the following corollary is directly acquired.

Corollary 2.12. *Let $\left(\widetilde{X}, d_S, E\right)$ be a complete parametric soft $S-$metric space and $f_{\varphi}:\left(\widetilde{X}, d_S, E\right) \rightarrow\left(\widetilde{X}, d_S, E\right)$ be a surjective soft self-mapping. If there is soft real* *numbers* $\widetilde{r} \geq \widetilde{0}$ *and* $\widetilde{k} > \widetilde{1}$ *such that*

$$d_S\left(f_\varphi(x_a), f_\varphi(x_a), f_\varphi(y_b), \widetilde{t}\right) \geq \widetilde{k} d_S\left(x_a, x_a, y_b, \widetilde{t}\right)$$
$$+\widetilde{r} \max\left\{d_S\left(f_\varphi(x_a), f_\varphi(x_a), x_a, \widetilde{t}\right), d_S\left(f_\varphi(y_b), f_\varphi(y_b), y_b, \widetilde{t}\right)\right\}$$

for each $x_a, y_b \in SP(\widetilde{X})$ *and* $\widetilde{t} > \widetilde{0}$, *then* f_φ *has a unique fixed soft point in* $SP(\widetilde{X})$.

Proof. The proof is obvious. □

Remark 2.13. If we take $\widetilde{k} = \widetilde{m}$ and $\widetilde{r} = \widetilde{s} = \widetilde{0}$, then the following corollary is directly obtained.

Corollary 2.14. *Let* $\left(\widetilde{X}, d_S, E\right)$ *be a complete parametric soft* S*-metric space and* $f_\varphi : \left(\widetilde{X}, d_S, E\right) \to \left(\widetilde{X}, d_S, E\right)$ *be a surjective soft self-mapping. If there is soft real number* $\widetilde{m} > \widetilde{1}$ *such that*

$$d_S\left(f_\varphi(x_a), f_\varphi(x_a), f_\varphi(y_b), \widetilde{t}\right) \geq \widetilde{m} d_S\left(x_a, x_a, y_b, \widetilde{t}\right)$$

for each $x_a, y_b \in SP(\widetilde{X})$ *and* $\widetilde{t} > \widetilde{0}$, *then* f_φ *has a unique fixed soft point in* $SP(\widetilde{X})$.

Proof. The existence of the soft point is obtained by Theorem 2.10. Then we only prove that the uniqueness of the soft point. Suppose z_a^* be another soft point in $SP(\widetilde{X})$. Then from the condition of corollary, we have

$$\begin{aligned} d_S\left(x_a^*, x_a^*, z_a^*, \widetilde{t}\right) &= d_S\left(f_\varphi(x_a^*), f_\varphi(x_a^*), f_\varphi(z_a^*), \widetilde{t}\right) \\ &\geq \widetilde{m} d_S\left(x_a^*, x_a^*, z_a^*, \widetilde{t}\right), \end{aligned}$$

which implies $d_S\left(x_a^*, x_a^*, z_a^*, \widetilde{t}\right) = \widetilde{0}$, since $\widetilde{m} > \widetilde{1}$. Thus $x_a^* = z_a^*$ is taken. □

Note that we give Corollary 2.14 as a generalization of Corollary 2.12.

Corollary 2.15. *Let* $\left(\widetilde{X}, d_S, E\right)$ *be a complete parametric soft* S*-metric space and* $f_\varphi : \left(\widetilde{X}, d_S, E\right) \to \left(\widetilde{X}, d_S, E\right)$ *be a surjective soft self-mapping. If there is a positive integer* n *and a soft real number* $\widetilde{k} > \widetilde{1}$ *such that*

$$d_S\left(f_\varphi^n(x_a), f_\varphi^n(x_a), f_\varphi^n(y_b), \widetilde{t}\right) \geq \widetilde{k} d_S\left(x_a, x_a, y_b, \widetilde{t}\right)$$

for each $x_a, y_b \in SP(\widetilde{X})$ *and* $\widetilde{t} > \widetilde{0}$, *then* f_φ *has a unique soft point in* $SP(\widetilde{X})$.

Proof. From Corollary 2.14, it is obvious that f_φ^n has a unique soft point as x_a^* in $SP(\widetilde{X})$. Also

$$f_\varphi(x_a^*) = f_\varphi\left(f_\varphi^n(x_a^*)\right) = f_\varphi^n\left(f_\varphi(x_a^*)\right).$$

Then $f_\varphi(x_a^*)$ is a soft point of f_φ^n and $f_\varphi(x_a^*) = x_a^*$ is obtained. Since the soft mappings f_φ and f_φ^n have the same soft point, the proof is completed. □

3. Conclusions

We have introduced namely parametric soft S-metric space which is based on soft point of soft sets. Later we give some kind of interesting fixed point theorem. We show that f_{φ} has a unique soft point under different conditions on a complete parametric soft S-metric space.

Acknowledgements. The authors would like to thank the referees for their valuable comments and suggestions.

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