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Abstract. In this paper, the concept of fuzzy F-spaces is introduced and studied. Several characterizations of fuzzy F-spaces are established. It is established that fuzzy F- spaces in which fuzzy G_δ-sets are fuzzy open, are fuzzy normal spaces and fuzzy F-spaces in which fuzzy G_δ- sets are fuzzy closed, are fuzzy F$'$-spaces. It is established that fuzzy F-spaces are not fuzzy hyperconnected spaces. A condition for fuzzy F$'$-spaces to become fuzzy F-spaces is obtained. The conditions under which fuzzy globally disconnected spaces and fuzzy perfectly disconnected spaces become fuzzy F-spaces are also obtained.

2020 AMS Classification: 54A40, 03E72, 54G05

Keywords: Fuzzy G_δ-set, Fuzzy F_σ-set, Fuzzy dense set, Fuzzy residual set, Fuzzy globally disconnected space, Fuzzy P-space, Fuzzy Oz space, Fuzzy basically disconnected space.

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1. Introduction

The notion of fuzzy sets as a new approach to a mathematical representation of vagueness in everyday language, was introduced by Zadeh [1] in his classical paper in 1965. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. In 1968, Chang [2] introduced the concept of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

In the recent years, there has been a growing trend to introduce and study different forms of fuzzy spaces in fuzzy topology. In classical topology, F-spaces are introduced by Gillman and Henriksen [3], in which disjoint cozero subsets of X are contained in disjoint zero sets. Motivated on these lines, the concept of fuzzy F-space is introduced in this paper. Several characterizations of fuzzy F-Spaces are established. The conditions for fuzzy globally disconnected spaces, fuzzy perfectly disconnected spaces, fuzzy basically disconnected spaces and fuzzy submaximal spaces to become fuzzy F-spaces are obtained. It is established that fuzzy F-spaces are not fuzzy hyperconnected spaces. Also a condition under which fuzzy F′-space becomes a fuzzy F-space is obtained. It is established that fuzzy F-spaces in which fuzzy G_δ-sets are fuzzy open, are fuzzy normal spaces and fuzzy F-spaces in which fuzzy G_δ-sets are fuzzy closed, are fuzzy F′-spaces. It is established that fuzzy F-spaces and fuzzy P-spaces, are fuzzy normal spaces.

In recent years, the topological space theory has been embedding in the soft set theory to obtain some interesting applications [4, 5, 6, 7]. Many authors redefined the classical topological concepts via soft topological structure. Recently, Şenel et al. [8] applied the concept of octahedron sets proposed by Lee et al. [9] to multi-criteria group decision making problems. Al-Shami [10, 11, 12] successfully applied some theoretical soft topological concepts to information Science and decision-making problems. On these lines, there is a need and scope of investigation considering different types of fuzzy spaces such as fuzzy F-spaces, fuzzy F′-spaces for applying some fuzzy topological concepts to information Science and decision-making problems.

2. Preliminaries

Some basic notions and results used in the sequel, are given in order to make the exposition self-contained. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval $[0, 1]$. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1. ([2]) Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The *interior*, the *closure* and the *complement* of λ are defined respectively as follows:

(i) $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$,
(ii) $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$,
(iii) $\lambda'(x) = 1 - \lambda(x)$ for all $x \in X$.

For a family $\{\lambda_i/i \in I\}$ of fuzzy sets in (X, T), the *union* $\psi = \vee_i(\lambda_i)$ and the *intersection* $\delta = \wedge_i(\lambda_i)$, are defined respectively as

(iv) $\psi(x) = sup_i\{\lambda_i(x)/x \in X\}$.
(v) $\delta(x) = inf_i\{\lambda_i(x)/x \in X\}$.

Lemma 2.2. ([13]) *For a fuzzy set λ of a fuzzy topological space X,*
(1) $1 - int(\lambda) = cl(1 - \lambda)$,
(2) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.3. A fuzzy set λ in a fuzzy topological space (X, T) is called a:

(i) *fuzzy regular open set* in X, if $\lambda = intcl(\lambda)$,
fuzzy regular closed set in X, if $\lambda = clint(\lambda)$ [13],
(ii) *fuzzy G_δ-set* in X, if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$, where $\lambda_i \in T$ for $i \in J$,
fuzzy F_σ-set in X, if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$, where $1 - \lambda_i \in T$ for $i \in J$ [14],

(iii) *fuzzy dense set*, if there exist no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$, i.e., $cl(\lambda) = 1$ in X [15],
(iv) *fuzzy first category set*, if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$, where each λ_i is a fuzzy nowhere dense set in X. Any other fuzzy set in X is said to be of *fuzzy second category set* [15],
(v) *fuzzy residual set*, if $1 - \lambda$ is a fuzzy first category set in X [16],
(vi) *fuzzy σ-nowhere dense set*, if λ is a fuzzy F_σ-set in X such that $int(\lambda) = 0$ [17],
(vii) *fuzzy Baire dense set*, if for a non-zero fuzzy open set μ in X, $\lambda \wedge \mu$ is a fuzzy second category set in X [18],
(viii) *fuzzy Baire set* in X, if $\lambda = \mu \wedge \delta$, where μ is a fuzzy open and δ is an fuzzy residual set in X [19].

Definition 2.4. A fuzzy topological space (X, T) is called a:

(i) *fuzzy extremally disconnected space*, if the closure of every fuzzy open set is a fuzzy open set in X [20],
(ii) *fuzzy P-space*, if each fuzzy G_δ-set in X, is fuzzy open in X [21],
(iii) *fuzzy submaximal space*, if for each fuzzy set λ in X such that $cl(\lambda) = 1$, $\lambda \in T$ in X [14],
(iv) *fuzzy hyperconnected*, if every non-null fuzzy open set in X is fuzzy dense in X [22],
(v) *fuzzy Baire space*, if $int(\bigvee_{i=1}^{\infty} \lambda_i) = 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in X [16],
(vi) *fuzzy perfectly disconnected space*, if for any two non-zero fuzzy sets λ and μ in X with $\lambda \leq 1 - \mu$, $cl(\lambda) \leq 1 - cl(\mu)$ in X [23],
(vii) *fuzzy normal space*, if every pair of disjoint fuzzy closed sets F and G in X, there exists disjoint fuzzy open sets U and V in X such that $F \leq U$ and $G \leq V$ [24],
(viii) *fuzzy F'-space*, if $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in X, then $cl(\lambda) \leq 1 - cl(\mu)$ in X [25],
(ix) *fuzzy maximal space*, if (X, T) is the fuzzy extremally disconnected space and fuzzy submaximal space [26],
(x) *fuzzy Oz-space* if each fuzzy regular closed set is a fuzzy G_δ-set in X [27].

Theorem 2.5. ([13])
(1) *The closure of an fuzzy open set is an fuzzy regular closed set.*
(2) *The interior of an fuzzy closed set is an fuzzy regular open set.*

Theorem 2.6. ([28]) *Let (X, T) be a fuzzy topological space. Then the following are equivalent:*
(1) *X is a fuzzy Baire space,*
(2) *each non-zero fuzzy open set is a fuzzy second category set in X.*

Theorem 2.7. ([29]) *If λ is a fuzzy residual set in a fuzzy topological space (X, T), then there exists a fuzzy G_δ-set μ in X such that $\mu \leq \lambda$.*

Theorem 2.8. ([26]) *If λ is a fuzzy open set in the fuzzy Baire space (X, T), then λ is a fuzzy Baire dense set in X.*

Theorem 2.9. ([30]) *If λ is a fuzzy first category set in the fuzzy globally disconnected space (X, T), then λ is a fuzzy F_σ-set in X.*

Theorem 2.10. ([18]) *If λ is a fuzzy Baire dense set in a fuzzy topological space (X, T), then there is no fuzzy F_σ-set μ with $int(\mu) = 0$ such that $\lambda < \mu$.*

Theorem 2.11. ([18]) *If λ is a fuzzy Baire dense set in a fuzzy topological space (X, T), then there exist fuzzy second category sets δ_1 and δ_2 in X such that $\delta_1 < \lambda < \delta_2$.*

Theorem 2.12. ([18]) *If λ is a fuzzy Baire dense set in a fuzzy topological space (X, T), then there is no fuzzy residual set δ in X such that $1 - \lambda > \delta$.*

Theorem 2.13. ([31]) *If λ is a fuzzy residual set in the fuzzy submaximal space (X, T), then λ is a fuzzy G_δ-set in X.*

Theorem 2.14. ([27]) *If λ is a fuzzy open set in the fuzzy Oz-space (X, T) such that $cl(\lambda) \neq 1$, then $cl(\lambda)$ is a fuzzy G_δ-set in X.*

Theorem 2.15. ([26]) *If λ is a fuzzy Baire set in the fuzzy maximal space (X, T), then λ is a fuzzy G_δ-set in X.*

Theorem 2.16. ([21]) *A fuzzy topological space (X, T) is a fuzzy basically disconnected space $\Leftrightarrow$ for all fuzzy open set λ and fuzzy G_δ-set μ in X such that $\lambda \leq \mu$, $cl(\lambda) \leq int(\mu)$.*

3. Fuzzy F-spaces

Definition 3.1. A fuzzy topological space (X, T) is called a *fuzzy F-space*, if for any two fuzzy F_σ-sets λ and μ in X with $\lambda \leq 1 - \mu$, there exists fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1 - \beta$.

Example 3.2. Let $X = \{a, b, c\}$. Consider the fuzzy sets λ, μ and γ defined in X as follows:

$$\lambda(a) = 0.4,\ \lambda(b) = 0.5,\ \lambda(c) = 0.7;$$
$$\mu(a) = 0.6,\ \mu(b) = 0.7,\ \mu(c) = 0.5;$$
$$\gamma(a) = 0.6,\ \gamma(b) = 0.5,\ \gamma(c) = 0.5.$$

Then $T = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \wedge \mu, \lambda \vee \mu, 1\}$ is a fuzzy topology on X. On computation, the fuzzy F_σ-sets in X are $1 - (\lambda \wedge \mu)$, $1 - \gamma$ and fuzzy G_δ-sets are $\lambda \wedge \mu$, γ. It is found that $[1 - (\lambda \wedge \mu)] \leq 1 - (1 - \gamma)$, for fuzzy F_σ-sets $1 - (\lambda \wedge \mu)$ and $1 - \gamma$ in X. Also $1 - (\lambda \wedge \mu) \leq \gamma$, $1 - \gamma \leq \lambda \wedge \mu$ and $\gamma \leq 1 - (\lambda \wedge \mu)$, where $\lambda \wedge \mu$ and γ are fuzzy G_δ-sets in X. Thus (X, T) is a fuzzy F-space.

Example 3.3. Let $X = \{a, b, c\}$. Consider the fuzzy sets λ, μ, γ and δ defined in X as follows:

$$\lambda(a) = 0.5,\ \lambda(b) = 0.4,\ \lambda(c) = 0.6;$$
$$\mu(a) = 0.6,\ \mu(b) = 0.5,\ \mu(c) = 0.4;$$
$$\gamma(a) = 0.4,\ \gamma(b) = 0.6;,\ \gamma(c) = 0.5;$$
$$\delta(a) = 0.6.\ \delta(b) = 0.4,\ \delta(c) = 0.5.$$

Then $T = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda \wedge \gamma, \mu \wedge \gamma, \lambda \vee (\mu \wedge \gamma), \lambda \wedge (\mu \vee \gamma), \mu \vee (\lambda \wedge \gamma), \mu \wedge (\lambda \vee \gamma), \gamma \vee (\lambda \wedge \mu), \gamma \wedge (\lambda \vee \mu), \lambda \vee \mu \vee \gamma, \lambda \wedge \mu \wedge \gamma, 1\}$ is a fuzzy topology on X. On computation, the fuzzy F_σ-sets in X are $\delta,\ 1 - (\lambda \wedge \mu),\ 1 - (\lambda \wedge \mu \wedge \gamma)$ and $1 - [\mu \vee (\lambda \wedge \gamma)]$. The fuzzy G_δ-sets in X) are $\delta,\ \lambda \wedge \mu,\ \lambda \wedge \mu \wedge \gamma$ and $\mu \vee (\lambda \wedge \gamma)$. It is found that $\delta \leq 1 - [\mu \vee (\lambda \wedge \gamma)]$ for the fuzzy F_σ-sets $\delta,\ 1 - [\mu \vee (\lambda \wedge \gamma)]$ in X. Also $\delta \leq \mu \vee (\lambda \wedge \gamma),\ 1 - [\mu \vee (\lambda \wedge \gamma)] \leq \delta$, where $\mu \vee (\lambda \wedge \gamma)$ and δ are fuzzy G_δ-sets in (X, T). But $\mu \vee (\lambda \wedge \gamma) \geq 1 - \delta$, implies that X) is not a fuzzy F-space.

Proposition 3.4. *If $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in a fuzzy F-space (X, T), then there exists fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha \leq 1 - \beta \leq 1 - \mu$.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in a fuzzy F-space (X, T). Then there exists fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1 - \beta$. Now $\mu \leq \beta$ implies that $1 - \beta \leq 1 - \mu$. Thus $\lambda \leq \alpha \leq 1 - \beta \leq 1 - \mu$. □

Corollary 3.5. *If $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in a fuzzy F-space (X, T), then*

(1) *there exists a fuzzy G_δ-set α in X such that $\lambda \leq \alpha \leq 1 - \mu$,*
(2) *there exists a fuzzy F_σ-set δ in X such that $\lambda \leq \delta \leq 1 - \mu$.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where λ, μ are fuzzy F_σ-sets in a fuzzy F-space (X, T). Then by the Proposition 3.4, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha \leq 1 - \beta \leq 1 - \mu$.

(1) $\lambda \leq \alpha \leq 1 - \beta \leq 1 - \mu$ implies that $\lambda \leq \alpha \leq 1 - \mu$, where α is a fuzzy G_δ-set in X.

(2) $\lambda \leq \alpha \leq 1 - \beta \leq 1 - \mu$ implies that $\lambda \leq 1 - \beta \leq 1 - \mu$ in X. Since β is a fuzzy G_δ-set, $1 - \beta$ is a fuzzy F_σ-set in X. Let $\delta = 1 - \beta$. Then $\lambda \leq \delta \leq 1 - \mu$, where δ is a fuzzy F_σ-set in X. □

Proposition 3.6. *If $\lambda \leq 1 - \mu$, where λ is a fuzzy open F_σ-set and μ is a fuzzy F_σ-set in a fuzzy F-space (X, T), then there exists a fuzzy open set δ in X such that $\lambda \leq \delta \leq 1 - cl(\mu)$.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where λ is a fuzzy open F_σ-set and μ is a fuzzy F_σ-set in a fuzzy F-space (X, T). Then by Proposition 3.4, there exists fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha \leq 1 - \beta \leq 1 - \mu$. Thus $int(\lambda) \leq int(\alpha) \leq int(1 - \beta) \leq int(1 - \mu)$. Since λ is a fuzzy open set in X, $int(\lambda) = \lambda$.. So $\lambda \leq int(\alpha) \leq 1 - cl(\beta) \leq 1 - l(\mu)$. Let $\delta = int(\alpha)$ or $1 - cl(\beta)$. Then δ is a fuzzy open set in X. Thus there exists a fuzzy open set δ in X such that $\lambda \leq \delta \leq 1 - cl(\mu)$. □

Proposition 3.7. *If $\lambda \leq 1 - \mu$, where λ is a fuzzy open F_σ-set and μ is a fuzzy F_σ-set in a fuzzy F-space (X, T), then $cl(\mu) \neq 1$.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where λ is a fuzzy open F_σ-set and μ is a fuzzy F_σ-et in a fuzzy F-space (X, T). Then by Proposition 3.6, there exists a fuzzy open set δ in X such that $\lambda \leq \delta \leq 1 - cl(\mu)$. Thus $int[1 - cl(\mu)] \neq 0$.. So $1 - cl(cl(\mu)) \neq 0$. Since $cl(cl(\mu)) = cl(\mu)$, $1 - cl(\mu) \neq 0$. Hence $cl(\mu) \neq 1$. □

Proposition 3.8. *If $\lambda + \mu \leq 1$, where λ and μ are fuzzy F_σ-sets in a fuzzy F- space (X, T), then there exists fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha + \beta \leq 1$.*

Proof. Suppose that $\lambda + \mu \leq 1$, where λ and μ are fuzzy F_σ-sets in a fuzzy F-space (X, T). Then $\lambda \leq 1 - \mu$. Since X is a fuzzy F-space, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$ and $\mu \leq \beta$ such that $\alpha \leq 1 - \beta$. Thus $\lambda \leq \alpha$ and $\mu \leq \beta$ and $\alpha + \beta \leq 1$. □

Proposition 3.9. *If $\lambda \leq 1 - \mu$, where λ and μ are fuzzy σ-nowhere dense sets in a fuzzy F-space (X, T), then there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$ and $\mu \leq \beta$ and $\alpha \leq 1 - \beta$.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where λ and μ are fuzzy σ-nowhere dense sets in a fuzzy F-space (X, T). Then λ and μ are fuzzy F_σ-sets with $int(\lambda) = 0$ and $int(\mu) = 0$. Since X is a fuzzy F space, for the fuzzy F_σ-sets λ and μ with $\lambda \leq 1 - \mu$, there exists fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1 - \beta$. □

Proposition 3.10. *If (X, T) is a fuzzy F-space, in which fuzzy open sets are fuzzy F_σ-sets, then for any fuzzy set λ defined on X, then there exists fuzzy G_δ-sets α and β in (X, T) such that $int(\lambda) \leq \beta \leq 1 - \alpha \leq cl(\lambda)$.*

Proof. Let λ be a fuzzy set in X. Then clearly, $1 - cl(\lambda) \leq 1 - int(\lambda)$. By hypothesis, the fuzzy open sets $1 - cl(\lambda)$ and $int(\lambda)$ are fuzzy F_σ-sets in X. Since X is a fuzzy F-space, there exist fuzzy G_δ-sets α and β in X such that $1 - cl(\lambda) \leq \alpha$, $int(\lambda) \leq \beta$ and $\alpha \leq 1 - \beta$. Now $1 - \alpha \leq cl(\lambda)$ and $\beta \leq 1 - \alpha$ implies that $int(\lambda) \leq \beta \leq 1 - \alpha \leq cl(\lambda)$. □

Proposition 3.11. *If $\lambda \leq \mu$, where λ is a fuzzy F_σ-set and μ is a fuzzy G_δ-set in a fuzzy F-space (X, T), then there exists fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha \leq 1 - \beta \leq \mu$.*

Proof. Suppose that $\lambda \leq \mu$, where λ is a fuzzy F_σ-set and μ is a fuzzy G_δ-set in X. Then $\lambda \leq 1 - (1 - \mu)$ in X. Since μ is a fuzzy G_δ-set, $1 - \mu$ is a fuzzy F_σ-set in X. Since X is a fuzzy F-space, by Proposition 3.4, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha \leq 1 - \beta \leq 1 - (1 - \mu)$. Thus $\lambda \leq \alpha \leq 1 - \beta \leq \mu$. □

Proposition 3.12. *If $\lambda \leq \mu$, where λ is a fuzzy F_σ-set and μ is a fuzzy G_δ-set in a fuzzy F-space (X, T), then*
(1) *$\bigwedge \alpha_i \leq \mu$, where each α_i is a fuzzy G_δ-set in X,*
(2) *$\lambda \leq \bigvee \delta_i$, where each δ_i is a fuzzy F_σ-set in X.*

Proof. Suppose that $\lambda \leq \mu$, where λ is a fuzzy F_σ-set and μ is a fuzzy G_δ-set in X. Then by Proposition 3.11, there exist fuzzy G_δ-sets α_1 and β_1 in X such that $\lambda \leq \alpha_1 \leq 1 - \beta_1 \leq \mu$, where λ is a fuzzy F_σ-set and α_1 is a fuzzy G_δ-set in X. By Proposition 3.11, there exist fuzzy G_δ-sets α_2 and β_2 in X such that $\lambda \leq \alpha_2 \leq 1 - \beta_2 \leq \alpha_1 \leq \mu$. Thus $\lambda \leq \alpha_2 \leq \alpha_1 \leq \mu$ and $1 - \beta_2 \leq \alpha_1 \leq 1 - \beta_1 \leq \mu$. This implies that $1 - \beta_2 \leq 1 - \beta_1 \leq \mu$. Now $\lambda \leq \alpha_2$, where λ is a fuzzy F_σ-set and α_2 is a fuzzy G_δ-set in X. By Proposition 3.11, there exist fuzzy G_δ-sets α_3 and β_3 in X such that $\lambda \leq \alpha_3 \leq 1 - \beta_3 \leq \alpha_2$. So $\lambda \leq \alpha_3 \leq \alpha_2 \leq \alpha_1$ and β_3 in X such that $\lambda \leq \alpha_3 \leq \alpha_2 \leq \alpha_1$ and $1 - \beta_3 \leq \alpha_2$. This implies that $1 - \beta_3 \leq 1 - \beta_2 \leq 1 - \beta_1 \leq \mu$. Proceeding on these lines, we can find fuzzy G_δ-sets $(\alpha_i)'s$ and $(\beta_j)'s$ in X such that $\lambda \leq, \cdots, \leq \alpha_3 \leq \alpha_2 \leq \alpha_1$ and $\lambda \leq, \cdots, \leq 1 - \beta_2 \leq 1 - \beta_1 \leq \mu$. Hence (1) and (2) hold. □

Proposition 3.13. *If δ and η are fuzzy G_δ-sets such that $1-\eta \leq \delta$ in a fuzzy F-space (X,T), then there exists fuzzy G_δ-sets α and β in X such that $1-\eta \leq \beta \leq 1-\alpha \leq \delta$.*

Proof. Suppose that $1-\eta \leq \delta$, where δ and η are fuzzy G_δ-sets in X. Then $1-\delta \leq 1-(1-\eta)$, where $1-\delta$ and $1-\eta$ are fuzzy F_σ-sets in X. Since X is a fuzzy F-space, there exist fuzzy G_δ-sets α and β in X such that $1-\delta \leq \alpha$, $1-\eta \leq \beta$ and $\alpha \leq 1-\beta$. This implies that $1-\delta \leq \alpha \leq 1-\beta \leq \eta$. Thus $1-(1-\delta) \geq 1-\alpha \geq 1-(1-\beta) \geq 1-\eta$. So $1-\eta \leq \beta \leq 1-\alpha \leq \delta$. □

Proposition 3.14. *If λ and μ are fuzzy residual sets such that $\lambda \leq 1-\mu$ in a fuzzy F-space (X,T), then there exist fuzzy G_δ-sets α and β in X such that $\alpha \leq \lambda \leq 1-mu \leq 1-\beta$.*

Proof. Let λ and μ be fuzzy residual sets in X. Then by the Theorem 2.7, there exist fuzzy G_δ-sets α and β in X such that $\alpha \leq \lambda$ and $\beta \leq \mu$. Now $\beta \leq \mu$ implies that $1-\mu \leq 1-\beta$. Since $\lambda \leq 1-\mu$. Thus $\alpha \leq \lambda \leq 1-\mu \leq 1-\beta$, and $\alpha \leq 1-\beta$, where $1-\beta$ is a fuzzy F_σ-set and α is a fuzzy G_δ-set in X. □

4. Fuzzy F- spaces and other fuzzy topological spaces

Proposition 4.1. *If $\lambda \leq 1-\mu$, where λ is a fuzzy open F_σ-set and μ is a fuzzy F_σ-set in a fuzzy Baire and fuzzy F-space (X,T), then*

(1) *there exists a fuzzy Baire dense set δ in X) such that $\lambda \leq \delta \leq 1-cl(\mu)$,*
(2) *$1-cl(\mu)$ is also fuzzy Baire dense set in X,*
(3) *there is no fuzzy σ-nowhere dense set η in X such that $1-cl(\mu) < \eta$,*
(4) *there exist fuzzy second category sets δ_1 and δ_2 in X such that $\delta_1 < 1-cl(\mu) < \delta_2$,*
(5) *there is no fuzzy residual set η in X such that $1-\delta > \eta$.*

Proof. (1) Suppose that $\lambda \leq 1-\mu$, where λ is a fuzzy open F_σ-set and μ is a fuzzy F_σ-set in X. Since X is a fuzzy F-space, by Proposition 3.6, there exists fuzzy open set δ in X such that $\lambda \leq \delta \leq 1-cl(\mu)$. Since X is a fuzzy Baire space, by Theorem 2.8, δ is a fuzzy Baire dense set in X). Then there exists a fuzzy Baire dense set δ in X such that $\lambda \leq \delta \leq 1-cl(\mu)$.

(2) Since X is a fuzzy Baire space, by Theorem 2.8, $1-cl(\mu)$ is a fuzzy Baire dense set in X.

(3) By (2), $1-cl(\mu)$ is a fuzzy Baire dense set in X. Then by Theorem 2.10, there is no fuzzy F_σ-set η in X with $int(\eta)=0$ such that $1-cl(\mu) < \eta$. Since a fuzzy F_σ-set η with $int(\eta)=0$ is a fuzzy σ-nowhere dense set in X, there is no fuzzy σ nowhere dense set η in X such that $1-cl(\mu) < \eta$.

(4) By (2), $1-cl(\mu)$ is a fuzzy Baire dense set in X. Then by Theorem 2.11, there exists fuzzy second category sets δ_1 and δ_2 in X such that $\delta_1 < 1-cl(\mu) < \delta_2$.

(5) By (1), there exists a fuzzy Baire dense set δ in X such that $\lambda \leq \delta \leq 1-cl(\mu)$. Then by Theorem 2.12, for the fuzzy Baire dense set δ in X, there is no fuzzy residual set η in X such that $1-\delta > \eta$. □

Proposition 4.2. *If $\lambda \leq 1-\mu$, where λ and μ are fuzzy first category sets in a fuzzy globally disconnected space and fuzzy F-space (X,T), then there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha \leq 1-\beta \leq 1-\mu$.*

Proof. Let λ and μ be fuzzy first category sets in X such that $\lambda \leq 1-\mu$. Since X is a fuzzy globally disconnected space, by Theorem 2.9, λ and μ are fuzzy F_σ-sets in X. Since X is the fuzzy F-space and $\lambda \leq 1-\mu$ in (X,T), by Proposition 3.4, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha \leq 1-\beta \leq 1-\mu$. □

The following proposition gives a condition for fuzzy globally disconnected spaces to become fuzzy F-spaces.

Proposition 4.3. *If there exists fuzzy residual sets α and β in a fuzzy globally disconnected space (X,T) such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1-\beta$, for the fuzzy F_σ-sets λ and μ, then X is a fuzzy F-space.*

Proof. Suppose that $\lambda \leq 1-\mu$, where λ and μ are fuzzy F_σ-sets in X. By hypothesis, there exists fuzzy residual sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1-\beta$. Now $\mu \leq \beta$ implies that $1-\beta \leq 1-\mu$. Then $\lambda \leq \alpha \leq 1-\beta \leq 1-\mu$. Since X is a fuzzy globally disconnected space, by Theorem 2.9, the fuzzy first category sets $1-\alpha$ and $1-\beta$ are fuzzy F_σ-sets in X. Thus the fuzzy residual sets α and β are fuzzy G_δ-sets in X. So for the fuzzy F_σ-sets λ and μ such that $\lambda \leq 1-\mu$, there exists fuzzy G_δ-sets α and β such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1-\beta$ in (X,T). Hence X is a fuzzy F-space. □

The following proposition gives a condition for fuzzy submaximal spaces to become fuzzy F-spaces.

Proposition 4.4. *If there exists fuzzy residual sets α and β in a fuzzy submaximal space (X,T) such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1-\beta$ for the fuzzy F_σ-sets λ and μ in X, then (X,T) is a fuzzy F-space.*

Proof. Suppose that $\lambda \leq 1-\mu$, where λ and μ are fuzzy F_σ-sets in X. By hypothesis, there exist fuzzy residual sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1-\beta$. Then $1-\beta \leq 1-\mu$. Thus $\lambda \leq \alpha \leq 1-\beta \leq 1-\mu$. Since X is a fuzzy submaximal space, by Theorem 2.13, the fuzzy residual sets α and β are fuzzy G_δ-sets in X. So for the fuzzy F_σ-sets λ and μ in X such that $\lambda \leq 1-mu$, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1-\beta$ in X. Hence X is a fuzzy F-space. □

Proposition 4.5. *If $\lambda \leq 1-\mu$, where λ is a fuzzy open F_σ-set and μ is a fuzzy F_σ-set in a fuzzy Baire and fuzzy F-space (X,T), then there exists fuzzy second category set δ in X such that $\lambda \leq \delta \leq 1-cl(\mu)$.*

Proof. Suppose that $\lambda \leq 1-\mu$, where λ is a fuzzy open F_σ-set and μ is a fuzzy F_σ-set in X. Then, by Proposition 3.6, there exists a fuzzy open set δ in X such that $\lambda \leq \delta \leq 1-cl(\mu)$. Since X is a fuzzy Baire space, by Theorem 2.6, δ is a fuzzy second category set in X. Thus there exists a fuzzy second category set δ in X such that $\lambda \leq \delta \leq 1-cl(\mu)$. □

Proposition 4.6. *If (X,T) is a fuzzy F-space, then X is not a fuzzy hyperconnected space.*

Proof. Suppose that $\lambda \leq 1-\mu$, where λ is a fuzzy open F_σ-set and μ is a fuzzy F_σ-set in X. Since (X,T) is a fuzzy F-space, by Proposition 3.6, there exists a fuzzy open set δ in X such that $\lambda \leq \delta \leq 1 - cl(\mu)$. Since $int(\mu) < cl(\mu)$, $1 - cl(\mu) < 1 - int(\mu)$ and $1 - int(\mu)$ is a fuzzy closed set in X. Then $\lambda \leq \delta \leq 1 - int(\mu)$ implies that $cl(\delta) \neq 1$. Thus δ is not a fuzzy dense set in X. So X is not a fuzzy hyperconnected space. □

Proposition 4.7. *If $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in a fuzzy F-space and fuzzy P-space (X, T), then $cl(\lambda) \neq 1$ and $cl(\mu) \neq 1$.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where the fuzzy sets λ and μ are fuzzy F_σ-sets in X. Since X is a fuzzy F-space, by Corollary 3.5, we have

(i) there exists fuzzy G_δ-set α in X such that $\lambda \leq \alpha \leq 1 - \mu$,
(ii) there exists fuzzy F_σ-set δ in X such that $\lambda \leq \delta \leq 1 - \mu$.

From (ii), $\lambda \leq \delta \leq 1 - \mu$, implies that $cl(\lambda) \leq cl(\delta) \leq cl(1 - \mu)$. Since X is a fuzzy P-space, the fuzzy F_σ-set δ is a fuzzy closed set in X. Then $cl(\lambda) \leq \delta \leq cl(1 - \mu)$. Thus $cl(\lambda) \neq 1$.

Also $\lambda \leq \alpha \leq 1 - \mu$ implies that $int(\lambda) \leq int(\alpha) \leq int(1 - \mu)$. Then $int(\lambda) \leq int(\alpha) \leq 1 - cl(\mu)$ in X. Since (X, T) is a fuzzy P-space, the fuzzy G_δ-set α is a fuzzy open set in X. Thus $int(\lambda) \leq \alpha \leq 1 - cl(\mu)$. So $int[1 - cl(\mu)] \neq 0$. This implies $1 - lcl(\mu) \neq 0$. Hence $1 - cl(\mu) \neq 0$. Therefore $cl(\mu) \neq 1$. □

The following proposition gives a condition for fuzzy F-spaces to become fuzzy normal spaces.

Proposition 4.8. *If (X, T) is a fuzzy F-space in which fuzzy G_δ-sets in X are fuzzy open in X, then X is a fuzzy normal space.*

Proof. Let γ and δ be disjoint fuzzy closed sets in X. Then $\gamma \wedge \delta = 0$. This implies that $\gamma \leq 1 - \delta$. Now consider the fuzzy sets $\lambda = \gamma \vee [\bigvee_{i=0}^{\infty} \theta_i]$ and $\mu = \delta \vee [\bigvee_{i=0}^{\infty} \theta_i]$, where each θ_i is a fuzzy closed set in X. Then λ and μ are fuzzy F_σ-sets in X. Now $\gamma \leq \lambda$ and $\delta \leq \mu$ implies that $\gamma \leq \lambda$ and $1 - \mu \leq 1 - \delta$. Thus $\gamma \leq 1 - \delta$ implies that $\lambda \leq 1 - \mu$. Since X is a fuzzy F-space, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1 - \beta$, in X. By hypothesis, α and β are fuzzy open sets in X. Thus for the fuzzy closed sets γ and δ such that $\gamma \leq 1 - \delta$, there exists fuzzy open sets α and β in X such that $\gamma \leq \alpha$, $\delta \leq \beta$ and $\alpha \leq 1 - \beta$. So X is a fuzzy normal space. □

Proposition 4.9. *If (X, T) is a fuzzy F-space and fuzzy P-space, then (X, T) is a fuzzy normal space.*

Proof. The proof follows from Proposition 4.8 and from the definition of a fuzzy P-space. □

The following proposition gives a condition for fuzzy F-spaces to become fuzzy F $'$-spaces.

Proposition 4.10. *If (X, T) is a fuzzy F-space in which fuzzy G_δ-sets are fuzzy closed sets in X, then X is a fuzzy F'-space.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in X. Since X is a fuzzy F-space, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1 - \beta$. Now $\lambda \leq \alpha$ implies that $cl(\lambda) \leq cl(\alpha)$. Also $\mu \leq \beta$ implies that

$cl(\mu) \leq cl(\beta)$. Then $1 - cl(\beta) \leq 1 - cl(\mu)$. By hypothesis, α and β are fuzzy closed sets in X. Thus $cl(\lambda) \leq cl(\alpha) = \alpha \leq 1 - \beta = 1 - cl(\beta) \leq 1 - cl(\mu)$. So $cl(\lambda) \leq 1 - cl(\mu)$. Hence (X, T) is a fuzzy F′-space. □

The following proposition gives a condition for fuzzy F'-spaces to become fuzzy F-spaces.

Proposition 4.11. *If (X, T) is a fuzzy F'-space in which the closure of fuzzy F_σ-sets are fuzzy G_δ-sets, then X is a fuzzy F-space.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in X. Since X is a fuzzy F′-space, $cl(\lambda) \leq 1 - cl(\mu)$. Now $\lambda \leq cl(\lambda)$ and $\mu \leq cl(\mu)$. Let $\alpha = cl(\lambda)$ and $\beta = cl(\mu)$. Then by hypothesis, α and β are fuzzy G_δ-sets in X. Thus for the fuzzy F_σ-sets λ and μ in X with $\lambda \leq 1 - \mu$, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1 - \beta$. So X is a fuzzy F-space. □

Proposition 4.12. *If (X, T) is a fuzzy F'-space and fuzzy Oz-space in which fuzzy F_σ-sets are fuzzy G_δ-sets, then X is a fuzzy F-space.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in X. Since X is a fuzzy F′-space, $cl(\lambda) \leq 1 - cl(\mu)$. Then $cl(\lambda) \neq 1$ and $cl(\mu) \neq 1$ [For, if $cl(\lambda) = 1$, then $1 \leq 1 - cl(\mu)$. Thus $cl(\mu) = 0$. So $\mu = 0$. This is a contradiction. Also if $cl(\mu) = 1$. then $cl(\lambda) = 0$ and $\lambda = 0$. This is a contradiction]. Now $\lambda \leq cl(\lambda)$ and $\mu \leq cl(\mu)$. Let $\alpha = cl(\lambda)$ and $\beta = cl(\mu)$. Then by hypothesis, λ and μ are fuzzy open sets in X. Since X is a fuzzy Oz-space, by Theorem 2.14, for the fuzzy open sets λ and μ, $cl(\lambda)$ and $cl(\mu)$ are fuzzy G_δ-sets in X. Thus α and β are fuzzy G_δ-sets in X. So for the fuzzy F_σ-sets λ and μ in (X, T) with $\lambda \leq 1 - \mu$, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1 - \beta$. Hence X is a fuzzy F-space. □

The following proposition gives a condition for fuzzy perfectly disconnected spaces to become fuzzy F- spaces.

Proposition 4.13. *If (X, T) is a fuzzy perfectly disconnected space in which the closure of fuzzy F_σ-sets are fuzzy G_δ-sets, then X is a fuzzy F-space.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in X. Since X is a fuzzy perfectly disconnected space, for the fuzzy sets λ and μ with $\lambda \leq 1 - \mu$, $cl(\lambda) \leq 1 - cl(\mu)$. Now $\lambda \leq cl(\lambda)$ and $\mu \leq cl(\mu)$. Let $\alpha = cl(\lambda)$ and $\beta = cl(\mu)$. Then $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1 - \beta$. Thus by hypothesis, α and β are fuzzy G_δ-sets in X. So for the fuzzy F_σ-sets λ and μ in X with $\lambda \leq 1 - \mu$, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1 - \beta$. Hence X is a fuzzy F-space. □

Proposition 4.14. *Let (X, T) be a fuzzy maximal space. If there exist fuzzy Baire sets α, β in X and fuzzy F_σ-sets λ, μ in X such that $\lambda \leq \alpha$, $\mu \leq \beta$, $\alpha \leq 1 - \beta$ and $\lambda \leq 1 - \mu$, then X is a fuzzy F-space.*

Proof. Suppose that α and β are fuzzy Baire sets in X such that $\lambda \leq \alpha$, $\mu \leq \beta$ and $\alpha \leq 1 - \beta$ for the fuzzy F_σ-sets λ and μ in X with $\lambda \leq 1 - \mu$. Since X is a fuzzy maximal space, by Theorem 2.16, α and β are fuzzy G_δ-sets in X. Then for the fuzzy F_σ-sets λ and μ in X with $\lambda \leq 1 - \mu$, there exist fuzzy G_δ-sets α and β in X such that $\lambda \leq \alpha$ and $\mu \leq \beta$ and $\alpha \leq 1 - \beta$. Thus (X, T) is a fuzzy F-space. □

Proposition 4.15. *If (X, T) is a fuzzy basically disconnected space in which fuzzy open sets are fuzzy F_σ-sets, then (X, T) is a fuzzy F-space.*

Proof. Suppose that δ be a fuzzy set in X. Now $int(\delta) \leq cl(\delta)$ and $int(\delta) \leq 1 - [1 - cl(\delta)]$. By hypothesis, the fuzzy open sets $int(\delta)$ and $1 - cl(\delta)$ are fuzzy F_σ-sets in X and $1 - [1 - cl(\delta)]$ is a fuzzy G_δ-set in X. Since X is a fuzzy basically disconnected space, by Theorem 2.15, $int(\delta) \leq 1 - [1 - cl(\delta)]$ implies that $clint(\delta) \leq int\{1 - [1 - cl(\delta)]\}$. Then $clint(\delta) \leq 1 - cl[1 - cl(\delta)]$. Now $int(\delta) \leq clint(\delta) \leq 1 - cl[1 - cl(\delta)] \leq 1 - [1 - cl(\delta)]$. Let $\lambda = int(\delta)$ and $\mu = 1 - cl(\delta)$. Then λ and μ are fuzzy F_σ-sets in X. Let $\theta = clint(\delta)$ and $\eta = cl[1 - cl(\delta)$. Then,θ and η are fuzzy G_δ-sets in X. Thus for fuzzy F_σ-sets λ and μ in X with $\lambda \leq 1 - \mu$, there exist fuzzy G_δ-sets θ and η in X such that $\lambda \leq \theta$, $\mu \leq \eta$ and $\theta \leq 1 - \eta$. So X is a fuzzy F-space. □

5. Conclusion

In this paper the concept of fuzzy F-space is introduced and several characterizations of fuzzy F-spaces are established. The conditions for fuzzy globally disconnected spaces, fuzzy perfectly disconnected spaces, fuzzy basically disconnected spaces and fuzzy submaximal spaces to become fuzzy F-spaces are obtained. It is established that fuzzy F-spaces are not fuzzy hyperconnected spaces. Also a condition under which fuzzy F′-spaces become fuzzy F-spaces is obtained. It is established that fuzzy F-spaces in which fuzzy G_δ-sets are fuzzy open, are fuzzy normal spaces and fuzzy F-spaces in which fuzzy G_δ-sets are fuzzy closed, are fuzzy F′-spaces. It is established that fuzzy F-spaces and fuzzy P-spaces, are fuzzy normal spaces.

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