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# On hybrid simple ordered hypersemigroups

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Abstract. Hybrid structures, which combine soft sets and fuzzy sets, offer a valuable approach to studying ordered hypersemigroups by utilizing hybrid hyperideals. In this paper, we propose the concept of hybrid interior hyperideals in ordered hypersemigroups. Additionally, we investigate a specific subset of an ordered hypersemigroup and establish its relationship with a hybrid hyperideal. Furthermore, we introduce the notion of hybrid simple ordered hypersemigroups and explore their connection to simple ordered hypersemigroups. To conclude, we characterize simple ordered hypersemigroups in terms of hybrid interior hyperideals.

2020 AMS Classification: 20N20

Keywords: Ordered hypersemigroup, Simple ordered hypersemigroup, Hybrid structure, Hybrid interior hyperideal.

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## 1. Introduction

The theory of fuzzy sets, introduced by Zadeh [1] in 1965, is the most appropriate theory for dealing with uncertainty. Following its initial introduction, fuzzy sets have garnered considerable attention from researchers across various scientific disciplines. This versatile concept has found applications in diverse fields such as artificial intelligence, biology, chemistry, decision theory, and mathematics (See [2, 3, 4, 5, 6, 7]). The wide-ranging applications of fuzzy sets underscore their significance as a fundamental and pervasive concept in modern scientific research. However, recognizing the limitations of conventional theoretical approaches in handling uncertainties, Molodtsov [8] introduced the concept of soft sets to address these challenges effectively. Soft sets provide a robust framework for dealing with uncertainties, offering a more flexible and practical approach compared to fuzzy sets. By overcoming the limitations of conventional theoretical approaches, soft sets have become a valuable tool for managing uncertainties in various fields of study. Soft set theory has many applications in several branches of both pure and applied sciences (See [9, 10, 11]).

In certain scenarios, fuzzy sets and soft sets may encounter limitations when used independently to solve complex problems. Recognizing this, researchers have explored the potential benefits of combining these two mathematical tools. Researchers aim to develop hybrid frameworks that can effectively address the problems by integrating the strengths of fuzzy sets and soft sets. Combining fuzzy sets and soft sets offers the potential to enhance problem-solving capabilities by leveraging the complementary aspects of each approach. By integrating these two tools, researchers strive to create more comprehensive and potent mathematical models to accommodate uncertainties and imprecise information better, improving problem-solving outcomes in various domains.

In 2001, Maji et al. [12] introduced the concept of fuzzy soft sets as a generalization of fuzzy sets and soft sets. This theory has gained significant attention and has been extensively studied due to its wide range of applications in various scientific disciplines (See [13, 14, 15, 16]). Another one that we focus on in this paper is the notion of hybrid structures, defined by Jun et al. [17]. Hybrid structures serve as an extension of fuzzy sets and soft sets. They were initially proposed to study $BCK$- and $BCI$-algebras, marking their first application. Consequently, hybrid structures have primarily been employed to examine the properties of various algebraic structures: semigroups, ordered semigroups, near-rings, modules, and hypersemigroups (See [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]). These illustrate the advantage of hybrid structures in investigating algebraic structures in various directions.

In this paper, our focus is on the application of hybrid structures to a specific type of ordered hyperalgebra known as ordered hypersemigroups. To provide context, we will briefly discuss the study of ordered hypersemigroups. The concept of hyperalgebras, introduced by Marty [29], expands upon the notion of ordinary algebras by mapping pairs of elements to nonempty sets instead of individual elements. This extension highlights the broader scope of hyperalgebras. Hypersemigroups (also known as semihypergroups or multisemigroups) are a particular type of hyperalgebra that satisfies specific conditions. Ordered hypersemigroups, introduced by Heidari and Davvaz, is an ordered hyperalgebra that extend hypersemigroups to a higher level by considering a partial order preserved by a hyperoperation (See [30]). The concept of ordered semihypergroups theory was enriched by the work of many researchers, for example, [31, 32, 33, 34]. In particular, the hyperideal theory on semihypergroups and ordered hypersemigroups can be seen in [31, 32, 35, 36].

The concept of fuzzy simple ordered semigroups was introduced by Kehayopulu and Tsingelis [37], who established that an ordered semigroup is simple if and only if it is fuzzy simple. They also characterized simple ordered semigroups in terms of fuzzy interior ideals. Building upon their work, this paper aims to extend the concept of fuzzy interior ideals in ordered semigroups to hybrid interior hyperideals in ordered hypersemigroups. In this study, we introduce the notions of hybrid interior hyperideals and hybrid simple ordered hypersemigroups in the context of ordered hypersemigroups. We find a connection between hyperideals and hybrid hyperideals in ordered hypersemigroups, especially interior hyperideals and hybrid interior hyperideals. Furthermore, we provide a characterization of simple ordered hypersemigroups using hybrid interior hyperideals. By exploring the interplay between hybrid structures and ordered hypersemigroups, we enhance our understanding of the properties and structures of ordered hypersemigroups.

## 2. Preliminaries

In this section, we will recall the basic terms and definitions of ordered hypersemigroups and hybrid structures that we will use in this paper. Throughout this paper, we will use the concepts of ordered hypersemigroups introduced by Kehayopulu [38].

**Definition 2.1** ([38])**.** A *hypergroupoid* is a nonempty set $H$ with a hyperoperation

$$\circ\colon H \times H \to \mathcal{P}^*(H) \mid (a, b) \mapsto a \circ b$$

on $H$ and an operation

$$* : \mathcal{P}^*(H) \times \mathcal{P}^*(H) \to \mathcal{P}^*(H) \mid (A, B) \mapsto A * B$$

on $\mathcal{P}^*(H)$ (induced by the hyperoperation $\circ$) defined by

$$A * B = \bigcup_{a \in A, b \in B} (a \circ b)$$

for all $\varnothing \neq A,\ B \subseteq H$.

We note here that:

(1) $\mathcal{P}^*(H)$ is the set of all nonempty subsets of $H$,
(2) $\{x\} * \{y\} = x * y$,
(3) $A \subseteq B$ implies $A * C \subseteq B * C$ and $C * A \subseteq C * B$ for any nonempty subsets $A,\ B$ and $C$ of $H$.

**Definition 2.2** ([39, Proposition 4])**.** A hypergroupoid $(H; \circ)$ is called a *hypersemigroup*, if

$$\{x\} * (y \circ z) = (x \circ y) * \{z\} \tag{2.1}$$

for every $x,\ y,\ z \in H$.

For convenience, Equation (2.1) could be identified as

$$x * (y * z) = (x * y) * z.$$

Let $(H; \leq)$ be a partial order set. We define a relation $\preceq$ on $\mathcal{P}^*(H)$ as follows: for two nonempty subsets $A$ and $B$ of $H$,

$$A \preceq B := \{(x, y) \in A \times B \mid \forall x \in A,\ \exists y \in B \text{ such that } x \leq y\}.$$

**Definition 2.3** ([38])**.** The structure $(H; \circ, \leq)$ is called an *ordered hypersemigroup*, if the following conditions are satisfied:

(i) $(H; \circ)$ is a hypersemigroup,
(ii) $(H; \leq)$ is a partial order set,
(iii) for $a,\ b,\ c \in H$, if $a \leq b$, then $a * c \preceq b * c$ and $c * a \preceq c * b$.

For simplicity, we denote an ordered hypersemigroup $(H; \circ, \leq)$ by its carrier set as the boldface $\mathbf{H}$.

A nonempty subset $A$ of $H$ is called a *hypersubsemigroup* of $\mathbf{H}$, if $A * A \subseteq A$.

**Definition 2.4** ([38]). Let **H** be an ordered hypersemigroup. A nonempty subset $A$ of $H$ is called a *left* (resp. *right*) hyperideal of **H**, if the following conditions holds:

(i) $H * A \subseteq A$ (resp., $A * H \subseteq A$),
(ii) for $a \in H$ and $b \in A$, if $a \leq b$, then $a \in A$.

A nonempty subset $A$ of $H$ is called a *two-sided hyperideal*, or simply *hyperideal* of **H**, if it is both a left and a right hyperideal of **H**.

**Definition 2.5** ([38]). Let **H** be an ordered hypersemigroup. A hypersubsemigroup $A$ of $H$ is called an *interior hyperideal* of **H**, if the following conditions holds:

(i) $H * A * H \subseteq A$,
(ii) for $a \in H$ and $b \in A$, if $a \leq b$, then $a \in A$.

Let $A$ be a nonempty subset of $H$. Define

$$(A] := \{x \in H \mid \ x \leq a \ \text{ for some } a \in A\}.$$

If $A$ and $B$ are nonempty subsets of $H$, then we obtain:

(1) $A \subseteq (A]$,
(2) $(A] \cup (B] \subseteq (A \cup B]$,
(3) $((A] * (B]] = (A * B]$,
(4) $(A] * (B] \subseteq (A * B]$.

Note that conditions (ii) in Definition 2.4 and 2.5 are equivalent to $A = (A]$.

Now, let us recall the concept of hybrid structures. Let $I = [0, 1]$ be the unit interval, $H$ a set of parameters and $\mathcal{P}(U)$ the set of all subsets of an initial universe set $U$.

**Definition 2.6** ([18]). A *hybrid structure* in $H$ over $U$ is defined to be a mapping

$$f := (f^*, f^+)\colon H \to \mathcal{P}(U) \times I \mid x \mapsto (f^*(x), f^+(x)),$$

where

$$f^*\colon H \to \mathcal{P}(U) \text{ and } f^+\colon H \to I$$

are mappings.

A hybrid structure in $H$ over $U$ encompasses both a soft set and a fuzzy set. Consequently, any soft set $f^*$ can be regarded as a hybrid structure $(f^*, 0)$, where $0\colon H \to \{0\}$, as well as any fuzzy set $(\varnothing, f^+)$, where $\varnothing\colon H \to \{\varnothing\}$. The versatility of hybrid structures allows for a comprehensive and unified treatment of both soft sets and fuzzy sets within a single framework. This enables researchers to seamlessly apply and explore the properties and applications of these mathematical tools in various domains.

Let $A$ be a nonempty subset of $H$. We denote by $\chi_A := (\chi_A^*, \chi_A^+)$ the *characteristic hybrid structure* of $A$ in $H$ over $U$ and is defined to be a hybrid structure

$$\chi_A\colon H \to \mathcal{P}(U) \times I \mid x \mapsto (\chi_A^*(x), \chi_A^+(x)),$$

where

$$\chi_A^*(x) := \begin{cases} U & \text{if } x \in A \\ \varnothing & \text{otherwise} \end{cases} \quad \text{and} \quad \chi_A^+(x) := \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{otherwise.} \end{cases}$$

## 3. Hybrid hyperdeals in ordered hypersemigroups

In this section, we recall the concept of hybrid left and hybrid right hyperideals in ordered hypersemigroups. We also define a new kind of hybrid hyperideals called hybrid interior hyperideals. Some characterizations of such hybrid hyperideals are provided in terms of a particular set.

**Definition 3.1.** Let $\mathbf{H}$ be an ordered hypersemigroup. A hybrid structure $f := (f^*, f^+)$ in $H$ over $U$ is called a *hybrid hypersubsemigroup* in $\mathbf{H}$ over $U$, if for every $x,\ y \in H$,

(i) $\bigcap\limits_{a\in x*y} f^*(a) \supseteq f^*(x) \cap f^*(y)$,

(ii) $\bigvee\limits_{a\in x*y} f^+(a) \leq \max\{f^+(x), f^+(y)\}$.

**Example 3.2.** Let $H = \{0, 1, 2, 3, 4, 5\}$ be an ordered hypersemigroup with the hyperoperation $\circ$ and the order relation $\leq$ on $H$ defined as follows:

| $\circ$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 0 | $\{0\}$ | $\{0\}$ | $\{0\}$ | $\{0\}$ | $\{0\}$ | $\{0\}$ |
| 1 | $\{0\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ |
| 2 | $\{0\}$ | $\{1\}$ | $\{1,2\}$ | $\{3\}$ | $\{1\}$ | $\{1\}$ |
| 3 | $\{0\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{1,2\}$ | $\{3\}$ |
| 4 | $\{0\}$ | $\{1\}$ | $\{4\}$ | $\{5\}$ | $\{1\}$ | $\{1\}$ |
| 5 | $\{0\}$ | $\{1\}$ | $\{1\}$ | $\{1\}$ | $\{4\}$ | $\{5\}$ |

and $\leq := \{(1, 2)\} \cup \Delta_H$, where $\Delta_H := \{(x, x) \mid x \in H\}$ is the identity relation on $H$. Then $\mathbf{H} := (H; \circ, \leq)$ is an ordered hypersemigroup. Define a hybrid structure $f := (f^*, f^+)$ in $H$ over $U$ as follows:

| $H$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f^*(x)$ | $\mathbb{Z}$ | $2\mathbb{Z}$ | $8\mathbb{N}$ | $4\mathbb{N}$ | $8\mathbb{N}$ | $4\mathbb{Z}$ |
| $f^+(x)$ | 0.2 | 0.5 | 0.9 | 0.7 | 0.9 | 0.6 |

By careful calculation, $f$ is a hybrid hypersubsemigroup in $\mathbf{H}$ over $U = \mathbb{Z}$.

Let $\mathbf{H}$ be an ordered hypersemigroup. A hybrid structure $f := (f^*, f^+)$ in $H$ over $U$ is said to be *convex*, if for any $x,\ y \in H$ such that $x \leq y$, we have $f^*(x) \supseteq f^*(y)$ and $f^+(x) \leq f^+(y)$.

**Definition 3.3** ([40, 41])**.** Let $\mathbf{H}$ be an ordered hypersemigroup. A convex hybrid structure $f := (f^*, f^+)$ in $H$ over $U$ is called a *hybrid left* (*resp. right*) *hyperideal* in $\mathbf{H}$ over $U$, if for every $x,\ y \in H$,

(i) $\bigcap\limits_{a\in x*y} f^*(a) \supseteq f^*(y)$ (resp. $\bigcap\limits_{a\in x*y} f^*(a) \supseteq f^*(x)$),

(ii) $\bigvee\limits_{a\in x*y} f^+(a) \leq f^+(y)$ (resp. $\bigvee\limits_{a\in x*y} f^+(a) \leq f^+(x)$).

A hybrid structure $f$ is called a *hybrid hyperideal* in $\mathbf{H}$ over $U$, if it is both a hybrid left and a hybrid right hyperideal in $\mathbf{H}$ over $U$. Sometimes, we call a hybrid hyperideal in $\mathbf{H}$ over $U$ by a *hybrid two-sided hyperideal* in $\mathbf{H}$ over $U$.

**Example 3.4.** Let $H = \{a, b, c\}$. We define the hyperoperation $\circ$ and the relation $\leq$ on $H$ as follows:

| $\circ$ | $a$ | $b$ | $c$ |
|---|---|---|---|
| $a$ | $\{a\}$ | $\{a\}$ | $\{a\}$ |
| $b$ | $\{a\}$ | $\{a\}$ | $\{a\}$ |
| $c$ | $\{a\}$ | $\{a, b\}$ | $\{c\}$ |

and $\leq := \{(a, b)\} \cup \Delta_H$. Then $\mathbf{H} := (H; \circ, \leq)$ is an ordered hypersemigroup. Let $U = \mathbb{N}$. Define a hybrid structure $f := (f^*, f^+)$ in $H$ over $U$ as follows:

| $H$ | $a$ | $b$ | $c$ |
|---|---|---|---|
| $f^*(x)$ | $2\mathbb{N}$ | $\mathbb{N}$ | $4\mathbb{N}$ |
| $f^+(x)$ | 0.7 | 0.2 | 0.8 |

By careful calculation, $f$ is a hybrid left hyperideal in $\mathbf{H}$ over $U$.

**Example 3.5.** Let $H = \{a, b, c\}$. We define the hyperoperation $\circ$ and the relation $\leq$ on $H$ as follows:

| $\circ$ | $a$ | $b$ | $c$ |
|---|---|---|---|
| $a$ | $\{a\}$ | $\{a\}$ | $\{a\}$ |
| $b$ | $\{a\}$ | $\{a\}$ | $\{a\}$ |
| $c$ | $H$ | $H$ | $H$ |

and $\leq := \{(a, c), (b, c)\} \cup \Delta_H$. Then $\mathbf{H} := (H; \circ, \leq)$ is an ordered hypersemigroup. Let $U = \mathbb{N}$. Define a hybrid structure $f := (f^*, f^+)$ in $H$ over $U$ as follows:

| $H$ | $a$ | $b$ | $c$ |
|---|---|---|---|
| $f^*(x)$ | $\mathbb{N}$ | $2\mathbb{N}$ | $4\mathbb{N}$ |
| $f^+(x)$ | 0.2 | 0.7 | 0.8 |

By careful calculation, $f$ is a hybrid right hyperideal in $\mathbf{H}$ over $U$.

Now, we introduce a new type of hybrid hyperideals in ordered hypersemigroups which is called hybrid interior hyperideals.

**Definition 3.6.** Let $\mathbf{H}$ be an ordered hypersemigroup. A convex hybrid hypersubsemigroup $f := (f^*, f^+)$ in $\mathbf{H}$ over $U$ is called a *hybrid interior hyperideal* in $\mathbf{H}$ over $U$, if for every $x,\ y,\ z \in H$,

(i) $\bigcap_{a \in x*y*z} f^*(a) \supseteq f^*(y)$,

(ii) $\bigvee_{a \in x*y*z} f^+(a) \leq f^+(y)$.

**Example 3.7.** Let $H = \{0, a, b, c\}$. We define the hyperoperation $\circ$ and the relation $\leq$ on $H$ as follows:

| $\circ$ | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $0$ | $\{0\}$ | $\{0\}$ | $\{0\}$ | $\{0\}$ |
| $a$ | $\{0\}$ | $\{0\}$ | $\{0\}$ | $\{a, b\}$ |
| $b$ | $\{0\}$ | $\{0\}$ | $\{a, b\}$ | $\{a, b\}$ |
| $c$ | $\{a, b\}$ | $\{a, b\}$ | $\{c\}$ | $\{c\}$ |

and $\leq := \{(a,b)\} \cup \Delta_H$. Then $\mathbf{H} := (H; \circ, \leq)$ is an ordered hypersemigroup. Let $U = [0,1]$. Define a hybrid structure $f := (f^*, f^+)$ in $H$ over $U$ as follows:

| $H$ | $0$ | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $f^*(x)$ | $[0, 0.8]$ | $[0, 0.5]$ | $[0, 0.3]$ | $[0, 0.1]$ |
| $f^+(x)$ | $0.4$ | $0.4$ | $0.4$ | $0.4$ |

By careful calculation, $f$ is a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

The concept of hybrid hyperideals closely connects with hybrid interior hyperideals, as demonstrated by the following result.

**Proposition 3.8.** *Let $\mathbf{H}$ be an ordered hypersemigroup. Any hybrid hyperideal in $\mathbf{H}$ over $U$ is a hybrid interior hyperideal in $\mathbf{H}$ over $U$.*

*Proof.* Let $f := (f^*, f^+)$ be a hybrid hyperideal in $\mathbf{H}$ over $U$. It is clear that $f$ is convex. Let $x,\ y,\ z \in H$. Then we have

$$\bigcap_{u_1 \in x*y*z} f^*(u_1) \supseteq \bigcap_{u_2 \in x*y} f^*(u_2) \supseteq f^*(y)$$

and

$$\bigvee_{u_1 \in x*y*z} f^*(u_1) \leq \bigvee_{u_2 \in x*y} f^*(u_2) \leq f^*(y).$$

Thus $f$ is a hybrid interior hyperideal in $\mathbf{H}$ over $U$. $\square$

Let $\mathbf{H}$ be an ordered hypersemigroup, $a \in H$, and $f := (f^*, f^+)$ a hybrid structure in $H$ over $U$. We denote by $I(a,f)$ the subset of $H$ defined as follows:

$$I(a,f) := \{b \in H \mid f^*(b) \supseteq f^*(a) \text{ and } f^+(b) \leq f^+(a)\}. \tag{3.1}$$

We see that $I(a,f) \neq \varnothing$, since $a \in I(a,f)$.

The following proposition establishes a relationship between hybrid right hyperideals and the set defined above.

**Proposition 3.9.** *Let $\mathbf{H}$ be an ordered hypersemigroup and $f := (f^*, f^+)$ a hybrid structure in $H$ over $U$. If $f$ is a hybrid right hyperideal of $\mathbf{H}$, then the set $I(a,f)$ is a right hyperideal of $\mathbf{H}$ for every $a \in H$.*

*Proof.* Let $a \in H$. We show that $I(a,f)$ is a right hyperideal of $\mathbf{H}$. Let $x,\ y \in H$ such that $x \in I(a,f)$. For any given $u \in x * y$, we have

$$f^*(u) \supseteq \bigcap_{z \in x*y} f^*(z) \supseteq f^*(x) \supseteq f^*(a)$$

and

$$f^+(u) \leq \bigvee_{z \in x*y} f^+(z) \leq f^+(x) \leq f^+(a).$$

This means that $u \in I(a,f)$. Then $I(a,f) * H \subseteq I(a,f)$. Let $x, y \in H$ such that $x \leq y$ with $y \in I(a,f)$. Then $f^*(x) \supseteq f^*(y) \supseteq f^*(a)$ and $f^+(x) \leq f^+(y) \leq f^+(a)$. It follows that $x \in I(a,f)$. Thus $I(a,f)$ is a right hyperideal of $\mathbf{H}$. $\square$

However, the converse of Proposition 3.9, in general, is not true as the following example.

**Example 3.10.** Let $H = \{a, b, c, d\}$ with the hyperoperation $\circ$ and the order relation $\leq$ below:

| $\circ$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $\{a\}$ | $\{a\}$ | $\{a\}$ | $\{a\}$ |
| $b$ | $\{a\}$ | $\{a\}$ | $\{a\}$ | $\{a\}$ |
| $c$ | $\{a\}$ | $\{a\}$ | $\{a, b\}$ | $\{a\}$ |
| $d$ | $\{a\}$ | $\{a\}$ | $\{a, b\}$ | $\{a, b\}$ |

and $\leq := \{(b, a)\} \cup \Delta_H$, where $\Delta_H$ is the identity relation on $H$. One can check that $(H; \circ, \leq)$ is an ordered hypersemigroup. Let $U = \{1, 2, 3\}$. Define a hybrid structure $f := (f^*, f^+)$ in $H$ over $U$ as follows:

| $H$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $f^*(x)$ | $U$ | $\{1, 2\}$ | $\{2\}$ | $\{1\}$ |
| $f^+(x)$ | 0.4 | 0.5 | 0.3 | 0.2 |

It is easy to verify that $I(x, f)$ is a right hyperideal of $\mathbf{H}$ for all $x \in H$. Nevertheless, $f$ is not a hybrid right hyperideal in $\mathbf{H}$ over $U$ since $b \leq a$ but

$$f^*(b) \not\supseteq f^*(a) \qquad \text{and} \qquad f^+(b) \not\leq f^+(a).$$

Similarly, we obtain the following result.

**Proposition 3.11.** *Let $\mathbf{H}$ be an ordered hypersemigroup and $f := (f^*, f^+)$ a hybrid structure in $H$ over $U$. If $f$ is a hybrid left hyperideal in $\mathbf{H}$ over $U$, then the set $I(a, f)$ is a left hyperideal of $\mathbf{H}$ for every $a \in H$.*

By Proposition 3.9 and 3.11, we obtain the following corollary.

**Corollary 3.12.** *Let $\mathbf{H}$ be an ordered hypersemigroup and $f := (f^*, f^+)$ a hybrid structure in $H$ over $U$. If $f$ is a hybrid hyperideal in $\mathbf{H}$ over $U$, then the set $I(a, f)$ is a hyperideal of $\mathbf{H}$ for every $a \in H$.*

The set defined in (3.1) is also related to the connection between interior hyperideals and hybrid interior hyperideals.

**Proposition 3.13.** *Let $\mathbf{H}$ be an ordered hypersemigroup and $f := (f^*, f^+)$ a hybrid structure in $H$ over $U$. If $f$ is a hybrid interior hyperideal in $\mathbf{H}$ over $U$, then the set $I(a, f)$ is an interior hyperideal of $\mathbf{H}$ for every $a \in H$.*

*Proof.* Let $a \in H$. Suppose that $x,\ y \in I(a, f)$. For any given $u \in x * y$, we have

$$f^*(u) \supseteq \bigcap_{z \in x*y} f^*(z) \supseteq f^*(x) \cap f^*(y) \supseteq f^*(a)$$

and

$$f^+(u) \leq \bigvee_{z \in x*y} f^+(z) \leq \max\{f^+(x), f^+(y)\} \leq f^+(a).$$

This means that $u \in I(a, f)$. Then $I(a, f)$ is a hypersubsemigroup of $\mathbf{H}$. Now let $x,\ y,\ z \in H$ such that $y \in I(a, f)$. For any given $u \in x * y * z$, we have

$$f^*(u) \supseteq \bigcap_{t \in x*y*z} f^*(t) \supseteq f^*(y) \supseteq f^*(a)$$

and
$$f^+(u) \leq \bigvee_{t\in x*y*z} f^+(t) \leq f^+(y) \leq f^+(a).$$
This means that $u \in I(a,f)$. Then $H * I(a,f) * H \subseteq I(a,f)$. Let $x, y \in H$ such that $x \leq y$ with $y \in I(a,f)$. Then $f^*(x) \supseteq f^*(y) \supseteq f^*(a)$ and $f^+(x) \leq f^+(y) \leq f^+(a)$. It follows that $x \in I(a,f)$. Thus $I(a,f)$ is an interior hyperideal of **H**. □

The converse of Proposition 3.13, in general, may not true. Consider Example 3.10, we see that $I(x,f)$ is an interior hyperideal of **H** of any $x \in H$. However, $f$ is not a hybrid interior hyperideal in **H** over $U$ since $f$ is not convex.

By the above observation, we obtain the converse of Proposition 3.9, 3.11 and 3.13, and Corollary 3.12 present as follows. Moreover, it is also a characterization of hybrid hyperideals we defined by the set presented in (3.1).

**Corollary 3.14.** *Let* **H** *be an ordered hypersemigroup, and* $f := (f^*, f^+)$ *a convex hybrid structure in* $H$ *over* $U$*. Then the following statements are equivalent:*

(1) $f$ *is a hybrid interior (resp. left, right, two-sided) hyperideal in* **H** *over* $U$*,*
(2) $I(a,f)$ *is an interior (resp. left, right, two-sided) hyperideal of* **H** *for all* $a \in H$*.*

*Proof.* We prove only for interior hyperideal and hybrid interior hyperideal. For other cases can be done similarly.

(1)⇒(2): This is clear by Proposition 3.9, 3.11 and 3.13, and Corollary 3.12.

(2)⇒(1): Let $x,\ y,\ z \in H$. By our presumption, $I(y,f)$ is an interior hyperideal of **H** and $y \in I(y,f)$. It follows that $x * y * z \subseteq I(y,f)$. Then any given $u \in x*y*z$, we have $u \in I(y,f)$. Thus $f^*(u) \supseteq f^*(y)$ and $f^+(u) \leq f^+(y)$. Since $u$ is arbitrary,
$$\bigcap_{u\in x*y*z} f^*(u) \supseteq f^*(y) \qquad \text{and} \qquad \bigvee_{u\in x*y*z} f^+(u) \leq f^+(y).$$
Finally, suppose that $x,\ y \in H$ such that $x \leq y$. Since $f$ is convex,
$$f^*(x) \supseteq f^*(y) \qquad \text{and} \qquad f^+(x) \leq f^+(y).$$
Altogether, $f$ is a hybrid interior hyperideal in **H** over $U$. □

On the other hand, we characterize hyperideals by hybrid hyperideals by the characteristic hybrid structure.

**Proposition 3.15.** *Let* $H$ *be an ordered hypersemigroup and* $\varnothing \neq A \subseteq H$*. Then the following statements are equivalent:*

(1) $A$ *is an interior (resp. left, right, two-sided) hyperideal of* **H***,*
(2) $\chi_A := (\chi^*_A, \chi^+_A)$ *is a hybrid interior (resp. left, right, two-sided) hyperideal in* **H** *over* $U$*.*

*Proof.* We prove only for interior hyperideal and hybrid interior hyperideal. We recommend referring to the reference [40] for more comprehensive information on other cases.

(1)⇒(2): Firstly, we show that $\chi_A$ is convex. Let $x,\ y \in H$ such that $x \leq y$. If $y \in A$, then $x \in A$. This means that $\chi^*_A(a) = \chi^*_A(y)$ and $\chi^+_A(a) = \chi^+_A(y)$. If $y \notin A$, then $\chi^*_A(x) \supseteq \varnothing = \chi^*_A(y)$ and $\chi^+_A(x) \leq 1 = \chi^*_A(y)$. Thus $\chi_A$ is a convex hybrid structure. Let $x,\ y,\ z \in H$. If $y \notin A$, then

$$\bigcap_{u\in x*y*z} \chi^*_A(u) \supseteq \varnothing = \chi^*_A(y) \qquad \text{and} \qquad \bigvee_{u\in x*y*z} \chi^+_A(u) \leq 1 = \chi^+_A(y).$$

Suppose that $y \in A$. Then $x * y * z \subseteq H * A * H \subseteq A$. This means that for any $u \in x * y * z$, we have $\chi^*_A(u) = U = \chi^*_A(y)$ and $\chi^+_A(u) = 0 = \chi^+_A(y)$. Since $u$ is arbitrary, we have

$$\bigcap_{u\in x*y*z} \chi^*_A(u) = U = \chi^*_A(y) \qquad \text{and} \qquad \bigvee_{u\in x*y*z} \chi^+_A(u) = 0 = \chi^+_A(y).$$

So $\chi_A$ is a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

(2)⇒(1): Let $x,\ a,\ y \in H$. If $a \in A$, then $x * a * y \subseteq H * A * H$. Since $U \supseteq \bigcap_{c\in x*a*y} \chi^*_A(c) \supseteq U = \chi^*_A(a)$. It follows that $\bigcap_{c\in x*a*y} \chi^*_A(c) = U$. Thus $c \in A$. Similarly, $\bigvee_{c\in x*a*y} \chi^+_A(c) = 0$ implies $c \in A$. So $H * A * H \subseteq A$. Let $x,\ y \in H$ such that $x \leq y$ with $y \in A$. Then $U \supseteq \chi^*_A(x) \supseteq f^*_A(y) = U$. It follows that $\chi^*_A(x) = U$ and $\chi^+_A(x) = 0$. Thus $x \in A$. So $A$ is an interior hyperideal of $\mathbf{H}$. □

## 4. Hybrid simple ordered hypersemigroups

In the final section of this paper, we introduce an approach to understand and characterize simple ordered hypersemigroups using hybrid structures. We propose a concept called hybrid simple ordered hypersemigroups which allows for a deeper comprehension of simple ordered hypersemigroups in hybridization. Through our investigation, we establish that simple ordered hypersemigroups are equivalent to hybrid simple ordered hypersemigroups, further highlighting the coherence between these two concepts. Moreover, we characterize simple ordered hypersemigroups by using hybrid interior hyperideals.

An ordered hypersemigroup $\mathbf{H}$ is called *simple*, if does not contain proper hyperideals. That is, for any hyperideal $A$ of $\mathbf{H}$, we have $A = H$ (See [42, 43]).

**Definition 4.1.** An ordered hypersemigroup $\mathbf{H}$ is called *hybrid simple*, if every hybrid hyperideal in $\mathbf{H}$ over $U$ is a constant function. That is, for every hybrid hyperideal $f := (f^*, f^+)$ in $\mathbf{H}$ over $U$, we have $f^*(a) = f^*(b)$ and $f^+(a) = f^+(b)$ for all $a, b \in H$.

**Example 4.2.** Let $H = \{a, b\}$ with the hyperoperation $\circ$ and the order relation $\leq$ below:

| $\circ$ | $a$ | $b$ |
|---|---|---|
| $a$ | $\{a\}$ | $\{a\}$ |
| $b$ | $\{b\}$ | $\{b\}$ |

and $\leq := \{(a, b)\} \cup \Delta_H$, where $\Delta_H$ is the identity relation on $H$. One can check that $(H; \circ, \leq)$ is an ordered hypersemigroup. Let $U = \{1, 2, 3\}$. Define a subset J($\mathbf{H}$) of hybrid structure in $H$ over $U$ as follows.

$$\{f := (f^*, f^+) \mid f^*(x) = f^*(y) \text{ and } f^+(x) = f^+(y) \text{ for all } x, y \in H\}.$$

We can examine that J($\mathbf{H}$) is the set of all hybrid hyperideals in $\mathbf{H}$ over $U$. To see this, let $f := (f^*, f^+)$ be a hybrid hyperideal in $\mathbf{H}$ over $U$. This means that $f^*(a) \supseteq$

$f^*(b)$ and $f^+(a) \leq f^+(b)$. Without loss of generality, suppose that $f^+(b) > f^+(a)$. Then by the hybrid right hyperideality of $f$, we obtain $\bigvee_{u \in a*b} f^+(u) > f^+(a)$. This terminates the right hyperideality of $f$. Thus $f^+(a) = f^+(b)$. Similarly, we have $f^*(a) = f^*(b)$. So $\mathbf{H}$ is hybrid simple.

The following result illustrates the coincidence of simple ordered hypersemigroups and hybrid simple ordered hypersemigroups

**Theorem 4.3.** *Let $\mathbf{H}$ be an ordered hypersemigroup. Then the following conditions are equivalent:*

(1) *$\mathbf{H}$ is simple,*
(2) *$\mathbf{H}$ is hybrid simple.*

*Proof.* (1)⇒(2): Let $f := (f^*, f^+)$ be a hybrid hyperideal in $\mathbf{H}$ over $U$. Let $a,\ b \in H$. Then by Corollary 3.12, the set $I(a, f)$ is a hyperideal of $\mathbf{H}$. Since $\mathbf{H}$ is simple, we have $I(a, f) = H$. Since $b \in H = I(a, f)$, we have $f^*(b) \supseteq f^*(a)$ and $f^+(b) \leq f^+(a)$. Similarly, since $a \in H = I(b, f)$, we have $f^*(a) \supseteq f^*(b)$ and $f^+(a) \leq f^+(b)$. Thus we have $f^*(a) = f^*(b)$ and $f^+(a) = f^+(b)$. This means that $f$ is a constant function. So $\mathbf{H}$ is hybrid simple.

(2)⇒(1): Suppose $H$ contains proper hyperideals. Let $I$ be a hyperideal of $\mathbf{H}$ such that $I \neq H$. By Lemma 3.15, $\chi_I$ is a hybrid hyperideal in $\mathbf{H}$ over $U$. Let $x \in H$. Since $\mathbf{H}$ is hybrid simple, the hybrid hyberideal $\chi_I$ is a constant function. That is, $\chi_I^*(x) = \chi_I^*(b)$ and $\chi_I^+(x) = \chi_I^+(b)$ for every $b \in H$. Now, let $a \in I$. Then we have $\chi_I^*(x) = \chi_I^*(a) = U$ and $\chi_I^+(x) = \chi_I^+(a) = 0$. Thus $x \in I$. So we have $H \subseteq I$. Hence $H = I$. This is a contradiction. Therefore $\mathbf{H}$ is a simple ordered hypersemigroup. □

We proceed to characterize simple ordered hypersemigroups by utilizing the notion of hybrid interior hyperideals. However, before delving into this characterization, we present an auxiliary lemma to aid our analysis.

**Lemma 4.4.** *Let $\mathbf{H}$ be an ordered hypersemigroup. Then the following conditions are equivalent:*

(1) *$\mathbf{H}$ is simple,*
(2) *$H = (H * a * H]$ for every $a \in H$.*

We now characterize simple ordered hypersemigroups in terms of hybrid interior hyperideals as the following theorem.

**Theorem 4.5.** *Let $\mathbf{H}$ be an ordered hypersemigroup. Then the following conditions are equivalent:*

(1) *$H$ is simple,*
(2) *every hybrid interior hyperideal in $\mathbf{H}$ over $U$ is a constant function.*

*Proof.* (1)⇒(2): Let $f := (f^*, f^+)$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$. Suppose $a,\ b \in H$. Since $\mathbf{H}$ is simple and $b \in H$, by Lemma 4.4, we have $H = (H * b * H]$. Since $a \in H$, we have $a \in (H * b * H]$. Then there exists $c \in x * b * y$ for some $x,\ y \in H$ such that $a \leq c$. Since $a,\ c \in H$ and $f$ is a hybrid interior hyperideal in $\mathbf{H}$ over $U$, we have

$$f^*(a) \supseteq f^*(c) \supseteq \bigcap_{d \in x*b*y} f^*(d) \supseteq f^*(b)$$

and

$$f^+(a) \leq f^+(c) \leq \bigvee_{d \in x*b*y} f^+(d) \leq f^+(b).$$

In a similar way, we prove that $f^*(b) \supseteq f^*(a)$ and $f^+(b) \leq f^+(a)$. Thus $f^*(a) = f^*(b)$ and $f^+(a) = f^+(b)$. This shows that $f$ is a constant function.

(2)⇒(1): Let $f := (f^*, f^+)$ be a hybrid hyperideal in $\mathbf{H}$ over $U$. Then by Proposition 3.8, $f$ is a hybrid interior hyperideal in $\mathbf{H}$ over $U$. By hypothesis, $f$ is a constant function. That is, $\mathbf{H}$ is hybrid simple. Thus by Theorem 4.3, $\mathbf{H}$ is simple □

As a consequence, we have the following corollary.

**Corollary 4.6.** *Let* $\mathbf{H}$ *be an ordered hypersemigroup. Then the following conditions are equivalent:*

(1) $\mathbf{H}$ *is simple,*
(2) $H = (H * a * H]$ *for every* $a \in H$*,*
(3) $\mathbf{H}$ *is hybrid simple,*
(4) *every hybrid interior hyperideal in* $\mathbf{H}$ *over* $U$ *is a constant function.*

**Example 4.7.** Let $H = \{a, b, c, d\}$ with the hyperoperation $\circ$ and the order relation $\leq$ below:

| $\circ$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| $a$ | $\{a\}$ | $\{b\}$ | $\{c\}$ | $\{d\}$ |
| $b$ | $\{b\}$ | $\{a, c\}$ | $\{b, c\}$ | $\{d\}$ |
| $c$ | $\{c\}$ | $\{b, c\}$ | $\{a, b\}$ | $\{d\}$ |
| $d$ | $\{d\}$ | $\{d\}$ | $\{d\}$ | $H$ |

and $\leq := \Delta_H$, where $\Delta_H$ is the identity relation on $H$. One can check that $(H; \circ, \leq)$ is an ordered hypersemigroup. Let $U = \{1, 2, 3\}$. Define a subset $\mathrm{In}(\mathbf{H})$ of hybrid structure in $H$ over $U$ as follows.

$$\{f := (f^*, f^+) \mid f^*(x) = f^*(y) \text{ and } f^+(x) = f^+(y) \text{ for all } x, y \in H\}.$$

We can examine that $\mathrm{In}(\mathbf{H})$ is the set of all hybrid interior hyperideals in $\mathbf{H}$ over $U$. To see this, let $f := (f^*, f^+)$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$. Without loss of generality, let us divide our consideration into four cases.

**Case 1:** If $f^+(a) > f^+(x)$ for all $x \in H - \{a\}$, then we separate into three cases.

(1) If $f^+(a) > f^+(b)$, then we can see that $\bigvee_{u \in b*b*b} f^+(u) > f^+(b)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.
(2) If $f^+(a) > f^+(c)$, then we can see that $\bigvee_{u \in a*c*c} f^+(u) > f^+(c)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

(3) If $f^+(a) > f^+(d)$, then we can see that $\bigvee_{u \in d*d*d} f^+(u) > f^+(d)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

**Case 2:** If $f^+(b) > f^+(x)$ for all $x \in H - \{b\}$, then we separate into three cases.

(1) If $f^+(b) > f^+(a)$, then we can see that $\bigvee_{u \in b*a*a} f^+(u) > f^+(a)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

(2) If $f^+(b) > f^+(c)$, then we can see that $\bigvee_{u \in a*c*b} f^+(u) > f^+(c)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

(3) If $f^+(b) > f^+(d)$, then we can see that $\bigvee_{u \in d*d*d} f^+(u) > f^+(d)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

**Case 3:** If $f^+(c) > f^+(x)$ for all $x \in H - \{c\}$, then we separate into three cases.

(1) If $f^+(c) > f^+(a)$, then we can see that $\bigvee_{u \in c*a*a} f^+(u) > f^+(a)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

(2) If $f^+(c) > f^+(b)$, then we can see that $\bigvee_{u \in a*b*c} f^+(u) > f^+(b)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

(3) If $f^+(c) > f^+(d)$, then we can see that $\bigvee_{u \in d*d*d} f^+(u) > f^+(d)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

**Case 4:** If $f^+(d) > f^+(x)$ for all $x \in H - \{d\}$, then we separate into three cases.

(1) If $f^+(d) > f^+(a)$, then we can see that $\bigvee_{u \in d*a*d} f^+(u) > f^+(a)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

(2) If $f^+(d) > f^+(b)$, then we can see that $\bigvee_{u \in d*b*d} f^+(u) > f^+(b)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

(3) If $f^+(d) > f^+(c)$, then we can see that $\bigvee_{u \in d*c*d} f^+(u) > f^+(c)$. This contradicts to the fact that $f$ be a hybrid interior hyperideal in $\mathbf{H}$ over $U$.

Similarly, we can do this procedure for the map $f^*$. This means that $\mathrm{In}(\mathbf{H})$ is the set of all hybrid interior hyperideals in $\mathbf{H}$ over $U$. Since any hybrid interior hyperideal in $\mathbf{H}$ over $U$ is a constant, by Corollary 4.6, $\mathbf{H}$ is simple.

## 5. Conclusion

The concepts of hybrid interior hyperideals in ordered hypersemigroups and hybrid simple ordered hypersemigroups are introduced. We studied the relationship between hybrid hyperideals and special sets in an ordered hypersemigroup. Finally, we proved coincident of the simple ordered hypersemigroups and the hybrid simple ordered hypersemigroups, we also characterized simple ordered hypersemigroups by some properties of hybrid interior hyperideals. In our future work, we use the concept of hybrid interior hyperideals to classify the classes of ordered hypersemigroups, and we apply the concept of hybrid simple to characterize ordered hypersemigroups. Moreover, will apply this concept to the theory of hypersemirings, hypergroups, BCK-hyperstructure, etc.

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