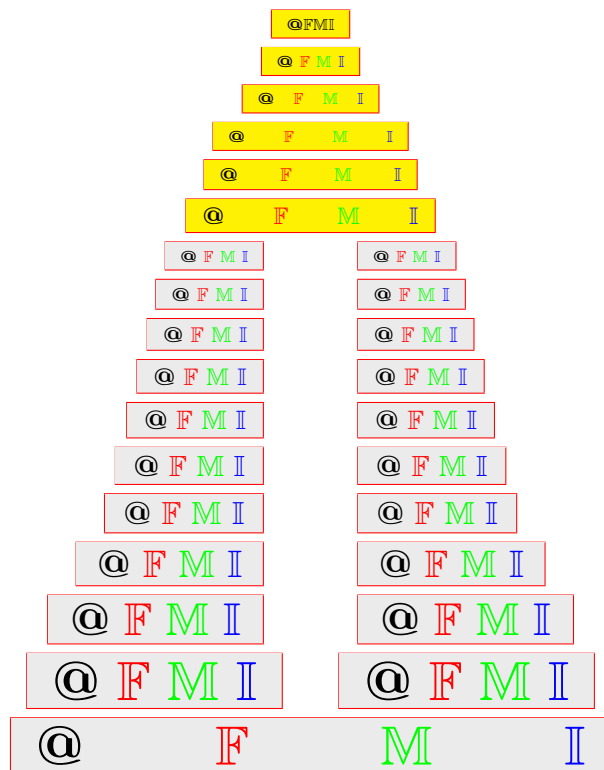


Pair-wise continuity in soft bitopological ordered spaces

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ABSTRACT. This paper extends soft bitopological spaces to soft bitopological ordered spaces by introducing and analyzing properties of increasing (decreasing, balancing) soft pairwise continuous (open, closed, homeomorphism) maps, denoted as xSP -continuous (open, closed, homeomorphism) maps, where x can take values I , D , or B . The investigation explores the interrelationships among these concepts, establishing equivalent conditions for each. Additionally, the study offers a comparative analysis of these notions. This research aims to contribute to the advancement of soft bitopological ordered spaces by providing a comprehensive understanding of their fundamental properties and relationships, paving the way for further exploration and applications in the field.

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1. INTRODUCTION

In 1965, Nachbin [1] introduced the concept of a topological ordered space as a triple (X, τ, \leq) , where τ is a topological structure and \leq is a partial order relation on a non-empty set X . He established several topological ordered notions based on this structure. In 1999, Molotdov [2] proposed the idea of soft sets as a means of dealing with uncertainties and vagueness in real-world situations and phenomena. He highlighted the advantages of soft set theory over fuzzy theory and probability theory and explored its applications in various fields. The use of soft sets in overcoming incomplete data has motivated researchers to introduce soft operations between soft sets and apply them in decision-making, information science, mathematics, and related disciplines. To incorporate soft sets into topology studies, Shabir and Naz

[3] formulated the notion of soft topological spaces in 2011 by drawing an analogy with the definition of topology. El-Shafei et al. [4] introduced the concept of soft topological ordered spaces as a generalization of topological ordered spaces, along with the notions of increasing and decreasing soft sets and maps. They further extended this concept by introducing the notions of soft $x\alpha$ -continuous, soft $x\alpha$ -open, soft $x\alpha$ -closed, and soft $x\alpha$ -homeomorphism maps for $x = I, D$, and B via soft topological ordered spaces. El-Sheikh et al. [5] generalized the notion of soft topological spaces by introducing the concept of supra soft topological spaces, while Ittanagi [6] introduced the notion of soft bitopological space defined over an initial universal set with a fixed set of parameters. Şenel [7] presented the soft topology generated by L -soft sets. Additionally, in a 2016, Şenel [8] proposed a new approach to hausdorff space theory via the soft sets. Şenel et al. [9] introduced soft topological subspaces in 2015. Furthermore, Şenel et al. [10] explored soft closed sets on soft bitopological space in 2014. In 2020, Şenel et al. [11] investigated distance and similarity measures for octahedron sets proposed by Lee et al. [12]. Kandil et al. [13] provided some structures on soft bitopological spaces and defined some basic notions such as pairwise open and closed soft sets, pairwise soft closure, interior, kernel operators, and related topics. They also studied pairwise soft continuous mappings and open and closed soft mappings between two soft bitopological spaces. El-Sheikh et al. [14] established the concept of a soft bitopological ordered space, which comprises a soft bitopological space with a partial order relation. They introduced and studied various concepts related to increasing and decreasing pairwise open and closed soft sets, increasing and decreasing total and partial pairwise soft neighborhoods, and increasing, decreasing, and balancing pairwise open soft neighborhoods. They also defined the concept of increasing and decreasing pairwise soft closure and interior.

The main objective of this research is to introduce and examine the notions of xSP -continuous, xSP -open, xSP -closed mappings and xSP -homeomorphism for $x = I; D; B$ in soft bitopological ordered spaces. The study aims to provide a comprehensive understanding of these concepts by exploring their equivalent conditions and establishing their relationships. Several examples are presented to demonstrate the connections between these maps and to show that they are more powerful than their P -soft counterparts. Moreover, the research characterizes each of these maps and emphasizes the crucial role of extended soft topologies in studying the links between these maps and their corresponding maps in soft bitopological ordered spaces.

2. PRELIMINARIES

In the remaining part of this section, we will introduce some important definitions and results that will be necessary in the following sections.

Definition 2.1 ([2]). Let X be a universe set and let E be a fixed set of parameters. If $G_E : E \rightarrow 2^X$ is a function, then an ordered pair (G, E) is called a *soft set*, where 2^X is the power set of X . The set of all soft sets over X is denoted by $P(X)^E$.

Definition 2.2 ([15]). Let $F_E, G_E \in P(X)^E$. Then:

- (i) F_E is called a *null soft set*, denoted by Φ , if $F(e) = \emptyset \forall e \in E$,
- (ii) F_E is called an *absolute soft set*, denoted by X_E , if $F(e) = X \forall e \in E$,
- (iii) F_E is called a *soft subset* of G_E , denoted by $F_E \subseteq G_E$, if $F(e) \subseteq G(e) \forall e \in E$,

(iv) F_E and G_E are said to be equal, denoted by $F_E = G_E$, if $F_E \subseteq G_E$ and $G_E \subseteq F_E$,

(v) The union of F_E and G_E , denoted by $F_E \sqcup G_E$, is a soft set H_E over X defined as: $H(e) = F(e) \cup G(e) \forall e \in E$,

(vi) The intersection of F_E and G_E , denoted by $F_E \sqcap G_E$, is a soft set H_E over X defined as: $H(e) = F(e) \cap G(e) \forall e \in E$,

(vii) The difference of F_E and G_E , denoted by $F_E - G_E$, is a soft set H_E over X defined as: $H(e) = F(e) - G(e) \forall e \in E$,

(viii) The complement of F_E , denoted by F_E^c , is a soft set over X defined as: $F^c(e) = (F(e))^c \forall e \in E$.

Definition 2.3 ([16]). Let $\phi : X \rightarrow Y$ and $\psi : E \rightarrow K$ be two mappings. Then the mapping ϕ_ψ is called a *soft mapping from X to Y* , denoted by $\phi_\psi : P(X)^E \rightarrow P(Y)^K$. The mapping ϕ_ψ itself is defined as follows: for any soft set $G_E \in P(X)^E$, ϕ_ψ maps it to a soft set $F_K \in P(Y)^K$ such that:

$$F_K(k) = \{y \in Y : y = \phi(x), x \in X, \text{ and } k = \psi(e) \text{ } e \in E\}.$$

Let $G_E \in P(X)^E$ and let $F_K \in P(Y)^K$. Then

(i) the image of G_E under ϕ_ψ , denoted by $\phi_\psi(G_E)$, is the soft set over Y defined as follows: for each $k \in K$,

$$\phi_\psi(G_E)(k) = \begin{cases} \bigcup_{e \in \psi^{-1}(k)} G_E(e) & \text{if } \psi^{-1}(k) \neq \emptyset \\ \emptyset & \text{otherwise,} \end{cases}$$

(ii) the inverse image of a soft set F_K under ϕ_ψ , denoted by $\phi_\psi^{-1}(F_K)$, is the soft set over X defined as follows: for each $e \in E$,

$$\phi_\psi^{-1}(F_K)(e) = \phi^{-1}(F_K(\psi(e))).$$

From Definition 2.4, we can easily see that $F_K(k) = \phi_\psi(G_E)(k)$ for each $k \in K$ if and only if for each $y \in Y$, the following conditions hold:

- (1) there exist $x \in X$, $e \in E$ such that $y = \phi(x)$ and $k = \psi(e)$, $x \in G_E(e)$,
- (2) there exist $e \in E$, $x \in G_E(e)$ such that $k = \psi(e)$ and $y = \phi(x)$.

Definition 2.4 ([17]). Let $P(X)^E$ and $P(Y)^K$ be two families of soft sets over X and Y , respectively. A soft mapping $\phi_\psi : P(X)^E \rightarrow P(Y)^K$ is called *soft surjective* (*injective*) *mapping*, if ϕ, ψ are surjective (injective) mappings, respectively. A soft mapping which is a soft surjective and soft injective mapping is called a *soft bijection mapping*.

Proposition 2.5 ([16]). Consider $\phi_\psi : P(X)^E \rightarrow P(Y)^K$ is a soft map and let G_E and H_K be two soft subsets of $P(X)^E$ and $P(Y)^K$, respectively. Then we have the following results:

- (1) $G_E \subseteq \phi_\psi^{-1}(\phi_\psi(G_E))$ and the equality relation holds if ϕ_ψ is injective,
- (2) $\phi_\psi(\phi_\psi^{-1}(H_K)) \subseteq H_K$ and the equality relation holds if ϕ_ψ is surjective.

Definition 2.6 ([3]). A *soft topology* on a set X is a collection τ of soft sets over X that satisfies the following conditions:

- (i) both the universal set X_E and the empty set Φ belong to τ ,
- (ii) the union of any collection of soft sets in τ belongs to τ ,

(iii) the intersection of any two soft sets in τ belongs to τ .

The elements of τ are called *open soft sets* in X and the complement of an open soft set is called a *closed soft set* in X .

Definition 2.7 ([18]). A *partial order relation* \leq on a set X satisfies three properties: reflexivity, antisymmetry, and transitivity. The equality relation on X is a special case of a partial order relation, denoted by \blacktriangle , where $\blacktriangle = \{(a, a) : a \in X\}$ and it satisfies all three properties of a partial order relation.

Definition 2.8 ([1]). A triple (X, τ, \leq) is said to be a *topological ordered space*, if (X, τ) is a topological space and (X, \leq) is a partially ordered set.

Definition 2.9 ([4]). A triple (X, E, \leq) is said to be a *partially ordered soft space*, where \leq is a partial order relation on X .

Definition 2.10 ([4]). An *increasing soft operator* i and a *decreasing soft operator* d can be defined on a partially ordered soft space (X, E, \leq) as follows: for each $F_E \in P(X)^E$,

(i) $i(F_E) = (iF)_E$, where iF is a mapping of E into X given by

$$iF(\alpha) = i(F(\alpha)) = \{a \in X : b \leq a \text{ for some } b \in F(\alpha)\}.$$

(ii) $d(F_E) = (dF)_E$, where dF is a mapping of E into X given by

$$dF(\alpha) = d(F(\alpha)) = \{a \in X : a \leq b \text{ for some } b \in F(\alpha)\}.$$

Definition 2.11 ([4]). Let (X, E, \leq) be a partially ordered soft space and let F_E be a soft set over X . Then F_E is said to be:

- (i) *increasing*, if F_E is equal to its own image under i ,
- (ii) *decreasing*, if F_E is equal to its own image under d ,
- (iii) *balancing*, if it is both increasing and decreasing.

Proposition 2.12 ([4]). Let (X, E, \leq) be a partially ordered soft space, and let $\{F_E^\beta : \beta \in \Omega\}$ be a collection of increasing (resp. decreasing) soft sets in (X, E, \leq) . Then

- (1) $\sqcup_{\beta \in \Omega} F_E^\beta$ is increasing (resp. decreasing),
- (2) $\sqcap_{\beta \in \Omega} F_E^\beta$ is increasing (resp. decreasing).

Definition 2.13 ([4]). A quadrable system (X, τ, E, \leq) is said to be a *soft topological ordered space*, if (X, τ, E) is a soft topological space and (X, \leq) is a partially ordered set.

Definition 2.14 ([4]). Let (X, τ, E, \leq) be a soft topological ordered space. A soft set G_E over X is called:

- (i) an *increasing open soft* (briefly, *IO-soft*), if G_E is open and increasing,
- (ii) a *decreasing open soft set* (briefly, *DO-soft set*), if G_E is open and decreasing,
- (iii) a *balancing open soft set* (briefly, *BO-soft set*), if G_E is an *IO-soft* and *DO-soft set*.

Definition 2.15 ([19]). A soft map $\phi_\psi : (X, \tau, E, \leq_1) \rightarrow (Y, \eta, K, \leq_2)$ is said to be *IS* (resp. *DS*, *BS*)-*continuous*, if the inverse image of each open soft subset of Y is a soft *IO* (resp. *DO*, *BO*)-soft subset of X .

Definition 2.16 ([6]). A quadruple (X, τ_1, τ_2, E) is said to be a *soft bitopological space*, where τ_1, τ_2 are arbitrary soft topologies on X with a fixed set of parameter E .

Definition 2.17 ([13]). A soft set F_E in a soft bitopological space (X, τ_1, τ_2, E) is said to be a pairwise open soft (briefly, *PO-soft set*), if it can be expressed as the union of a τ_1 -open soft set F_E^1 and a τ_2 -open soft set F_E^2 , i.e., $F_E = F_E^1 \sqcup F_E^2$. On the other hand, F_E is said to be a *pairwise closed soft* (briefly, *PC-soft set*), if its complement in X , denoted by $X - F_E$, is a *PO-soft set*.

Definition 2.18 ([13]). The following concepts are defined for a subset G_E of (X, τ_1, τ_2, E) .

(i) The *pairwise soft closure* of G_E , denoted by $cl_{12}^s(G_E)$, is the intersection of all *PC-soft sets* containing G_E .

(ii) The *pairwise soft interior* of G_E , denoted by $int_{12}^s(G_E)$, is the union of all *PO-soft sets* which are contained in G_E .

Definition 2.19 ([20]). A soft mapping $\phi_\psi : (X, \tau_1, \tau_2, E) \rightarrow (Y, \eta_1, \eta_2, K)$ is said to be:

(i) *pairwise soft continuous* (briefly, *P-soft continuous*), if the inverse image of any *PO-soft set* in (Y, η_1, η_2, K) is a *PO-soft set* in (X, τ_1, τ_2, E) ,

(ii) *pairwise soft open* (briefly, *P-soft open*), if the image of any *PO-soft set* in (X, τ_1, τ_2, E) is a *PO-soft set* in (Y, η_1, η_2, K) ,

(iii) *pairwise soft closed* (briefly, *P-soft closed*), if the image of any *PC-soft set* in (X, τ_1, τ_2, E) is a *PC-soft set* in (Y, η_1, η_2, K) ,

(iv) *pairwise soft homeomorphism* (briefly, *P-soft homeomorphism*), if it is bijective, *P-soft continuous* and *P-soft open*.

Definition 2.20 ([14]). A system $(X, \tau_1, \tau_2, E, \leq)$ is said to be a *soft bitopological ordered space*, if (X, τ_1, τ_2, E) is a soft bitopological space and (X, \leq) is a partially ordered set.

Definition 2.21 ([14]). Let $(X, \tau_1, \tau_2, E, \leq)$ be a soft bitopological ordered space and let G_E be a soft set over X . Then G_E is said to be:

(i) an *increasing* (resp. a *decreasing*) *pairwise open* (briefly, *IPO*) (resp. briefly, *DPO*)-*soft set*, if it can be expressed as $G_E = G_E^1 \sqcup G_E^2$, where $G_E^\beta \in \tau_\beta$ and is increasing (decreasing) for $\beta = 1, 2$.

(ii) an *increasing* (resp. *decreasing*) *pairwise closed* (briefly, *IPC*) (resp. briefly, *DPC*)-*soft set*, if it can be expressed as $G_E = G_E^1 \cap G_E^2$, where $G_E^\beta \in \tau_\beta^c$ and is increasing (decreasing) for $\beta = 1, 2$.

(iii) a *balancing pairwise open* (resp. *closed*) (briefly, *BPO*) (resp. briefly, *BPC*)-*soft set*, if it is both increasing pairwise open (resp. closed) and decreasing pairwise open (resp. closed).

Definition 2.22 ([14]). Let $(X, \tau_1, \tau_2, E, \leq)$ be a soft bitopological ordered space and let G_E be a soft set over X .

(i) The *increasing* (resp. *decreasing*, *balancing*) *pairwise soft closure* of G_E , denoted by $Icl_{12}^s(G_E)$ (resp. $Dcl_{12}^s(G_E)$, $Bcl_{12}^s(G_E)$), is the intersection of all *IPO* (resp. *DPO*, *BPO*)-*soft sets* containing G_E .

(ii) The *increasing* (resp. *decreasing, balancing*) *pairwise soft interior* of G_E , denoted by $\text{Int}_{12}^s(G_E)$ (resp. $\text{Dint}_{12}^s(G_E)$, $\text{Bint}_{12}^s(G_E)$), is the union of all *IPO* (*DPO*, *BPO*)-soft sets contained in G_E .

3. *ISP* (*DSP*, *BSP*)-CONTINUOUS MAPPINGS

In this section, we introduce a different kind of a definition of soft continuity in a soft bitopological ordered space.

Definition 3.1. A soft mapping $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ is said to be *ISP* (resp. *DSP*, *BSP*)-continuous, if $\phi_\psi^{-1}(G_E)$ is an *IPO* (resp. *DPO*, *BPO*)-soft set in X , whenever G_E is a *PO*-soft set in Y .

Example 3.2. Let $E = \{\alpha_1, \alpha_2\}$ and $\leq = \blacktriangle \cup \{(a, b), (b, c), (a, c)\}$ be a partial order relation on $X = \{a, b, c\}$ and let $\tau_1 = \{X_E, \Phi, G_E\}$ and $\tau_2 = \{X_E, \Phi, F_E\}$, where $G_E = \{(\alpha_1, \{b\}), (\alpha_2, \{a\})\}$, $F_E = \{(\alpha_1, \{a, c\}), (\alpha_2, \emptyset)\}$. Then clearly, $(X, \tau_1, \tau_2, E, \leq)$ is a soft bitopological ordered space. Consider ϕ and ψ are identity mappings. Since G_E is a *PO*-soft set and $\phi_\psi^{-1}(G)(\alpha_1) = \phi^{-1}(G(\psi(\alpha_1))) = \phi^{-1}(G(\alpha_1)) = \phi^{-1}(\{b\}) = \{b\}$, $\phi_\psi^{-1}(G_E)$ is not an element in $x\text{POS}(X, \tau_1, \tau_2)_E$, $x = I; D; B$. Then ϕ_ψ is *P*-soft continuous but it is not *xSP*-continuous for $x = I; D; B$.

Example 3.3. Let $E = \{\alpha_1, \alpha_2\} = K$ and $X = \{a, b, c\} = Y$, $\leq_1 = \blacktriangle \cup \{(a, c)\}$, $\leq_2 = \blacktriangle \cup \{(a, b), (b, c), (a, c)\}$ and let $\tau_1 = \{X_E, \Phi, G_E\} = \eta_1$ and $\tau_2 = \{X_E, \Phi, F_E\} = \eta_2$, where $G_E = \{(\alpha_1, \{c\}), (\alpha_2, \{a, c\})\}$, $F_E = \{(\alpha_1, \{c\}), (\alpha_2, \{b, c\})\}$. Then clearly, $(X, \tau_1, \tau_2, E, \leq_1)$ and $(Y, \eta_1, \eta_2, K, \leq_2)$ are soft bitopological ordered spaces and $\tau_{12} = \eta_{12} = \{X_E, \Phi, G_E, F_E, H_E\}$, where $H_E = \{(\alpha_1, \{c\}), (\alpha_2, X)\}$. Consider ϕ and ψ are identity mappings. Then we have

$$\begin{aligned}\phi_\psi^{-1}(G)(\alpha_1) &= \phi^{-1}(G(\psi(\alpha_1))) = \phi^{-1}(G(\alpha_1)) = \phi^{-1}(\{c\}) = \{c\}, \\ \phi_\psi^{-1}(G)(\alpha_2) &= \phi^{-1}(G(\psi(\alpha_2))) = \phi^{-1}(G(\alpha_2)) = \phi^{-1}(\{a, c\}) = \{a, c\}, \\ \phi_\psi^{-1}(F)(\alpha_1) &= \phi^{-1}(F(\psi(\alpha_1))) = \phi^{-1}(F(\alpha_1)) = \phi^{-1}(\{c\}) = \{c\}, \\ \phi_\psi^{-1}(F)(\alpha_2) &= \phi^{-1}(F(\psi(\alpha_2))) = \phi^{-1}(F(\alpha_2)) = \phi^{-1}(\{b, c\}) = \{b, c\}, \\ \phi_\psi^{-1}(H)(\alpha_1) &= \phi^{-1}(H(\psi(\alpha_1))) = \phi^{-1}(H(\alpha_1)) = \phi^{-1}(\{c\}) = \{c\}, \\ \phi_\psi^{-1}(H)(\alpha_2) &= \phi^{-1}(H(\psi(\alpha_2))) = \phi^{-1}(H(\alpha_2)) = \phi^{-1}(X) = X.\end{aligned}$$

Thus $\phi_\psi^{-1}(G_E) = G_E$, $\phi_\psi^{-1}(F_E) = F_E$ and $\phi_\psi^{-1}(H_E) = H_E$ are *IPO*-soft sets in X . So ϕ_ψ is *ISP*-continuous but it is not *DSP*-continuous. However ϕ_ψ is not *BSP*-continuous.

Example 3.4. Let $E = \{\alpha_1, \alpha_2\} = K$ and $X = \{a, b, c\} = Y$, $\leq_1 = \leq_2 = \blacktriangle \cup \{(a, b), (b, c), (a, c)\}$ and let $\tau_1 = \{X_E, \Phi, G_E^1\}$, $\tau_2 = \{X_E, \Phi, G_E^2\}$, and $\eta_1 = \{Y_K, \Phi, F_K^1\}$, $\eta_2 = \{Y_K, \Phi, F_K^2\}$, where $G_E^1 = \{(\alpha_1, \{a\}), (\alpha_2, \emptyset)\}$, $G_E^2 = \{(\alpha_1, \emptyset), (\alpha_2, \{a, b\})\}$, $F_K^1 = \{(\alpha_1, \{a, b\}), (\alpha_2, \{b\})\}$, $F_K^2 = \{(\alpha_1, \emptyset), (\alpha_2, \{b\})\}$. Then clearly, $(X, \tau_1, \tau_2, E, \leq_1)$ and $(Y, \eta_1, \eta_2, K, \leq_2)$ are soft bitopological ordered spaces and $\tau_{12} = \{X_E, \Phi, G_E^1, G_E^2, G_E^3\}$ and $\eta_{12} = \{Y_K, \Phi, F_K^1, F_K^2\}$, where $G_E^3 = \{(\alpha_1, \{a\}), (\alpha_2, \{a, b\})\}$.

Define: $\phi(a) = b, \phi(b) = a, \phi(c) = c$ and $\psi(\alpha_1) = \alpha_2, \psi(\alpha_2) = \alpha_1$. Then we have

$$\phi_\psi^{-1}(F^1)(\alpha_1) = \phi^{-1}(F^1(\psi(\alpha_1))) = \phi^{-1}(F^1(\alpha_2)) = \phi^{-1}(\{b\}) = \{a\},$$

$$\phi_{\psi}^{-1}(F^1)(\alpha_2) = \phi^{-1}(F^1(\psi(\alpha_2))) = \phi^{-1}(F^1(\alpha_1)) = \phi^{-1}(\{a, b\}) = \{a, b\},$$

Then, $\phi_{\psi}^{-1}(F_K^1) = G_E^3$ is a *DPO*-soft set in X .

$$\phi_{\psi}^{-1}(F^2)(\alpha_1) = \phi^{-1}(F^2(\psi(\alpha_1))) = \phi^{-1}(F^2(\alpha_2)) = \phi^{-1}(\{b\}) = \{a\},$$

$$\phi_{\psi}^{-1}(F^2)(\alpha_2) = \phi^{-1}(F^2(\psi(\alpha_2))) = \phi^{-1}(F^2(\alpha_1)) = \phi^{-1}(\emptyset) = \emptyset.$$

Thus $\phi_{\psi}^{-1}(F_K^1) = G_E^3$ and $\phi_{\psi}^{-1}(F_K^2) = G_E^1$ are *DPO*-soft sets in X . So ϕ_{ψ} is *DSP*-continuous, but it is not *ISP*-continuous map. However ϕ_{ψ} is not *BSP*-continuous.

Remark 3.5. The following diagram shows the relation between *ISP*-continuous, *DSP*-continuous and *BSP*-continuous mappings. For a soft mapping $\phi_{\psi} : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ where $\phi : X \rightarrow Y, \psi : E \rightarrow K$, we have the diagram:

$$\begin{array}{ccc} \phi_{\psi} \text{ is } ISP\text{-continuous} & \leftrightarrow & \phi_{\psi} \text{ is } DSP\text{-continuous} \\ & \searrow \quad \swarrow & \\ & \phi_{\psi} \text{ is } P\text{-soft continuous} & \\ & \swarrow \quad \searrow & \\ & \phi_{\psi} \text{ is } BSP\text{-continuous} & \end{array}$$

Theorem 3.6. Let $\phi_{\psi} : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ be a soft mapping. The following statements are equivalent:

- (1) ϕ_{ψ} is *ISP*-continuous,
- (2) $\phi_{\psi}(Icl_{12}^s(G_E)) \subseteq cl_{12}^s(\phi_{\psi}(G_E))$ for any $G_E \in P(X)^E$,
- (3) $Icl_{12}^s(\phi_{\psi}^{-1}(F_K)) \subseteq \phi_{\psi}^{-1}(cl_{12}^s(F_K))$ for any $F_K \in P(Y)^K$,
- (4) for any *PC*-soft subset M_K of $(Y, \eta_1, \eta_2, K, \leq_2)$, $\phi_{\psi}^{-1}(M_K)$ is *DPC*-soft subset of $(X, \tau_1, \tau_2, E, \leq_1)$.

Proof. (1) \Rightarrow (2): Suppose (1) holds and let $G_E \in P(X)^E$. Since $Y - cl_{12}^s(\phi_{\psi}(G_E))$ is *PO*-soft set in Y and ϕ_{ψ} is *ISP*-continuous, $\phi_{\psi}^{-1}(Y - cl_{12}^s(\phi_{\psi}(G_E)))$ is *IPO*-soft set in X . Then clearly, $X - \phi_{\psi}^{-1}(Y - cl_{12}^s(\phi_{\psi}(G_E)))$ is *DPC*-soft set in X . Since $X - \phi_{\psi}^{-1}(Y - cl_{12}^s(\phi_{\psi}(G_E))) = \phi_{\psi}^{-1}(cl_{12}^s(\phi_{\psi}(G_E)))$, $\phi_{\psi}^{-1}(cl_{12}^s(\phi_{\psi}(G_E)))$ is *DPC*-soft set in X . Since $G_E \subseteq \phi_{\psi}^{-1}(cl_{12}^s(\phi_{\psi}(G_E)))$ and $Icl_{12}^s(G_E)$ is the smallest *DPC*-soft set containing G_E in X , we have

$$Icl_{12}^s(G_E) \subseteq \phi_{\psi}^{-1}(cl_{12}^s(\phi_{\psi}(G_E))), \quad \phi_{\psi}(Icl_{12}^s(G_E)) \subseteq \phi_{\psi}(\phi_{\psi}^{-1}(cl_{12}^s(\phi_{\psi}(G_E)))).$$

Thus $\phi_{\psi}(Icl_{12}^s(G_E)) \subseteq cl_{12}^s(\phi_{\psi}(G_E))$.

(2) \Rightarrow (3): Suppose (2) holds, and let $F_K \in P(Y)^K$ and let $H_E = \phi_{\psi}^{-1}(F_K)$. Then clearly, $\phi_{\psi}(H_E) = \phi_{\psi}(\phi_{\psi}^{-1}(F_K)) \subseteq F_K$. Thus $cl_{12}^s(\phi_{\psi}(H_E)) \subseteq cl_{12}^s(F_K)$. By the condition (2), we have

$$Icl_{12}^s(\phi_{\psi}^{-1}(F_K)) = Icl_{12}^s(H_E) \subseteq \phi_{\psi}^{-1}(\phi_{\psi}(Icl_{12}^s(H_E))) \subseteq \phi_{\psi}^{-1}(cl_{12}^s(\phi_{\psi}(H_E))).$$

But $\phi_{\psi}^{-1}(cl_{12}^s(\phi_{\psi}(H_E))) \subseteq \phi_{\psi}^{-1}(cl_{12}^s(F_K))$. So $Icl_{12}^s(\phi_{\psi}^{-1}(F_K)) \subseteq \phi_{\psi}^{-1}(cl_{12}^s(F_K))$.

(3) \Rightarrow (4): Suppose (3) holds and let M_K be any *PC*-soft subset of Y . Then clearly, $Icl_{12}^s(\phi_{\psi}^{-1}(M_K)) \subseteq \phi_{\psi}^{-1}(cl_{12}^s(M_K))$. Thus $\phi_{\psi}^{-1}(M_K)$ is a *DPC*-soft set of X .

(4) \Rightarrow (1): Suppose (4) holds and let N_K be a *PO*-soft subset of Y . Then $\phi_{\psi}^{-1}(Y - N_K)$ is a *DPC*-soft set of X , since $Y - N_K$ is a *PC*-soft subset of Y . But $X -$

$\phi_\psi^{-1}(N_K) = \phi_\psi^{-1}(Y - N_K)$. Thus $X - \phi_\psi^{-1}(N_K)$ is a *DPC*-soft set of X . So $\phi_\psi^{-1}(N_K)$ is an *IPO*-soft set of X . Hence ϕ_ψ is *ISP*-continuous. \square

The following two theorems characterized *DSP*-continuous and *BSP*-continuous mappings, whose proofs are similar to as that of the above Theorem 3.6.

Theorem 3.7. For a soft mapping $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$, the following statements are equivalent:

- (1) ϕ_ψ is *DSP*-continuous,
- (2) $\phi_\psi(Dcl_{12}^s(G_E)) \subseteq cl_{12}^s(\phi_\psi(G_E))$ for any $G_E \in P(X)^E$,
- (3) $Dcl_{12}^s(\phi_\psi^{-1}(F_K)) \subseteq \phi_\psi^{-1}(cl_{12}^s(F_K))$ for any $F_K \in P(Y)^K$,
- (4) for any *PC*-soft subset M_K of $(Y, \eta_1, \eta_2, K, \leq_2)$, $\phi_\psi^{-1}(M_K)$ is *IPC*-soft subset of $(X, \tau_1, \tau_2, E, \leq_1)$.

Theorem 3.8. For a soft mapping $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$, the following statements are equivalent:

- (1) ϕ_ψ is *BSP*-continuous,
- (2) $\phi_\psi(Bcl_{12}^s(G_E)) \subseteq cl_{12}^s(\phi_\psi(G_E))$ for any $G_E \in S(X)^E$,
- (3) $Bcl_{12}^s(\phi_\psi^{-1}(F_K)) \subseteq \phi_\psi^{-1}(cl_{12}^s(F_K))$ for any $F_K \in S(Y)^K$.
- (4) for any *P*-closed soft subset M_K of $(Y, \eta_1, \eta_2, K, \leq_2)$, $\phi_\psi^{-1}(M_K)$ is *BPC*-soft subset of $(X, \tau_1, \tau_2, E, \leq_1)$.

Theorem 3.9. Let $(X, \tau_1, \tau_2, E, \leq_1)$ and $(Y, \eta_1, \eta_2, K, \leq_2)$ are soft bitopological ordered spaces and $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ be a soft mapping. Then ϕ_ψ is an *ISP*-continuous if and only if $\phi_\psi : (X, \tau_1, E, \leq_1) \rightarrow (Y, \eta_1, K, \leq_2)$ and $\phi_\psi : (X, \tau_2, E, \leq_1) \rightarrow (Y, \eta_2, K, \leq_2)$ are *IS*-continuous mappings.

Proof. Suppose ϕ_ψ is an *ISP*-continuous mapping and let G_E be a *PO*-soft set over Y . Then there exist $G_E^1 \in \eta_1$ and $G_E^2 \in \eta_2$ such that $G_E = G_E^1 \sqcup G_E^2$. Since ϕ_ψ is an *ISP*-continuous mapping, $\phi_\psi^{-1}(G_E) = \phi_\psi^{-1}(G_E^1 \sqcup G_E^2) = \phi_\psi^{-1}(G_E^1) \sqcup \phi_\psi^{-1}(G_E^2)$. In this case, $\phi_\psi^{-1}(G_E^1)$ is τ_1 -increasing and $\phi_\psi^{-1}(G_E^2)$ is τ_2 -increasing. Thus $\phi_\psi : (X, \tau_1, E, \leq_1) \rightarrow (Y, \eta_1, K, \leq_2)$ and $\phi_\psi : (X, \tau_2, E, \leq_1) \rightarrow (Y, \eta_2, K, \leq_2)$ are *IS*-continuous mappings.

Conversely, suppose $\phi_\psi : (X, \tau_1, E, \leq_1) \rightarrow (Y, \eta_1, K, \leq_2)$ and $\phi_\psi : (X, \tau_2, E, \leq_1) \rightarrow (Y, \eta_2, K, \leq_2)$ are *IS*-continuous mappings and let $G_E^1 \in \eta_1, G_E^2 \in \eta_2$. Then there exists a *PO*-soft set G_E such that $G_E = G_E^1 \sqcup G_E^2$. Since $\phi_\psi : (X, \tau_1, E, \leq_1) \rightarrow (Y, \eta_1, K, \leq_2)$ and $\phi_\psi : (X, \tau_2, E, \leq_1) \rightarrow (Y, \eta_2, K, \leq_2)$ are *IS*-continuous mappings, $\phi_\psi^{-1}(G_E^1)$ is *IO*-soft set in τ_1 and $\phi_\psi^{-1}(G_E^2)$ is *IO*-soft set in τ_2 . Thus $\phi_\psi^{-1}(G_E^1) \sqcup \phi_\psi^{-1}(G_E^2) = \phi_\psi^{-1}(G_E^1 \sqcup G_E^2) = \phi_\psi^{-1}(G_E)$ is an *IPO*-soft set. So ϕ_ψ is an *ISP*-continuous mapping. \square

The following two theorems characterized *DSP*-continuous and *BSP*-continuous mappings, whose proofs are similar to as that of the above Theorem 3.9.

Theorem 3.10. Let $(X, \tau_1, \tau_2, E, \leq_1)$ and $(Y, \eta_1, \eta_2, K, \leq_2)$ are soft bitopological ordered spaces and $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ be a soft mapping. Then ϕ_ψ is an *DSP*-continuous if and only if $\phi_\psi : (X, \tau_1, E, \leq_1) \rightarrow (Y, \eta_1, K, \leq_2)$ and $\phi_\psi : (X, \tau_2, E, \leq_1) \rightarrow (Y, \eta_2, K, \leq_2)$ are *DS*-continuous mappings.

Theorem 3.11. Let $(X, \tau_1, \tau_2, E, \leq_1)$ and $(Y, \eta_1, \eta_2, K, \leq_2)$ are soft bitopological ordered spaces and $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ be a soft mapping. Then ϕ_ψ is an *BSP*-continuous if and only if $\phi_\psi : (X, \tau_1, E, \leq_1) \rightarrow (Y, \eta_1, K, \leq_2)$ and $\phi_\psi : (X, \tau_2, E, \leq_1) \rightarrow (Y, \eta_2, K, \leq_2)$ are *BS*-continuous mappings.

Theorem 3.12. Let $(X, \tau_1, \tau_2, E, \leq_1)$, $(Y, \eta_1, \eta_2, K, \leq_2)$ and $(Z, \delta_1, \delta_2, L, \leq_3)$ are soft bitopological ordered spaces. If $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ and $\Delta_\beta : (Y, \eta_1, \eta_2, K, \leq_2) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ are *ISP*-continuous mappings, then $\Delta_\beta \circ \phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ is *ISP*-continuous.

Proof. Let $W_L \in \delta_{12}$ be a *PO*-soft set over Z and let us show that $(\Delta_\beta \circ \phi_\psi)^{-1}(W_L)$ is an *IPO*-soft set in X . Since $(\Delta_\beta \circ \phi_\psi)^{-1}(W_L) = \phi_\psi^{-1}(\Delta_\beta^{-1}(W_L))$ and Δ_β is *ISP*-continuous, $\Delta_\beta^{-1}(W_L)$ is an *IPO*-soft set in Y . On the other hand, since ϕ_ψ is *ISP*-continuous mapping, $\phi_\psi^{-1}(\Delta_\beta^{-1}(W_L))$ is an *IPO*-soft set in X . Then $\Delta_\beta \circ \phi_\psi$ is *ISP*-continuous mapping. \square

The following two theorems characterized *DSP*-continuous and *BSP*-continuous mappings, whose proofs are similar to as that of the above Theorem 3.12.

Theorem 3.13. Let $(X, \tau_1, \tau_2, E, \leq_1)$, $(Y, \eta_1, \eta_2, K, \leq_2)$ and $(Z, \delta_1, \delta_2, L, \leq_3)$ are soft bitopological ordered spaces. If $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ and $\Delta_\beta : (Y, \eta_1, \eta_2, K, \leq_2) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ are *DSP*-continuous mappings, then $\Delta_\beta \circ \phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ is *DSP*-continuous.

Theorem 3.14. Let $(X, \tau_1, \tau_2, E, \leq_1)$, $(Y, \eta_1, \eta_2, K, \leq_2)$ and $(Z, \delta_1, \delta_2, L, \leq_3)$ are soft bitopological ordered spaces. If $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ and $\Delta_\beta : (Y, \eta_1, \eta_2, K, \leq_2) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ are *BSP*-continuous mappings, then $\Delta_\beta \circ \phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ is *BSP*-continuous.

4. *ISP* (*DSP*, *BSP*)-OPEN AND *ISP* (*DSP*, *BSP*)-CLOSED MAPPINGS

In this section, we introduce the concepts of *ISP*-open and *ISP*-closed mappings, as well as *DSP*-open and *DSP*-closed maps, and *BSP*-open and *BSP*-closed maps. We then establish the relationships among these concepts and provide equivalent conditions for each type of soft map. Finally, we investigate the interrelations between these soft maps and their counterparts on bitopological ordered spaces.

Definition 4.1. A soft mapping $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ is called *ISP* (resp. *SP*, *BSP*)-open, if $\phi_\psi(G_E)$ is *IPO* (resp. *DPO*, *BPO*)-soft set in Y , whenever G_E is a *PO*-soft set in X .

Example 4.2. From Example 3.2, $\tau_{12} = \{X_E, \Phi, G_E, F_E, H_E\}$, where $H_E = \{(\alpha_1, X), (\alpha_2, \{a\})\}$. Then clearly, G_E , F_E and H_E are *PO*-soft sets in X . Moreover, we have

$$\begin{aligned}\phi_\psi(G)(\alpha_1) &= \cup_{e \in \psi^{-1}(\alpha_1)} \phi(G(e)) = \phi(G(\alpha_1)) = \phi(\{b\}) = \{b\}, \\ \phi_\psi(G)(\alpha_2) &= \cup_{e \in \psi^{-1}(\alpha_2)} \phi(G(e)) = \phi(G(\alpha_2)) = \phi(\{a\}) = \{a\}, \\ \phi_\psi(F)(\alpha_1) &= \cup_{e \in \psi^{-1}(\alpha_1)} \phi(F(e)) = \phi(F(\alpha_1)) = \phi(\{a, c\}) = \{a, c\}, \\ \phi_\psi(F)(\alpha_2) &= \cup_{e \in \psi^{-1}(\alpha_2)} \phi(F(e)) = \phi(F(\alpha_2)) = \phi(\emptyset) = \emptyset, \\ \phi_\psi(H)(\alpha_1) &= \cup_{e \in \psi^{-1}(\alpha_1)} \phi(H(e)) = \phi(H(\alpha_1)) = \phi(\{c\}) = \{c\}, \\ \phi_\psi(H)(\alpha_2) &= \cup_{e \in \psi^{-1}(\alpha_2)} \phi(H(e)) = \phi(H(\alpha_2)) = \phi(X) = X.\end{aligned}$$

Thus ϕ_ψ is P -soft open mapping but it is not xSP -open for $x = I; D; B$.

Example 4.3. Let $X, Y, \tau_1, \tau_2, \eta_1, \eta_2, E, K, \phi, \psi$ as in Example 3.3. Consider $\leq_1 = \blacktriangle \cup \{(a, b), (a, c)\}$ and $\leq_2 = \blacktriangle \cup \{(a, c)\}$. Then we get

$$\begin{aligned}\phi_\psi(G)(\alpha_1) &= \phi(G(\alpha_1)) = \phi(\{c\}) = \{c\}, \\ \phi_\psi(G)(\alpha_2) &= \phi(G(\alpha_2)) = \phi(\{a, c\}) = \{a, c\}, \\ \phi_\psi(F)(\alpha_1) &= \phi(F(\alpha_1)) = \phi(\{c\}) = \{c\}, \\ \phi_\psi(F)(\alpha_2) &= \phi(F(\alpha_2)) = \phi(\{b, c\}) = \{b, c\}, \\ \phi_\psi(M)(\alpha_1) &= \phi(M(\alpha_1)) = \phi(\{c\}) = \{c\}, \\ \phi_\psi(M)(\alpha_2) &= \phi(M(\alpha_2)) = \phi(X) = Y.\end{aligned}$$

Thus $\phi_\psi(G_E) = G_E$, $\phi_\psi(G_E) = G_E$ and $\phi_\psi(M_E) = H_E$ are IPO -soft sets in Y . So ϕ_ψ is an ISP -open mapping, but it is not a DSP -open mapping. However ϕ_ψ is not BSP -open.

Example 4.4. In Example 3.4, consider $\leq_1 = \leq_2 = \blacktriangle \cup \{(b, a), (a, c), (b, c)\}$, $\eta_1 = \{Y_K, \Phi, F_K^1, F_K^3\}$, where $F_K^3 = \{(\alpha_1, \{a, b\}), (\alpha_2, \emptyset)\}$ and $\eta_{12} = \{Y_K, \Phi, F_K^1, F_K^2, F_K^3\}$. Then we have

$$\begin{aligned}\phi_\psi(G^1)(\alpha_1) &= \phi(G^1(\alpha_2)) = \phi(\emptyset) = \emptyset, \\ \phi_\psi(G^1)(\alpha_2) &= \phi(G(\alpha_1)) = \phi(\{a\}) = \{b\}, \\ \phi_\psi(G^2)(\alpha_1) &= \phi(G^2(\alpha_2)) = \phi(\{a, b\}) = \{a, b\}, \\ \phi_\psi(G^2)(\alpha_2) &= \phi(G^2(\alpha_1)) = \phi(\emptyset) = \emptyset, \\ \phi_\psi(G^3)(\alpha_1) &= \phi(G^3(\alpha_2)) = \phi(\{a, b\}) = \{a, b\}, \\ \phi_\psi(G^3)(\alpha_3) &= \phi(G^2(\alpha_1)) = \phi(\{a\}) = \{b\}.\end{aligned}$$

Thus $\phi_\psi(G_E^1) = F_K^2$, $\phi_\psi(G_E^2) = F_K^3$ and $\phi_\psi(G_E^3) = F_K^1$ are DPO -soft sets in Y . So ϕ_ψ is a DSP -open mapping, but it is not an ISP -open mapping. However ϕ_ψ is not BSP -open.

Remark 4.5. For a soft mapping $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$, We have the following diagram:

$$\begin{array}{ccc}\phi_\psi \text{ is } ISP\text{-open} & \leftrightarrow & \phi_\psi \text{ is } DSP\text{-open} \\ & \searrow & \swarrow \\ & \phi_\psi \text{ is } P\text{-soft open} & \\ & \swarrow \quad \uparrow \quad \searrow & \\ & \phi_\psi \text{ is } BSP\text{-open.} & \end{array}$$

Theorem 4.6. Let $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ be a soft mapping. The following statements are equivalent:

- (1) ϕ_ψ is ISP -open,
- (2) $\phi_\psi(int_{12}^s(G_E)) \subseteq Int_{12}^s(\phi_\psi(G_E))$ for any $G_E \in P(X)^E$,
- (3) $int_{12}^s(\phi_\psi^{-1}(F_K)) \subseteq \phi_\psi^{-1}(Int_{12}^s(F_K))$ for any $F_K \in P(Y)^K$.

Proof. (1) \Rightarrow (3): Suppose (1) holds and let $F_K \in P(Y)^K$. Since $\text{int}_{12}^s(\phi_\psi^{-1}(F_K))$ is a PO -soft set in X and ϕ_ψ is ISP -open map, $\phi_\psi(\text{int}_{12}^s(\phi_\psi^{-1}(F_K)))$ is an IPO -soft set in Y . Also, $\phi_\psi(\text{int}_{12}^s(\phi_\psi^{-1}(F_K))) \subseteq \phi_\psi(\phi_\psi^{-1}(F_K)) \subseteq F_K$. Then $\phi_\psi(\text{int}_{12}^s(\phi_\psi^{-1}(F_K))) \subseteq \text{Iint}_{12}^s(F_K)$, since $\text{Iint}_{12}^s(F_K)$ is the largest IPO -soft set contained in F_K . Thus $\text{int}_{12}^s(\phi_\psi^{-1}(F_K)) \subseteq \phi_\psi^{-1}(\text{Iint}_{12}^s(F_K))$.

(3) \Rightarrow (2): Suppose (3) holds and let $G_E \in P(X)^E$. Replacing F_K by $\phi_\psi(G_E)$ in (3), we get $\text{int}_{12}^s(\phi_\psi^{-1}(\phi_\psi(G_E))) \subseteq \phi_\psi^{-1}(\text{Iint}_{12}^s(\phi_\psi(G_E)))$. Since $\text{int}_{12}^s(G_E) \subseteq \text{int}_{12}^s(\phi_\psi^{-1}(\phi_\psi(G_E)))$, $\text{int}_{12}^s(G_E) \subseteq \phi_\psi^{-1}(\text{Iint}_{12}^s(\phi_\psi(G_E)))$. Thus we have

$$\phi_\psi(\text{int}_{12}^s(G_E)) \subseteq \phi_\psi(\phi_\psi^{-1}(\text{Iint}_{12}^s(\phi_\psi(G_E)))) \subseteq \text{Iint}_{12}^s(\phi_\psi(G_E)).$$

So $\phi_\psi(\text{int}_{12}^s(G_E)) \subseteq \text{Iint}_{12}^s(\phi_\psi(G_E))$.

(2) \Rightarrow (1): Suppose (2) holds and let G_E be any PO -soft set in X . Then we have

$$\phi_\psi(G_E) = \phi_\psi(\text{int}_{12}^s(G_E)) \subseteq \text{Iint}_{12}^s(\phi_\psi(G_E)).$$

Thus $\phi_\psi(G_E)$ is an IPO -soft set in Y . So ϕ_ψ is ISP -open mapping. \square

The following two theorems characterized DSP -open and BSP -open mappings, whose proofs are similar to as that of the above Theorem 4.6.

Theorem 4.7. Let $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ be a soft mapping. The following statements are equivalent:

- (1) ϕ_ψ is DSP -open,
- (2) $\phi_\psi(\text{int}_{12}^s(G_E)) \subseteq \text{Dint}_{12}^s(\phi_\psi(G_E))$ for any $G_E \in P(X)^E$.
- (3) $\text{int}_{12}^s(\phi_\psi^{-1}(F_K)) \subseteq \phi_\psi^{-1}(\text{Dint}_{12}^s(F_K))$ for any $F_K \in P(Y)^K$.

Theorem 4.8. Let $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ be a soft mapping. The following statements are equivalent:

- (1) ϕ_ψ is a BSP -open mapping,
- (2) $\phi_\psi(\text{int}_{12}^s(G_E)) \subseteq \text{Bint}_{12}^s(\phi_\psi(G_E))$ for any $G_E \in S(X)^E$,
- (3) $\text{int}_{12}^s(\phi_\psi^{-1}(F_K)) \subseteq \phi_\psi^{-1}(\text{Bint}_{12}^s(F_K))$ for any $F_K \in S(Y)^K$.

Theorem 4.9. Let $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ and $\Delta_\beta : (Y, \eta_1, \eta_2, K, \leq_2) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ be two soft mappings. If ϕ_ψ and Δ_β are ISP -open, then $\Delta_\beta \circ \phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ is ISP -open mapping.

Proof. Suppose ϕ_ψ and Δ_β are ISP -open. Let G_E be a PO -soft set over X and let us show that $(\Delta_\beta \circ \phi_\psi)(G_E)$ is an IPO -soft set in Z . Since $(\Delta_\beta \circ \phi_\psi)(G_E) = \Delta_\beta(\phi_\psi(G_E))$ and ϕ_ψ is ISP -open, $\phi_\psi(G_E)$ is an IPO -soft set in Y . On the other hand, since Δ_β is ISP -open, $\Delta_\beta(\phi_\psi(G_E))$ is an IPO -soft set in Z . Thus $\Delta_\beta \circ \phi_\psi$ is ISP -open mapping. \square

The following two theorems characterized DSP -open and BSP -open mapping, whose proofs are similar to as that of the above Theorem 4.9.

Theorem 4.10. Let $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ and $\Delta_\beta : (Y, \eta_1, \eta_2, K, \leq_2) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ be two soft mappings. If ϕ_ψ and Δ_β are DSP -open, then $\Delta_\beta \circ \phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ is DSP -open mapping.

Theorem 4.11. Let $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ and $\Delta_\beta : (Y, \eta_1, \eta_2, K, \leq_2) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ be two soft mappings. If ϕ_ψ and Δ_β are BSP-open, then $\Delta_\beta \circ \phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Z, \delta_1, \delta_2, L, \leq_3)$ is BSP-open mapping.

Definition 4.12. A soft mapping $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ is called ISP (resp. DSP, BSP)-closed, if $\phi_\psi(G_E)$ is IPC (resp. DPC, BPC)-soft set in Y , whenever G_E is a PC-soft set in X .

Example 4.13. From Example 4.2, we have $G_E^c = \{(\alpha_1, \{a, c\}), (\alpha_2, \{b, c\})\}$, $F_E^c = \{(\alpha_1, \{b\}), (\alpha_2, X)\}$, $H_E^c = \{(\alpha_1, \emptyset), (\alpha_2, \{b, c\})\}$. Since G_E^c , F_E^c and H_E^c are PC-soft set in X , we have

$$\begin{aligned}\phi_\psi(G^c)(\alpha_1) &= \bigcup_{e \in \psi^{-1}(\alpha_1)} \phi(G^c(e)) = \phi(G^c(\alpha_1)) = \phi(\{a, c\}) = \{a, c\}, \\ \phi_\psi(G^c)(\alpha_2) &= \bigcup_{e \in \psi^{-1}(\alpha_2)} \phi(G^c(e)) = \phi(G^c(\alpha_2)) = \phi(\{b, c\}) = \{b, c\}, \\ \phi_\psi(F^c)(\alpha_1) &= \bigcup_{e \in \psi^{-1}(\alpha_1)} \phi(F^c(e)) = \phi(F^c(\alpha_1)) = \phi(\{b\}) = \{b\}, \\ \phi_\psi(F^c)(\alpha_2) &= \bigcup_{e \in \psi^{-1}(\alpha_2)} \phi(F^c(e)) = \phi(F^c(\alpha_2)) = \phi(X) = X, \\ \text{then } \phi_\psi(H^c)(\alpha_1) &= \bigcup_{e \in \psi^{-1}(\alpha_1)} \phi(H^c(e)) = \phi(H^c(\alpha_1)) = \phi(\emptyset) = \emptyset, \\ \phi_\psi(H^c)(\alpha_2) &= \bigcup_{e \in \psi^{-1}(\alpha_2)} \phi(H^c(e)) = \phi(H^c(\alpha_2)) = \phi(\{b, c\}) = \{b, c\}.\end{aligned}$$

Then clearly, ϕ_ψ is P -soft closed map, but it is not xSP -closed for $x = I; D; B$.

Example 4.14. From Example 4.3, we have $G_E^c = \{(\alpha_1, \{a, b\}), (\alpha_2, \{b\})\}$, $F_E^c = \{(\alpha_1, \{a, b\}), (\alpha_2, \{a\})\}$, $H_E^c = \{(\alpha_1, \{a, b\}), (\alpha_2, \emptyset)\}$. Then we have

$$\begin{aligned}\phi_\psi(G^c)(\alpha_1) &= \bigcup_{e \in \psi^{-1}(\alpha_1)} \phi(G^c(e)) = \phi(G^c(\alpha_1)) = \phi(\{a, b\}) = \{a, b\}, \\ \phi_\psi(G^c)(\alpha_2) &= \bigcup_{e \in \psi^{-1}(\alpha_2)} \phi(G^c(e)) = \phi(G^c(\alpha_2)) = \phi(\{b\}) = \{b\}, \\ \phi_\psi(F^c)(\alpha_1) &= \bigcup_{e \in \psi^{-1}(\alpha_1)} \phi(F^c(e)) = \phi(F^c(\alpha_1)) = \phi(\{a, b\}) = \{a, b\}, \\ \phi_\psi(F^c)(\alpha_2) &= \bigcup_{e \in \psi^{-1}(\alpha_2)} \phi(F^c(e)) = \phi(F^c(\alpha_2)) = \phi(\{a\}) = \{a\}, \\ \phi_\psi(H^c)(\alpha_1) &= \bigcup_{e \in \psi^{-1}(\alpha_1)} \phi(H^c(e)) = \phi(H^c(\alpha_1)) = \phi(\{a, b\}) = \{a, b\}, \\ \phi_\psi(H^c)(\alpha_2) &= \bigcup_{e \in \psi^{-1}(\alpha_2)} \phi(H^c(e)) = \phi(H^c(\alpha_2)) = \phi(\emptyset) = \emptyset.\end{aligned}$$

Thus $\phi_\psi(G_E^c) = G_E^c$, $\phi_\psi(F_E^c) = F_E^c$ and $\phi_\psi(H_E^c) = H_E^c$ are an IPC-soft sets in Y . So ϕ_ψ is ISP-closed, but it is not a DSP-closed mapping. However ϕ_ψ is not BSP-closed.

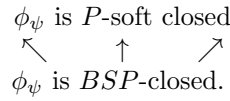
Example 4.15. From Example 4.4 consider $\leq_2 = \blacktriangle \cup \{(a, b)\}$, we have $G_E^{1c} = \{(\alpha_1, \{b, c\}), (\alpha_2, X)\}$, $G_E^{2c} = \{(\alpha_1, X), (\alpha_2, \{c\})\}$, $G_E^{3c} = \{(\alpha_1, \{b, c\}), (\alpha_2, \{c\})\}$, $F_K^{1c} = \{(\alpha_1, \{c\}), (\alpha_2, \{a, c\})\}$, $F_K^{2c} = \{(\alpha_1, X), (\alpha_2, \{a, c\})\}$, $F_K^{3c} = \{(\alpha_1, \{c\}), (\alpha_2, X)\}$. Then

$$\begin{aligned}\phi_\psi(G^{1c})(\alpha_1) &= \bigcup_{e \in \psi^{-1}(\alpha_1)} \phi(G^{1c}(e)) = \phi(G^{1c}(\alpha_2)) = \phi(X) = X, \\ \phi_\psi(G^{1c})(\alpha_2) &= \bigcup_{e \in \psi^{-1}(\alpha_2)} \phi(G^{1c}(e)) = \phi(G^c(\alpha_1)) = \phi(\{b, c\}) = \{a, c\}, \\ \phi_\psi(G^{2c})(\alpha_1) &= \bigcup_{e \in \psi^{-1}(\alpha_1)} \phi(G^{2c}(e)) = \phi(G^{2c}(\alpha_2)) = \phi(\{c\}) = \{c\}, \\ \phi_\psi(G^{2c})(\alpha_2) &= \bigcup_{e \in \psi^{-1}(\alpha_2)} \phi(G^{2c}(e)) = \phi(G^{2c}(\alpha_1)) = \phi(X) = X, \\ \phi_\psi(G^{3c})(\alpha_1) &= \bigcup_{e \in \psi^{-1}(\alpha_1)} \phi(G^{3c}(e)) = \phi(G^{3c}(\alpha_2)) = \phi(\{c\}) = \{c\}, \\ \phi_\psi(G^{3c})(\alpha_2) &= \bigcup_{e \in \psi^{-1}(\alpha_2)} \phi(G^{3c}(e)) = \phi(G^{3c}(\alpha_1)) = \phi(\{b, c\}) = \{a, c\}.\end{aligned}$$

Thus $\phi_\psi(G_E^{1c}) = F_K^{2c}$, $\phi_\psi(G_E^{2c}) = F_K^{3c}$ and $\phi_\psi(G_E^{3c}) = F_K^{1c}$ are DPC-soft set in Y . So ϕ_ψ is DSP-closed, but it is not an ISP-closed mapping. However ϕ_ψ is not BSP-closed.

Remark 4.16. For a soft mapping $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$, we have the following diagram:

$$\begin{array}{ccc}\phi_\psi \text{ is ISP-closed} & \leftrightarrow & \phi_\psi \text{ is DSP-closed} \\ & \searrow & \swarrow \\ & 216 & \end{array}$$



Theorem 4.17. Let $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ be a soft mapping. Then ϕ_ψ is an *ISP*-closed mapping if and only if $Icl_{12}^s(\phi_\psi(G_E)) \subseteq \phi_\psi(cl_{12}^s(G_E))$ for any $G_E \in P(X)^E$.

Proof. Necessity: Suppose ϕ_ψ is an *ISP*-closed mapping and let $G_E \in P(X)^E$. Then by the hypothesis, $\phi_\psi(cl_{12}^s(G_E))$ is an *IPC*-soft set in Y and $\phi_\psi(G_E) \subseteq \phi_\psi(cl_{12}^s(G_E))$. Thus $Icl_{12}^s(\phi_\psi(G_E)) \subseteq \phi_\psi(cl_{12}^s(G_E))$, since $Icl_{12}^s(\phi_\psi(G_E))$ is the smallest *IPC*-soft set containing $\phi_\psi(G_E)$ in Y .

Sufficiency: Suppose the necessary condition holds and let F_E be any *PC*-soft set in X . Then $\phi_\psi(F_E) \subseteq Icl_{12}^s(\phi_\psi(F_E)) \subseteq \phi_\psi(cl_{12}^s(F_E)) = \phi_\psi(F_E)$. Thus $\phi_\psi(F_E) = Icl_{12}^s(\phi_\psi(F_E))$. So $\phi_\psi(F_E)$ is an *IPC*-soft set in Y . So ϕ_ψ is an *ISP*-closed mapping. \square

The following two theorems characterized *DSP*-closed and *BSP*-closed mapping, whose proofs are similar to as that of the above Theorem 4.17.

Theorem 4.18. Let $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ be a soft mapping. Then ϕ_ψ is a *DSP*-closed mapping if and only if $Dcl_{12}^s(\phi_\psi(G_E)) \subseteq \phi_\psi(cl_{12}^s(G_E))$ for any $G_E \in P(X)^E$.

Theorem 4.19. Let $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ be a soft mapping. Then ϕ_ψ is a *BSP*-closed mapping if and only if $Bcl_{12}^s(\phi_\psi(G_E)) \subseteq \phi_\psi(cl_{12}^s(G_E))$ for any $G_E \in P(X)^E$.

Theorem 4.20. The following three statements hold for a bijective soft map $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$:

- (1) ϕ_ψ is *ISP* (resp. *DSP*, *BSP*)-open if and only if ϕ_ψ is *DSP* (resp. *ISP*, *BSP*)-closed.
- (2) ϕ_ψ is *ISP* (resp. *DSP*, *BSP*)-open if and only if ϕ_ψ^{-1} is *ISP* (resp. *DSP*, *BSP*)-continuous.
- (3) ϕ_ψ is *ISP* (resp. *DSP*, *BSP*)-closed if and only if ϕ_ψ^{-1} is *ISP* (resp. *DSP*, *BSP*)-continuous.

Proof. To summarize, we only give proofs of cases outside the parenthesis for the three statements above and the cases between parenthesis can be made similarly.

(1) Suppose ϕ_ψ is an *ISP*-open mapping and let G_E be a *PC*-soft set in X . Then G_E^c be a *PO*-soft set and $\phi_\psi(G_E^c)$ is an *IPO*-soft set in Y . Since ϕ_ψ is bijective, $\phi_\psi(G_E^c) = (\phi_\psi(G_E))^c$. Thus $\phi_\psi(G_E)$ is a *DPC*-soft set in Y . So ϕ_ψ is an *DSP*-closed mapping. The sufficiency condition can be proved in a similar manner.

(2) Suppose ϕ_ψ is an *ISP*-open mapping and let F_E be a *PC*-soft set in X . Then $\phi_\psi(F_E)$ is a *IPO*-soft set in Y . Since ϕ_ψ is bijective, $\phi_\psi(F_E) = (\phi_\psi^{-1})^{-1}(F_E)$. Thus $(\phi_\psi^{-1})^{-1}(F_E)$ is an *IPO*-soft set in Y . So ϕ_ψ^{-1} is an *ISP*-continuous mapping. The sufficiency condition can be proved in a similar manner.

(3) The proof of this statement comes immediately from (1) and (2). \square

5. ISP (DSP , BSP)-HOMEOMORPHISMS

The concepts of ISP (resp. DSP , BSP)-homeomorphisms are introduced and their main properties are discussed. Some examples are constructed to illustrate the relationships among the initiated soft mappings.

Definition 5.1. A soft mapping $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ is called ISP (resp. DSP , BSP)-homeomorphism, if it satisfies the following conditions:

- (i) ϕ_ψ is ISP (resp. DSP , BSP)-open,
- (ii) ϕ_ψ is ISP (resp. DSP , BSP)-closed,
- (iii) ϕ_ψ is ISP (resp. DSP , BSP)-continuous,
- (iv) ϕ_ψ^{-1} is ISP (resp. DSP , BSP)-continuous.

Example 5.2. From Example 3.2, 4.2, 4.13, we obtain that ϕ_ψ is not xSP -homeomorphism for $x = I; D; B$, but it is a P -soft homeomorphism.

Example 5.3. Let $E = \{\alpha_1, \alpha_2\} = K$ and $X = \{a, b, c, d\} = Y$, $\leq_1 = \blacktriangle \cup \{(c, b)\}$, $\leq_2 = \blacktriangle \cup \{(a, d)\}$ and let $\tau_1 = \eta_1 = \{X_E, \Phi, G_E\}$ and $\tau_2 = \eta_2 = \{X_E, \Phi, F_E\}$, where $G_E = \{(\alpha_1, \{a, d\}), (\alpha_2, \{b, c\})\}$, $F_E = \{(\alpha_1, \{a, b, d\}), (\alpha_2, \{b, c\})\}$. Then clearly, $(X, \tau_1, \tau_2, E, \leq_1)$ and $(Y, \eta_1, \eta_2, K, \leq_2)$ are soft bitopological ordered spaces and $\tau_{12} = \eta_{12} = \{X_E, \Phi, G_E, F_E\}$. Let us consider identity mappings ϕ and ψ .

- (i) It is obvious that

$$\phi_\psi(G_E)(\alpha_1) = \phi(\{a, d\}) = \{a, d\}, \quad \phi_\psi(G_E)(\alpha_2) = \phi(\{b, c\}) = \{b, c\},$$

$$\phi_\psi(F_E)(\alpha_1) = \phi(\{a, b, d\}) = \{a, b, d\}, \quad \phi_\psi(F_E)(\alpha_2) = \phi(\{b, c\}) = \{b, c\}.$$

Then $\phi_\psi(G_E) = G_E$ and $\phi_\psi(F_E) = F_E$ are IPO -soft sets in Y . Thus ϕ_ψ is ISP -open.

- (ii) It is clear that

$$\phi_\psi(G_E^c)(\alpha_1) = \phi(\{b, c\}) = \{b, c\}, \quad \phi_\psi(G_E^c)(\alpha_2) = \phi(\{a, d\}) = \{a, d\},$$

$$\phi_\psi(F_E^c)(\alpha_1) = \phi(\{c\}) = \{c\}, \quad \phi_\psi(F_E^c)(\alpha_2) = \phi(\{a, d\}) = \{a, d\}.$$

Then $\phi_\psi(G_E^c) = G_E^c$ and $\phi_\psi(F_E^c) = F_E^c$ are IPC -soft sets in Y . Thus ϕ_ψ is ISP -closed.

- (iii) We can easily prove that ϕ_ψ is ISP -continuous.

- (iv) We obtain easily the followings:

$$\phi_\psi^{-1}(G_E)(\alpha_1) = \phi^{-1}(G_E(\psi(\alpha_1))) = \phi^{-1}(G_E(\alpha_1)) = \phi^{-1}(\{a, d\}) = \{a, d\},$$

$$\phi_\psi^{-1}(G_E)(\alpha_2) = \phi^{-1}(G_E(\psi(\alpha_2))) = \phi^{-1}(G_E(\alpha_2)) = \phi^{-1}(\{b, c\}) = \{b, c\},$$

$$\phi_\psi^{-1}(F_E)(\alpha_1) = \phi^{-1}(F_E(\psi(\alpha_1))) = \phi^{-1}(F_E(\alpha_1)) = \phi^{-1}(\{a, b, d\}) = \{a, b, d\},$$

$$\phi_\psi^{-1}(F_E)(\alpha_2) = \phi^{-1}(F_E(\psi(\alpha_2))) = \phi^{-1}(F_E(\alpha_2)) = \phi^{-1}(\{b, c\}) = \{b, c\}.$$

Then $\phi_\psi^{-1}(G_E) = G_E$ and $\phi_\psi^{-1}(F_E) = F_E$ are IPO -soft sets in X . Thus ϕ_ψ^{-1} is ISP -continuous. So ϕ_ψ is an ISP -homeomorphism, but it is not a DSP -homeomorphism. However ϕ_ψ is not a BSP -homeomorphism.

Example 5.4. In Example 5.3, consider $\leq_1 = \blacktriangle \cup \{(b, c)\}$ and $\leq_2 = \blacktriangle \cup \{(d, a)\}$.

- (i) We obtain easily the followings:

$$\phi_\psi(G_E)(\alpha_1) = \phi(\{a, d\}) = \{a, d\}, \quad \phi_\psi(G_E)(\alpha_2) = \phi(\{b, c\}) = \{b, c\},$$

$$\phi_\psi(F_E)(\alpha_1) = \phi(\{a, b, d\}) = \{a, b, d\}, \quad \phi_\psi(F_E)(\alpha_2) = \phi(\{b, c\}) = \{b, c\}.$$

Then $\phi_\psi(G_E) = G_E$ and $\phi_\psi(F_E) = F_E$ are *DPO*-soft sets in Y . Thus ϕ_ψ is *DSP*-open.

(ii) It can be calculated:

$$\phi_\psi(G_E^c)(\alpha_1) = \phi(\{b, c\}) = \{b, c\}, \quad \phi_\psi(G_E^c)(\alpha_2) = \phi(\{a, d\}) = \{a, d\},$$

$$\phi_\psi(F_E^c)(\alpha_1) = \phi(\{c\}) = \{c\}, \quad \phi_\psi(F_E^c)(\alpha_2) = \phi(\{a, d\}) = \{a, d\}.$$

Then $\phi_\psi(G_E^c) = G_E^c$ and $\phi_\psi(F_E^c) = F_E^c$ are *DPC*-soft sets in Y . Thus ϕ_ψ is *DSP*-closed.

(iii) It is clear that ϕ_ψ is *DSP*-continuous.

(iv) We have

$$\phi_\psi^{-1}(G_E)(\alpha_1) = \phi^{-1}(G_E(\psi(\alpha_1))) = \phi^{-1}(G_E(\alpha_1)) = \phi^{-1}(\{a, d\}) = \{a, d\},$$

$$\phi_\psi^{-1}(G_E)(\alpha_2) = \phi^{-1}(G_E(\psi(\alpha_2))) = \phi^{-1}(G_E(\alpha_2)) = \phi^{-1}(\{b, c\}) = \{b, c\}.$$

Then $\phi_\psi^{-1}(G_E) = G_E$ and $\phi_\psi^{-1}(F_E) = F_E$ are *DPO*-soft sets in X . Thus ϕ_ψ is a *DSP*-homeomorphism, but it is not an *ISP*-homeomorphism. However ϕ_ψ is not a *BSP*-homeomorphism.

Theorem 5.5. *If a bijective soft mapping $\phi_\psi : (X, \tau_1, \tau_2, E, \leq_1) \rightarrow (Y, \eta_1, \eta_2, K, \leq_2)$ is *ISP* (resp. *DSP*, *BSP*)-continuous, then the following three statements are equivalent:*

- (1) ϕ_ψ is a *ISP* (resp. *DSP*, *BSP*)-homeomorphism,
- (2) ϕ_ψ^{-1} is *ISP* (resp. *DSP*, *BSP*)-continuous,
- (3) ϕ_ψ is *DSP* (resp. *ISP*, *BSP*)-closed.

Proof. (1) \Rightarrow (2): Suppose (1) holds. Then clearly, ϕ_ψ is *ISP* (resp. *DSP*, *BSP*)-open. Thus by Theorem 4.20 (2), ϕ_ψ^{-1} is *ISP* (resp. *DSP*, *BSP*)-continuous.

(2) \Rightarrow (3): The proof follows from Theorem 4.20 (3).

(3) \Rightarrow (1): It sufficient to prove that ϕ_ψ is *ISP* (resp. *DSP*, *BSP*)-open. This follows from Theorem 4.20 (1). \square

6. CONCLUSIONS

This paper introduces and defines the concepts of *xSP*-continuous, *xSP*-open, *xSP*-closed, and *xSP*-homeomorphism maps via soft bitopological ordered spaces. Relationships between the introduced soft maps and their counterparts in bitopological ordered spaces are established. These new soft ordered maps provide a basis for further developments in soft bitopological ordered spaces. Future research will introduce additional soft ordered bitopological concepts and explore their properties.

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REFERENCES

- [1] L. Nachbin, *Topology and ordered*, D. Van Nostrand Inc., Princeton, New Jersey 1965.
- [2] D. A. Molodtsov, *Soft set theory-first results*, *Comput. Math. Appl.* **37** (1999) 19–31.
- [3] M. Shabir and M. Naz, *On soft topological spaces*, *Comput. Math. Appl.* **61** (2011) 1786–1799.
- [4] T. M. Al-shami, M. E. El-Shafei and M. Abo-Elhamayel. *On soft topological ordered spaces*, *Journal of King Saud University-Science* **31** (4) (2018) 556–566.
- [5] S. A. El-Sheikh and A. M. Abd El-latif, *Decompositions of some types of supra soft sets and soft continuity*, *International Journal of Mathematics Trends and Technology* **9** (1) (2014) 37–56.
- [6] B. M. Ittanagi, *Soft bitopological spaces*, *International Journal of Computer Applications* **107** (7) (2014) 1–4.
- [7] G. Şenel, *Soft topology generated by L-soft sets*, *Journal of New Theory* **4** (24) (2018) 88–100.
- [8] G. Şenel, *A new approach to hausdorff space theory via the soft sets*, *Mathematical Problems in Engineering* **9** (2016) 1–6.
- [9] G. Şenel and N. Çağman, *Soft topological subspaces*, *Ann. Fuzzy Math. Inform.* **10** (4) (2015) 525–535.
- [10] G. Şenel and N. Çağman, *Soft closed sets on soft bitopological space*, *Journal of New Results in Science* **3** (5) (2014) 57–66.
- [11] Güzide Şenel, Jeong-Gon Lee and Kul Hur, *Distance and similarity measures for octahedron sets and their application to MCGDM Problems*, *Mathematics* **8** (2020) 1–16.
- [12] J. G. Lee, G. Şenel, P. K. Lim, J. Kim and K. Hur *Octahedron sets*, *Ann. Fuzzy Math. Inform.* **19** (3) (2020) 211–238.
- [13] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and S. A. Hazza, *Pairwise open (closed) soft sets in soft bitopological spaces*, *Ann. Fuzzy Math. Inform.* **11** (4) (2016) 571–588.
- [14] S. A. El-Sheikh, S. A. Kandil and S. H. Shalil, *On soft bitopological ordered spaces*, *Malaysian Journal of Mathematical Sciences*, accepted.
- [15] P. K. Maji, R. Biswas and A. R. Roy, *Soft set theory*, *Comput. Math. Appl.* **45** (2003) 555–562.
- [16] S. Nazmul and S. K. Samanta, *Neighbourhood properties of soft topological spaces*, *Ann. Fuzzy Math. Inform.* **6** (1) (2013) 1–15.
- [17] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, *Remarks on soft topological spaces*, *Ann. Fuzzy Math. Inform.* **3** (2) (2012) 171–185.
- [18] J. C. Kelly, *General Topology*, Springer Verlag 1975.
- [19] T. M. Al-shami, M. E. El-Shafei and M. Abo-Elhamayel. *On soft ordered maps*, *General Letters in Mathematics* **5** (3) (2018) 118–131.
- [20] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and S. A. Hazza, *Some types of pairwise soft open (continuous) mappings and some related results*, *South Asian Journal of Mathematics* **7** (2) (2017) 88–107.

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