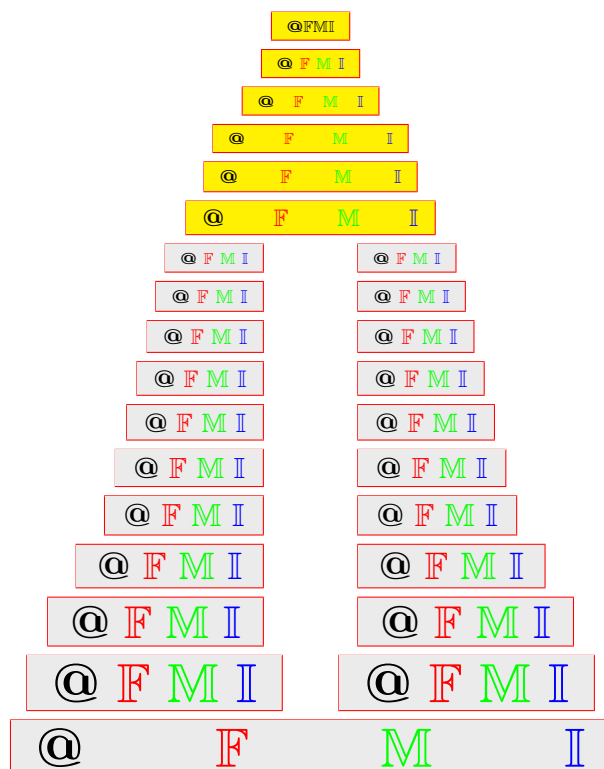


Fuzzy α - b -almost compact space

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ABSTRACT. This paper deals with some applications of fuzzy α - b -open set. Here we introduce fuzzy α - b -almost compactness and characterize this concept via fuzzy net and prefilterbase. Also we introduce fuzzy regularly α - b -open set which characterizes fuzzy α - b -almost compactness. It is shown that fuzzy α - b -almost compactness implies fuzzy almost compactness and the converse is true only on fuzzy α - b -regular space.

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Keywords: Fuzzy α - b -open set, Fuzzy α - b -regular space, Fuzzy regularly α - b -closed set, Fuzzy α - b -almost compact set (space), α - b -adherent point of a prefilterbase, α - b -cluster point of a fuzzy net.

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1. INTRODUCTION

Fuzzy α - b -open set was introduced in [1] using fuzzy α -open set as a basic tool. After introducing fuzzy compactness by Chang [2], many mathematicians have engaged themselves to introduce different types of fuzzy compactness. In [3], fuzzy almost compactness was introduced.

In this paper, we introduce the concept of fuzzy α - b -almost compactness which is weaker than fuzzy almost compactness. Here we use fuzzy net [4] and prefilterbase [5] to characterize fuzzy α - b -almost compactness.

In recent time, different types of fuzzy sets, viz., fuzzy soft set and fuzzy octahedron set are introduced and studied. A new branch of fuzzy topology is developed using these types of fuzzy sets. In this context we have to mention [6, 7, 8, 9, 10, 11].

2. PRELIMINARIES

In 1965, Zadeh [12] introduced a *fuzzy set* A which is a function from a non-empty set X into the closed interval $I = [0, 1]$, i.e., $A \in I^X$. The *support* of a fuzzy set A in X , denoted by $\text{supp}A$, is defined by $\text{supp}A = \{x \in X : A(x) \neq 0\}$. The fuzzy set with the singleton support $\{x\} \subseteq X$ and the value t ($0 < t \leq 1$) will be denoted by x_t . 0_X and 1_X are the constant fuzzy sets taking values 0 and 1 respectively in X . The *complement* of a fuzzy set A in X , denoted by $1_X \setminus A$, is defined by $(1_X \setminus A)(x) = 1 - A(x)$ for each $x \in X$. For any two fuzzy sets A, B in X , $A \leq B$ means $A(x) \leq B(x)$, for all $x \in X$ while AqB means A is quasi-coincident (q-coincident, for short) [4] with B , i.e., there exists $x \in X$ such that $A(x) + B(x) > 1$. The negation of these two statements will be denoted by $A \not\leq B$ and $A \not q B$ respectively. The *intersection* and the *union* of two fuzzy sets A, B in X , denoted by $A \wedge B$ and $A \vee B$, are fuzzy sets in X defined by: for each $x \in X$,

$$(A \wedge B)(x) = \min\{A(x), B(x)\} \text{ and } (A \vee B)(x) = \max\{A(x), B(x)\}.$$

Throughout this paper, (X, τ) or simply by X we shall mean an fts. For a fuzzy set A in X , clA and $\text{int}A$ will stand for the *fuzzy closure* and the *fuzzy interior* of A respectively (See [2]). A fuzzy set A in X is called a *fuzzy neighbourhood* (fuzzy nbd, for short) of a fuzzy point x_t , if there exists a fuzzy open set G in X such that $x_t \in G \leq A$. If, in addition, A is fuzzy open, then A is called a *fuzzy open nbd* of x_t (See [4]). A fuzzy set A is said to be a *fuzzy q-nbd* of a fuzzy point x_t in an fts X , if there is a fuzzy open set U in X such that $x_t q U \leq A$. If, in addition, A is fuzzy open, then A is called a *fuzzy open q-nbd* of x_t (See [4]).

A fuzzy set A in an fts (X, τ) is called a *fuzzy α -open set* in X , if $A \leq \text{int}(cl(\text{int}A))$. The complement of a fuzzy α -open set is called a *fuzzy α -closed set* in X . The union (the intersection) of all fuzzy α -open (resp. fuzzy α -closed) sets contained in (resp. containing) a fuzzy set A is called the *fuzzy α -interior* (resp. the *fuzzy α -closure* of A , denoted by $\alpha\text{int}A$ (resp. αclA) (See [13]).

Let $(D, \geq)$ be a directed set and X be an ordinary set. Let J denote the collection of all fuzzy points in X . A function $S : D \rightarrow J$ is called a *fuzzy net* in X [4]. It is denoted by $\{S_n : n \in (D, \geq)\}$. A non empty family $\mathcal{F}$ of fuzzy sets in X is called a *prefilterbase* on X , if (i) $0_X \notin \mathcal{F}$ and (ii) for any $U, V \in \mathcal{F}$, there exists $W \in \mathcal{F}$ such that $W \leq U \wedge V$ [5].

3. FUZZY α -b-OPEN SETS : SOME RESULTS

In this section we recall some definitions and results from [1, 2, 3, 14, 15] for ready references. Also some properties of fuzzy α -b-open sets are discussed here.

Definition 3.1 ([1]). A fuzzy set A in an fts (X, τ) called a *fuzzy α -b-open set* in X , if $A \leq cl(\alpha\text{int}(clA))$.

The complement of a fuzzy α -b-open set is called a *fuzzy α -b-closed set* in X . The collection of all fuzzy α -b-open (resp. fuzzy α -b-closed) sets in X is denoted by $FabO(X)$ (resp. $FabC(X)$).

Definition 3.2 ([1]). Let (X, τ) be an fts and $A \in I^X$. Then the *fuzzy α -b-closure* of A , denoted by $\alpha bclA$, is defined by

$$\alpha bclA = \bigwedge \{U \in I^X : A \leq U, U \in FabC(X)\}$$

and the fuzzy α - b -interior of A , denoted by $\alpha bint A$, is defined by

$$\alpha bint A = \bigvee \{G : G \leq A, G \in FabO(X)\}.$$

Definition 3.3 ([1]). A fuzzy set A in an fts (X, τ) is called a fuzzy α - b -nbd of a fuzzy point x_t in X , if there exists a fuzzy α - b -open set U in X such that $x_t \in U \leq A$. If, in addition, A is fuzzy α - b -open, then A is called a fuzzy α - b -open nbd of x_t .

Definition 3.4 ([1]). A fuzzy set A in an fts (X, τ) is called a fuzzy α - b - q -nbd of a fuzzy point x_t in X , if there exists a fuzzy α - b -open set U in X such that $x_t q U \leq A$. If, in addition, A is fuzzy α - b -open, then A is called a fuzzy α - b -open q -nbd of x_t .

Result 3.5 ([1]). The union (resp. intersection) of any two fuzzy α - b -open (resp. fuzzy α - b -closed) sets is also so.

Result 3.6 ([1]). $x_t \in \alpha bcl A$ if and only if every fuzzy α - b -open q -nbd U of x_t , $U q A$.

Result 3.7. $\alpha bcl(\alpha bcl A) = \alpha bcl A$ for any fuzzy set A in an fts X .

Proof. Let $A \in I^X$. Then clearly, $A \leq \alpha bcl A$. Thus we have

$$(3.1) \quad \alpha bcl A \leq \alpha bcl(\alpha bcl A).$$

Conversely, let $x_t \in \alpha bcl(\alpha bcl A)$. Assume that $x_t \notin \alpha bcl A$. Then there exists $U \in FabO(X)$ such that

$$(3.2) \quad x_t q U, U \not\leq A.$$

Since $x_t \in \alpha bcl(\alpha bcl A)$, $U q(\alpha bcl A)$. Thus there exists $y \in X$ such that $U(y) + (\alpha bcl A)(y) > 1$. So $U(y) + s > 1$, where $s = (\alpha bcl A)(y)$. Then $y_s \in \alpha bcl A$ and $y_s q U$ where $U \in FabO(X)$. By definition, $U q A$. This contradicts (3.2). Hence we get

$$(3.3) \quad \alpha bcl(\alpha bcl A) \leq \alpha bcl A.$$

Therefore combining (3.1) and (3.3), we get the result. $\square$

Result 3.8. $\alpha bcl(A \vee B) = \alpha bcl A \vee \alpha bcl B$ for any two fuzzy sets A, B in X .

Proof. It is clear that

$$(3.4) \quad \alpha bcl A \vee \alpha bcl B \leq \alpha bcl(A \vee B).$$

Conversely, let $x_t \in \alpha bcl(A \vee B)$ and let U be any α - b -open q -nbd of x_t . Then clearly, $U q(A \vee B)$. Thus there exists $y \in X$ such that $U(y) + \max\{A(y), B(y)\} > 1$. So either $U(y) + A(y) > 1$ or $U(y) + B(y) > 1$, i.e., either $U q A$ or $U q B$. Hence either $x_t \in \alpha bcl A$ or $x_t \in \alpha bcl B$. Therefore $x_t \in \alpha bcl A \vee \alpha bcl B$. $\square$

Result 3.9. For any fuzzy set A in an fts (X, τ) ,

- (1) $\alpha bcl(1_X \setminus A) = 1_X \setminus \alpha bint A$,
- (2) $\alpha bint(1_X \setminus A) = 1_X \setminus \alpha bcl A$.

Proof. (1) Let $x_t \in \alpha bcl(1_X \setminus A)$ and assume that $x_t \notin 1_X \setminus \alpha bint A$. Then $x_t q \alpha bint A$. Thus there exists a fuzzy α - b -open set B in X with $B \leq A$ such that $x_t q B$. So B is a fuzzy α - b -open q -nbd of x_t . By the assumption, $B q(1_X \setminus A) \Rightarrow A q(1_X \setminus A)$, which is absurd.

Conversely, let $x_t \in 1_X \setminus \alpha bint A$. Then $x_t \not\in \alpha bint A$. Thus $x_t \not\in U$ for any fuzzy α - b -open set U in X with $U \leq A$. So $x_t \in 1_X \setminus U$ which is fuzzy α - b -closed set in X with $1_X \setminus A \leq 1_X \setminus U$. Hence $x_t \in \alpha bcl(1_X \setminus A)$.

(2) Writing $1_X \setminus A$ for A in (1), we get the result. $\square$

Definition 3.10. Let A be a fuzzy set in an fts (X, τ) . A collection $\mathcal{U}$ of fuzzy sets in X is called a *fuzzy cover* of A , if $\sup\{U(x) : U \in \mathcal{U}\} = 1$ for each $x \in \text{supp} A$ [14]. If each member of $\mathcal{U}$ is fuzzy open (resp. fuzzy α - b -open), we call $\mathcal{U}$ is a *fuzzy open* [14] (resp. *fuzzy α - b -open*) cover of A . In particular, if $A = 1_X$, we get the definition of fuzzy cover of X [2].

Definition 3.11. A fuzzy cover $\mathcal{U}$ of a fuzzy set A in an fts (X, τ) is said to *have a finite* (resp. *finite proximate*) *subcover* $\mathcal{U}_0$, if $\mathcal{U}_0$ is a finite subcollection of $\mathcal{U}$ such that $\bigvee \mathcal{U}_0 \geq A$ [14] (resp. $\bigvee \{clU : U \in \mathcal{U}_0\} \geq A$ [15]). In particular, if $A = 1_X$, we get $\bigvee \mathcal{U}_0 = 1_X$ (resp. $\bigvee \{clU : U \in \mathcal{U}_0\} = 1_X$ [3]).

Definition 3.12 ([3]). An fts (X, τ) is called a *fuzzy almost compact space*, if every fuzzy open cover has a finite proximate subcover.

4. FUZZY α - b -ALMOST COMPACT SPACE : SOME CHARACTERIZATIONS

In this section, the concept of fuzzy α - b -almost compactness is introduced and studied by fuzzy α - b -open and fuzzy regularly α - b -open sets and characterize this space via fuzzy net and prefilter base.

Definition 4.1. A fuzzy set A in an fts (X, τ) is said to be a *fuzzy α - b -almost compact set*, if every fuzzy α - b -open cover $\mathcal{U}$ of A has a finite αb -proximate subcover, i.e., there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee \{\alpha bclU : U \in \mathcal{U}_0\} \geq A$. If, in addition, $A = 1_X$, we say that the fts X is fuzzy α - b -almost compact space.

Definition 4.2. A fuzzy point x_t in an fts X is said to *belong to* the αb -closure of a fuzzy set A in X , denoted by $x_t \in \alpha b-clA$, if for every fuzzy α - b -open q -nbd U of x_t , $\alpha bclU qA$.

Definition 4.3. Let x_t be a fuzzy point in an fts (X, τ) . A prefilterbase $\mathcal{F}$ on X is said to be:

(i) *αb -adhere at x_t* , written as $x_t \in \alpha b-ad \mathcal{F}$, if for each fuzzy α - b -open q -nbd U of x_t and each $F \in \mathcal{F}$, $Fq(\alpha bclU)$, i.e., $x_t \in \alpha b-clF$, for each $F \in \mathcal{F}$,

(ii) *αb -converge to x_t* , written as $\mathcal{F} \xrightarrow{\alpha b} x_t$, if to each fuzzy α - b -open q -nbd U of x_t , there corresponds some $F \in \mathcal{F}$ such that $F \leq \alpha bclU$.

Definition 4.4. Let x_t be a fuzzy point in an fts (X, τ) . A fuzzy net $\{S_n : n \in (D, \geq)\}$ is said to be:

(i) *αb -adhere at x_t* , denoted by $x_t \in \alpha b-ad(S_n)$, if for each fuzzy α - b -open q -nbd U of x_t and each $n \in D$, there exists $m \in D$ with $m \geq n$ such that $S_m q \alpha bclU$,

(ii) *αb -converge to x_t* , denoted by $S_n \xrightarrow{\alpha b} x_t$, if for each fuzzy α - b -open q -nbd U of x_t , there exists $m \in D$ such that $S_n q \alpha bclU$, for all $n \geq m (n \in D)$.

Theorem 4.5. For a fuzzy set A in an fts X , the following statements are equivalent:

- (1) A is a fuzzy α - b -almost compact set,

- (2) for every prefilterbase $\mathcal{B}$ in X , $[\bigwedge\{\alpha bcl B : B \in \mathcal{B}\}] \wedge A = 0_X$ implies that there exists a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigwedge\{\alpha bint B : B \in \mathcal{B}_0\} \not\leq A$,
 (3) for any family $\mathcal{F}$ of fuzzy α - b -closed sets in X with $\bigwedge\{F : F \in \mathcal{F}\} \wedge A = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigwedge\{\alpha bint F : F \in \mathcal{F}_0\} \not\leq A$,
 (4) every prefilterbase on X , each member of which is q -coincident with A , αb -adheres at some fuzzy point in A .

Proof. (1) $\Rightarrow$ (2): Suppose (1) holds and let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge\{\alpha bcl B : B \in \mathcal{B}\}] \wedge A = 0_X$. Then we have: for any $x \in \text{supp}A$,

$$\begin{aligned} & [\bigwedge\{\alpha bcl B : B \in \mathcal{B}\}](x) = 0 \\ & \Rightarrow 1 - [\bigwedge\{\alpha bcl B(x) : B \in \mathcal{B}\}] = 1 \\ & \Rightarrow \bigvee[(1_X \setminus \alpha bcl B)(x) : B \in \mathcal{B}] = 1 \\ & \Rightarrow \sup\{\alpha bint(1_X \setminus B)(x) : B \in \mathcal{B}\} = 1 \\ & \Rightarrow \{\alpha bint(1_X \setminus B) : B \in \mathcal{B}\} \text{ is a fuzzy } \alpha\text{-}b\text{-open cover of } A. \end{aligned}$$

By (1), there exists a finite αb -proximate subcover of it for A

$$\{\alpha bint(1_X \setminus B_1), \alpha bint(1_X \setminus B_2), \dots, \alpha bint(1_X \setminus B_n)\}.$$

Thus we get

$$\begin{aligned} A & \leq \bigvee_{i=1}^n \alpha bcl(\alpha bint(1_X \setminus B_i)) \\ & = \bigvee_{i=1}^n [1_X \setminus \alpha bint(\alpha bcl B_i)] \\ & = 1_X \setminus \bigwedge_{i=1}^n \alpha bint(\alpha bcl B_i). \end{aligned}$$

Thus $\bigwedge_{i=1}^n \alpha bint(\alpha bcl B_i) \leq 1_X \setminus A$. So $A \not\leq \bigwedge_{i=1}^n \alpha bint(\alpha bcl B_i)$. Hence $A \not\leq \bigwedge_{i=1}^n \alpha bint B_i$.

(2) $\Rightarrow$ (1): Suppose (2) holds and assume that there exists a fuzzy α - b -open cover $\mathcal{U}$ of A having no finite αb -proximate subcover for A . Then for every finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$, there exists $x \in \text{supp}A$ such that $\sup\{\alpha bcl U(x) : U \in \mathcal{U}_0\} < A(x)$, i.e., $1 - \sup\{\alpha bcl U(x) : U \in \mathcal{U}_0\} > 1 - A(x) \geq 0$, i.e., $\inf\{(1_X \setminus \alpha bcl U)(x) : U \in \mathcal{U}_0\} > 0$. Thus $\{\bigwedge_{U \in \mathcal{U}_0} (1_X \setminus \alpha bcl U) : \mathcal{U}_0 \text{ is a finite subcollection of } \mathcal{U}\} (= \mathcal{B}, \text{ say})$

is a prefilterbase in X . If there exists a finite subcollection $\{U_1, U_2, \dots, U_n\}$ (say) of $\mathcal{U}$ such that $\bigwedge_{i=1}^n \alpha bint(1_X \setminus \alpha bcl U_i) \not\leq A$, then Result 3.7, we have

$$\begin{aligned} A & \leq 1_X \setminus \bigwedge_{i=1}^n \alpha bint(1_X \setminus \alpha bcl U_i) \\ & = \bigvee_{i=1}^n [1_X \setminus \alpha bint(1_X \setminus \alpha bcl U_i)] \\ & = \bigvee_{i=1}^n \alpha bcl(\alpha bcl U_i) \end{aligned}$$

$$= \bigvee_{i=1}^n \alpha bcl U_i.$$

Thus $\mathcal{U}$ has a finite αb -proximate subcover for A , contradicts our hypothesis. So for every finite subcollection $\{ \bigwedge_{U \in \mathcal{U}_1} (1_X \setminus \alpha bcl U), \dots, \bigwedge_{U \in \mathcal{U}_k} (1_X \setminus \alpha bcl U) \}$ of $\mathcal{B}$, where $\mathcal{U}_1, \dots, \mathcal{U}_k$ are finite subset of $\mathcal{U}$, we have $[\bigwedge_{U \in \mathcal{U}_1 \vee \dots \vee \mathcal{U}_k} \alpha bint(1_X \setminus \alpha bcl U)] qA$. By (2), $[\bigwedge_{U \in \mathcal{U}} \alpha bcl(1_X \setminus \alpha bcl U)] \wedge A \neq 0_X$. Hence there exists $x \in \text{supp}A$, such that

$$\begin{aligned} & \inf_{U \in \mathcal{U}} [\alpha bcl(1_X \setminus \alpha bcl U)](x) > 0 \\ & \Rightarrow 1 - \inf_{U \in \mathcal{U}} [\alpha bcl(1_X \setminus \alpha bcl U)](x) < 1 \\ & \Rightarrow \sup_{U \in \mathcal{U}} [1_X \setminus \alpha bcl(1_X \setminus \alpha bcl U)](x) < 1 \\ & \Rightarrow \sup_{U \in \mathcal{U}} U(x) \leq \sup_{U \in \mathcal{U}} \alpha bint(\alpha bcl U)(x) < 1. \end{aligned}$$

This contradicts that $\mathcal{U}$ is a fuzzy α - b -open cover of A .

(1) $\Rightarrow$ (3): Suppose (1) holds and let $\mathcal{F}$ be a family of fuzzy α - b -closed sets in X such that $\bigwedge \{F : F \in \mathcal{F}\} \wedge A = 0_X$. Then for each $x \in \text{supp}A$ and for each positive integer n , there exists some $F_n \in \mathcal{F}$ such that $F_n(x) < 1/n$, i.e., $1 - F_n(x) > 1 - 1/n$, i.e., $\sup_{F \in \mathcal{F}} [(1_X \setminus F)(x)] = 1$. Thus $\{1_X \setminus F : F \in \mathcal{F}\}$ is a fuzzy α - b -open cover of A .

By (1), there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $A \leq \bigvee_{F \in \mathcal{F}_0} \alpha bcl(1_X \setminus F)$. So $1_X \setminus A \geq 1_X \setminus \bigvee_{F \in \mathcal{F}_0} \alpha bcl(1_X \setminus F) = \bigwedge_{F \in \mathcal{F}_0} (1_X \setminus \alpha bcl(1_X \setminus F)) = \bigwedge_{F \in \mathcal{F}_0} \alpha bint F$. Hence

$A \not\leq (\bigwedge_{F \in \mathcal{F}_0} \alpha bint F)$, where $\mathcal{F}_0$ is a finite subcollection of $\mathcal{F}$.

(3) $\Rightarrow$ (2): Suppose (3) holds and let $\mathcal{B}$ be a prefilterbase in X such that $[\bigwedge \{\alpha bcl B : B \in \mathcal{B}\}] \wedge A = 0_X$. Then the family $\mathcal{F} = \{\alpha bcl B : B \in \mathcal{B}\}$ is a family of fuzzy α - b -closed sets in X with $(\bigwedge F) \wedge A = 0_X$. Thus by (3), there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $[\bigwedge \{\alpha bint(\alpha bcl B) : B \in \mathcal{B}_0\}] /qA$. So $(\bigwedge_{B \in \mathcal{B}_0} \alpha bint B) \not\leq A$.

(1) $\Rightarrow$ (4): Suppose (1) holds and let $\mathcal{F}$ be a prefilterbase in X , each member of which is q -coincident with A . Assume that $\mathcal{F}$ do not αb -adhere at any fuzzy point in A . Then for each $x \in \text{supp}A$, there exists $n_x \in \mathcal{N}$ (the set of all natural numbers) such that $x_{1/n_x} \in A$. Thus there are a fuzzy α - b -open set $U_{n_x}^x$ and a member $F_{n_x}^x$ of $\mathcal{F}$ such that $x_{1/n_x} q U_{n_x}^x$ and $\alpha bcl U_{n_x}^x \not\leq F_{n_x}^x$. So $U_{n_x}^x(x) > 1 - 1/n_x$. Hence $\sup \{U_n^x(x) : n \in \mathcal{N}, n \geq n_x\} = 1$. Therefore $\{U_n^x : n \in \mathcal{N}, n \geq n_x, x \in \text{supp}A\}$ forms a fuzzy α - b -open cover of A . By (1), there exist finitely many points

$x_1, x_2, \dots, x_k \in \text{supp}A$ and $n_1, n_2, \dots, n_k \in \mathcal{N}$ such that $A \leq \bigvee_{i=1}^k \alpha bcl U_{n_{x_i}}^{x_i}$. Choose

$F \in \mathcal{F}$ such that $F \leq \bigwedge_{i=1}^k F_{n_{x_i}}^{x_i}$. Then $F \not\leq [\bigvee_{i=1}^k \alpha bcl U_{n_{x_i}}^{x_i}]$, i.e., $F \not\leq A$, a contradiction.

(4) $\Rightarrow$ (1): Suppose (4) holds and assume that there exist a fuzzy α - b -open cover $\mathcal{U}$ of A such that for every finite subset $\mathcal{U}_0$ of $\mathcal{U}$, $\bigvee \{abclU : U \in \mathcal{U}_0\} \not\geq A$. Then $\mathcal{F} = \{1_X \setminus \bigvee_{U \in \mathcal{U}_0} abclU : \mathcal{U}_0 \text{ is a finite subset of } \mathcal{U}\}$ is a prefilterbase on X such that FqA for each $F \in \mathcal{F}$. By (4), $\mathcal{F}$ αb -adheres at some fuzzy point $x_t \in A$. As $\mathcal{U}$ is a fuzzy cover of A , $\sup_{U \in \mathcal{U}} U(x) = 1$. Thus there exists $U_0 \in \mathcal{U}$ such that $U_0(x) > 1 - t$, i.e., $x_t q U_0$. As $x_t \in \alpha b\text{-ad}\mathcal{F}$ and $1_X \setminus abclU_0 \in \mathcal{F}$, we have $abclU_0 q (1_X \setminus abclU_0)$, a contradiction. $\square$

Theorem 4.6. For a fuzzy set A in an fts X , Consider the following statements:

- (1) every fuzzy net in A αb -adheres at some fuzzy point in A ,
- (2) every fuzzy net in A has a αb -convergent fuzzy subnet,
- (3) every prefilterbase in A αb -adheres at some fuzzy point in A ,
- (4) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge_{\alpha \in \Lambda} \alpha b\text{-cl}B_\alpha] \wedge A =$

0_X , there is a finite subset Λ_0 of Λ such that $(\bigwedge_{\alpha \in \Lambda_0} B_\alpha) \wedge A = 0_X$,

- (5) A is fuzzy α - b -almost compact set.

Then (1), (2) and (3) are equivalent, and (3) $\Rightarrow$ (4) and (4) $\Rightarrow$ (5).

Proof. (1) $\Rightarrow$ (2): Suppose (1) holds and let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net in A , where $(D, \geq)$ is a directed set, αb -adhere at a fuzzy point $x_\alpha \in A$. Let Q_{x_α} denote the set of the fuzzy α - b -closures of all fuzzy α - b -open q -nbds of x_α . For any $B \in Q_{x_\alpha}$, we can choose some $n \in D$ such that $S_n q B$. Let E denote the set of all ordered pairs (n, B) with the property that $n \in D$, $B \in Q_{x_\alpha}$ and $S_n q B$. Then $(E, \gg)$ is a directed set, where $(m, C) \gg (n, B)$ if and only if $m \geq n$ in D and $C \leq B$. Thus $T : (E, \gg) \rightarrow (X, \tau)$ given by $T(n, B) = S_n$ is a fuzzy subnet of $\{S_n : n \in (D, \geq)\}$. Let V be any fuzzy α - b -open q -nbd of x_α . Then there is $n \in D$ such that $(n, abclV) \in E$. Thus $S_n q (abclV)$. Now, for any $(m, U) \gg (n, abclV)$, $T(m, U) = S_m q U \leq abclV$. So $T(m, U) q (abclV)$. Hence $T \alpha b \rightarrow x_\alpha$.

(2) $\Rightarrow$ (1). Suppose (2) holds and assume that a fuzzy net $\{S_n : n \in (D, \geq)\}$ does not αb -adhere at a fuzzy point x_α . Then there is a fuzzy α - b -open q -nbd U of x_α and an $n \in D$ such that $S_m \not q (abclU)$ for all $m \geq n$. Thus obviously no fuzzy subnet of the fuzzy net can αb -converge to x_α . This is a contradiction.

(1) $\Rightarrow$ (3): Suppose (1) holds and let $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a prefilterbase in A . For each $\alpha \in \Lambda$, choose a fuzzy point $x_{F_\alpha} \in F_\alpha$ and construct the fuzzy net $S = \{x_{F_\alpha} : F_\alpha \in \mathcal{F}\}$ in A with $(\mathcal{F}, \gg)$ as domain, where for two members $F_\alpha, F_\beta \in \mathcal{F}$, $F_\alpha \gg F_\beta$ if and only if $F_\alpha \leq F_\beta$. By (1), the fuzzy net S αb -adheres at some fuzzy point x_t ($0 < t \leq 1$) $\in A$. Then for any fuzzy α - b -open q -nbd U of x_t and any $F_\alpha \in \mathcal{F}$, there exists $F_\beta \in \mathcal{F}$ such that $F_\beta \gg F_\alpha$ and $x_{F_\beta} q (abclU)$. Thus $F_\beta q (abclU)$. So $F_\alpha q (abclU)$. Hence $\mathcal{F}$ αb -adheres at x_t .

(3) $\Rightarrow$ (1): Suppose (3) holds and let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net in A . Consider the prefilterbase $\mathcal{F} = \{T_n : n \in D\}$ generated by the net, where $T_n = \{S_m : m \in D, m \geq n\}$. Then by (3), there exists a fuzzy point $a_\alpha \in A$ such that $\mathcal{F}$ αb -adheres at a_α . Thus for each fuzzy α - b -open q -nbd U of a_α and each

$F \in \mathcal{F}$, $Fq(abclU)$, i.e., $(abclU)qT_n$ for all $n \in D$. So the given fuzzy net αb -adheres at a_α .

(3) $\Rightarrow$ (4): Suppose (3) holds and let $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ be a family of fuzzy sets in X such that for every finite subset Λ_0 of Λ , $(\bigwedge_{\alpha \in \Lambda_0} B_\alpha) \wedge A \neq 0_X$. Then

$\mathcal{F} = \{(\bigwedge_{\alpha \in \Lambda_0} B_\alpha) \wedge A : \Lambda_0 \text{ is a finite subset of } \Lambda\}$ is a prefilterbase in A . By (3), $\mathcal{F}$ αb -adheres at some fuzzy point $a_t \in A$ ($0 < t \leq 1$). Thus for each $\alpha \in \Lambda$ and each fuzzy α - b -open q -nbd U of a_t , $B_\alpha q(abclU)$, i.e., $a_t \in \alpha b-clB_\alpha$ for each $\alpha \in \Lambda$. So $(\bigwedge_{\alpha \in \Lambda} \alpha b-clB_\alpha) \wedge A \neq 0_X$.

(4) $\Rightarrow$ (5): Suppose (4) holds and let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy α - b -open cover of a fuzzy set A . Then by (4), $A \wedge [\bigwedge_{\alpha \in \Lambda} (1_X \setminus U_\alpha)] = A \wedge [1_X \setminus \bigvee_{\alpha \in \Lambda} U_\alpha] = 0_X$.

If for some $\alpha \in \Lambda$, $1_X \setminus \alpha bclU_\alpha = 0_X$, then we are done. If $1_X \setminus \alpha bclU_\alpha (=B_\alpha, \text{ say}) \neq 0_X$, then for each $\alpha \in \Lambda$, $\mathcal{B} = \{B_\alpha : \alpha \in \Lambda\}$ is a family of non-null fuzzy sets. We show that $\bigwedge_{\alpha \in \Lambda} \alpha b-clB_\alpha \leq \bigwedge_{\alpha \in \Lambda} (1_X \setminus U_\alpha)$. In fact, let x_t ($0 < t \leq 1$)

be a fuzzy point such that $x_t \in \alpha b-clB_\alpha = \alpha b-cl(1_X \setminus \alpha bclU_\alpha)$. If $x_t q U_\alpha$, then $\alpha bclU_\alpha q (1_X \setminus \alpha bclU_\alpha)$, which is absurd. Thus $x_t \not q U_\alpha \Rightarrow x_t \in 1_X \setminus U_\alpha$. So $[\bigwedge_{\alpha \in \Lambda} \alpha b-clB_\alpha] \wedge A \leq A \wedge [\bigwedge_{\alpha \in \Lambda} (1_X \setminus U_\alpha)] = 0_X$. By (4), there exists a finite subset Λ_0 of Λ such

that $[\bigwedge_{\alpha \in \Lambda_0} B_\alpha] \wedge A = 0_X$, i.e., $A \leq 1_X \setminus \bigwedge_{\alpha \in \Lambda_0} B_\alpha = \bigvee_{\alpha \in \Lambda_0} (1_X \setminus B_\alpha) = \bigvee_{\alpha \in \Lambda_0} \alpha bclU_\alpha$. Hence (5) holds. $\square$

Definition 4.7. A fuzzy set A in an fts (X, τ) is said to be *fuzzy regularly α - b -open*, if $A = \alpha bint(\alpha bclA)$. The complement of such a set is called *fuzzy regularly α - b -closed*.

Note 4.8. It is clear from definitions that fuzzy regularly α - b -open set is fuzzy α - b -open set. But the converse need not be true, as it follows from the next example.

Example 4.9. Let $X = \{a, b\}$, $\tau = \{0_X, 1_X, A\}$, where $A(a) = 0.5, A(b) = 0.6$. Then (X, τ) is an fts. Here $F\alpha bO(X) = \{0_X, 1_X, U\}$, where $U \not\leq 1_X \setminus A$ and thus $F\alpha bC(X) = \{0_X, 1_X, 1_X \setminus U\}$, where $1_X \setminus U \not\leq A$. Consider the fuzzy set B defined by $B(a) = B(b) = 0.6$. Then clearly B is fuzzy α - b -open set, but not fuzzy regularly α - b -open set.

Definition 4.10. A fuzzy point x_α in X is said to be a *fuzzy αb -cluster point* of a prefilterbase $\mathcal{B}$, if $x_\alpha \in \alpha bclB$, for all $B \in \mathcal{B}$. If, in addition, $x_\alpha \in A$ for a fuzzy set A , then $\mathcal{B}$ is said to *have a fuzzy αb -cluster point* in A .

Theorem 4.11. A fuzzy set A in an fts (X, τ) is fuzzy α - b -almost compact if and only if for each prefilterbase $\mathcal{F}$ in X which is such that for each set of finitely many members $F_1, F_2, \dots, F_n$ from $\mathcal{F}$ and for any fuzzy regularly α - b -closed set C containing A , one has $(F_1 \wedge \dots \wedge F_n)qC$, $\mathcal{F}$ has a fuzzy αb -cluster point in A .

Proof. Suppose A is a fuzzy α - b -almost compact set and let $\mathcal{F}$ be a prefilterbase in X such that

$$(4.1) \quad [\bigwedge \{\alpha bcl F : F \in \mathcal{F}\}] \wedge A = 0_X.$$

Let $x \in \text{supp} A$. Consider any $n \in \mathcal{N}$ (the set of all natural numbers) such that $1/n < A(x)$, i.e., $x_{1/n} \in A$. Then by (4.1), $x_{1/n} \notin \alpha bcl F_x^n$ for some $F_x^n \in \mathcal{F}$. Thus there exists a fuzzy α - b -open q -nbd U_x^n of $x_{1/n}$ such that $U_x^n \not\leq F_x^n$. Now $U_x^n(x) > 1 - 1/n \Rightarrow \sup\{U_x^n(x) : 1/n < A(x), n \in \mathcal{N}\} = 1 \Rightarrow \mathcal{U} = \{U_x^n : x \in \text{supp} A, n \in \mathcal{N}\}$ forms a fuzzy α - b -open cover of A such that for U_x^n , there exists $F_x^n \in \mathcal{F}$ with $U_x^n \not\leq F_x^n$. Since A is fuzzy α - b -almost compact, there exist finitely many members

$$U_{x_1}^{n_1}, \dots, U_{x_k}^{n_k} \text{ of } \mathcal{U} \text{ such that } A \leq \bigvee_{i=1}^k \alpha bcl U_{x_i}^{n_i} = \alpha bcl \left(\bigvee_{i=1}^k U_{x_i}^{n_i} \right) \text{ (by Result 3.8) } (=U,$$

say). Now $F_{x_1}^{n_1}, \dots, F_{x_k}^{n_k} \in \mathcal{F}$ such that $U_{x_i}^{n_i} \not\leq F_{x_i}^{n_i}$ for $i = 1, 2, \dots, k$. Now U is a fuzzy regularly α - b -closed set containing A such that $U \not\leq (F_{x_1}^{n_1} \wedge \dots \wedge F_{x_k}^{n_k})$.

Conversely, let $\mathcal{B}$ be a prefilterbase in X having no fuzzy αb -cluster point in A . Then by the hypothesis, there is a fuzzy regularly α - b -closed set C containing A such that for some finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$, $(\bigwedge \mathcal{B}_0) \not\leq C$. Thus $(\bigwedge \mathcal{B}_0) \not\leq A$. By (2) $\Rightarrow$ (1) of Theorem 4.5, A is fuzzy α - b -almost compact set. $\square$

From Theorem 4.5, Theorem 4.6 and Theorem 4.11, we have the characterizations of fuzzy α - b -almost compact space as follows.

Theorem 4.12. *For an fts X , the following statements are equivalent:*

- (1) X is fuzzy α - b -almost compact,
- (2) every fuzzy net in X αb -adheres at some fuzzy point in X ,
- (3) every fuzzy net in X has a αb -convergent fuzzy subnet,
- (4) every prefilterbase in X αb -adheres at some fuzzy point in X ,
- (5) for every family $\{B_\alpha : \alpha \in \Lambda\}$ of non-null fuzzy sets with $[\bigwedge_{\alpha \in \Lambda} \alpha b-cl B_\alpha] = 0_X$,

there is a finite subset Λ_0 of Λ such that $(\bigwedge_{\alpha \in \Lambda_0} B_\alpha) = 0_X$,

- (6) for every prefilterbase $\mathcal{B}$ in X with $\bigwedge \{\alpha bcl B : B \in \mathcal{B}\} = 0_X$, there is a finite subcollection $\mathcal{B}_0$ of $\mathcal{B}$ such that $\bigwedge \{\alpha bint B : B \in \mathcal{B}_0\} = 0_X$,

- (7) for any family $\mathcal{F}$ of fuzzy α - b -closed sets in X with $\bigwedge \mathcal{F} = 0_X$, there exists a finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$ such that $\bigwedge \{\alpha bint F : F \in \mathcal{F}_0\} = 0_X$.

Theorem 4.13. *An fts X is fuzzy α - b -almost compact if and only if for any collection $\{F_\alpha : \alpha \in \Lambda\}$ of fuzzy α - b -open sets in X having finite intersection property $\bigwedge \{\alpha bcl F_\alpha : \alpha \in \Lambda\} \neq 0_X$.*

Proof. Suppose X is a fuzzy α - b -almost compact space and let $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ be a collection of fuzzy α - b -open sets in X with finite intersection property. Assume that $\bigwedge \{\alpha bcl F_\alpha : \alpha \in \Lambda\} = 0_X$. Then $\{1_X \setminus \alpha bcl F_\alpha : \alpha \in \Lambda\}$ is a fuzzy α - b -open cover of X . Thus by the hypothesis, there exists a finite subset Λ_0 of Λ such that $1_X = \bigvee \{\alpha bcl(1_X \setminus \alpha bcl F_\alpha) : \alpha \in \Lambda_0\} = \bigvee \{1_X \setminus \alpha bint(\alpha bcl F_\alpha) : \alpha \in \Lambda_0\} \leq \bigvee \{1_X \setminus F_\alpha : \alpha \in \Lambda_0\} = 1_X \setminus \bigwedge_{\alpha \in \Lambda_0} F_\alpha \Rightarrow \bigwedge_{\alpha \in \Lambda_0} F_\alpha = 0_X$ which contradicts the fact that $\mathcal{F}$ has finite intersection property.

Conversely, suppose the necessary condition holds and assume that X is not fuzzy α - b -almost compact space. Then there is a fuzzy α - b -open cover $\mathcal{F} = \{F_\alpha : \alpha \in \Lambda\}$ of X such that for every finite subset Λ_0 of Λ , $\bigvee\{\text{abcl}F_\alpha : \alpha \in \Lambda_0\} \neq 1_X$. Thus $1_X \setminus \bigvee\{\text{abcl}F_\alpha : \alpha \in \Lambda_0\} \neq 0_X$. So $\bigwedge_{\alpha \in \Lambda_0} (1_X \setminus \text{abcl}F_\alpha) \neq 0_X$. Hence $\{1_X \setminus \text{abcl}F_\alpha : \alpha \in \Lambda\}$ is a collection of fuzzy α - b -open sets with finite intersection property. By the hypothesis, $\bigwedge_{\alpha \in \Lambda} \text{abcl}(1_X \setminus \text{abcl}F_\alpha) \neq 0_X$, i.e., $1_X \setminus \bigvee_{\alpha \in \Lambda} \text{abint}(\text{abcl}F_\alpha) \neq 0_X$, i.e., $\bigvee_{\alpha \in \Lambda} \text{abint}(\text{abcl}F_\alpha) \neq 1_X$. Therefore $\bigvee_{\alpha \in \Lambda} F_\alpha \neq 1_X$. This is a contradiction as $\mathcal{F}$ is a fuzzy α - b -open cover of X . $\square$

Definition 4.14. Let $\{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy α - b -open sets in X , i.e., for each member n of a directed set $(D, \geq)$, S_n is a fuzzy α - b -open set in X . A fuzzy point x_α in X is said to be a *fuzzy αb -cluster point* of the fuzzy net, if for every $n \in D$ and every fuzzy α - b -open q -nbd V of x_α , there exists $m \in D$ with $m \geq n$ such that $S_m qV$.

Theorem 4.15. An fts X is fuzzy α - b -almost compact if and only if every fuzzy net of fuzzy α - b -open sets in X has a fuzzy αb -cluster point in X .

Proof. Suppose X is fuzzy α - b -almost compact and let $\mathcal{U} = \{S_n : n \in (D, \geq)\}$ be a fuzzy net of fuzzy α - b -open sets in a fuzzy α - b -almost compact space X . For each $n \in D$, let $F_n = \text{abcl}[\bigvee\{S_m : m \in D \text{ and } m \geq n\}]$. Then $\mathcal{F} = \{F_n : n \in D\}$ is a family of fuzzy α - b -closed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{\text{abint}F : F \in \mathcal{F}_0\} \neq 0_X$. By (1) $\Rightarrow$ (7) of Theorem 4.12, $\bigwedge_{n \in D} F_n \neq 0_X$. Let $x_\alpha \in \bigwedge_{n \in D} F_n$. Then $x_\alpha \in F_n$ for all $n \in D$. Thus for any fuzzy α - b -open q -nbd A of x_α and any $n \in D$, $Aq[\bigvee\{S_m : m \geq n\}]$. So there exists some $m \in D$ with $m \geq n$ and AqS_m . Hence x_α is a fuzzy αb -cluster point of $\mathcal{U}$.

Conversely, suppose the necessary condition holds and let $\mathcal{F}$ be a collection of fuzzy α - b -closed sets in X with the condition that for every finite subcollection $\mathcal{F}_0$ of $\mathcal{F}$, $\bigwedge\{\text{abint}F : F \in \mathcal{F}_0\} \neq 0_X$. Let $\mathcal{F}^*$ denote the family of all finite intersections of members of $\mathcal{F}$ directed by the relation ' $\gg$ ' such that for $F_1, F_2 \in \mathcal{F}^*$, $F_1 \gg F_2$ if and only if $F_1 \leq F_2$. Let $F^* = \text{abint}F$ for each $F \in \mathcal{F}^*$. Then $F^* \neq 0_X$. Consider the fuzzy net $\mathcal{U} = \{F^* : F \in (\mathcal{F}^*, \gg)\}$ of non-null fuzzy α - b -open sets of X . By the hypothesis, $\mathcal{U}$ has a fuzzy αb -cluster point, say x_α . We claim that $x_\alpha \in \bigwedge \mathcal{F}$. In fact, let $F \in \mathcal{F}$ be arbitrary and A be any fuzzy α - b -open q -nbd of x_α . Since $F \in \mathcal{F}^*$ and x_α is a fuzzy αb -cluster point of $\mathcal{U}$, there exists $G \in \mathcal{F}^*$ such that $G \gg F$ (i.e., $G \leq F$) and $G^* qA \Rightarrow GqA \Rightarrow FqA \Rightarrow x_\alpha \in \text{abcl}F = F$ for each $F \in \mathcal{F} \Rightarrow x_\alpha \in \bigwedge \mathcal{F} \Rightarrow \bigwedge \mathcal{F} \neq 0_X$. By (7) $\Rightarrow$ (1) of Theorem 4.12, X is a fuzzy α - b -almost compact space. $\square$

Definition 4.16. A fuzzy cover $\mathcal{U}$ by fuzzy α - b -closed sets of an fts (X, τ) is called a *fuzzy αb -cover* of X , if for each fuzzy point x_α ($0 < \alpha < 1$) in X , there exists $U \in \mathcal{U}$ such that U is a fuzzy α - b -open nbd of x_α .

Theorem 4.17. An fts (X, τ) is fuzzy α - b -almost compact if and only if every fuzzy αb -cover of X has a finite subcover.

Proof. Suppose X is fuzzy α - b -almost compact and let $\mathcal{U}$ be any fuzzy αb -cover of X . Then for each $n \in \mathcal{N}$ (the set of all natural numbers) with $n > 1$, there exist $U_x^n \in \mathcal{U}$ and a fuzzy α - b -open set V_x^n in X such that $x_{1-1/n} \leq V_x^n \leq U_x^n$. Thus $V_x^n(x) \geq 1 - 1/n$. So $\sup\{V_x^n(x) : n \in \mathcal{N}\} = 1$. Hence $\mathcal{V} = \{V_x^n : x \in X, n \in \mathcal{N}, n > 1\}$ is a fuzzy α - b -open cover of X . As X is fuzzy α - b -almost compact, there exist finitely many points $x_1, x_2, \dots, x_m \in X$ and $n_1, n_2, \dots, n_m \in \mathcal{N} \setminus \{1\}$ such that

$$1_X = \bigvee_{k=1}^m \alpha bcl V_{x_k}^{n_k} \leq \bigvee_{k=1}^m \alpha bcl U_{x_k}^{n_k} = \bigvee_{k=1}^m U_{x_k}^{n_k}.$$

Conversely, suppose the necessary condition holds and let $\mathcal{U}$ be fuzzy α - b -open cover of X . For any fuzzy point x_α ($0 < \alpha < 1$) in X , as $\sup_{U \in \mathcal{U}} U(x) = 1$, there exists $U_{x_\alpha} \in \mathcal{U}$ such that $U_{x_\alpha}(x) \geq \alpha$ ($0 < \alpha < 1$). Then $\mathcal{V} = \{\alpha bcl U : U \in \mathcal{U}\}$ is a fuzzy αb -cover of X and the rest is clear. $\square$

The following theorem gives a necessary condition for an fts to be fuzzy α - b -almost compact.

Theorem 4.18. *If an fts X is fuzzy α - b -almost compact, then every prefilterbase on X with at most one αb -adherent point is αb -convergent*

Proof. Suppose X is fuzzy α - b -almost compact and let $\mathcal{F}$ be a prefilterbase with at most one αb -adherent point in X . Then by Theorem 4.12, $\mathcal{F}$ has at least one αb -adherent point in X . Let x_α be the unique αb -adherent point of $\mathcal{F}$ and assume that $\mathcal{F}$ do not αb -converge to x_α . Then for some fuzzy α - b -open q -nbd U of x_α and for each $F \in \mathcal{F}$, $F \not\leq \alpha bcl U$. Thus $F \wedge \{1_X \setminus \alpha bcl U\} \neq 0_X$. So $\mathcal{G} = \{F \wedge (1_X \setminus \alpha bcl U) : F \in \mathcal{F}\}$ is a prefilterbase in X . Hence $\mathcal{G}$ has a αb -adherent point y_t (say) in X . Now $\alpha bcl U \not\leq G$ for all $G \in \mathcal{G}$ so that $x_\alpha \neq y_t$. Again, for each fuzzy α - b -open q -nbd V of y_t and each $F \in \mathcal{F}$, $\alpha bcl V q(F \wedge (1_X \setminus \alpha bcl U))$. Then $\alpha bcl V q F$. Thus y_t is a fuzzy αb -adherent point of $\mathcal{F}$, where $x_\alpha \neq y_t$. This contradicts the fact that x_α is the only fuzzy αb -adherent point of $\mathcal{F}$. $\square$

Some results on fuzzy α - b -almost compactness of an fts are given by the following theorem.

Theorem 4.19. *Let (X, τ) be an fts and $A \in I^X$. Then the following statements are true:*

- (1) *if A is fuzzy α - b -almost compact, then so is $\alpha bcl A$,*
- (2) *the union of two fuzzy α - b -almost compact sets is also so,*
- (3) *if X is fuzzy α - b -almost compact, then every fuzzy regularly α - b -closed set A in X is fuzzy α - b -almost compact.*

Proof. (1) Suppose A is fuzzy α - b -almost compact and let $\mathcal{U}$ be a fuzzy α - b -open cover of $\alpha bcl A$. Then $\mathcal{U}$ is also a fuzzy α - b -open cover of A . As A is fuzzy α - b -almost compact, there exists a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that

$$A \leq \bigvee \{\alpha bcl U : U \in \mathcal{U}_0\} = \alpha bcl \left\{ \bigvee U : U \in \mathcal{U}_0 \right\}.$$

Thus we have

$$\begin{aligned} \alpha bcl A &\leq \alpha bcl \{ \alpha bcl [\bigvee \{U : U \in \mathcal{U}_0\}] \} \\ &= \alpha bcl \{ \bigvee U : U \in \mathcal{U}_0 \} \end{aligned}$$

$$= \bigvee \{ \alpha bcl U : U \in \mathcal{U}_0 \}.$$

So $\alpha bcl A$ is fuzzy α - b -almost compact.

(2) Obvious.

(3) Suppose X is fuzzy α - b -almost compact and let $\mathcal{U} = \{U_\alpha : \alpha \in \Lambda\}$ be a fuzzy α - b -open cover of a fuzzy regularly α - b -closed set A in X . Then for each $x \notin \text{supp} A$, $A(x) = 0$, i.e., $(1_X \setminus A)(x) = 1$. Thus $\mathcal{U} \vee \{(1_X \setminus A)\}$ is a fuzzy α - b -open cover of X . Since X is fuzzy α - b -almost compact, there are finitely many members $U_1, U_2, \dots, U_n$ in $\mathcal{U}$ such that

$$1_X = (\alpha bcl U_1 \bigvee \dots \bigvee \alpha bcl U_n) \bigvee \alpha bcl (1_X \setminus A).$$

We claim that $\alpha bint A \leq \alpha bcl U_1 \bigvee \dots \bigvee \alpha bcl U_n$. If not, there exists a fuzzy point $x_t \in \alpha bint A$, but $x_t \notin (\alpha bcl U_1 \bigvee \dots \bigvee \alpha bcl U_n)$, i.e., $t > \max\{(\alpha bcl U_1)(x), \dots, (\alpha bcl U_n)(x)\}$. As $1_X = (\alpha bcl U_1 \bigvee \dots \bigvee \alpha bcl U_n) \bigvee \alpha bcl (1_X \setminus A)$, we have

$$[\alpha bcl (1_X \setminus A)](x) = 1.$$

Then $1 - \alpha bint A(x) = 1$. Thus $\alpha bint A(x) = 0$. So $x_t \notin \alpha bint A$. This is a contradiction. Hence by Results 3.7 and 3.8, we get

$$A = \alpha bcl(\alpha bint A) \leq \alpha bcl(\alpha bcl U_1 \bigvee \dots \bigvee \alpha bcl U_n) = \alpha bcl U_1 \bigvee \dots \bigvee \alpha bcl U_n.$$

Therefore A is a fuzzy α - b -almost compact set. $\square$

5. MUTUAL RELATIONSHIP

Here we establish the mutual relationship between fuzzy almost compactness [3] and fuzzy α - b -almost compactness. Then it is shown that fuzzy α - b -almost compactness implies fuzzy almost compactness, but converse is true in fuzzy α - b -regular space [1]. It is also established that fuzzy α - b -almost compactness remains invariant under fuzzy α - b -irresolute function [1].

Since for any fuzzy set A in an fts X , $\alpha bcl A \leq cl A$ (as every fuzzy closed set is fuzzy α - b -closed [1]), we can state the following theorem easily.

Theorem 5.1. *Every fuzzy α - b -almost compact space is fuzzy almost compact.*

To get the converse we have to recall the following definition and theorem for ready references.

Definition 5.2 ([1]). An fts (X, τ) is said to be *fuzzy α - b -regular*, if for each fuzzy α - b -closed set F in X and each fuzzy point x_α in X with $x_\alpha q (1_X \setminus F)$, there exists a fuzzy open set U in X and a fuzzy α - b -open set V in X such that $x_\alpha q U$, $F \leq V$ and $U \not\leq V$.

Theorem 5.3 ([1]). *An fts (X, τ) is fuzzy α - b -regular iff every fuzzy α - b -closed set is fuzzy closed.*

Theorem 5.4. *A fuzzy α - b -regular, fuzzy almost compact space X is fuzzy α - b -almost compact.*

Proof. Let $\mathcal{U}$ be a fuzzy α - b -open cover of a fuzzy α - b -regular, fuzzy almost compact space X . Then by Theorem 5.3, $\mathcal{U}$ is a fuzzy open cover of X . As X is fuzzy almost compact, there is a finite subcollection $\mathcal{U}_0$ of $\mathcal{U}$ such that $\bigvee \{cl U : U \in \mathcal{U}_0\} =$

$\bigvee\{abclU : U \in \mathcal{U}_0\}$ (by Theorem 5.3) $= 1_X$. Thus X is fuzzy α - b -almost compact space. $\square$

Next we recall the following definition and theorem for ready references.

Definition 5.5 ([1]). A function $f : X \rightarrow Y$ is said to be *fuzzy α - b -irresolute*, if the inverse image of every fuzzy α - b -open set in Y is fuzzy α - b -open in X .

Theorem 5.6 ([1]). For a function $f : X \rightarrow Y$, the following statements are equivalent:

- (1) f is fuzzy α - b -irresolute,
- (2) $f(abclA) \leq abcl(f(A))$ for all $A \in I^X$,
- (3) for each fuzzy point x_α in X and each fuzzy α - b -open q -nbd V of $f(x_\alpha)$ in Y , there exists a fuzzy α - b -open q -nbd U of x_α in X such that $f(U) \leq V$.

Theorem 5.7. Fuzzy α - b -irresolute image of a fuzzy α - b -almost compact space is fuzzy α - b -almost compact.

Proof. Let $f : X \rightarrow Y$ be fuzzy α - b -irresolute surjective function from a fuzzy α - b -almost compact space X onto an fts Y and let $\mathcal{V}$ be a fuzzy α - b -open cover of Y . Let $x \in X$ and $f(x) = y$. Since $\sup\{V(y) : V \in \mathcal{V}\} = 1$ for each $n \in \mathcal{N}$ (the set of all natural numbers), there exists some $V_x^n \in \mathcal{V}$ with $V_x^n(y) > 1 - 1/n$, i.e., $y_{1/n}qV_x^n$. Since f is fuzzy α - b -irresolute, by (1) $\Rightarrow$ (3) of Theorem 5.6, $f(U_x^n) \leq V_x^n$ for some fuzzy α - b -open set U_x^n in X q -coincident with $x_{1/n}$. Since $U_x^n(x) > 1 - 1/n$, $\sup\{U_x^n(x) : n \in \mathcal{N}\} = 1$. Thus $\mathcal{U} = \{U_x^n : n \in \mathcal{N}, x \in X\}$ is a fuzzy α - b -open cover of X . By fuzzy α - b -almost compactness of X , $\bigvee_{i=1}^k abclU_{x_i}^{n_i} = 1_X$ for some finite subcollection $\{U_{x_1}^{n_1}, \dots, U_{x_k}^{n_k}\}$ of $\mathcal{U}$. So we get

$$1_Y = f\left(\bigvee_{i=1}^k abclU_{x_i}^{n_i}\right) = \bigvee_{i=1}^k f(abclU_{x_i}^{n_i}) \leq \bigvee_{i=1}^k abcl(f(U_{x_i}^{n_i})).$$

Then by (1) $\Rightarrow$ (2) of Theorem 5.6, $\bigvee_{i=1}^k abcl(f(U_{x_i}^{n_i})) \leq \bigvee_{i=1}^k abclV_{x_i}^{n_i}$. Hence Y is a fuzzy α - b -almost compact space. $\square$

6. CONCLUSIONS

This paper is a continuation of [1]. The main goal of this paper is to establish the various results of fuzzy α - b -open sets and fuzzy covering properties. We further want to establish the inter-relations of various types of fuzzy covering properties.

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