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# Power aggregation operators on trapezoidal fuzzy multi-numbers and their applications to a zero-waste problem

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Abstract. In daily life, when conventional sets are insufficient in handling problems with multi-criterion, we use trapezoidal fuzzy multi-sets to solve decision-making problems in a lot of areas. Therefore, we introduce power aggregation operators on trapezoidal fuzzy multi-numbers as a novel way. By doing this, we aimed to introduce new operators on trapezoidal fuzzy multi-numbers which allow argument values to support each other in the aggregation process. Then, we described the properties of the operators and gave a formulation for the support function that is used in the operators. Moreover, we introduced two algorithms to solve multiple attribute group decision-making problems given with trapezoidal fuzzy multi-numbers. Lastly, we applied the introduced algorithms to a zero-waste problem to show the usage of the operators.

2020 AMS Classification: 03B52, 03E72

Keywords: Power aggregation operator, Trapezoidal fuzzy number, Trapezoidal fuzzy multi-numbers, Multiple attribute group decision-making, Zero-waste.

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## 1. Introduction

In 1965, the theory of fuzzy set was introduced by Zadeh [1] as an enlargement of the classical concept of a set of ambiguous information. Following of introduction of the theory, it was implemented in many areas. For example, Sahin et al. [2, 3] used fuzzy logic to conduct a study on education in 2021 and 2022. In time, a new concept of fuzzy set was introduced by Yager [4] which is called *fuzzy multi-sets* (fuzzy bags). The notion presents a new generalization of fuzzy sets. In addition, it gives complete information for some problems including situations in which each element has different membership values. Miyamoto [5] and Sebastian and Ramakrishnan [6, 7] expanded and studied in detailly the Yager's multi-sets and fuzzy multi-sets. Since some situations have multiple possibility of the same or different membership values, Uluçay et al. [8] developed TFM-numbers on the real number set $\mathbb{R}$. They are expansion of both fuzzy multi-sets and trapezoidal fuzzy numbers enabling the recurrent occurrences of any element. Later, many studies have been conducted by scientists. For example, Şahin et al. [9, 10] developed dice vector similarity measures and improved hybrid vector similarity measures of TFM-numbers in 2019. Then, Uluçay [11] introduced a new similarity function of TFM-numbers in 2020. In 2021, Deli and Keles [12] proposed distance measures on TFM-numbers and their application to multi-criteria decision-making problems. Then, Kesen and Deli [13] introduced weighted Bonferroni harmonic mean operator on TFM-numbers in 2022. In 2023, Şahin et al. [14] proposed a method by centroid point of TFM-numbers for multi-criteria decision-making problems. Deli and Kesen [15] introduced the Bonferroni geometric mean operator of TFM-numbers and they applied the operator to a multi-criteria decision-making problem. Then, Bakbak and Uluçay [16] proposed a paper related to the harmonic mean operator on intuitionistic TFM-numbers and they gave its application to architecture. In addition, Uluçay and Şahin [17] introduced some harmonic aggregation operators with TFM-numbers and their application of the law. Further, Kesen and Deli [18] conducted a study on Archimedean norms on TFM-numbers for multi-criteria decision-making problems. Lastly, Deli and Kesen [19] proposed a paper on Bonferroni arithmetic mean operator of TFM-numbers and gave an application to a multi-criteria decision-making problem.

By inspired Yager [20], in this work two aggregation operators are introduced: TFMPWA operator and TFMPWG operator. In addition, we mention their ordered types which are called TFMPOWA operator and TFMPOWG operator. They mainly cope with the information given with TFM-numbers. The main characteristic of these operators is that weight vectors depend on the arguments and that allows the values being aggregated to support and reinforce each other. In opposition to most aggregation operators, given operators incorporate information regarding the relationship between the values being combined. This is our main motivation for preparing the paper. Readers can find many papers about prominent aggregation operators in [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32].

Technological developments resulting from the Industrial Revolution increasing population and urbanization with innovations lead to differentiation of standards and consumption habits. This situation causes depletion of natural resources and climate change led to air, water, and soil pollution which threaten bios. Creating waste, consuming natural resources, putting pressure on the soil, and using energy and water, mean polluting the environment and creating additional costs for waste management. To prevent these negative results, the material should not be accepted as waste and It is compulsory to transition to an understanding that argues that it should be transformed into useful products as much as possible. This understanding is expressed as zero-waste. Zero-waste accepts the zero as a good potential resource to benefit rather than a problem. This acceptance is influenced by the notion of waste as a valuable resource requiring solutions for collection, separation, management, and recovery which were seen as a burden for industries and societies because of its effect on global warming and resource depletion. What is targeted with Zero-waste are preventing waste, more efficient resource usage, reducing the amount of waste, the establishment of an effective collection system, and the recycling of waste as a waste prevention approach that includes recycling. In the recycling and recovery process of waste disposal without recycling causes serious resource losses both material and energy. To leave a livable world to the next generations, we should adopt the zero-waste principle within the framework of sustainable development principles. Due to the importance of the topic, there are so many studies on zero-waste. For instance in 2018, Durgun and Durgun [33] reviewed the relationship between renewable energy consumption and economic development. Then, Onder [34] conducted a study related to circular economy as a new concept in the sustainable development approach. Okorie et al. [35] proposed a review of current research and future trends under digitization and the circular economy. Reike et al. [36], by focusing on history and resource value retention options, studied some debates on circular economy. In 2022, Uluçay and Okumus [37] proposed a study on combined fuzzy mathematical modeling and circular economy.

The organization of the paper is as follows: In the second section, we give some essential terms which are necessary for the next sections. In the third section, we introduce two aggregation techniques called TFMPWA operator and TFMPWG operator. Then, we investigate their properties and give their some special cases. In the section fourth, we introduce two algorithms to solve multiple attribute group decision-making problems given with TFM-numbers. In section five, we give a selection problem about to set a zero-waste factory which Turkiye government faced. The problem is containing five alternatives (plastic waste factory, paper waste factory, battery waste factory, organic waste factory, and glass waste factory) and four attributes (setup cost, human resource, adaptation period of public, and amount of waste) to be considered by decision-makers. We give this problem to see how effective introduced methods are. Then, in section six, we give a comparison table to show the compatibility of existing methods.

The following is the list of abbreviations.

Table 1. List of Abbreviations

| | |
|---|---|
| PA | power average |
| PG | power geometric |
| TFM | trapezoidal fuzzy multi |
| GTF | generalized trapezoidal fuzzy |
| FWA | fuzzy weighted average |
| FWG | fuzzy weighted geometric |
| TFMG | trapezoidal fuzzy multi geometric |
| TFMPG | trapezoidal fuzzy multi power geometric |
| TFMPA | trapezoidal fuzzy multi power average |
| TFMPWA | trapezoidal fuzzy multi power weighted average |
| TFMPWG | trapezoidal fuzzy multi power weighted geometric |
| TFMBHM | trapezoidal fuzzy multi Bonferroni harmonic mean |
| TFMPOWA | trapezoidal fuzzy multi power ordered weighted average |
| TFMPOWG | trapezoidal fuzzy multi power ordered weighted geometric |

## 2. Preliminaries

Here, we give some basic notions connected to fuzzy set, fuzzy number and fuzzy multi-set which are needful for the rest of the paper.

**Definition 2.1** ([1])**.** Let $A$ be a non-empty set. A *fuzzy set* F on $A$ is defined as:

$$F = \{\langle x, \mu_F(x)\rangle : x \in A\},$$

where $\mu_F : A \to [0,1]$ for $x \in A$.

In the context of this definition, the following two definitions are given:

**Definition 2.2** ([38])**.** *t-norms* are monotonic, commutative, and associative functions $t$ with two-valued mapping from $[0,1]\times[0,1]$ into $[0,1]$ and satisfying following conditions: for each $x \in [0,1]$,

(i) $t(0,0) = 0$, $t(\mu_{X_1}(x), 1) = t(1, \mu_{X_1}(x)) = \mu_{X_1}(x)$,
(ii) if $\mu_{X_1}(x) \leq \mu_{X_3}(x)$ and $\mu_{X_2}(x) \leq \mu_{X_4}(x)$, then $t(\mu_{X_1}(x), \mu_{X_2}(x)) \leq t(\mu_{X_3}x), \mu_{X_4}(x))$,
(iii) $t(\mu_{X_1}(x), \mu_{X_2}(x)) = t(\mu_{X_2}(x), \mu_{X_1}(x))$,
(iv) $t(\mu_{X_1}(x), t(\mu_{X_2}(x), \mu_{X_3}(x))) = t(t(\mu_{X_1}(x), \mu_{X_2})(x), \mu_{X_3}(x))$.

**Definition 2.3** ([38])**.** *s-norms* are monotonic, commutative, and associative functions $s$ with two-placed mapping from $[0,1]\times[0,1]$ into $[0,1]$ and satisfying following conditions: for each $x \in [0,1]$,

(i) $s(1,1) = 1$, $s(\mu_{X_1}(x), 0) = s(0, \mu_{X_1}(x)) = \mu_{X_1}(x)$,
(ii) if $\mu_{X_1}(x) \leq \mu_{X_3}(x)$ and $\mu_{X_2}(x) \leq \mu_{X_4}(x)$, then $s(\mu_{X_1}(x), \mu_{X_2}(x)) \leq s(\mu_{X_3}(x), \mu_{X_4}(x))$,
(iii) $s(\mu_{X_1}(x), \mu_{X_2}(x)) = s(\mu_{X_2}(x), \mu_{X_1}(x))$,
(iv) $s(\mu_{X_1}(x), s(\mu_{X_2}(x), \mu_{X_3}(x))) = s(s(\mu_{X_1}(x), \mu_{X_2})(x), \mu_{X_3}(x))$.

For example; $t_2(\mu_{X_1}(x), \mu_{X_2}(x)) = \mu_{X_1}(x)\mu_{X_2}(x)$ is a t-norm and $s_2(\mu_{X_1}(x), \mu_{X_2}(x)) = \mu_{X_1}(x) + \mu_{X_2}(x) - \mu_{X_1}(x)\mu_{X_2}(x)$ is a $s$-norm.

**Definition 2.4** ([39])**.** Let $\eta_T \in [0,1]$ and $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 \in R$ such that $\epsilon_1 \leq \epsilon_2 \leq \epsilon_3 \leq \epsilon_4$. Then a *GTF-number* $T = \langle(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4); \eta_T\rangle$ is a special fuzzy set on the real number set $\mathbb{R}$. Its membership functions are defined as follows:

$$\mu_T(x) = \begin{cases} (x-\epsilon_1)\eta_T/(\epsilon_2-\epsilon_1) & \epsilon_1 \le x < \epsilon_2 \\ \eta_T & \epsilon_2 \le x \le \epsilon_3 \\ (\epsilon_4 - x)\eta_T/(\epsilon_4-\epsilon_3) & \epsilon_3 < x \le \epsilon_4 \\ 0 & otherwise. \end{cases}$$

**Definition 2.5** ([6])**.** Let $X \neq \varnothing$. A *fuzzy multi-set* $T$ on $X$ is defined as

$$T = \{\langle x, \mu_T^1(x), \mu_T^2(x), \cdots, \mu_T^i(x), \cdots \rangle : x \in X\},$$

where $x \in X$ and $\mu_T^i : X \to [0,1]$ for all $i \in \{1, 2, \cdots, p\}$.

In the paper, $I_m$, $I_n$, $I_p$ and $I_t$ will be used instead of $\{1, 2, \cdots, m\}$, $\{1, 2, \cdots, n\}$, $\{1, 2, \cdots, p\}$ and $\{1, 2, \cdots, t\}$ respectively.

**Definition 2.6** ([8])**.** Let $\eta_T^i \in [0,1]$ $(i \in I_P)$ and $\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 \in R$ such that $\epsilon_1 \le \epsilon_2 \le \epsilon_3 \le \epsilon_4$. Then a *TFM-number* $T = \langle(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4); \eta_T^1, \eta_T^2, \cdots, \eta_T^P\rangle$ is a special fuzzy multi-set on the real number set $\mathbb{R}$. Its membership functions are defined as follows:

$$\mu_T^i(x) = \begin{cases} (x-\epsilon_1)\eta_T^i/(\epsilon_2-\epsilon_1) & \epsilon_1 \le x < \epsilon_2 \\ \eta_T^i & \epsilon_2 \le x \le \epsilon_3 \\ (\epsilon_4 - x)\eta_T^i/(\epsilon_4-\epsilon_3) & \epsilon_3 < x \le \epsilon_4 \\ 0 & otherwise. \end{cases}$$

**Definition 2.7** ([8])**.** Let $T_1 = \langle(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4); \eta_{T_1}^1, \eta_{T_1}^2, \cdots, \eta_{T_1}^P\rangle$ and $T_2 = \langle(\rho_1, \rho_2, \rho_3, \rho_4); \eta_{T_2}^1, \eta_{T_2}^2, \cdots, \eta_{T_2}^P\rangle$ be two TFM-numbers and $\gamma \neq 0$ be any real number. Then

(i) $T_1 \oplus T_2 = (\epsilon_1+\rho_1, \epsilon_2+\rho_2, \epsilon_3+\rho_3, \epsilon_4+\rho_4); s(\eta_{T_1}^1, \eta_{T_2}^1), s(\eta_{T_1}^2, \eta_{T_2}^2), \cdots, s(\eta_{T_1}^p, \eta_{T_2}^p)\rangle$,
(ii)

$$T_1 \otimes T_2 = \begin{cases} \langle(\epsilon_1\rho_1, \epsilon_2\rho_2, \epsilon_3\rho_3, \epsilon_4\rho_4); t(\eta_{T_1}^1 \eta_{T_2}^1), t(\eta_{T_1}^2, \eta_{T_2}^2), \cdots, t(\eta_{T_1}^p, \eta_{T_2}^p))\rangle & (\epsilon_4 > 0, \rho_4 > 0) \\ \langle(\epsilon_1\rho_4, \epsilon_2\rho_3, \epsilon_3\rho_2, \epsilon_4\rho_1); t(\eta_{T_1}^1, \eta_{T_2}^1), t(\eta_{T_1}^2, \eta_{T_2}^2), \cdots, t(\eta_{T_1}^p, \eta_{T_2}^p))\rangle & (\epsilon_4 < 0, \rho_4 > 0) \\ \langle(\epsilon_4\rho_4, \epsilon_3\rho_3, \epsilon_2\rho_2, \epsilon_1\rho_1); t(\eta_{T_1}^1, \eta_{T_2}^1), t(\eta_{T_1}^2, \eta_{T_2}^2), \cdots, t(\eta_{T_1}^p, \eta_{T_2}^p))\rangle & (\epsilon_4 < 0, \rho_4 < 0), \end{cases}$$

(iii) $\gamma T_1 = \langle(\gamma\epsilon_1, \gamma\epsilon_2, \gamma\epsilon_3, \gamma\epsilon_4); 1-(1-\eta_{T_1}^1)^\gamma, 1-(1-\eta_{T_1}^2)^\gamma, \cdots, 1-(1-\eta_{T_1}^p)^\gamma\rangle(\gamma \ge 0)$,
(iv) $T_1^\gamma = \langle(\epsilon_1^\gamma, \epsilon_2^\gamma, \epsilon_3^\gamma, \epsilon_4^\gamma); (\eta_{T_1}^1)^\gamma, (\eta_{T_1}^2)^\gamma, \cdots, (\eta_{T_1}^P)^\gamma\rangle(\gamma \ge 0)$.

Here, t-norms and s-norms are mappings that are given in Definitions 2.2 and 2.3, respectively.

Based on the complement of a fuzzy set given by Zimmermann [38], we can give following property for TFM-numbers.

**Definition 2.8.** Let $T = \langle(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4); \eta_T^1, \eta_T^2, \cdots, \eta_T^P\rangle$ be a TFM-number. Then the *complement* of $T$, denoted by $T^c$, is defined as follows:

$$T^c = \langle(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4); 1-\eta_T^1, 1-\eta_T^2, \cdots, 1-\eta_T^P\rangle$$

Note that if $\epsilon_1 \le \rho_1$, $\epsilon_2 \le \rho_2$, $\epsilon_3 \le \rho_3$, $\epsilon_4 \le \rho_4$, $\eta_{T_1}^1 \le \eta_{T_2}^1$, $\eta_{T_1}^2 \le \eta_{T_2}^2$,$\cdots$,$\eta_{T_1}^P \le \eta_{T_2}^P$ then, we said $T_1 \le T_2$.

**Definition 2.9** ([8]). Let $T_1 = \langle(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4); \eta^1_{T_1}, \eta^2_{T_1}, \cdots, \eta^P_{T_1}\rangle$ and $T_2 = \langle(\rho_1, \rho_2, \rho_3, \rho_4); \eta^1_{T_2}, \eta^2_{T_2}, \cdots, \eta^P_{T_2}\rangle$ be two TFM-numbers. Then the *Hamming distance* between $T_1$ and $T_2$ is defined as follows:

$$d(T_1, T_2) = \frac{1}{8p}\sum_{i=1}^{p}(|(1+\eta^i_{T_1})\epsilon_1 - (1+\eta^i_{T_2})\rho_1| + |(1+\eta^i_{T_1})\epsilon_2 - (1+\eta^i_{T_2})\rho_2| + |(1+\eta^i_{T_1})\epsilon_3 - (1+\eta^i_{T_2})\rho_3| + |(1+\eta^i_{T_1})\epsilon_4 - (1+\eta^i_{T_2})\rho_4|).$$

**Definition 2.10** ([8]). $T_1 = \langle(\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4); \eta^1_{T_1}, \eta^2_{T_1}, \cdots, \eta^P_{T_1}\rangle$ and $T_2 = \langle(\rho_1, \rho_2, \rho_3, \rho_4); \eta^1_{T_2}, \eta^2_{T_2}, \cdots, \eta^P_{T_2}\rangle$ be two TFM-numbers and d be Hamming distance between two TFM-numbers. Then the comparison of $T_1$ and $T_2$ defined as follows:

(i) if $d(T_1, r^+) < d(T_2, r^+)$, then $T_1$ is *bigger than* $T_2$, symbolised by $T_1 \succ T_2$,
(ii) if $d(T_1, r^+) = d(T_2, r^+)$ *and* $d(T_1, r^-) < d(T_2, r^-)$, then $T_2$ is *bigger than* $T_1$, symbolised by $T_1 \prec T_2$,
(iii) if $d(T_1, r^+) = d(T_2, r^+)$ *and* $d(T_1, r^-) = d(T_2, r^-)$, then $T_1$ is *similar to* $T_2$, symbolised by $T_1 \simeq T_2$,

where $r^+_A$ is positive ideal and $r^-_A$ is negative ideal and defined as follows, respectively:

$$r^+_A = \langle(a^+, b^+, c^+, d^+); (\eta^1_A)^+, (\eta^2_A)^+, \cdots, (\eta^P_A)^+\rangle = \langle(1,1,1,1); 1, 1, \cdots, 1\rangle,$$
$$r^-_A = \langle(a^-, b^-, c^-, d^-); (\eta^1_A)^-, (\eta^2_A)^-, \cdots, (\eta^P_A)^-\rangle = \langle(0,0,0,0); 0, 0, \cdots, 0\rangle.$$

**Definition 2.11** ([20]). Let $R_i$ $(i \in I_n)$ be a collection of non-negative real numbers. To aggregate this collection, the *PA operator* is defined as follows:

$$PA(R_1, R_2, \cdots, R_n) = \frac{\sum\limits_{i=1}^{n}(1+T(R_i))R_i}{\sum\limits_{i=1}^{n}(1+T(R_i))},$$

where $T(R_i) = \sum\limits_{j=1, i\neq j}^{n} Sup(R_i, R_j)$.

$Sup(R_i, R_j)$ is denoted as support from $R_i$ to $R_j$ and satisfies following properties:
(1) $Sup(R_i, R_j) \in [0, 1]$,
(2) $Sup(R_i, R_j) = Sup(R_j, R_i)$,
(3) $Sup(R_i, R_j) \geq Sup(R_x, R_y), \quad if \quad |R_i - R_j| < |R_x - R_y|$.

**Definition 2.12** ([26]). Let $R_i$ $(i \in I_n)$ be a collection of non-negative real numbers. To aggregate this collection, the *PG operator* is defined as follows:

$$PG(R_1, R_2, \cdots, R_n) = \prod_{i=1}^{n} R_i^{\frac{1+T(R_i)}{\sum\limits_{i=1}^{n}(1+T(R_i))}},$$

where $T(R_i) = \sum\limits_{j=1, i\neq j}^{n} Sup(R_i, R_j)$.

$Sup(R_i, R_j)$ is denoted as support from $R_i$ to $R_j$ and satisfies following properties:
(1) $Sup(R_i, R_j) \in [0, 1]$,
(2) $Sup(R_i, R_j) = Sup(R_j, R_i)$,

(3) $Sup(R_i, R_j) \geq Sup(R_x, R_y), \quad if \quad |R_i - R_j| < |R_x - R_y|.$

## 3. Power aggregation operators on TFM-numbers

In this section, we developed two TFM-Aggregation operators based on power aggregation operators for multi-criteria decision-making problems given with TFM information.

**Definition 3.1.** Let $F_i = \langle(\epsilon_i, \rho_i, \delta_i, \gamma_i); \eta^1_{F_i}, \eta^2_{F_i}, \cdots, \eta^P_{F_i}\rangle$ $(i \in I_n)$ be a collection of TFM-numbers and $\upsilon = (\upsilon_1, \upsilon_2, \cdots, \upsilon_n)^T$ weight vector of $F_i$ $(i \in I_n)$, where $\upsilon_i \geq 0$ $(i \in I_n)$ and $\sum\limits_{i=1}^{n} \upsilon_i = 1$. Then the *TFMPWA operator* is defined as follows:

$$TFMPWA(F_1, F_2, \cdots, F_n) = \frac{\bigoplus\limits_{i=1}^{n} \upsilon_i(1 + T(F_i))F_i}{\sum\limits_{i=1}^{n} \upsilon_i(1 + T(F_i))}, \tag{3.1}$$

where $T(F_i) = \sum\limits_{j=1, i\neq j}^{n} Sup(F_i, F_j)$, $Sup(F_i, F_j) = 1 - d(F_i, F_j)$ and $d(F_i, F_j)$ is Hamming distance [8] between $F_i$ and $F_j$.

**Theorem 3.2.** *Let $F_i = \langle(\epsilon_i, \rho_i, \delta_i, \gamma_i); \eta^1_{F_i}, \eta^2_{F_i}, \cdots, \eta^P_{F_i}\rangle$ $(i \in I_n)$ be a collection of TFM-numbers and $\upsilon = (\upsilon_1, \upsilon_2, \cdots, \upsilon_n)^T$ weight vector of $F_i$ $(i \in I_n)$, where $\upsilon_i \geq 0$ $(i \in I_n)$ and $\sum\limits_{i=1}^{n} \upsilon_i = 1$. Then the aggregated value by TFMPWA operator is a TFM-number and computed as follows:*

*For simplicity, let* $\frac{\upsilon_i(1+T(F_i))}{\sum\limits_{i=1}^{n} \upsilon_i(1+T(F_i))} = \varsigma_i$.

(3.2)

$$\begin{aligned} TFMPWA(F_1, F_2, \cdots, F_n) &= \bigoplus_{i=1}^{n} \varsigma_i F_i \\ &= \varsigma_1 F_1 \oplus \varsigma_2 F_2 \oplus, \cdots, \oplus \varsigma_n F_n \\ &= \Big\langle \big(\bigoplus_{i=1}^{n} \varsigma_i \epsilon_i, \bigoplus_{i=1}^{n} \varsigma_i \rho_i, \bigoplus_{i=1}^{n} \varsigma_i \delta_i, \bigoplus_{i=1}^{n} \varsigma_i \gamma_i\big); \\ & 1 - \prod_{i=1}^{n}(1 - \eta^1_{F_i})^{\varsigma_i}, 1 - \prod_{i=1}^{n}(1 - \eta^2_{F_i})^{\varsigma_i}, \cdots, 1 - \prod_{i=1}^{n}(1 - \eta^P_{F_i})^{\varsigma_i}\Big\rangle, \end{aligned}$$

*where* $T(F_i) = \sum\limits_{j=1, i\neq j}^{n} Sup(F_i, F_j)$.

*Proof.* By using the mathematical induction on $n$ and considering operational laws in Definition 2.7, we can prove the theorem. We need to show that:
(3.3)

$$\begin{aligned}TFMPWA(F_1, F_2, \cdots, F_n) &= \bigoplus_{i=1}^{n} \varsigma_i F_i \\ &= \varsigma_1 F_1 \oplus \varsigma_2 F_2 \oplus, \cdots, \oplus \varsigma_n F_n \\ &= \big\langle \big( \bigoplus_{i=1}^{n} \varsigma_i \epsilon_i, \bigoplus_{i=1}^{n} \varsigma_i \rho_i, \bigoplus_{i=1}^{n} \varsigma_i \delta_i, \bigoplus_{i=1}^{n} \varsigma_i \gamma_i \big); \\ & 1 - \prod_{i=1}^{n} (1-\eta_{F_i}^{1})^{\varsigma_i}, 1 - \prod_{i=1}^{n} (1-\eta_{F_i}^{2})^{\varsigma_i}, \cdots, 1 - \prod_{i=1}^{n} (1-\eta_{F_i}^{P})^{\varsigma_i} \big\rangle.\end{aligned}$$

(i) Suppose $n = 2$. Then we have

$$\begin{aligned}& TFMPWA(F_1, F_2) \\ &= \bigoplus_{i=1}^{2} \varsigma_i F_i \\ &= \varsigma_1 F_1 \oplus \varsigma_2 F_2 \\ &= \big\langle \big( \varsigma_1 \epsilon_1 + \varsigma_2 \epsilon_2, \varsigma_1 \rho_1 + \varsigma_2 \rho_2, \varsigma_1 \delta_1 + \varsigma_2 \delta_2, \varsigma_1 \gamma_1 + \varsigma_2 \gamma_2 \big); \\ & \qquad s(\varsigma_1 \eta_{F_1}^{1}, \varsigma_2 \eta_{F_2}^{1}), s(\varsigma_1 \eta_{F_1}^{2}, \varsigma_2 \eta_{F_2}^{2}) \big\rangle \\ &= \big\langle \big( \bigoplus_{i=1}^{2} \varsigma_i \epsilon_i, \bigoplus_{i=1}^{2} \varsigma_i \rho_i, \bigoplus_{i=1}^{2} \varsigma_i \delta_i, \bigoplus_{i=1}^{2} \varsigma_i \gamma_i \big); \\ & \quad 1 - \prod_{i=1}^{2} (1-\eta_{F_i}^{1})^{\varsigma_i}, 1 - \prod_{i=1}^{2} (1-\eta_{F_i}^{2})^{\varsigma_i}, \cdots, 1 - \prod_{i=1}^{2} (1-\eta_{F_i}^{P})^{\varsigma_i} \big\rangle.\end{aligned}$$

Thus the Equation (3.3) is right.

(ii) Suppose the Equation (3.3) is right for $n = k$, i.e.,

$$\begin{aligned}& TFMPWA(F_1, F_2, ..., F_n) \\ &= \bigoplus_{i=1}^{k} \varsigma_i F_i \\ &= \varsigma_1 F_1 \oplus \varsigma_2 F_2 \oplus ... \oplus \varsigma_k F_k \\ &= \big\langle \big( \bigoplus_{i=1}^{k} \varsigma_i \epsilon_i, \bigoplus_{i=1}^{k} \varsigma_i \rho_i, \bigoplus_{i=1}^{k} \varsigma_i \delta_i, \bigoplus_{i=1}^{k} \varsigma_i \gamma_i \big); \\ & \quad 1 - \prod_{i=1}^{k} (1-\eta_{F_i}^{1})^{\varsigma_i}, 1 - \prod_{i=1}^{k} (1-\eta_{F_i}^{2})^{\varsigma_i}, \cdots, 1 - \prod_{i=1}^{k} (1-\eta_{F_i}^{P})^{\varsigma_i} \big\rangle.\end{aligned}$$

Now, we need to prove it is true for $n = k+1$ as well. Then from the Equation (3.2) for $n = k+1$, we obtain the following:

$$
\begin{aligned}
TFMPWA(F_1, F_2, ..., F_{k+1}) &= \bigoplus_{i=1}^{k} \varsigma_i F_i + \varsigma_{k+1} F_{k+1} \\
&= \varsigma_1 F_1 \oplus \varsigma_2 F_2 \oplus ... \oplus \varsigma_n F_n \oplus \varsigma_{k+1} F_{k+1} \\
&= \big\langle \big( \bigoplus_{i=1}^{k} \varsigma_i \epsilon_i + \varsigma_{k+1} \epsilon_{k+1}, \bigoplus_{i=1}^{k} \varsigma_i \rho_i + \varsigma_{k+1} \rho_{k+1}, \\
&\bigoplus_{i=1}^{k} \varsigma_i \delta_i + \varsigma_{k+1} \delta_{k+1}, \bigoplus_{i=1}^{k} \varsigma_i \gamma_i + \varsigma_{k+1} \gamma_{k+1} \big); \\
&s(1 - \prod_{i=1}^{k} (1 - \eta^1_{F_i})^{\varsigma_i}, \varsigma_{k+1} \eta^1_{F_{k+1}}), \\
&s(1 - \prod_{i=1}^{k} (1 - \eta^2_{F_i})^{\varsigma_i}, \varsigma_{k+1} \eta^2_{F_{k+1}}), \cdots, \\
&s(1 - \prod_{i=1}^{k} (1 - \eta^P_{F_i})^{\varsigma_i}, \varsigma_{k+1} \eta^P_{F_{k+1}}) \big\rangle \\
&= \bigoplus_{i=1}^{k+1} \varsigma_i F_i.
\end{aligned}
$$

Thus the Equation (3.2) holds for $n = k+1$. So the Equation (3.2) holds for all $n$. □

**Note 3.3.** Let $F_i = \langle (\epsilon_i, \rho_i, \delta_i, \gamma_i); \eta^1_{F_i}, \eta^2_{F_i}, \cdots, \eta^P_{F_i} \rangle$ $(i \in I_n)$ be a collection of TFM-numbers. If $\upsilon = (1/n, 1/n, \cdots, 1/n)^T$, then TFMPWA operator (3.1) converted into a TFMPA operator:

(3.4)

$$
TFMPA(F_1, F_2, \cdots, F_n) = \frac{((1 + T(F_1))F_1) \oplus ((1 + T(F_2))F_2) \oplus, \cdots, \oplus((1 + T(F_n))F_n)}{\sum_{i=1}^{n} (1 + T(F_i))},
$$

where $T(F_i) = \sum_{j=1, i \neq j}^{n} Sup(F_i, F_j)$.

**Proposition 3.4.** *Let $F_i = \langle (\epsilon_i, \rho_i, \delta_i, \gamma_i); \eta^1_{F_i}, \eta^2_{F_i}, \cdots, \eta^P_{F_i} \rangle$ $(i \in I_n)$ be a collection of TFM-numbers. TFMPWA operator has the following desirable properties:*

(1) (**Commutativity**) *if $(\dot{F}_1, \dot{F}_2, \cdots, \dot{F}_n)$ is any permutation of $(F_1, F_2, \cdots, F_n)$, then $TFMPWA(F_1, F_2, \cdots, F_n) = TFMPWA(\dot{F}_1, \dot{F}_2, \cdots, \dot{F}_n)$,*

(2) (**Idempotency**) *if $F_i = F$ for all i, then $TFMPWA(F_1, F_2, \cdots, F_n) = F$,*

(3) (**Boundedness**) *$F^- \leq TFMPWA(F_1, F_2, \cdots, F_n) \leq F^+$,*

*where*

$$
\begin{aligned}
F^+ = \langle (&\max\{\epsilon_i\}_{i \in I_n}, \max\{\rho_i\}_{i \in I_n}, \max\{\delta_i\}_{i \in I_n}, \max\{\gamma_i\}_{i \in I_n}); \\
&\max\{\eta^1_{F_i}\}_{i \in I_n}, \max\{\eta^2_{F_i}\}_{i \in I_n}, \cdots, \max\{\eta^P_{F_i}\}_{i \in I_n} \rangle
\end{aligned}
$$

*and*

$$
F^- = \langle (\min\{\epsilon_i\}_{i \in I_n}, \min\{\rho_i\}_{i \in I_n}, \min\{\delta_i\}_{i \in I_n}, min\{\gamma_i\}_{i \in I_n});
$$

$$\min\{\eta^1_{F_i}\}_{i\in I_n}, \min\{\eta^2_{F_i}\}_{i\in I_n}, \cdots, \min\{\eta^P_{F_i}\}_{i\in I_n}\rangle.$$

*Proof.* (1) Suppose $(\dot{F_1}, \dot{F_2}, \cdots, \dot{F_n})$ is any permutation of $(F_1, F_2, \cdots, F_n)$. Then we get

$$\begin{aligned} TFMPWA(F_1, F_2, \cdots, F_n) &= \frac{\bigoplus_{i=1}^{n} \upsilon_i(1+T(F_i))F_i}{\sum_{i=1}^{n} \upsilon_i(1+T(F_i))} \\ &= \frac{\bigoplus_{i=1}^{n} \upsilon_i(1+T(\dot{F_i}))\dot{F_i}}{\sum_{i=1}^{n} \upsilon_i(1+T(\dot{F_i}))} \\ &= TFMPWA(\dot{F_1}, \dot{F_2}, \cdots, \dot{F_n}). \end{aligned}$$

(2) Suppose $F_i = F$ for all i. Then we get

$$\begin{aligned} TFMPWA(F_1, F_2, \cdots, F_n) &= \frac{\bigoplus_{i=1}^{n} \upsilon_i(1+T(F_i))F_i}{\sum_{i=1}^{n} \upsilon_i(1+T(F_i))} \\ &= \frac{\bigoplus_{i=1}^{n} \upsilon_i(1+T(F))F}{\sum_{i=1}^{n} \upsilon_i(1+T(F))} \\ &= \frac{(1+T(F))F \sum_{i=1}^{n} \upsilon_i}{(1+T(F)) \sum_{i=1}^{n} \upsilon_i} \\ &= \frac{(1+T(F))F}{(1+T(F))} \\ &= TFMPWA(F, F, \cdots, F) \\ &= F. \end{aligned}$$

(3) Let's denote

$$\epsilon_b = \max\{\epsilon_i\}_{i\in I_n},\ \epsilon_l = \min\{\epsilon_i\}_{i\in I_n},$$

$$\rho_b = \max\{\rho_i\}_{i\in I_n},\ \rho_l = \min\{\rho_i\}_{i\in I_n},$$

$$\delta_b = \max\{\delta_i\}_{i\in I_n},\ \delta_l = \min\{\delta_i\}_{i\in I_n},$$

$$\gamma_b = \max\{\gamma_i\}_{i\in I_n},\ \gamma_l = \min\{\gamma_i\}_{i\in I_n}.$$

Similarly,

$$\eta^1_{F_b} = \max\{\eta^1_{F_i}\}_{i\in I_n},\ \eta^1_{F_l} = \min\{\eta^1_{F_i}\}_{i\in I_n},$$

$$\eta^2_{F_b} = \max\{\eta^2_{F_i}\}_{i\in I_n},\ \eta^2_{F_l} = \min\{\eta^2_{F_i}\}_{i\in I_n},$$

$$\vdots$$

$$\eta^P_{F_b} = \max\{\eta^P_{F_i}\}_{i\in I_n},\ \eta^P_{F_l} = \min\{\eta^P_{F_i}\}_{i\in I_n}.$$

and let

$$F^+ = \langle(\epsilon_b, \rho_b, \delta_b, \gamma_b); \eta^1_{F_b}, \eta^2_{F_b}, \cdots, \eta^P_{F_b}\rangle$$

and

$$F^- = \langle(\epsilon_l, \rho_l, \delta_l, \gamma_l); \eta^1_{F_l}, \eta^2_{F_l}, \cdots, \eta^P_{F_l}\rangle.$$

Since $\epsilon_l \le \epsilon_i \le \epsilon_b, \rho_l \le \rho_i \le \rho_b, \delta_l \le \delta_i \le \delta_b$ and $\gamma_l \le \gamma_i \le \gamma_b$,

$$\eta^1_{F_l} \leq \eta^1_{F_i} \leq \eta^1_{F_b},\ \eta^2_{F_l} \leq \eta^2_{F_i} \leq \eta^2_{F_b}, \cdots, \eta^P_{F_l} \leq \eta^P_{F_i} \leq \eta^P_{F_b},$$

we get

$$(3.5)\quad \begin{aligned} &\bigoplus_{i=1}^{n} \upsilon_i \epsilon_l \leq \bigoplus_{i=1}^{n} \upsilon_i \epsilon_i \leq \bigoplus_{i=1}^{n} \upsilon_i \epsilon_b, \bigoplus_{i=1}^{n} \upsilon_i \rho_l \leq \bigoplus_{i=1}^{n} \upsilon_i \rho_i \leq \bigoplus_{i=1}^{n} \upsilon_i \rho_b, \\ &\bigoplus_{i=1}^{n} \upsilon_i \delta_l \leq \bigoplus_{i=1}^{n} \upsilon_i \delta_i \leq \bigoplus_{i=1}^{n} \upsilon_i \delta_b \ \ and \ \ \bigoplus_{i=1}^{n} \upsilon_i \gamma_l \leq \bigoplus_{i=1}^{n} \upsilon_i \gamma_i \leq \bigoplus_{i=1}^{n} \upsilon_i \gamma_b. \end{aligned}$$

On the other hand, we get

$$1-\eta^1_{F_b} \leq 1-\eta^1_{F_i} \leq 1-\eta^1_{F_l}, 1-\eta^2_{F_b} \leq 1-\eta^2_{F_i} \leq 1-\eta^2_{F_l}, \cdots, 1-\eta^P_{F_b} \leq 1-\eta^P_{F_i} \leq 1-\eta^P_{F_l}.$$

Then we have

$$(3.6)\quad \begin{aligned} &1-\prod_{i=1}^{n}(1-\eta^1_{F_l})^{\upsilon_i} \leq 1-\prod_{i=1}^{n}(1-\eta^1_{F_i})^{\upsilon_i} \leq 1-\prod_{i=1}^{n}(1-\eta^1_{F_b})^{\upsilon_i}, \\ &1-\prod_{i=1}^{n}(1-\eta^2_{F_l})^{\upsilon_i} \leq 1-\prod_{i=1}^{n}(1-\eta^2_{F_i})^{\upsilon_i} \leq 1-\prod_{i=1}^{n}(1-\eta^2_{F_b})^{\upsilon_i}, \cdots, \\ &1-\prod_{i=1}^{n}(1-\eta^P_{F_l})^{\upsilon_i} \leq 1-\prod_{i=1}^{n}(1-\eta^P_{F_i})^{\upsilon_i} \leq 1-\prod_{i=1}^{n}(1-\eta^P_{F_b})^{\upsilon_i}. \end{aligned}$$

Thus from inequalities in (3.5) and (3.6), we get

$$F^- \leq TFMPWA(F_1, F_2, \cdots, F_n) \leq F^+.$$

□

**Definition 3.5.** Let $F_i = \langle(\epsilon_i, \rho_i, \delta_i, \gamma_i); \eta^1_{F_i}, \eta^2_{F_i}, \cdots, \eta^P_{F_i}\rangle$ $(i \in I_n)$ be a collection of TFM-numbers and $\upsilon = (\upsilon_1, \upsilon_2, \cdots, \upsilon_n)^T$ weight vector of $F_i$ $(i \in I_n)$, where $\upsilon_i \geq 0$ $(i \in I_n)$ and $\sum\limits_{i=1}^{n} \upsilon_i = 1$. Then the *TFMPOWA operator* is defined as follows:

$$TFMPOWA(F_1, F_2, \cdots, F_n) = \frac{\sum\limits_{i=1}^{n} \upsilon_i(1 + T(F_{\sigma(i)}))F_{\sigma(i)}}{\sum\limits_{i=1}^{n} \upsilon_i(1 + T(F_{\sigma(i)}))},$$

where $F_{\sigma(i)}$ is the $i^{th}$ largest of the trapezoidal fuzzy sets $(F_1, F_2, \cdots, F_n)$ and $\upsilon_i$ is the collection of weights such that

$$(3.7)\quad \upsilon_i = f\left(\frac{R_i}{TV}\right) - f\left(\frac{R_{i-1}}{TV}\right), R_i = \sum_{j=1}^{i} V_{\sigma(j)}, TV = \sum_{i=1}^{n} V_{\sigma(i)}, V_{\sigma(i)} = 1 + T(F_{\sigma(i)})$$

and $T(F_{\sigma(i)})$ denotes the support of the $i^{th}$ largest trapezoidal fuzzy variable $F_{\sigma(i)}$ by all the other trapezoidal fuzzy variables, i.e.,

$$(3.8)\quad T(F_{\sigma(i)}) = \sum_{j=1, i\neq j}^{n} Sup(T(F_{\sigma(i)}), T(F_{\sigma(j)})),$$

where $Sup(T(F_{\sigma(i)}), T(F_{\sigma(j)}))$ represents the support of $j^{th}$ largest trapezoidal fuzzy variable $F_{\sigma(i)})$ for the $i^{th}$ largest trapezoidal fuzzy variable $F_{\sigma(j)})$ and $f : [0, 1] \rightarrow$

$[0,1]$ is a basic unit interval monotonic function which satisfies the following properties:

(i) $f(0)=0$,
(ii) $f(1)=1$,
(iii) $f(x)\geq f(y)$, if $x>y$.

**Definition 3.6.** Let $F_i=\langle(\epsilon_i,\rho_i,\delta_i,\gamma_i);\eta^1_{F_i},\eta^2_{F_i},\cdots,\eta^P_{F_i}\rangle$ $(i\in I_n)$ be a collection of TFM-numbers and $\upsilon=(\upsilon_1,\upsilon_2,\cdots,\upsilon_n)^T$ weight vector of $F_i$ $(i\in I_n)$, where $\upsilon_i\geq 0$ $(i\in I_n)$ and $\sum\limits_{i=1}^{n}\upsilon_i=1$. Then the *aggregated value* by the TFMPWG operator is defined as follows:

(3.9)

$$TFMPWG(F_1,F_2,...,F_n)=\bigotimes_{i=1}^{n}(F_i)^{\frac{1}{n-1}\left(1-\frac{\upsilon_i(1+T(F_i))}{\sum\limits_{i=1}^{n}\upsilon_i(1+T(F_i))}\right)}$$

$$=F_1^{\frac{1}{n-1}\left(1-\frac{\upsilon_1(1+T(F_1))}{\sum\limits_{i=1}^{n}\upsilon_i(1+T(F_i))}\right)}\otimes F_2^{\frac{1}{n-1}\left(1-\frac{\upsilon_2(1+T(F_2))}{\sum\limits_{i=1}^{n}\upsilon_i(1+T(F_i))}\right)}\otimes,\cdots,\otimes F_n^{\frac{1}{n-1}\left(1-\frac{\upsilon_n(1+T(F_n))}{\sum\limits_{i=1}^{n}\upsilon_i(1+T(F_i))}\right)},$$

where $T(F_i)=\sum\limits_{j=1,i\neq j}^{n}Sup(F_i,F_j)$ and $Sup(F_i,F_j)=1-d(F_i,F_j)$.

**Theorem 3.7.** *Let $F_i=\langle(\epsilon_i,\rho_i,\delta_i,\gamma_i);\eta^1_{F_i},\eta^2_{F_i},\cdots,\eta^P_{F_i}\rangle$ $(i\in I_n)$ be a collection of TFM-numbers and $\upsilon=(\upsilon_1,\upsilon_2,\cdots,\upsilon_n)^T$ weight vector of $F_i$ $(i\in I_n)$, where $\upsilon_i\geq 0$ $(i\in I_n)$ and $\sum\limits_{i=1}^{n}\upsilon_i=1$. Then the aggregated value by the TFMPWG is a TFM-number and computed as follows:*

*For simplicity, let* $\frac{1}{n-1}\left(1-\frac{\upsilon_i(1+T(F_i))}{\sum\limits_{i=1}^{n}\upsilon_i(1+T(F_i))}\right)=\zeta_i$

$$\begin{aligned}TFMPWG(F_1,F_2,\cdots,F_n)=&\bigotimes_{i=1}^{n}{F_i}^{\zeta_i}\\ =&\langle(\bigotimes_{i=1}^{n}{\epsilon_i}^{\zeta_i},\bigotimes_{i=1}^{n}{\rho_i}^{\zeta_i},\bigotimes_{i=1}^{n}{\delta_i}^{\zeta_i},\bigotimes_{i=1}^{n}{\gamma_i}^{\zeta_i});\\ &\bigotimes_{i=1}^{n}{\eta^1_{F_i}}^{\zeta_i},\bigotimes_{i=1}^{n}{\eta^2_{F_i}}^{\zeta_i},\cdots,\bigotimes_{i=1}^{n}{\eta^P_{F_i}}^{\zeta_i}\rangle.\end{aligned}$$

*Proof.* The proof can be done similar to Theorem 3.2. □

**Note 3.8.** Let $F_i=\langle(\epsilon_i,\rho_i,\delta_i,\gamma_i);\eta^1_{F_i},\eta^2_{F_i},\cdots,\eta^P_{F_i}\rangle$ $(i\in I_n)$ be a collection of TFM-numbers. If $\upsilon=(1/n,1/n,\cdots,1/n)^T$, then the TFMPWG operator (3.9) converted into TFMPG operator:

$$TFMPG(F_1, F_2, \cdots, F_n) = F_1^{\frac{1}{n-1}\left(1-\frac{1+T(F_1)}{\sum\limits_{i=1}^{n}(1+T(F_i))}\right)}$$

$$\otimes F_2^{\frac{1}{n-1}\left(1-\frac{1+T(F_2)}{\sum\limits_{i=1}^{n}(1+T(F_i))}\right)} \otimes, \cdots, \otimes F_n^{\frac{1}{n-1}\left(1-\frac{1+T(F_n)}{\sum\limits_{i=1}^{n}(1+T(F_i))}\right)} \tag{3.10}$$

**Proposition 3.9.** *Let $F_i = \langle(\epsilon_i, \rho_i, \delta_i, \gamma_i); \eta^1_{F_i}, \eta^2_{F_i}, \cdots, \eta^P_{F_i}\rangle$ $(i \in I_n)$ be a collection of TFM-numbers. TFMPWG operator has the following desirable properties:*

(1) (**Commutativity**) *if $(\dot{F}_1, \dot{F}_2, \cdots, \dot{F}_n)$ is any permutation of $(F_1, F_2, ..., F_n)$, then $TFMPWG(F_1, F_2, \cdots, F_n) = TFMPWG(\dot{F}_1, \dot{F}_2, \cdots, \dot{F}_n)$,*

(2) (**Idempotency** *if $F_i = F$ for all i, then $TFMPWG(F_1, F_2, \cdots, F_n) = F$,*

(3) (**Boundedness**) *$F^- \leq TFMPWG(F_1, F_2, \cdots, F_n) \leq F^+$,*

*where* $F^+ = \langle(max\{\epsilon_i\}_{i\in I_n}, max\{\rho_i\}_{i\in I_n}, max\{\delta_i\}_{i\in I_n}, max\{\gamma_i\}_{i\in I_n});$
$max\{\eta^1_{F_i}\}_{i\in I_n}, max\{\eta^2_{F_i}\}_{i\in I_n}, \cdots, max\{\eta^P_{F_i}\}_{i\in I_n}\rangle$

*and*

$F^- = \langle(min\{\epsilon_i\}_{i\in I_n}, min\{\rho_i\}_{i\in I_n}, min\{\delta_i\}_{i\in I_n}, min\{\gamma_i\}_{i\in I_n});$
$min\{\eta^1_{F_i}\}_{i\in I_n}, min\{\eta^2_{F_i}\}_{i\in I_n}, \cdots, min\{\eta^P_{F_i}\}_{i\in I_n}\rangle.$

*Proof.* The proof can be done similar to Proposition 3.4. □

**Definition 3.10.** Let $F_i = \langle(\epsilon_i, \rho_i, \delta_i, \gamma_i); \eta^1_{F_i}, \eta^2_{F_i}, ..., \eta^P_{F_i}\rangle$ $(i \in I_n)$ be a collection of TFM-numbers and $\upsilon = (\upsilon_1, \upsilon_2, \cdots, \upsilon_n)^T$ weight vector of $F_i$ $(i \in I_n)$, where $\upsilon_i \geq 0$ $(i \in I_n)$ and $\sum\limits_{i=1}^{n} \upsilon_i = 1$. Then the *TFMPOWG operator* is defined as follows:

$$TFMPOWG(F_1, F_2, \cdots, F_n) = \prod_{i=1}^{n}(F_{\sigma(i)})^{\frac{\upsilon_i(1+T(F_{\sigma(i)}))}{\sum\limits_{i=1}^{n}\upsilon_i(1+T(F_{\sigma(i)}))}}, \tag{3.11}$$

where $\upsilon_i$ $(i \in I_n)$ is the collection of weights which satisfies conditions (3.7) and (3.8).

Especially, if $f(x) = x$, then the TFMPOWA operator (3.11) converted into the TFMPA operator (3.10).

## 4. Two approaches to multi-attribute group decision-making with trapezoidal fuzzy multi information

In this section, by inspiring Xu [40], two approaches to solve multi-attribute group decision-making problem introduced.

Let alternative's set be $Z = \{Z_i | i \in I_m\}$ and attributes' set be $G = \{G_j | j \in I_n\}$. The weight vector of attributes is $\upsilon = (\upsilon_1, \upsilon_2, \cdots, \upsilon_n)$ satisfying $\upsilon_j \geq 0$ $(j \in I_n)$, $\sum\limits_{j=1}^{n} \upsilon_j = 1$. Moreover, let decision maker's set be $D = \{D_k | (k \in I_t)\}$ whose weight vector $\tau = (\tau_1, \tau_2, \cdots, \tau_t)$ with $\tau_k \geq 0$ $(k \in I_t)$, $\sum\limits_{k=1}^{t} \tau_k = 1$.

Approach I

**Step 1.** Construct the decision matrix of each expert.

**Step 2.** Normalise each attribute value $F_{ij}^{(k)}$ in the matrix $F_k$ into a corresponding element in the matrix $\tilde{R}_k = (\tilde{r}_{ij}^{(k)})_{mxn}$ such that

$$(\tilde{r}_{ij}^{(k)}) = \begin{cases} \langle(\epsilon_{ij}, \rho_{ij}, \delta_{ij}, \gamma_{ij}); \eta_{F_{ij}}^1, \eta_{F_{ij}}^2, \cdots, \eta_{F_{ij}}^k\rangle & \text{for benefit attribute } G_j \\ \langle(\epsilon_{ij}, \rho_{ij}, \delta_{ij}, \gamma_{ij}); 1-\eta_{F_{ij}}^1, 1-\eta_{F_{ij}}^2, \cdots, 1-\eta_{F_{ij}}^k\rangle & \text{for cost attribute } G_j. \end{cases} \tag{4.1}$$

**Step 3.** Support measure is calculated as follows:

$$Sup(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)}) = 1 - d(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)}), (l \in I_t). \tag{4.2}$$

Here $d(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)})$ is calculated with Hamming distance in Definition 2.9 and utilize weights $\tau = (\tau_1, \tau_2, ..., \tau_t)$ of the decision makers $D_k (k \in I_t)$ to find the weighted support $T(\tilde{r}_{ij}^{(k)})$ of the trapezoidal fuzzy preference value $(\tilde{r}_{ij}^{(k)})$ by other trapezoidal fuzzy preference value $(\tilde{r}_{ij}^{(l)})$ ($l \in I_t$ and $l \neq k$)

$$T(\tilde{r}_{ij}^{(k)}) = \sum_{l=1, l\neq k}^{t} \tau_l Sup(\tilde{r}_{ij}^{(k)}, \tilde{r}_{ij}^{(l)})$$

and weights $\eta_{ij}^{(k)}$ $(k \in I_t)$ of the trapezoidal fuzzy preference value $\tilde{r}_{ij}^{(k)}$ $(k \in I_t)$ is calculated as follows:

$$\eta_{ij}^{(k)} = \frac{\tau_k(1 + T(r_{ij}^{(k)}))}{\sum_{k=1}^{t} \tau_k(1 + T(r_{ij}^{(k)}))} \ (k \in I_t),$$

where $\eta_{ij}^{(k)} \geq 0$ $(k \in I_t)$ and $\sum_{k=1}^{t} \eta_{ij}^{(k)} = 1$.

**Step 4.** Use the matrix $\tilde{R}_k$ and find the TFMPWA operator:

$$\sum_{k=1}^{t} \frac{\tau_k(1 + T(\tilde{r}_{ij}^{(k)}))\tilde{r}_{ij}^{(k)}}{\sum_{k=1}^{t} \tau_k(1 + T(r_{ij}^{(k)}))} = \sum_{k=1}^{t} \eta_{ij}^{(k)} \tilde{r}_{ij}^{(k)} \ (i \in I_m, j \in I_n)$$

or the TFMPWG operator:

$$\prod_{k=1}^{t} (\tilde{r}_{ij}^{(k)})^{\frac{\tau_k(1+T(r_{ij}^{(k)}))}{\sum_{k=1}^{t} \tau_k(1+T(r_{ij}^{(k)}))}} = \prod_{k=1}^{t} (\tilde{r}_{ij}^{(k)})^{\eta_{ij}^{(k)}} \ (i \in I_m, j \in I_n).$$

**Step 5.** By using the FWA operator, find the aggregation of all trapezoidal fuzzy preference value $\tilde{r}_{ij}$ $(j \in I_n)$:

$$\tilde{r}_i = FWA_\upsilon = \sum_{j=1}^{n} \upsilon_j \tilde{r}_{ij} \ (i \in I_m)$$

or FWG operator:

$$\tilde{r}_i = FWG_\upsilon = \prod_{j=1}^{n} \tilde{r}_{ij}^{\upsilon_j} \ (i \in I_m)$$

**Step 6.** Find the Hamming distance between $\tilde{r}_i$ and positive ideal (or negative ideal) given in Definition 2.10.

**Step 7.** Rank all the alternatives. If Hamming distance between $\tilde{r}_i$ and positive ideal taken consideration, then the bigger result, the better alternative. If Hamming distance between $\tilde{r}_i$ and negative ideal taken consideration, then the smaller result, the better alternative.

Approach II

**Step 1.** Same as Approach I.
**Step 2.** Same as Approach I.
**Step 3.** Support measure is calculated as follows:

$$Sup(\tilde{r}_{ij}^{\sigma(k)}, \tilde{r}_{ij}^{\sigma(l)}) = 1 - d(\tilde{r}_{ij}^{(\sigma(k)}, \tilde{r}_{ij}^{(\sigma(l)})$$

which remarks the support of $l^{th}$ largest trapezoidal fuzzy preference value $r_{ij}^{l}$ for the $k^{th}$ largest trapezoidal fuzzy preference value $\tilde{r}_{ij}^{k}$ of $\tilde{r}_{ij}^{k}$.

**Step 4.** Find the value of the support $T(\tilde{r}_{ij}^{(k)})$ of the $k^{th}$ largest trapezoidal fuzzy preference value $\tilde{r}_{ij}^{(k)}$ $(l \in I_t$ and $l \neq k)$

$$T(\tilde{r}_{ij}^{\sigma(k)}) = \sum_{l=1, l\neq k}^{t} Sup(\tilde{r}_{ij}^{\sigma(k)}, \tilde{r}_{ij}^{\sigma(l)})$$

and use equalities in (3.7) to find the weights $\upsilon_{ij}^{(k)}$ $(k \in I_t)$ related to the $k^{th}$ largest trapezoidal fuzzy preference value $\tilde{r}_{ij}^{(k)}$, where

$$\upsilon_{ij}^{(k)} = f\left(\frac{Q_{ij}^{(k)}}{TV_{ij}}\right) - f\left(\frac{Q_{ij}^{(k-1)}}{TV_{ij}}\right),$$

$$Q_{ij}^{(k)} = \sum_{l=1}^{k} \eta_{ij}^{\sigma(l)}, TV_{ij} = \sum_{l=1}^{t} \eta_{ij}^{\sigma(l)}, \eta_{ij}^{\sigma(l)} = 1 + T(\tilde{r}_{ij}^{\sigma(l)}),$$

where $\upsilon_{ij}^{(k)} \geq 0$ $(k \in I_t)$ $\quad and \quad$ $\sum_{k=1}^{t} \upsilon_{ij}^{(k)} = 1$.

**Step 5.** To convert all the individual decision matrices $\tilde{R}_k (k \in I_t)$ into the collective decision matrix $\tilde{R} = (\tilde{r}_{ij})_{mxn}$,
Utilise TFMPOWA operator:

$$TFMPOWA = \sum_{k=1}^{t} \upsilon_{ij}^{(k)} \tilde{r}_{ij}^{(k)} \ (i \in I_m \quad and \quad j \in I_n)$$

or the TFMPOWG operator:

$$TFMPOWG = \prod_{k=1}^{t} (\tilde{r}_{ij}^{(k)})^{\upsilon_{ij}^{(k)}} \ (i \in I_m \quad and \quad j \in I_n).$$

**Step 6.** Same as Approach I.
**Step 7.** Same asApproach I.

## 5. Application

Here, we give an illustrative example to show the efficiency of the proposed operators.

Table 2. TFM-numbers response to linguistic terms

| Linguistic terms | TFM-numbers |
|---|---|
| Absolutely high(AH) | $\langle(0.70, 0.80, 0.90, 1.00); 0.7, 0.8, 0.9, 0.2\rangle$ |
| Very Very High(VVH) | $\langle(0.50, 0.60, 0.70, 0.80); 0.1, 0.7, 0.8, 0.9\rangle$ |
| Very High(VH) | $\langle(0.45, 0.55, 0.65, 0.75); 0.7, 0.8, 0.6, 0.3\rangle$ |
| High(H) | $\langle(0.40, 0.45, 0.50, 0.55); 0.8, 0.9, 0.3, 0.6\rangle$ |
| Fairly high(FH) | $\langle(0.30, 0.35, 0.40, 0.45); 0.6, 0.1, 0.8, 0.4\rangle$ |
| Medium(M) | $\langle(0.25, 0.30, 0.35, 0.40); 0.4, 0.5, 0.6, 0.8\rangle$ |
| Fairly low(FL) | $\langle(0.15, 0.20, 0.25, 0.30); 0.4, 0.6, 0.2, 0.5\rangle$ |
| Low(L) | $\langle(0.10, 0.20, 0.20, 0.30); 0.3, 0.4, 0.8, 0.1\rangle$ |
| Very Low(VL) | $\langle(0.10, 0.15, 0.15, 0.20); 0.2, 0.4, 0.5, 0.3\rangle$ |
| Very Very Low(VVL) | $\langle(0.05, 0.10, 0.15, 0.20); 0.2, 0.3, 0.4, 0.1\rangle$ |
| Absolutely low(AL) | $\langle(0.01, 0.05, 0.10, 0.15); 0.1, 0.2, 0.3, 0.4\rangle$ |

5.1. **An Illustrative Example.**

**Example 5.1.** In Turkiye, awareness of zero-waste has been growing recently as in other developing countries. As a result of this growth, wastes created by people have been collected more than ever. This issue triggers Turkish government to take action for recycling the collected wastes instead of releasing them to nature. As the first step of the action, the Government wants to set a zero-waste factory in the biggest city of the country, Istanbul. The government has five possible zero-waste factories to be chosen and four attributes to consider. List of five possible alternatives as follows:

(1) $Z_1$ (plastic waste),
(2) $Z_2$ (paper waste),
(3) $Z_3$ (battery waste),
(4) $Z_4$ (organic waste),
(5) $Z_5$ (glass waste).

The government will decide by considering the attributes given below:

(1) $G_1$ is setup cost,
(2) $G_2$ is human resource,
(3) $G_3$ is adaptation period of public,
(4) $G_4$ is amount of waste.

Their weight vector is (0.3,0.1,0.2,0.4).
Three decision makers $D_k$ $(k \in I_3)$ will take part in decision process. Their weight vector is $\tau = (0.4, 0.3, 0.3)$. Trapezoidal fuzzy decision matrices are shown in Tables 6, 7 and 8.

**Step 1.** We constructed decision matrices of each decision maker according to Table 2 [13] linguistically (in Tables 3, 4 and 5) and numerically (in Tables 6, 7 and 8):

Table 3. Decision Matrix $F_1$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | VL | FL | FH | VH |
| $Z_2$ | VVL | L | M | H |
| $Z_3$ | AH | VVH | L | AL |
| $Z_4$ | M | VVL | VVH | M |
| $Z_5$ | VVH | AH | H | L |

Table 4. Decision Matrix $F_2$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | VVL | FH | AL | VVL |
| $Z_2$ | VVH | L | FH | VVH |
| $Z_3$ | AL | AH | VVL | M |
| $Z_4$ | VH | VL | H | L |
| $Z_5$ | FH | FL | M | AH |

Table 5. Decision Matrix $F_3$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | VVL | FL | L | VL |
| $Z_2$ | H | M | M | FH |
| $Z_3$ | AH | VVL | H | AH |
| $Z_4$ | FH | VL | VVH | L |
| $Z_5$ | VH | AL | AL | M |

Table 6. Decision Matrix $F_1$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | $\langle(0.10, 0.15, 0.15, 0.20); 0.2, 0.4, 0.5, 0.3\rangle$ | $\langle(0.15, 0.20, 0.25, 0.30); 0.4, 0.6, 0.2, 0.5\rangle$ | $\langle(0.30, 0.35, 0.40, 0.45); 0.6, 0.1, 0.8, 0.4\rangle$ | $\langle(0.45, 0.55, 0.65, 0.75); 0.7, 0.8, 0.6, 0.3\rangle$ |
| $Z_2$ | $\langle(0.05, 0.10, 0.15, 0.20); 0.2, 0.3, 0.4, 0.1\rangle$ | $\langle(0.10, 0.20, 0.20, 0.30); 0.3, 0.4, 0.8, 0.1\rangle$ | $\langle(0.25, 0.30, 0.35, 0.40); 0.4, 0.5, 0.6, 0.8\rangle$ | $\langle(0.40, 0.45, 0.50, 0.55); 0.8, 0.9, 0.3, 0.6\rangle$ |
| $Z_3$ | $\langle(0.70, 0.80, 0.90, 1.00); 0.7, 0.8, 0.9, 0.2\rangle$ | $\langle(0.50, 0.60, 0.70, 0.80); 0.1, 0.7, 0.8, 0.9\rangle$ | $\langle(0.10, 0.20, 0.20, 0.30); 0.3, 0.4, 0.8, 0.1\rangle$ | $\langle(0.01, 0.05, 0.10, 0.15); 0.1, 0.2, 0.3, 0.4\rangle$ |
| $Z_4$ | $\langle(0.25, 0.30, 0.35, 0.40); 0.4, 0.5, 0.6, 0.8\rangle$ | $\langle(0.05, 0.10, 0.15, 0.20); 0.2, 0.3, 0.4, 0.1\rangle$ | $\langle(0.50, 0.60, 0.70, 0.80); 0.1, 0.7, 0.8, 0.9\rangle$ | $\langle(0.25, 0.30, 0.35, 0.40); 0.4, 0.5, 0.6, 0.8\rangle$ |
| $Z_5$ | $\langle(0.50, 0.60, 0.70, 0.80); 0.1, 0.7, 0.8, 0.9\rangle$ | $\langle(0.70, 0.80, 0.90, 1.00); 0.7, 0.8, 0.9, 0.2\rangle$ | $\langle(0.40, 0.45, 0.50, 0.55); 0.8, 0.9, 0.3, 0.6\rangle$ | $\langle(0.10, 0.20, 0.20, 0.30); 0.3, 0.4, 0.8, 0.1\rangle$ |

Table 7. Decision Matrix $F_2$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | $\langle(0.05, 0.10, 0.15, 0.20); 0.2, 0.3, 0.4, 0.1\rangle$ | $\langle(0.30, 0.35, 0.40, 0.45); 0.6, 0.1, 0.8, 0.4\rangle$ | $\langle(0.01, 0.05, 0.10, 0.15); 0.1, 0.2, 0.3, 0.4\rangle$ | $\langle(0.05, 0.10, 0.15, 0.20); 0.2, 0.3, 0.4, 0.1\rangle$ |
| $Z_2$ | $\langle(0.50, 0.60, 0.70, 0.80); 0.1, 0.7, 0.8, 0.9\rangle$ | $\langle(0.10, 0.20, 0.20, 0.30); 0.3, 0.4, 0.8, 0.1\rangle$ | $\langle(0.30, 0.35, 0.40, 0.45); 0.6, 0.1, 0.8, 0.4\rangle$ | $\langle(0.50, 0.60, 0.70, 0.80); 0.1, 0.7, 0.8, 0.9\rangle$ |
| $Z_3$ | $\langle(0.01, 0.05, 0.10, 0.15); 0.1, 0.2, 0.3, 0.4\rangle$ | $\langle(0.70, 0.80, 0.90, 1.00); 0.7, 0.8, 0.9, 0.2\rangle$ | $\langle(0.05, 0.10, 0.15, 0.20); 0.2, 0.3, 0.4, 0.1\rangle$ | $\langle(0.25, 0.30, 0.35, 0.40); 0.4, 0.5, 0.6, 0.8\rangle$ |
| $Z_4$ | $\langle(0.45, 0.55, 0.65, 0.75); 0.7, 0.8, 0.6, 0.3\rangle$ | $\langle(0.10, 0.15, 0.15, 0.20); 0.2, 0.4, 0.5, 0.3\rangle$ | $\langle(0.40, 0.45, 0.50, 0.55); 0.8, 0.9, 0.3, 0.6\rangle$ | $\langle(0.10, 0.20, 0.20, 0.30); 0.3, 0.4, 0.8, 0.1\rangle$ |
| $Z_5$ | $\langle(0.30, 0.35, 0.40, 0.45); 0.6, 0.1, 0.8, 0.4\rangle$ | $\langle(0.15, 0.20, 0.25, 0.30); 0.4, 0.6, 0.2, 0.5\rangle$ | $\langle(0.25, 0.30, 0.35, 0.40); 0.4, 0.5, 0.6, 0.8\rangle$ | $\langle(0.70, 0.80, 0.90, 1.00); 0.7, 0.8, 0.9, 0.2\rangle$ |

Table 8. Decision Matrix $F_3$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | $\langle(0.05,0.10,0.15,0.20);0.2,0.3,0.4,0.1\rangle$ | $\langle(0.15,0.20,0.25,0.30);0.4,0.6,0.2,0.5\rangle$ | $\langle(0.10,0.20,0.20,0.30);0.3,0.4,0.8,0.1\rangle$ | $\langle(0.10,0.15,0.15,0.20);0.2,0.4,0.5,0.3\rangle$ |
| $Z_2$ | $\langle(0.40,0.45,0.50,0.55);0.8,0.9,0.3,0.6\rangle$ | $\langle(0.25,0.30,0.35,0.40);0.4,0.5,0.6,0.8\rangle$ | $\langle(0.25,0.30,0.35,0.40);0.4,0.5,0.6,0.8\rangle$ | $\langle(0.30,0.35,0.40,0.45);0.6,0.1,0.8,0.4\rangle$ |
| $Z_3$ | $\langle(0.70,0.80,0.90,1.00);0.7,0.8,0.9,0.2\rangle$ | $\langle(0.05,0.10,0.15,0.20);0.2,0.3,0.4,0.1\rangle$ | $\langle(0.40,0.45,0.50,0.55);0.8,0.9,0.3,0.6\rangle$ | $\langle(0.70,0.80,0.90,1.00);0.7,0.8,0.9,0.2\rangle$ |
| $Z_4$ | $\langle(0.30,0.35,0.40,0.45);0.6,0.1,0.8,0.4\rangle$ | $\langle(0.10,0.15,0.15,0.20);0.2,0.4,0.5,0.3\rangle$ | $\langle(0.50,0.60,0.70,0.80);0.1,0.7,0.8,0.9\rangle$ | $\langle(0.10,0.20,0.20,0.30);0.3,0.4,0.8,0.1\rangle$ |
| $Z_5$ | $\langle(0.45,0.55,0.65,0.75);0.7,0.8,0.6,0.3\rangle$ | $\langle(0.01,0.05,0.10,0.15);0.1,0.2,0.3,0.4\rangle$ | $\langle(0.01,0.05,0.10,0.15);0.1,0.2,0.3,0.4\rangle$ | $\langle(0.25,0.30,0.35,0.40);0.4,0.5,0.6,0.8\rangle$ |

**Step 2.** Since $G_1$ and $G_3$ are cost attributes, normalized decision matrix $\tilde{R}_k$ is constructed. The results are shown in Tables 9, 10 and 11:

Table 9. Decision Matrix $\tilde{R}_1$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | $\langle(0.10,0.15,0.15,0.20);0.8,0.6,0.5,0.7\rangle$ | $\langle(0.15,0.20,0.25,0.30);0.4,0.6,0.2,0.5\rangle$ | $\langle(0.30,0.35,0.40,0.45);0.4,0.9,0.2,0.6\rangle$ | $\langle(0.45,0.55,0.65,0.75);0.7,0.8,0.6,0.3\rangle$ |
| $Z_2$ | $\langle(0.05,0.10,0.15,0.20);0.8,0.7,0.6,0.9\rangle$ | $\langle(0.10,0.20,0.20,0.30);0.3,0.4,0.8,0.1\rangle$ | $\langle(0.25,0.30,0.35,0.40);0.6,0.5,0.4,0.2\rangle$ | $\langle(0.40,0.45,0.50,0.55);0.8,0.9,0.3,0.6\rangle$ |
| $Z_3$ | $\langle(0.70,0.80,0.90,1.00);0.3,0.2,0.1,0.8\rangle$ | $\langle(0.50,0.60,0.70,0.80);0.1,0.7,0.8,0.9\rangle$ | $\langle(0.10,0.20,0.20,0.30);0.7,0.6,0.2,0.9\rangle$ | $\langle(0.01,0.05,0.10,0.15);0.1,0.2,0.3,0.4\rangle$ |
| $Z_4$ | $\langle(0.25,0.30,0.35,0.40);0.6,0.5,0.4,0.2\rangle$ | $\langle(0.05,0.10,0.15,0.20);0.2,0.3,0.4,0.1\rangle$ | $\langle(0.50,0.60,0.70,0.80);0.9,0.3,0.2,0.1\rangle$ | $\langle(0.25,0.30,0.35,0.40);0.4,0.5,0.6,0.8\rangle$ |
| $Z_5$ | $\langle(0.50,0.60,0.70,0.80);0.9,0.3,0.2,0.1\rangle$ | $\langle(0.70,0.80,0.90,1.00);0.7,0.8,0.9,0.2\rangle$ | $\langle(0.40,0.45,0.50,0.55);0.2,0.1,0.7,0.4\rangle$ | $\langle(0.10,0.20,0.20,0.30);0.3,0.4,0.8,0.1\rangle$ |

Table 10. Decision Matrix $\tilde{R}_2$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | $\langle(0.05,0.10,0.15,0.20);0.8,0.7,0.6,0.9\rangle$ | $\langle(0.30,0.35,0.40,0.45);0.6,0.1,0.8,0.4\rangle$ | $\langle(0.01,0.05,0.10,0.15);0.9,0.8,0.7,0.6\rangle$ | $\langle(0.05,0.10,0.15,0.20);0.2,0.3,0.4,0.1\rangle$ |
| $Z_2$ | $\langle(0.50,0.60,0.70,0.80);0.9,0.3,0.2,0.1\rangle$ | $\langle(0.10,0.20,0.20,0.30);0.3,0.4,0.8,0.1\rangle$ | $\langle(0.30,0.35,0.40,0.45);0.4,0.9,0.2,0.6\rangle$ | $\langle(0.50,0.60,0.70,0.80);0.1,0.7,0.8,0.9\rangle$ |
| $Z_3$ | $\langle(0.01,0.05,0.10,0.15);0.9,0.8,0.7,0.6\rangle$ | $\langle(0.70,0.80,0.90,1.00);0.7,0.8,0.9,0.2\rangle$ | $\langle(0.05,0.10,0.15,0.20);0.8,0.7,0.6,0.9\rangle$ | $\langle(0.25,0.30,0.35,0.40);0.4,0.5,0.6,0.8\rangle$ |
| $Z_4$ | $\langle(0.45,0.55,0.65,0.75);0.3,0.2,0.4,0.7\rangle$ | $\langle(0.10,0.15,0.15,0.20);0.2,0.4,0.5,0.3\rangle$ | $\langle(0.40,0.45,0.50,0.55);0.2,0.1,0.7,0.4\rangle$ | $\langle(0.10,0.20,0.20,0.30);0.3,0.4,0.8,0.1\rangle$ |
| $Z_5$ | $\langle(0.30,0.35,0.40,0.45);0.4,0.9,0.2,0.6\rangle$ | $\langle(0.15,0.20,0.25,0.30);0.4,0.6,0.2,0.5\rangle$ | $\langle(0.25,0.30,0.35,0.40);0.6,0.5,0.4,0.2\rangle$ | $\langle(0.70,0.80,0.90,1.00);0.7,0.8,0.9,0.2\rangle$ |

Table 11. Decision Matrix $\tilde{R}_3$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | $\langle(0.05,0.10,0.15,0.20);0.8,0.7,0.6,0.9\rangle$ | $\langle(0.15,0.20,0.25,0.30);0.4,0.6,0.2,0.5\rangle$ | $\langle(0.10,0.20,0.20,0.30);0.7,0.6,0.2,0.9\rangle$ | $\langle(0.10,0.15,0.15,0.20);0.2,0.4,0.5,0.3\rangle$ |
| $Z_2$ | $\langle(0.40,0.45,0.50,0.55);0.2,0.1,0.7,0.4\rangle$ | $\langle(0.25,0.30,0.35,0.40);0.4,0.5,0.6,0.8\rangle$ | $\langle(0.25,0.30,0.35,0.40);0.6,0.5,0.4,0.2\rangle$ | $\langle(0.30,0.35,0.40,0.45);0.6,0.1,0.8,0.4\rangle$ |
| $Z_3$ | $\langle(0.70,0.80,0.90,1.00);0.3,0.2,0.1,0.8\rangle$ | $\langle(0.05,0.10,0.15,0.20);0.2,0.3,0.4,0.1\rangle$ | $\langle(0.40,0.45,0.50,0.55);0.2,0.1,0.7,0.4\rangle$ | $\langle(0.70,0.80,0.90,1.00);0.7,0.8,0.9,0.2\rangle$ |
| $Z_4$ | $\langle(0.30,0.35,0.40,0.45);0.4,0.9,0.2,0.6\rangle$ | $\langle(0.10,0.15,0.15,0.20);0.2,0.4,0.5,0.3\rangle$ | $\langle(0.50,0.60,0.70,0.80);0.9,0.3,0.2,0.1\rangle$ | $\langle(0.10,0.20,0.20,0.30);0.3,0.4,0.8,0.1\rangle$ |
| $Z_5$ | $\langle(0.45,0.55,0.65,0.75);0.3,0.2,0.4,0.7\rangle$ | $\langle(0.01,0.05,0.10,0.15);0.1,0.2,0.3,0.4\rangle$ | $\langle(0.01,0.05,0.10,0.15);0.9,0.8,0.7,0.6\rangle$ | $\langle(0.25,0.30,0.35,0.40);0.4,0.5,0.6,0.8\rangle$ |

**Step 3.** Use equations (4.1) and (4.2) to find weight $\eta_{ij}^{(k)}(i \in I_5, j \in I_4, k \in I_3)$ related to attribute values $\tilde{r}_{ij}^{(k)}(i \in I_5, j \in I_4, k \in I_3)$ which are given in the matrices $\eta^{(k)} = (\eta_{ij}^{(k)})_{5x4}$ $(k \in I_3)$ and given in Tables 12, 13 and 14 respectively.

Table 12. Weight Matrix $\eta^{(1)}$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | 0.385 | 0.389 | 0.384 | 0.372 |
| $Z_2$ | 0.383 | 0.389 | 0.387 | 0.392 |
| $Z_3$ | 0.399 | 0.403 | 0.391 | 0.391 |
| $Z_4$ | 0.388 | 0.385 | 0.389 | 0.382 |
| $Z_5$ | 0.388 | 0.367 | 0.387 | 0.394 |

Table 13. Weight Matrix $\eta^{(2)}$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | 0.307 | 0.301 | 0.305 | 0.313 |
| $Z_2$ | 0.304 | 0.310 | 0.304 | 0.303 |
| $Z_3$ | 0.279 | 0.308 | 0.309 | 0.324 |
| $Z_4$ | 0.301 | 0.308 | 0.300 | 0.309 |
| $Z_5$ | 0.304 | 0.321 | 0.313 | 0.284 |

Table 14. Weight Matrix $\eta^{(3)}$

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | 0.307 | 0.310 | 0.312 | 0.315 |
| $Z_2$ | 0.314 | 0.301 | 0.309 | 0.306 |
| $Z_3$ | 0.321 | 0.289 | 0.300 | 0.285 |
| $Z_4$ | 0.311 | 0.308 | 0.311 | 0.309 |
| $Z_5$ | 0.308 | 0.312 | 0.300 | 0.322 |

**Step 4.** Use the TFMPWA and TFMPWG operators to find the aggregation of all individual decision matrices into the collective decision matrix in Tables 15 and 16.

TABLE 15. Decision Matrix $\tilde{R}$ (TFMPWA)

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | $\langle(0.069, 0.119, 0.150, 0.200); 0.800, 0.664, 0.564, 0.847\rangle$ | $\langle(0.195, 0.245, 0.295, 0.345); 0.469, 0.489, 0.473, 0.472\rangle$ | $\langle(0.149, 0.212, 0.246, 0.312); 0.720, 0.810, 0.407, 0.740\rangle$ | $\langle(0.214, 0.283, 0.336, 0.405); 0.445, 0.582, 0.513, 0.243\rangle$ |
| $Z_2$ | $\langle(0.148, 0.362, 0.427, 0.492); 0.750, 0.452, 0.549, 0.658\rangle$ | $\langle(0.331, 0.230, 0.245, 0.330); 0.332, 0.432, 0.754, 0.428\rangle$ | $\langle(0.189, 0.315, 0.365, 0.415); 0.547, 0.694, 0.345, 0.352\rangle$ | $\langle(0.324, 0.465, 0.520, 0.593); 0.610, 0.727, 0.673, 0.702\rangle$ |
| $Z_3$ | $\langle(0.507, 0.590, 0.676, 0.762); 0.594, 0.457, 0.338, 0.757\rangle$ | $\langle(0.432, 0.517, 0.603, 0.688); 0.370, 0.662, 0.778, 0.642\rangle$ | $\langle(0.175, 0.244, 0.275, 0.344); 0.645, 0.533, 0.519, 0.829\rangle$ | $\langle(0.285, 0.345, 0.409, 0.473); 0.423, 0.537, 0.665, 0.544\rangle$ |
| $Z_4$ | $\langle(0.326, 0.391, 0.456, 0.521); 0.463, 0.651, 0.344, 0.520\rangle$ | $\langle(0.081, 0.131, 0.150, 0.200); 0.200, 0.363, 0.464, 0.229\rangle$ | $\langle(0.460, 0.555, 0.630, 0.725); 0.813, 0.245, 0.404, 0.203\rangle$ | $\langle(0.157, 0.238, 0.257, 0.338); 0.330, 0.440, 0.739, 0.493\rangle$ |
| $Z_5$ | $\langle(0.424, 0.509, 0.593, 0.678); 0.686, 0.597, 0.268, 0.499\rangle$ | $\langle(0.308, 0.374, 0.442, 0.510); 0.472, 0.615, 0.642, 0.371\rangle$ | $\langle(0.236, 0.283, 0.333, 0.383); 0.655, 0.523, 0.627, 0.419\rangle$ | $\langle(0.319, 0.403, 0.447, 0.531); 0.476, 0.586, 0.795, 0.464\rangle$ |

TABLE 16. Decision Matrix $\tilde{R}$ (TFMPWG)

| | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $Z_1$ | $\langle(0.065, 0.117, 0.150, 0.200); 0.800, 0.660, 0.559, 0.817\rangle$ | $\langle(0.185, 0.237, 0.288, 0.339); 0.452, 0.350, 0.304, 0.467\rangle$ | $\langle(0.076, 0.162, 0.211, 0.284); 0.610, 0.765, 0.293, 0.681\rangle$ | $\langle(0.141, 0.214, 0.259, 0.327); 0.319, 0.473, 0.499, 0.213\rangle$ |
| $Z_2$ | $\langle(0.193, 0.276, 0.349, 0.418); 0.537, 0.294, 0.451, 0.358\rangle$ | $\langle(0.132, 0.226, 0.237, 0.327); 0.327, 0.428, 0.734, 0.187\rangle$ | $\langle(0.264, 0.314, 0.365, 0.415); 0.530, 0.598, 0.324, 0.279\rangle$ | $\langle(0.392, 0.455, 0.517, 0.579); 0.390, 0.426, 0.545, 0.599\rangle$ |
| $Z_3$ | $\langle(0.213, 0.369, 0.487, 0.588); 0.408, 0.295, 0.172, 0.738\rangle$ | $\langle(0.285, 0.391, 0.485, 0.574); 0.222, 0.571, 0.679, 0.300\rangle$ | $\langle(0.122, 0.206, 0.241, 0.318); 0.501, 0.367, 0.409, 0.706\rangle$ | $\langle(0.095, 0.197, 0.281, 0.354); 0.273, 0.400, 0.514, 0.411\rangle$ |
| $Z_4$ | $\langle(0.316, 0.378, 0.439, 0.501); 0.429, 0.456, 0.322, 0.410\rangle$ | $\langle(0.077, 0.128, 0.150, 0.200); 0.200, 0.358, 0.459, 0.197\rangle$ | $\langle(0.468, 0.550, 0.633, 0.715); 0.573, 0.216, 0.291, 0.152\rangle$ | $\langle(0.142, 0.233, 0.248, 0.335); 0.335, 0.436, 0.717, 0.221\rangle$ |
| $Z_5$ | $\langle(0.414, 0.496, 0.577, 0.658); 0.501, 0.370, 0.248, 0.314\rangle$ | $\langle(0.114, 0.216, 0.301, 0.376); 0.319, 0.474, 0.394, 0.333\rangle$ | $\langle(0.114, 0.205, 0.276, 0.337); 0.443, 0.309, 0.588, 0.364\rangle$ | $\langle(0.233, 0.338, 0.367, 0.463); 0.419, 0.523, 0.754, 0.238\rangle$ |

**Step 5.** By using the decision information in Tables 15 and 16, TFMPWA-TFMPWG operators and weight vector of the attributes $\upsilon=$ (0.3,0.1,0.2,0.4), we access to the overall preference values of the alternatives. The aggregating results are presented in Tables 17 and 18.

TABLE 17. The overall preference values of the alternatives (TFMPWA)

| | |
|---|---|
| $Z_1$ | $\langle(0.155, 0.216, 0.258, 0.319); 0.645, 0.645, 0.506, 0.635\rangle$ |
| $Z_2$ | $\langle(0.245, 0.381, 0.438, 0.502); 0.629, 0.630, 0.598, 0.613\rangle$ |
| $Z_3$ | $\langle(0.344, 0.416, 0.482, 0.555); 0.525, 0.529, 0.576, 0.697\rangle$ |
| $Z_4$ | $\langle(0.263, 0.337, 0.383, 0.457); 0.509, 0.477, 0.564, 0.431\rangle$ |
| $Z_5$ | $\langle(0.333, 0.408, 0.468, 0.543); 0.586, 0.581, 0.642, 0.458\rangle$ |

TABLE 18. The overall preference values of the alternatives (TFMPWG)

| | |
|---|---|
| $Z_1$ | $\langle(0.101, 0.171, 0.213, 0.275); 0.495, 0.558, 0.442, 0.435\rangle$ |
| $Z_2$ | $\langle(0.264, 0.339, 0.396, 0.464); 0.448, 0.408, 0.478, 0.392\rangle$ |
| $Z_3$ | $\langle(0.142, 0.257, 0.339, 0.423); 0.341, 0.372, 0.364, 0.529\rangle$ |
| $Z_4$ | $\langle(0.216, 0.301, 0.338, 0.418); 0.381, 0.376, 0.450, 0.244\rangle$ |
| $Z_5$ | $\langle(0.223, 0.328, 0.389, 0.473); 0.435, 0.420, 0.482, 0.291\rangle$ |

**Step 6.** According to the results shown in Tables 17 and 18, Definitions 3.1 and 3.6, the ordering of the alternatives is shown below:

**for TFMPWA**

- $$\begin{aligned} d(Z_1, r^+) = \frac{1}{32}(&|(1+0.645)0.155-(1+1)1| + |(1+0.645)0.155-(1+1)1| \\ &+ |(1+0.506)0.155-(1+1)1| + |(1+0.635)0.155-(1+1)1| \\ &+ |(1+0.645)0.216-(1+1)1| + |(1+0.645)0.216-(1+1)1| \\ &+ |(1+0.506)0.216-(1+1)1| + |(1+0.635)0.216-(1+1)1| \\ &+ |(1+0.645)0.258-(1+1)1| + |(1+0.645)0.258-(1+1)1| \\ &+ |(1+0.506)0.258-(1+1)1| + |(1+0.635)0.258-(1+1)1| \\ &+ |(1+0.645)0.319-(1+1)1| + |(1+0.645)0.319-(1+1)1| \\ &+ |(1+0.506)0.319-(1+1)1| + |(1+0.635)0.319-(1+1)1|) \\ = 0.807& \end{aligned}$$
- $d(Z_2, r^+) = 0.683$
- $d(Z_3, r^+) = 0.645$
- $d(Z_4, r^+) = 0.731$
- $d(Z_5, r^+) = 0.657$

Then we get $Z_1 > Z_4 > Z_2 > Z_5 > Z_3$ and the best alternative is $Z_1$ that is, the government should set plastic waste factory.

**for TFMPWG**

- $d(Z_1, r^+) = 0.859$

- $d(Z_2, r^+) = 0.738$
- $d(Z_3, r^+) = 0.797$
- $d(Z_4, r^+) = 0.783$
- $d(Z_5, r^+) = 0.751$

Therefore we get $Z_1 > Z_3 > Z_4 > Z_5 > Z_2$ and the best alternative is $Z_1$, i.e., the government should set plastic waste factory.

## 6. Comparison Table

Table 19. Some rankings in terms of different methods and proposed methods of Example 5.1

| Methods | Operator | Ranking |
|---|---|---|
| Proposed Method 1 | $TFMPWA$ | $Z_1 > Z_4 > Z_2 > Z_5 > Z_3$ |
| Proposed Method 2 | $TFMPWG$ | $Z_1 > Z_3 > Z_4 > Z_5 > Z_2$ |
| Method of Uluçay et al. [8] | $TFMG_v$ | $Z_5 > Z_3 > Z_4 > Z_1 > Z_2$ |
| Method of Şahin et al. [9] | $D_v$ | $Z_3 > Z_5 > Z_1 > Z_4 > Z_2$ |
| Method of Uluçay [11] | $S_v$ | $Z_4 > Z_3 > Z_1 > Z_5 > Z_2$ |
| Method of Deli and Keleş [12] | $S^i(Z_i)$ | $Z_5 > Z_3 > Z_4 > Z_1 > Z_2$ |
| Method of Kesen and Deli [13] | $TFMBHM_v^{(1,1)}$ | $Z_1 > Z_3 > Z_5 > Z_2 > Z_4$ |

In the Table 19, we gave a brief comparison of introduced operators with some existing operators such as the weighted Bonferroni harmonic mean operator given by Kesen and Deli [13], distance measure operator proposed by Deli and Keleş [12], TFM weighted geometric operator introduced by Uluçay et al. [8], weighted dice vector similarity operator submitted by Şahin et al. [9] and vector similarity operator given by Uluçay [11] based on Example 5.1 including a zero-waste problem having five alternatives and four attributes and given under trapezoidal fuzzy multi-environment. Zero-waste has been drawing attention all around the world for saving nature and decreasing the destruction of the natural surroundings. This is the reason why we chose the zero-waste problem. As for the operators we introduced, we used them efficiently on the problem. If the comparison table is analyzed, results of the proposed aggregation methods present a new perspective to decision-making process and are compatible with the existing methods. Therefore, decision-makers can easily use the proposed methods to solve decision-making problems with multiple criteria.

## 7. Conclusion and Future Scope of Studies

In this article, we proposed TFMPWA operator and TFMPWG operator to find the solution of a zero-waste decision-making problem. By using these operators, this article aimed to investigate how to solve zero-waste group decision-making problem with multi-criterion by using trapezoidal fuzzy multi-numbers and to access a solution of the problem. In the paper, a zero-waste problem including five alternatives and four attributes was handled. In consideration of the given data in the problem, introduced operators were efficiently utilized to have a solution. The results we obtained may help to decision-makers to find the best alternative in a decision-making problem. In addition, they are consistent with the existing methods. As seen in the application, the main advantage of the operators is allowing the argument values to support each other in the aggregation process. As for limitation, since the operators don't contain parameters, decision-makers lack flexibility during the decision-making process.

In the future, the operators will be applied to other zero-waste problems given with triangular intuitionistic fuzzy multi-numbers, trapezoidal intuitionistic fuzzy multi-numbers, and trapezoidal neutrosophic fuzzy multi-numbers. In addition, the operators will be extended to bipolar soft sets and bipolar complex fuzzy sets and applications. in the daily life that are given under the bipolar soft sets and the bipolar complex fuzzy sets will be given.

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