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Abstract. In this paper, we introduce the notion of a fuzzy tri-quasi ideal and a fuzzy soft tri-quasi ideal of Γ-semirings. We characterize the regular Γ-semiring in terms of fuzzy tri-quasi ideals and fuzzy soft tri-quasi ideals of a Γ-semiring and study some of their properties.

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1. Introduction

Semiring is an algebraic structure which is a common generalization of rings and distributive lattices, was first introduced by Vandiver [1] in 1934. In 1995, Rao [2] introduced the notion of a Γ-semiring as a generalization of a Γ-ring, a ring, a ternary semiring and a semiring. We know that the notion of a one sided ideal of any algebraic structure is a generalization of notion of an ideal. The quasi-ideals are the generalization of the left ideals, and right ideals whereas the bi-ideals are the generalization of quasi-ideals. Steinfeld [3] first introduced the notion of quasi-ideals for semigroups and then for rings. Iséki [4] introduced the concept of quasi-ideals for a semiring. Rao [5], Rao et al. [6] and Rao [7] introduced the notion of a left (right) bi-quasi ideal of a semiring, a Γ-semiring, a Γ-semigroup and studied the properties of left bi-quasi ideals and characterized a left bi-quasi simple Γ-semiring and a regular Γ-semiring using left bi-quasi ideals of Γ-semirings. Also, Rao [8, 9, 10] studied ideals of an ordered Γ-semiring, he [11] dealt with fuzzy soft bi-ideals, fuzzy soft quasi-ideals and fuzzy soft interior ideals in an ordered Γ-semirings.

Zadeh [12] developed the fuzzy set theory in 1965. Many papers on fuzzy sets appeared, showing the importance of the concept and its applications to logic, set theory, taxonomy, linguistics, automata theory, control theory, group theory, ring theory, real analysis, topology, and measure theory. Rosenfeld [13] introduced the fuzzification of an algebraic structure and the notion of fuzzy subgroups in 1971. Soft set theory was introduced by Molodtsov [14] as a generalization of classical and fuzzy set theory. Soft set theory is a mathematical frame work that allows for flexible handling of uncertainity. Fuzzy soft set theory is an extension of fuzzy set theory, which has more flexibility in handling uncertainty. In fuzzy soft set theory, each element is associated with a set of values rather than a single degree membership, as in fuzzy set theory. Fuzzy soft set theory has many applications in decision-making, image processing, data analysis, and pattern recognition. Rao [11, 15, 16], Rao et al. [17] and Saeid et al. [18] studied fuzzy soft Γ-semiring and fuzzy soft k-ideal over Γ-semiring, T-fuzzy ideals of ordered Γ-semirings, fuzzy tri-ideals, fuzzy soft tri-ideals, and fuzzy soft quasi-interior ideals over semirings. In this paper, we introduce the notion of a fuzzy soft tri-quasi ideal of a Γ-semiring to generalize fuzzy soft ideals, characterize a regular Γ-semiring in terms of fuzzy soft tri-quasi ideals, and study some of their properties.

2. Preliminaries

In this section, we recall some fundamental concepts and definitions necessary for this paper.

Definition 2.1 ([2])**.** A set M together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) is called a *semiring*, provided

(i) addition is a commutative operation,
(ii) multiplication distributes over addition both from the left and from the right,
(iii) there exists $0 \in M$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in M$.

Definition 2.2 ([2])**.** Let M and Γ be two non-empty sets. Then M is called a Γ-*semigroup*, if there exists a mapping $M \times \Gamma \times M \to M$ (the image of (x, α, y) will be denoted by $x\alpha y; x, y \in M, \alpha \in \Gamma$) satisfying the following axiom:

$$x\alpha(y\beta z) = (x\alpha y)\beta z, \text{ for all } x,\ y,\ z \in M,\ \alpha,\ \beta \in \Gamma.$$

Definition 2.3 ([2])**.** Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then M is a Γ-*semiring*, if there exists a mapping $M \times \Gamma \times M \to M$ (the image of (x, α, y) will be denoted by $x\alpha y; x, y \in M; \alpha \in \Gamma$) satisfying the following axioms: for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$,

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$,
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$,
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$.
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Definition 2.4 ([8])**.** A Γ-semiring M is said to *have zero element*, if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = x\alpha 0 = 0$ for all $x \in M, \alpha \in \Gamma$.

Definition 2.5 ([8])**.** Let M be a Γ-semiring. An element $a \in M$ is said to be a *regular element* of M, if there exist $x \in M, \alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$. If every element of M is regular, then M is said to be a *regular* Γ-*semiring*

$+$	a	b	c
a	a	b	c
b	b	b	b
c	c	b	a

$\cdot$	a	b	c
a	a	a	a
b	a	b	b
c	a	c	c

Table 2.1

Example 2.6. Let $M = \{a, b, c\}$. Define the binary operation $+, \cdot$ by the following tables:
and $\Gamma = M$. Then the ternary operation $x\alpha y$ as the usual semiring multiplication. Thus M is a regular Γ-semiring.

Definition 2.7 ([8])**.** An element $a \in M$ is said to be *idempotent* of M, if $a = a\alpha a$ for some $\alpha \in \Gamma$. If every element of M is idempotent of M, then M is said to be an *idempotent* Γ*-semiring*.

Definition 2.8 ([8])**.** A Γ-semiring M is called a *left* (*right*) *simple* Γ*-semiring*, if M has no proper left (right) ideal of M

Theorem 2.9 ([15])**.** *Let M be a Γ-semiring. M is a regular Γ-semiring if and only if $A\Gamma B = A \cap B$ for any right ideal A and left ideal B of M.*

Definition 2.10 ([16])**.** A non-empty subset A of a Γ-semiring M is said to be *tri-quasi ideal* of M, if A is a subsemiring of M and $A\Gamma A\Gamma M\Gamma A\Gamma A \subseteq A$, i.e., $x\alpha y\beta z\alpha x\beta y \in A$ for all $x,\ y \in A$, $z \in M$, $\alpha,\ \beta \in \Gamma$.

Definition 2.11 ([16])**.** A non-empty subset A of a Γ-semiring M is said to be *left*(*right*)*quasi-interior ideal* of M, if A is a subsemiring of M and $A\Gamma M\Gamma A\Gamma M \subseteq A$ ($M\Gamma A\Gamma M\Gamma A \subseteq A$). If A is both a left quasi-interior ideal and right quasi-interior ideal, then A is called a *quasi-interior ideal* of M.

Definition 2.12 ([15])**.** Let M be a non-empty set. A mapping $f : M \to [0, 1]$ is called a *fuzzy subset* of M.

Definition 2.13 ([15])**.** Let f be a fuzzy subset of a non-empty set M, for $t \in [0, 1]$ the set $f_t = \{x \in M \mid f(x) \geq t\}$ is called a *level subset* of M with respect to f.

Definition 2.14 ([15])**.** Let A be a non-empty subset of M. The characteristic function of A, denoted by $\chi_{_A}$, is a fuzzy subset of M defined as follows:

$$\chi_{_A}(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

For any $a,\ b \in [0, 1]$, we will denote $max\{a, b\} = a \vee b$ and $min\{a, b\} = a \wedge b$.

Definition 2.15 ([15])**.** Let f and g be fuzzy subsets of a Γ-semiring M. Then $f \circ g, f + g, f \cup g, f \cap g$, are defined by: for each $z \in M$,

$$f \circ g(z) = \begin{cases} \bigvee_{z=x\alpha y}[f(x) \wedge g(y)] \text{ if } z = x\alpha y \text{ for some } x, y \in M, \alpha \in \Gamma \\ \\ 0 \qquad\qquad\qquad\qquad \text{otherwise,} \end{cases} \tag{2.1}$$

$$f + g(z) = \begin{cases} \bigvee_{z=x+y}[f(x) \wedge g(y)] \text{ if } z = x + y \text{ for some } x, y \in M, \alpha \in \Gamma \\ \\ 0 \qquad\qquad\qquad\qquad \text{otherwise,} \end{cases} \tag{2.2}$$

(2.3) $$f \cup g(z) = f(z) \vee g(z),\ f \cap g(z) = f(z) \wedge g(z).$$

Definition 2.16 ([17]). A fuzzy subset μ of a Γ-semiring M is called a *fuzzy right(left)quasi interior-ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,

(i) $\mu(x+y) \geq \mu(x) \wedge \mu(y)$,
(ii) $\mu(x\alpha y) \geq \mu(x) \wedge \mu(y)$,
(iii) $\mu \circ \chi_{_M} \circ \mu \circ \chi_{_M} \subseteq \mu \ \big(\chi_{_M} \circ \mu \circ \chi_{_M} \circ \mu \subseteq \mu\big)$.

A fuzzy subset μ of a Γ-semiring M is called a *fuzzy quasi interior-ideal* of M, if it is a fuzzy right quasi interior-ideal and a fuzzy left quasi interior-ideal of M.

Definition 2.17 ([17]). A fuzzy subset μ of a Γ-semiring M is called a *fuzzy left (right) tri-ideal* of M, if it satisfies the following conditions: for all $x, y \in M, \alpha \in \Gamma$,

(i) $\mu(x+y) \geq \mu(x) \wedge \mu(y)$,
(ii) $\mu(x\alpha y) \geq \mu(x) \wedge \mu(y)$,
(iii) $\mu \circ \chi_{_M} \circ \mu \circ \mu \subseteq \mu \ \big(\mu \circ \mu \circ \chi_{_M} \circ \mu \subseteq \mu\big)$.

A fuzzy subset μ of a Γ-semiring M is called a *fuzzy tri-ideal* of M, if it is a left tri-ideal and a right tri-ideal of M.

Definition 2.18 ([18]). Let U be an initial universe set and E be the set of parameters. Let $P(U)$ denotes the power set of U. A pair (μ, E) is called a *soft set over* U, where μ is a mapping given by $\mu : E \to P(U)$.

Definition 2.19 ([16]). Let U be an initial universe set, E be the set of parameters, and $A \subseteq E$. A pair (μ, A) is called a *fuzzy soft set over* U, where $\mu : A \to [0,1]^U$ is a mapping, $[0,1]^U$ denotes the collection of all fuzzy subsets of U and for each $a \in A, \mu(a) = \mu_a$ is a fuzzy subset of M.

Definition 2.20 ([16]). Let $(\mu, A), (\lambda, B)$ be fuzzy soft sets over U. Then (μ, A) is said to be a *fuzzy soft subset* of (λ, B), denoted by $(\mu, A) \subseteq (\lambda, B)$, if $A \subseteq B$ and $\mu_a \subseteq \lambda_a$ for all $a \in A$.

Definition 2.21 ([16]). Let M be a Γ-semiring, E be a parameters set and $A \subseteq E$. Let $\mu : A \to [0,1]^M$ be a mapping. Then (μ, A) is called a *fuzzy soft* Γ*-semiring over* M, if for each $a \in A$, μ_a is the fuzzy subsemiring of M, i.e., for all $x, y \in M, \alpha \in \Gamma$,

(i) $\mu_a(x+y) \geq \mu_a(x) \wedge \mu_a(y)$,
(ii) $\mu_a(x\alpha y) \geq \mu_a(x) \wedge \mu_a(y)$.

And (μ, A) is called a *fuzzy soft left(right) ideal over* M, if for each $a \in A$, the corresponding fuzzy subset $\mu_a : M \to [0,1]$ is a fuzzy left(right) ideal of M. i.e., for all $x, y \in M, \alpha \in \Gamma$,

(i) $\mu_a(x+y) \geq \mu_a(x) \wedge \mu_a(y)$,
(ii) $\mu_a(x\alpha y) \geq \mu_a(y)$ $(\mu_a(y))$.

Theorem 2.22 ([18]). *Let M be a Γ-semiring, E be a parameters set, $A \subseteq E$ and (μ, A) be a fuzzy soft set over M. Then a fuzzy soft set (μ, A) over M is a fuzzy soft subsemiring if and only if $\mu_a \circ \mu_a \subseteq \mu_a$ for all $a \in A$.*

Theorem 2.23 ([18]). *Let M be a Γ-semiring, E be a parameters set, $A \subseteq E$ and (μ, A) be a fuzzy soft set over M. If (μ, A) is a fuzzy soft right(left) ideal over M, then $\mu_a \circ \chi_{_M}$ $(\chi_{_M} \circ \mu_a) \subseteq \mu_a$ for all $a \in A$.*

3. Fuzzy (soft) tri-quasi ideals over Γ-semirings

In this section, we introduce the notion of a fuzzy(soft) tri-quasi ideal as a generalization of a fuzzy(soft) bi-ideal, fuzzy(soft) quasi ideal, fuzzy(soft) interior ideal, over Γ-semirings and study their properties.

Definition 3.1. A fuzzy subset μ of a Γ-semiring M is called a *fuzzy tri-quasi ideal* of M, if for all $x, y \in M$, $\alpha \in \Gamma$,

(i) $\mu(x+y) \geq \mu(x) \wedge \mu(y)$, $\mu(x\alpha y) \geq \mu(x) \wedge \mu(y)$
(ii) $\mu \circ \mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$.

Example 3.2. Let $M = \{0, a, b, c\}$ and $\Gamma = \{\alpha, \beta\}$. Define the operations + and the ternary operation by the following tables:

+	0	a	b	c
0	0	a	b	c
a	a	a	c	a
b	b	c	b	b
c	c	a	b	c

+	α	β
α	α	α
β	α	β

α/β	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	b	c
c	0	c	c	c

Table 3.1

Then $(M, +)$ and $(\Gamma, +)$ are additive semigroups. Thus M is a Γ-semiring. Let $B = \{0, b\}$. Since $b\alpha c = c \notin B$, B is not a right ideal of M. Moreover, we can see that B is not an ideal (resp. a left ideal, a quasi-ideal, a bi-ideal and an interior ideal) of M but is a tri-quasi ideal of M.

(1) Define $\mu : M \to [0, 1]$ by $\mu(0) = 1, \mu(a) = 0.7, \mu(b) = 0.4, \mu(c) = 0.2$. Then μ is a fuzzy tri-quasi ideal of M.

(2) Define $\mu : M \to [0, 1]$ such that $\mu(x) = \begin{cases} 1 & \text{if } x \in B \\ 0 & \text{if } x \notin B. \end{cases}$

Then μ is a fuzzy tri-quasi ideal of M.

Definition 3.3. Let M be a Γ-semiring and E be a parameters set, $A \subseteq E$ and $\mu : A \to \ I^M$ be a mapping. Then (μ, A) is called a *fuzzy soft tri-quasi ideal over* M, if it satisfies the following conditions: for each $a \in A$, any $x,\ y \in M$ and each $\alpha \in \Gamma$,

(i) $\mu_a(x+y) \geq \mu_a(x) \wedge \mu_a(y)$,
(ii) $\mu_a(x\alpha y) \geq \mu(x) \wedge \mu(y)$,
(iii) $\mu_a \circ \mu_a \circ \chi_M \circ \mu_a \circ \mu_a \subseteq \mu_a$.

Example 3.4. Let $M = \{a, b, c, d\}$, and $\Gamma = \{\alpha, \beta\}$. Let The binary operation + and the ternary operation be defined by the following tables:

+	a	b	c	d
a	a	b	c	d
b	b	b	c	d
c	c	c	c	d
d	d	c	c	d

+	α	β
α	α	α
β	α	β

α/β	a	b	c	d
a	a	a	a	a
b	a	b	b	b
c	a	b	c	c
d	a	b	c	d

Table 3.2

Then clearly, $(M,+)$ and $(\Gamma,+)$ are additive semigroups. Thus M is a Γ-semiring and $B=\{a,b\}$ is a tri-quasi ideal of M.

Let $A=\{e_1,e_2,e_3\}\subseteq E$. Define $f_{e_i}:M\to[0,1]$, where $F=\{f_{e_i}\}, i=1,2,3$ as the following table:

	a	b	c	d
f_{e1}	0.9	0.4	0.3	0.1
f_{e2}	1	0.7	0.4	0.2
f_{e3}	0.8	0.5	0.2	0.3

Table 3.3

Then (F,A) is a fuzzy soft tri-quasi ideal over M.

Theorem 3.5. *Let I be a non-empty subset of a Γ-semiring M. Then I is a tri-quasi ideal of M if and only if χ_I is a fuzzy tri-quasi ideal of M.*

Proof. Suppose I is a tri-quasi ideal of M. Obviously, χ_I is a fuzzy Γ-subsemiring of M. Then $I\Gamma I\Gamma M\Gamma I\Gamma I\subseteq I$. Thus

$$\begin{aligned}\chi_I\circ\chi_I\circ\chi_M\circ\chi_I\circ\chi_I &= \chi_{I\Gamma I\Gamma M\Gamma I\Gamma I}\\ &\subseteq \chi_I.\end{aligned}$$

So χ_I is a fuzzy tri-quasi ideal of the Γ-semiring M.

Conversely, suppose χ_I is a fuzzy tri-quasi ideal of M. Then I is a Γ-subsemiring of M. Thus $\chi_I\circ\chi_I\circ\chi_M\circ\chi_I\circ\chi_I\subseteq\chi_I$. So $\chi_{I\Gamma I\Gamma M\Gamma I\Gamma I}\subseteq\chi_I$. Hence $I\Gamma I\Gamma M\Gamma I\Gamma I\subseteq I$. Therefore I is a tri-quasi ideal of M. □

Theorem 3.6. *Let M be a Γ-semiring and μ be a non-empty fuzzy subset of M. A fuzzy subset μ is a fuzzy tri-quasi ideal of M if and only if μ_t is a fuzzy tri-quasi ideal of M for every $t\in[0,1]$, where $\mu_t\neq\phi$.*

Proof. Suppose μ is a fuzzy tri-quasi ideal of M, let $\mu_t\neq\phi$ for each $t\in[0,1]$ and let $a,\ b\in\mu_t$. Then clearly, $\mu(a)\geq t,\ \mu(b)\geq t$. Thus we have

$$\mu(a+b)\geq\mu(a)\wedge\mu(b)\geq t,\ \mu(a\alpha b)\geq\mu(a)\wedge\mu(b)\geq t.$$

So $a+b,\ a\alpha b\in\mu_t$.

Let $x\in\mu_t\Gamma\mu_t\Gamma M\Gamma\mu_t\Gamma\mu_t$. Then $x=b\alpha a\beta y\gamma d\delta c$, where $y\in M,a,b,c,$ $d\in\mu_t,\alpha,\beta,\gamma,\delta\in\Gamma$. Thus $\mu\circ\mu\circ\chi_M\circ\mu\circ\mu(x)\geq t$. So $\mu(x)\geq\mu\circ\mu\circ\chi_M\circ\mu\circ\mu(x)\geq t$. Hence $x\in\mu_t$. Therefore μ_t is a fuzzy tri-quasi ideal of M.

Conversely, suppose μ_t is a fuzzy tri-quasi ideal of M for all $t\in Im(\mu)$. Let $x,\ y\in M,\alpha\in\Gamma,\mu(x)=t_1,\mu(y)=t_2$ and $t_1\geq t_2$. Then $x,\ y\in\mu_{t_2}$. Thus we get

$$\mu(x+y)\geq t_2=t_1\wedge t_2=\mu(x)\wedge\mu(y),$$

$$\mu(x\alpha y)\geq t_2=t_1\wedge t_2=\mu(x)\wedge\mu(y.$$

So $x+y\in\mu_{t_2},\ x\alpha y\in\mu_{t_2}$.

On the other hand $\mu_l\Gamma\mu_l\Gamma M\Gamma\mu_l\Gamma\mu_l\subseteq\mu_l$ for all $l\in Im(\mu)$. Let $t=\bigwedge\{Im(\mu)\}$. Then $\mu_t\Gamma\mu_t\Gamma M\Gamma\mu_t\Gamma\mu_t\subseteq\mu_t$. Thus $\mu\circ\mu\circ\chi_M\circ\mu\circ\mu\subseteq\mu$. So μ is a fuzzy tri-quasi ideal of M. □

Theorem 3.7. *Every fuzzy right ideal of a* Γ*-semiring* M *is a fuzzy tri-quasi ideal of* M.

Proof. Let μ be a fuzzy right ideal of M and $x \in M$. Then we have

$$\begin{aligned} \mu \circ \chi_M(x) &= \bigvee_{x=a\alpha b,\ a,b\in M,\ \alpha\in\Gamma} [\mu(a) \wedge \chi_{_M}(b)] \\ &= \bigvee_{x=a\alpha b,\ a,b\in M,\ \alpha\in\Gamma} \mu(a) \\ &\leq \bigvee_{x=a\alpha b,\ a,b\in M,\ \alpha\in\Gamma} \mu(a\alpha b) \\ &= \mu(x). \end{aligned}$$

Thus $\mu \circ \chi_{_M}(x) \leq \mu(x)$. On the other hand, we get

$$\begin{aligned} \mu \circ \mu \circ \chi_{_M}(x) &= \bigvee_{x=a\alpha b,\ a,b\in M,\ \alpha\in\Gamma} [\mu(a) \wedge \mu \circ \chi_{_M}(b)] \\ &\leq \bigvee_{x=a\alpha b,\ a,b\in M,\ \alpha\in\Gamma} [\mu(a) \wedge \mu(b)] \\ &= \mu \circ \mu(x) \leq \mu(x), \end{aligned}$$

$$\begin{aligned} \mu \circ \mu \circ \chi_{_M} \circ \mu \circ \mu(x) &= \bigvee_{x=u\alpha v,\ u,v\in M,\ \alpha\in\Gamma} [\mu \circ \mu \circ \chi_M(u) \wedge \mu \circ \mu(v)] \\ &\leq \bigvee_{x=u\alpha v,\ u,v\in M,\ \alpha\in\Gamma} [\mu(u) \wedge \mu(v)] \\ &= \mu \circ \mu(x) \leq \mu(x). \end{aligned}$$

So μ is a fuzzy tri-quasi ideal of M. □

Corollary 3.8. *Every fuzzy (left) ideal of a* Γ*-semiring* M *is a fuzzy tri-quasi ideal of* M.

Definition 3.9 ([17]). Let M be a Γ-semiring and E be a parameters set and $A \subseteq E$. Let $\mu : A \to [0,1]^M$ be a mapping. Then (μ, A) is called a *fuzzy soft left (right) tri-ideal* of M, if it satisfis the following conditions: for each $a \in A$, any $x, y \in M$ and each $\alpha \in \Gamma$,

(i) $\mu_a(x + y) \geq \mu_a(x) \wedge \mu_a(y)$,
(ii) $\mu_a(x\alpha y) \geq \mu_a(x) \wedge \mu_a(y)$,
(ii) $\mu_a \circ \chi_{_M} \circ \mu_a \circ \mu_a \subseteq \mu_a$ $\big(\mu_a \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a\big)$.

A fuzzy soft subset(μ, A) of a Γ-semiring M is called a *fuzzy soft tri-ideal* of M, if it is both fuzzy soft left tri-ideal and fuzzy soft right tri-ideal of M.

The following theorem follows from the proof of Theorem 3.7.

Theorem 3.10. *Let* M *be a* Γ*-semiring,* E *be a parameters set and* $A \subseteq E$*. If* (μ, A) *is a fuzzy soft right tri-ideal over* M*, then* (μ, A) *is a fuzzy soft tri-quasi ideal over* M.

Corollary 3.11. *Every fuzzy soft (left) ideal of a Γ-semiring M is a fuzzy soft tri-quasi ideal over M.*

Definition 3.12 ([18])**.** Let (μ, A), (λ, B) be fuzzy soft sets. The *intersection* of (μ, A) and (λ, B), denoted by $(\mu, A) \cap (\lambda, B) = (\gamma, C)$, where $C = A \cup B$, for all $c \in C, \gamma(c) = \gamma_c$, is a fuzzy subset of M defined as follows:

$$\gamma_c = \begin{cases} \mu_c & \text{if } c \in A \setminus B \\ \lambda_c & \text{if } c \in B \setminus A \\ \mu_c \cap \lambda_c & \text{if } c \in A \cap B. \end{cases}$$

Theorem 3.13. *Let M be a Γ-semiring, E be a parameters set and A, $B \subseteq E$. If (μ, A) and (λ, B) are fuzzy soft tri-quasi ideals over M, then $(\mu, A) \cap (\lambda, B)$ is a fuzzy soft tri-quasi ideal over M.*

Proof. Suppose (μ, A) and (λ, B) are fuzzy soft tri-quasi ideal of M. By Definition 3.12, we have that $(\mu, A) \cap (\lambda, B) = (\gamma, C)$, where $C = A \cup B$.

Case (i): If $c \in A \setminus B$, then $\gamma_c = \mu_c$. Thus γ_c is a fuzzy tri-quasi ideal of M, since (μ, A) is a fuzzy soft tri-quasi ideal over M.

Case (ii): If $c \in B \setminus A$, then $\gamma_c = \lambda_c$. Thus γ_c is a fuzzy tri-quasi ideal of M, since (λ, B) is a fuzzy soft tri-quasi ideal over M.

Case (iii): If $c \in A \cap B$, then $\gamma_c = \mu_c \cap \lambda_c$. Then μ_c and λ_c are fuzzy tri-quasi ideals of M. Let x, $y \in M$ and $\alpha \in \Gamma$. Thus we have

$$\begin{aligned} \mu_c \cap \lambda_c(x+y) &= \mu_c(x+y) \wedge \lambda_c(x+y) \\ &\geq (\mu_c(x) \wedge \mu_c(y)) \wedge (\lambda_c(x) \wedge \lambda_c(y)) \\ &= (\mu_c(x) \wedge \lambda_c(x)) \wedge (\mu_c(y) \wedge \lambda_c(y)) \\ &= \mu_c \cap \lambda_c(x) \wedge \mu_c \cap \lambda_c(y), \\ \mu_c \cap \lambda_c(x\alpha y) &= \mu_c(x\alpha y) \wedge \lambda_c(x\alpha y) \\ &\geq (\mu_c(x) \wedge \mu_c(y)) \wedge (\lambda_c(x) \wedge \lambda_c(y)) \\ &= (\mu_c(x) \wedge \lambda_c(x)) \wedge (\mu_c(y) \wedge \lambda_c(y)) \\ &= \mu_c \cap \lambda_c(x) \wedge \mu_c \cap \lambda_c(y). \end{aligned}$$

So $\mu_c \cap \lambda_c$ is a Γ-subsemiring of M.

On the other hand,

$$\begin{aligned} &((\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c))(x) \\ &= \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [(\mu_c \cap \lambda_c)(a) \wedge (\mu_c \cap \lambda_c)(b)] \\ &= \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [(\mu_c(a) \wedge \lambda_c(a)) \wedge (\mu_c(b) \wedge \lambda_c(b))] \\ &= \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [\mu_c(a) \wedge \lambda_c(a)] \wedge \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [\mu_c(b) \wedge \lambda_c(b)] \\ &= \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [\mu_c(a) \wedge \mu_c(b)] \wedge \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [\lambda_c(a) \wedge \lambda_c(b)] \\ &= \mu_c \circ \mu_c(x) \wedge \lambda_c \circ \lambda_c(x) \\ &= (\mu_c \circ \mu_c) \cap (\lambda_c \circ \lambda_c)(x). \end{aligned}$$

Then we get

$$\begin{aligned} &\chi_M \circ (\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c)(x) \\ &= \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [\chi_M(a), (\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c)(b)] \\ &= \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [\chi_M(a) \wedge (\mu_c \circ \mu_c) \cap (\lambda_c \cap \lambda_c)(b)] \\ &= \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [\chi_M(a) \wedge ((\mu_c \circ \mu_c)(b) \wedge (\lambda_c \circ \lambda_c)(b))] \\ &= \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [\chi_M(a) \wedge (\mu_c \circ \mu_c)(b) \wedge (\chi_M(a) \wedge (\lambda_c \circ \lambda_c)(b)] \\ &= \bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [\chi_M(a) \wedge (\mu_c \circ \mu_c)(b)] \wedge (\bigvee_{x=a\alpha b, a,b \in M, \alpha \in \Gamma} [\chi_M(a) \wedge (\lambda_c \circ \lambda_c)(b)] \\ &= (\chi_M \circ \mu_c \circ \mu_c)(x) \wedge (\chi_M \circ \lambda_c \circ \lambda_c)(x) \end{aligned}$$

$= [(\chi_M \circ \mu_c \circ \mu_c) \cap (\chi_M \circ \lambda_c \circ \lambda_c)](x)$.
Thus $(\chi_M \circ (\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c)) = (\chi_M \circ \mu_c \circ \mu_c) \cap (\chi_M \circ \lambda_c \circ \lambda_c)$. Furthermore, we have

$$\begin{aligned}
&(\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c) \circ \chi_M \circ (\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c)(x)\\
&= \bigvee_{x=a\alpha b, a,b\in M,\alpha\in\Gamma}[\mu_c \circ \mu_c \cap \lambda_c \circ \lambda_c(a) \wedge \chi_M \circ (\mu_c \cap \lambda_c) \circ (\mu_c \cap \lambda_c)(b)]\\
&= \bigvee_{x=a\alpha b, a,b\in M,\alpha\in\Gamma}[\mu_c \circ \mu_c \cap \lambda_c \circ \lambda_c(a) \wedge \chi_M \circ \mu_c \circ \mu_c \cap \chi_M \circ \lambda_c \circ \lambda_c(b)]\\
&= \bigvee_{x=a\alpha b, a,b\in M,\alpha\in\Gamma}[(\mu_c \circ \mu_c(a) \wedge \lambda_c \circ \lambda_c(a)) \wedge (\chi_M \circ \mu_c \circ \mu_c(b) \wedge \chi_M \circ \lambda_c \circ \lambda_c(b))]\\
&= \bigvee_{x=a\alpha b, a,b\in M,\alpha\in\Gamma}[(\mu_c \circ \mu_c(a) \wedge \chi_M \circ \mu_c \circ \mu_c(b)) \wedge (\lambda_c \circ \lambda_c(a) \wedge \chi_M \circ \lambda_c \circ \lambda_c(b))]\\
&= \bigvee_{x=a\alpha b, a,b\in M,\alpha\in\Gamma}[\mu_c \circ \mu_c(a) \wedge \chi_M \circ \mu_c \circ \mu_c(b)]\\
&\qquad \wedge \bigvee_{x=a\alpha b, a,b\in M,\alpha\in\Gamma}[\lambda_c \circ \lambda_c(a) \wedge \chi_M \circ \lambda_c \circ \lambda_c(b)]\\
&= \mu_c \circ \mu_c \circ \chi_M \circ \mu_c \circ \mu_c(x) \wedge \lambda_c \circ \lambda_c \circ \chi_M \circ \lambda_c \circ \lambda_c(x)\}\\
&= \mu_c \circ \mu_c \circ \chi_M \circ \mu_c \circ \mu_c \cap \lambda_c \circ \lambda_c \circ \chi_M \circ \lambda_c \circ \lambda_c(x).
\end{aligned}$$

This implies that

$$\begin{aligned}
&\mu_c \cap \lambda_c \circ \mu_c \cap \lambda_c \circ \chi_M \circ \mu_c \cap \lambda_c \circ \mu_c \cap \lambda_c\\
&= \mu_c \circ \mu_c \circ \chi_M \circ \mu_c \circ \mu_c \cap \lambda_c \circ \lambda_c \circ \chi_M \circ \lambda_c \circ \lambda_c\\
&\subseteq \mu_c \cap \lambda_c.
\end{aligned}$$

So $\mu_c \cap \lambda_c \circ \mu_c \cap \lambda_c \circ \chi_M \circ \mu_c \cap \lambda_c \circ \mu_c \cap \lambda_c \subseteq \mu_c \cap \lambda_c$.
This implies that $\mu_c \cap \lambda_c$ is a fuzzy tri-quasi ideal of M. Hence γ_c is a fuzzy tri-quasi ideal of M. Therefore $(\mu, A) \cap (\lambda, B)$ is a fuzzy soft tri-quasi ideal over M. □

Corollary 3.14. *Let M be a Γ-semiring. If μ and λ are tri-quasi ideals of M, then $\mu \cap \lambda$ is a tri-quasi ideal of M.*

Theorem 3.15. *Let M be a regular Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft tri-quasi ideal over M, then (μ, A) is a fuzzy soft ideal over M.*

Proof. Suppose (μ, A) is a fuzzy soft tri-quasi ideal over M. Then clearly,

$$\mu_a \circ \mu_a \circ \chi_M \circ \mu_a \circ \mu_a \subseteq \mu_a \text{ for all } a \in A.$$

Let $z \in M$. Since M is regular, there exists $x \in M, \alpha, \beta \in \Gamma$ such that $z = z\alpha x\beta z$. Then $\mu_a \circ \mu_a \circ \chi_M \circ \mu_a \circ \mu_a(z) \leq \mu_a(z)$. By Definition 2.15, we have

$$\bigvee_{z=z\alpha x\beta z, x\in M,\alpha,\beta\in\Gamma}[\mu_a \circ \mu_a \circ \chi_M(z) \wedge \mu_a \circ \mu_a(x\beta z)] \leq \mu_a(z).$$

Thus we get
$\bigvee_{z=z\alpha x\beta z, x\in M,\alpha,\beta\in\Gamma}(\bigvee_{z=z\alpha x\beta z, x\in M,\alpha,\beta\in\Gamma}[\mu_a(z\alpha x) \wedge \mu_a \circ \chi_M(z)] \wedge \mu_a \circ \mu_a(x\beta z))$
$\leq \mu_a(z)$. So $\mu_a \circ \chi_M(z) \leq \mu_a(z)$. Similarly, $\chi_M \circ \mu_a(z) \leq \mu_a(z)$. Hence (μ, A) is a fuzzy soft ideal over M. □

Definition 3.16 ([18]). Let M be a Γ-semiring and E be a parameters set and $A \subseteq E$. Let $\mu : A \to [0, 1]^M$. Then (μ, A) is called a *fuzzy soft left (right) quasi interior ideal over M*, if it satisfies the following conditions: for each $a \in A$, any $x,\ y \in M$ and each $\alpha \in \Gamma$,

(i) $\mu_a(x + y) \geq \mu_a(x) \wedge \mu_a(y)$,
(ii) $\mu_a(x\alpha y) \geq \mu_a(x) \wedge \mu_a(y)$,
(iii) $\chi_M \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a$ $(\mu_a \circ \chi_M \circ \mu_a \circ \chi_M \subseteq \mu_a)$.

A fuzzy soft subset(μ, A) of a Γ-semiring M is called a *fuzzy soft quasi-interior ideal*, if it is both fuzzy soft left quasi-interior ideal and fuzzy soft right quasi-interior ideal of M.

Theorem 3.17. *Let M be a Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft left-ideal over M, then (μ, A) is a fuzzy soft left quasi interior ideal over M.*

Proof. Suppose (μ, A) is a fuzzy soft left-ideal over M. Then for each $a \in A, \mu_a$ is a left-ideal of M. Let $z \in M, \alpha \in \Gamma$. Then we have

$$\begin{aligned}\chi_M \circ \mu_a \circ \chi_M \circ \mu_a(z) &= \bigvee_{z=l\alpha m, l,m\in M} [\chi_M \circ \mu_a(l) \wedge \chi_M \circ \mu_a(m)] \\ &\leq \bigvee_{z=l\alpha m, l,m\in M} [\mu_a(l) \wedge \mu_a(m)] \\ &= \mu_a(z).\end{aligned}$$

Thus $\chi_M \circ \mu_a \circ \chi_M \circ \mu_a(z) \leq \mu_a(z)$. So $\chi_M \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a$ for all $a \in A$. Hence (μ, A) is a fuzzy soft left quasi interior ideal over M. □

Corollary 3.18. *Let M be a Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft (right) ideal over M, then (μ, A) is a fuzzy soft (right) quasi interior ideal over M.*

Definition 3.19. Let M be a Γ-semiring and E be a parameters set and $A \subseteq E$. Let $\mu : A \to [0,1]^M$ be a mapping. Then (μ, A) is called a *fuzzy soft left (right) tri-ideal over M*, if it satisfies the following conditions: for each $a \in A$, any $x, y \in M$ and each $\alpha \in \Gamma$,

(i) $\mu_a(x + y) \geq \mu_a(x) \wedge \mu_a(y)$,
(ii) $\mu_a(x\alpha y) \geq \mu_a(x) \wedge \mu_a(y)$,
(iii) $\mu_a \circ \chi_M \circ \mu_a \circ \mu_a \subseteq \mu_a$ $\left(\mu_a \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a\right)$.

A fuzzy soft subset (μ, A) of a Γ-semiring M, is called a *fuzzy soft tri-ideal*, if it is both a fuzzy soft left tri-ideal and a fuzzy soft right tri-ideal of M.

Theorem 3.20. *Let M be a Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft left quasi interior ideal over M, then (μ, A) is a fuzzy soft right tri-ideal over M.*

Proof. Suppose (μ, A) is a fuzzy soft left quasi interior ideal over M and let $a \in A$. Then $\chi_M \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a$. Thus $\mu_a \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \chi_M \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a$. So (μ, A) is a fuzzy soft right tri-ideal over M. □

Corollary 3.21. *Let M be a Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft quasi interior ideal over M, then (μ, A) is a fuzzy soft tri-ideal over M.*

Definition 3.22 ([18])**.** Let M be a Γ-semiring and E be a parameters set and $A \subseteq E$. Let $\mu : A \to [0,1]^M$ be a mapping. Then (μ, A) is called a *fuzzy soft bi-ideal over M*, if it satisfies the following conditions: for each $a \in A$, any $x, y \in M$ and each $\alpha \in \Gamma$,

(i) $\mu_a(x+y) \geq \mu_a(x) \wedge \mu_a(y)$,
(ii) $\mu_a(x\alpha y) \geq \mu_a(x) \wedge \mu_a(y)$,
(iii) $\mu_a \circ \chi_{_M} \circ \mu_a \subseteq \mu_a$.

Theorem 3.23. *Let M be a Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft left-ideal over M, then (μ, A) is a fuzzy soft bi-deal over M.*

Proof. Suppose (μ, A) is a fuzzy soft left-ideal over M. Then for each $a \in A, \mu_a$ is a left-ideal of M. Let $z \in M, \alpha \in \Gamma$. Then we have

$$\begin{aligned}
\chi_{_M} \circ \mu_a(z) &= \bigvee_{z=l\alpha m, l,m\in M} [\chi_{_M}(l) \wedge \mu_a(m) \\
&= \bigvee_{z=l\alpha m, l,m\in M} [1 \wedge \mu_a(m)] \\
&= \bigvee_{z=l\alpha m, l,m\in M} \mu_a(m) \\
&\leq \bigvee_{z=l\alpha m, l,m\in M} \mu_a(l\alpha m) \\
&= \mu_a(z), \\
\mu_a \circ \chi_{_M} \circ \mu_a(z) &= \bigvee_{z=l\alpha m, l,m\in M} [\mu_a(l) \wedge \chi_{_M} \circ \mu_a(m)] \\
&\leq \bigvee_{z=l\alpha m, l,m\in M} [\mu_a(l) \wedge \mu_a(m)] \\
&= \mu_a(z).
\end{aligned}$$

Thus $\mu_a \circ \chi_{_M} \circ \mu_a(z) \leq \mu_a(z)$. So $\mu_a \circ \chi_{_M} \circ \mu_a \subseteq \mu_a$. Hence (μ, A) is a fuzzy soft bi-deal over M. □

Theorem 3.24. *Let M be a Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft bi-ideal over M, then (μ, A) is a fuzzy soft tri-deal over M.*

Proof. Suppose (μ, A) is a fuzzy soft bi-ideal over M. Then $\mu_a \circ \chi_{_M} \circ \mu_a \subseteq \mu_a$ for all $a \in A$. Let $z \in M$, $\alpha \in \Gamma$. Then we get

$$\begin{aligned}
\mu_a \circ \chi_{_M} \circ \mu_a \circ \mu_a(z) &= \bigvee_{z=l\alpha m, l,m\in M} [\mu_a \circ \chi_{_M} \circ \mu_a(l) \wedge \mu_a(m)\}] \\
&\leq \bigvee_{z=l\alpha m, l,m\in M} [\mu_a(l) \wedge \mu_a(m) \\
&= \mu_a \circ \mu_a(z) \\
&\leq \mu_a(z).
\end{aligned}$$

Thus (μ, A) is a fuzzy soft left tri-ideal over M. On the other hand, we have

$$\begin{aligned}\mu_a \circ \mu_a \circ \chi_{_M} \circ \mu_a(z) &= \bigvee_{z=l\alpha m, l,m\in M} [\mu_a(l) \wedge \mu_a \circ \chi_{_M} \circ \mu_a(m)] \\ &\leq \bigvee_{z=l\alpha m, l,m\in M} [\mu_a(l) \wedge \mu_a(m)] \\ &= \mu_a \circ \mu_a(z) \\ &\leq \mu_a(z).\end{aligned}$$

So (μ, A) is a fuzzy soft tri-ideal over M. □

Theorem 3.25. *Let M be a Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft left tri-ideal over M, then (μ, A) is a fuzzy soft tri-quasi ideal over M.*

Proof. Suppose (μ, A) is a fuzzy soft left tri-ideal over M. Then for each $a \in A$,

$$\mu_a \circ \chi_{_M} \circ \mu_a \circ \mu_a \subseteq \mu_a.$$

Thus for each $z \in M$ and each $\alpha \in \Gamma$, we get

$$\begin{aligned}\mu_a \circ \mu_a \circ \chi_{_M} \circ \mu_a \circ \mu_a(z) &= \bigvee_{z=l\alpha m, l,m\in M} [\mu_a(l) \wedge \mu_a \circ \chi_{_M} \circ \mu_a \circ \mu_a(m)] \\ &\leq \bigvee_{z=l\alpha m, l,m\in M} [\mu_a(l), \mu_a(m)] \\ &= \mu_a(z).\end{aligned}$$

So $\mu_a \circ \mu_a \circ \chi_{_M} \circ \mu_a \circ \mu_a \subseteq \mu_a$. Hence (μ, A) is a fuzzy soft tri-quasi deal over M. □

Corollary 3.26. *Let M be a Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft (right) tri-ideal over M, then (μ, A) is a fuzzy soft tri-quasi ideal over M.*

Theorem 3.27. *Let M be a regular Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft $\Gamma-$subsemiring ,then the following are equivalent:*
(1) *(μ, A) is a fuzzy soft ideal over M,*
(2) *(μ, A) is a fuzzy soft quasi-interior ideal over M,*
(3) *(μ, A) is a fuzzy soft tri-ideal over M,*
(4) *(μ, A) is a fuzzy soft tri-quasi ideal over M.*

Proof. (1) ⇒ (2): The proof follows from Corollary 3.18.
(2) ⇒ (3): The proof is obvious from Corollary 3.21.
(3) ⇒ (4): The proof is straightforward from Corollary 3.26.
(4) ⇒ (1): The proof is clear from Theorem 3.15. □

We obtain the following relationship among fuzzy soft ideals, fuzzy soft quasi-interior ideals, fuzzy soft tri-ideals and fuzzy soft tri-quasi ideals.

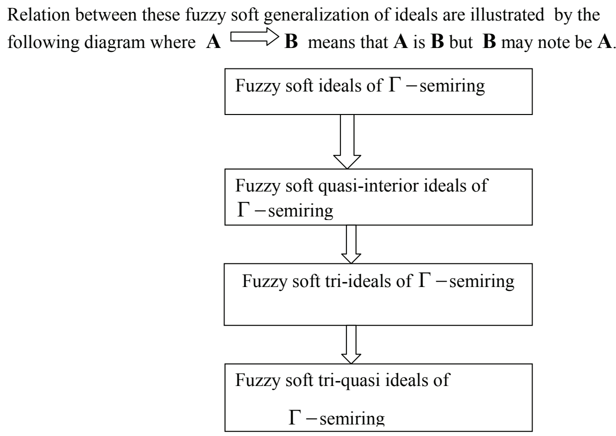

Figure 1. FSTQIGSR.eps

4. Conclusion

In this paper, as a generalization of ideals, we introduced the notion of fuzzy soft tri-quasi ideals of a $\Gamma-$semiring and characterized the regular Γ-semiring in terms of fuzzy soft tri-quasi ideal and studied some of their properties. We proved, M is a Γ-semiring, E be a parameters set and $A \subseteq E$. If (μ, A) is a fuzzy soft tri-ideal over M, then (μ, A) is a fuzzy soft tri-quasi ideal over M. In continuity of this paper, we study fuzzy soft prime, maximal and minimal tri-quasi ideals of Γ-semirings.

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