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Abstract. In this paper, the notion of fuzzy regular Oz-space is introduced by means of fuzzy regular closed sets and fuzzy regular G_δ sets. It is established that fuzzy regular Oz-spaces are fuzzy Oz-spaces and fuzzy regular Oz and fuzzy P-spaces are fuzzy extremally disconnected spaces and fuzzy Moscow spaces. A condition under which fuzzy regular Oz-spaces become weak fuzzy Oz-spaces is also established. It is obtained that fuzzy regular Oz and weak fuzzy P-spaces become fuzzy pseudo P-spaces if fuzzy regular Oz and weak fuzzy P-spaces have non-zero fuzzy regular G_δ-sets.

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Keywords: Fuzzy regular G_δ-set, Fuzzy regular F_σ-set, Fuzzy Baire dense set, Fuzzy pseudo P-space, Fuzzy extremally disconnected space, Weak fuzzy P-space, Fuzzy Baire space.

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1. Introduction

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by Zadeh [1] in 1965. Fuzzy set theory provides us with a framework which is wider than that of classical set theory. Various mathematical structures, whose features emphasize the effects of ordered structure, can be developed on the theory and the potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. In 1968, Chang [2] introduced the concept of fuzzy topological spaces and his work paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Blair [3], Scepin [4] and Terada [5] independently introduced the concept of Oz-spaces, also known as perfectly κ-normal spaces which is analogous to that of normality in classical topology. Chigogidze [6] investigated weak Oz-spaces under the name of "almost Scepin spaces". The concept of Moscow spaces in classical topology was introduced by Arhangel'skii [7] and the notion of Moscow spaces generalizes the notion of perfectly κ-normal spaces. The notion of quasi-Oz-spaces was introduced by Kim [8] which generalizes the concept of Oz-spaces. In classical topology, Lane [9] investigated the topological spaces in which each regular closed subset is an intersection of a sequence of closed neighbourhoods, under the name of perfectly mildly normal spaces.

In recent years,there has been a growing trend among many fuzzy topologists to introduce and study various types of fuzzy topological spaces. The purpose of this paper is to introduce the concept of fuzzy regular Oz-spaces and study its properties and applications. Several characterizations of fuzzy regular Oz-spaces are established. It is obtained that fuzzy regular Oz-spaces are fuzzy Oz-spaces and fuzzy regular Oz and fuzzy P-spaces are fuzzy extremally disconnected spaces. A condition under which fuzzy regular Oz-spaces become weak fuzzy Oz-spaces is obtained. It is established that fuzzy regular Oz and weak fuzzy P-spaces are fuzzy Moscow spaces. A condition under which fuzzy semi-open sets become fuzzy pre-open sets in fuzzy regular Oz spaces is also obtained. A condition for fuzzy regular Oz and weak fuzzy P-spaces to become fuzzy Baire spaces is also obtained in this paper. It is obtained that fuzzy regular Oz and weak fuzzy P-spaces become fuzzy pseudo P-spaces if fuzzy regular Oz and weak fuzzy P-spaces have non-zero fuzzy regular G_δ-sets.

In recent years, the topological space theory has been embedding in the soft set theory to obtain some interesting applications [10, 11, 12, 13]. Many authors redefined the classical topological concepts via soft topological structure. Recently, Şenel et al. [14] applied the concept of octahedron sets proposed by Lee et al. [15] to multi-criteria group decision making problems. Al-Shami [16] successfully applied some theoretical soft topological concepts to information Science and decision-making problems. On these lines, there is a need and scope of investigation considering different types of fuzzy spaces such as fuzzy F-spaces, fuzzy Oz-spaces, for applying some fuzzy topological concepts to information science and decision-making problems.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang(1968). Let X be a non-empty set and I, the unit interval $[0, 1]$. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 ([2])**.** A *fuzzy topology* is a family T of fuzzy sets in X which satisfies the following conditions:

(i) $0_X \in T$ and $1_X \in T$,
(ii) if $A, B \in T$, then $A \wedge B \in T$,
(iii) if $A_i \in T$ for each $i \in J$, then $\bigvee_{i \in J} A_i \in T$.

The pair (X, T) is called a *fuzzy topological space* or fts for short. Every member of

T is called a *T-open fuzzy set*. The complement of a T-open fuzzy set is called a *T-closed fuzzy set*.

Definition 2.2 ([2])**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in X. The *interior*, the *closure* and the *complement* of λ are defined respectively as follows:
(i) $int(\lambda) = \bigvee\{\mu|\mu \leq \lambda,\ \mu \in T\}$,
(ii) $cl(\lambda) = \bigwedge\{\mu|\lambda \leq \mu,\ 1-\mu \in T\}$,
(iii) $\lambda'(x) = 1 - \lambda(x)$ for all $x \in X$.
For a family $\{\lambda_i|i \in J\}$ of fuzzy sets in X, the *union* $\psi = \bigvee_{i\in J} \lambda_i$ and the *intersection* $\delta = \bigwedge_{i\in J} \lambda_i$, are defined respectively as follows: for each $x \in X$,
(iv) $\psi(x) = sup_{i\in J}\lambda_i(x)$,
(v) $\delta(x) = inf_{i\in J}\lambda_i(x)$.

Lemma 2.3 ([17])**.** *For a fuzzy set λ of a fuzzy topological space X,*
(1) $1 - int(\lambda) = cl(1 - \lambda)$,
(2) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.4. A fuzzy set λ in a fuzzy topological space (X, T) is called a:
(i) *fuzzy regular-open set* in X if $\lambda = intcl(\lambda)$ and *fuzzy regular-closed set* in X if $\lambda = clint(\lambda)$ [17],
(ii) *fuzzy G_δ-set* in X if $\lambda = \bigwedge_{i=1}^{\infty} \lambda_i$, where $\lambda_i \in T$ and *fuzzy F_σ-set* in X if $\lambda = \bigvee_{i=1}^{\infty} \mu_i$, where $1 - \mu_i \in T$ [18],
(iii) *fuzzy semi-open set* in X if $\lambda \leq clint(\lambda)$ and *fuzzy semi-closed set* in (X, T) if $intcl(\lambda) \leq \lambda$ [17],
(iv) *fuzzy pre-open set* in X if $\lambda \leq intcl(\lambda)$ and *fuzzy pre-closed set* in X if $clint(\lambda) \leq \lambda$ [19],
(v) *fuzzy regular G_δ-set* in X if $\lambda = \bigwedge_{i=1}^{\infty} int(\lambda_i)$, where $1 - \lambda_i \in T$ and *fuzzy regular F_σ-set* in X if $\lambda = \bigvee_{i=1}^{\infty} cl(\mu_i)$, where $\mu_i \in T$ [20].

Definition 2.5. A fuzzy set λ in a fuzzy topological space (X, T) is called a:
(i) *fuzzy dense set* in X if there exists no fuzzy closed set μ in X such that $\lambda < \mu < 1$, i.e., $cl(\lambda) = 1$ [21],
(ii) *fuzzy nowhere dense set* in X if there exists no non-zero fuzzy open set μ in X such that $\mu < cl(\lambda)$, i.e., $intcl(\lambda) = 0$ [21],
(iii) *fuzzy somewhere dense set* in X if there exists a non-zero fuzzy open set μ in X such that $\mu < cl(\lambda)$, i.e., $intcl(\lambda) \neq 0$ [22],
(iv) *fuzzy first category set* in X if $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$, where λ_i's are fuzzy nowhere dense sets in X. Any other fuzzy set in (X, T) is said to be of *fuzzy second category* in X [21],
(v) *fuzzy Baire dense set* in X if for a non-zero fuzzy open set μ in X, $\lambda \wedge \mu$ is a fuzzy second category set in X [23].

Definition 2.6. A fuzzy topological space (X, T) is called a:
(i) *fuzzy P-space* if each fuzzy G_δ-set in X is fuzzy open in X [24],
(ii) *fuzzy almost P-space* if for every non-zero fuzzy G_δ-set λ in X, $int(\lambda) \neq 0$ [25],
(iii) *weak fuzzy P-space* if $\bigwedge_{i=1}^{\infty} \lambda_i$ is a fuzzy regular open set in X, where $\lambda_i's$ are fuzzy regular open sets in X [26],

(iv) *fuzzy hyperconnected space* if every non-null fuzzy open set in X is fuzzy dense in X [27],

(v) *fuzzy extremally disconnected space* if the closure of every fuzzy open set in X is fuzzy open in X [28],

(vi) *fuzzy Oz-space* if each fuzzy regular closed set in X is a fuzzy G_δ-set in X [29],

(vii) *weak fuzzy Oz-space* if for each fuzzy F_σ-set δ in X, $cl(\delta)$ is a fuzzy G_δ-set in X [29],

(viii) *fuzzy quasi-Oz-space* if for a fuzzy regular closed set λ in X, there exists a fuzzy G_δ-set μ in X such that $\lambda = clint(\mu)$ [30],

(ix) *fuzzy Moscow space* if for each fuzzy open set λ in X, $cl(\lambda) = \bigvee_{i=1}^{\infty} \delta_i$, where δ_i's are fuzzy G_δ-sets in X [31],

(x) *fuzzy pseudo P-space* if (X, T) is a fuzzy almost P-space and weak fuzzy P-space [32],

(xi) *fuzzy Baire space* if $int(\bigvee_{i=1}^{\infty} \lambda_i) = 0$, where $\lambda_i^{\prime}s$ are fuzzy nowhere dense sets in X[33].

Theorem 2.7 ([17]). *In a fuzzy topological space,*

(1) *the closure of a fuzzy open set is a fuzzy regular closed set,*

(2) *the interior of a fuzzy closed set is a fuzzy regular open set.*

Theorem 2.8 ([20]). *A fuzzy set λ is a fuzzy regular G_δ-set in a fuzzy topological space (X, T) if and only if $1 - \lambda$ is a fuzzy regular F_σ-set in X.*

Theorem 2.9 ([20]). *If λ is a fuzzy regular G_δ-set in a fuzzy topological space (X, T), then λ is a fuzzy G_δ-set in X.*

Theorem 2.10 ([20]). *If λ is a fuzzy regular F_σ-set in a fuzzy topological space (X, T), then λ is a fuzzy F_σ-set in X.*

Theorem 2.11 ([22]). *If λ is a fuzzy somewhere dense set in a fuzzy topological space (X, T), then there exists a fuzzy regular closed set η in X such that $\eta \leq cl(\lambda)$.*

Theorem 2.12 ([30]). *If a fuzzy topological space (X, T) is a fuzzy Oz-space, then X is a fuzzy quasi-Oz-space.*

Theorem 2.13 ([26]). *A fuzzy topological space (X, T) is weak fuzzy P-space if and only if each fuzzy regular G_δ-set is a fuzzy regular open set in X.*

Theorem 2.14 ([31]). *If a fuzzy topological space (X, T) is a fuzzy extremally disconnected space, then X is a fuzzy Moscow space.*

Theorem 2.15 ([32]). *If μ is a fuzzy regular F_σ-set in a fuzzy pseudo P-space (X, T), then μ is a fuzzy Baire dense set in X.*

Theorem 2.16 ([32]). *If there exists a fuzzy regular F_σ-set μ in a fuzzy pseudo P-space (X, T) such that $\mu < \delta$, for a fuzzy open set δ in X, then X is a fuzzy Baire space.*

Theorem 2.17 ([23]). *If λ is fuzzy Baire dense set in a fuzzy topological space (X, T), then there exists a fuzzy second category set δ such that $\delta \leq \lambda$.*

Theorem 2.18 ([23]). *If λ is fuzzy Baire dense set in a fuzzy topological space (X, T). Then, $cl(\lambda)$ is a fuzzy second category set in X.*

Theorem 2.19 ([26]). *If λ is a fuzzy regular G_δ-set in a weak fuzzy P-space (X, T), then λ is not a fuzzy dense set in X.*

Theorem 2.20 ([32]). *If $(\lambda_i)'s$ are fuzzy semi-open sets in the fuzzy pseudo P-space (X, T), then $\bigvee_{i=1}^{\infty} cl(\lambda_i)$ is an fuzzy regular closed set in X.*

3. Fuzzy regular Oz-spaces

Motivated by the works of Lane[9] on perfectly mildly normal spaces, the notion of regular Oz-spaces in fuzzy setting is defined as follows.

Definition 3.1. A fuzzy topological space (X, T) is called a *fuzzy regular Oz-space* if each fuzzy regular closed set λ in (X, T) is a fuzzy regular G_δ-set in X.

That is, (X, T) is a fuzzy regular Oz-space if for a fuzzy regular closed set λ in X, λ is a countable intersection of interior of fuzzy open sets and $\lambda = \bigwedge_{i=1}^{\infty} \mathrm{int}(\lambda_i)$, where $1 - \lambda_i \in T$.

Remark 3.2. It should be noted that fuzzy regular G_δ-sets in fuzzy topological spaces are fuzzy G_δ-sets [20] and fuzzy G_δ-sets need not be fuzzy open sets whereas fuzzy G_δ-sets in P-spaces alone are fuzzy open sets. Hence fuzzy regular closed sets in fuzzy regular Oz-spaces need not be fuzzy open.

Example 3.3. Let $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets α, β and γ are defined on X as follows:

$\alpha : X \to I$ is defined by $\alpha(a) = 0.4; \alpha(b) = 0.6; \alpha(c) = 0.4$,

$\beta : X \to I$ is defined by $\beta(a) = 0.5; \beta(b) = 0.4; \beta(c) = 0.5$,

$\gamma : X \to I$ is defined by $\gamma(a) = 0.6; \gamma(b) = 0.5; \gamma(c) = 0.7$.

Then $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \vee [\alpha \wedge \gamma], 1\}$ is a fuzzy topology on X. By computation, one can find that $cl(\alpha) = 1 - \beta$; $cl(\beta) = 1 - (\alpha \vee \beta)$; $cl(\gamma) = 1$; $cl(\alpha \vee \beta) = 1 - \beta$; $cl(\alpha \vee \gamma) = 1$; $cl(\alpha \wedge \beta) = 1 - (\alpha \vee \beta)$; $cl(\alpha \wedge \gamma) = 1 - (\beta \vee [\alpha \wedge \gamma])$ and $cl(\beta \vee [\alpha \wedge \gamma]) = 1 - (\beta \vee [\alpha \wedge \gamma])$. Also one can find that $int(1 - \alpha) = \beta$; $int(1 - \beta) = \alpha \vee \beta$; $int(1 - \gamma) = 0$; $int(1 - [\alpha \vee \beta]) = \beta$; $int(1 - [\alpha \vee \gamma]) = 0$; $int(1 - [\alpha \wedge \beta]) = \alpha \vee \beta$; $int(1 - [\alpha \wedge \gamma]) = \beta \vee [\alpha \wedge \gamma]$ and $int(1 - [\beta \vee [\alpha \wedge \gamma]) = \beta \vee [\alpha \wedge \gamma]$. The fuzzy regular closed sets in (X, T) are $1 - \beta$, $1 - (\alpha \vee \beta)$ and $1 - (\beta \vee [\alpha \wedge \gamma])$ and the fuzzy regular G_δ-sets in (X, T) are $\beta, \alpha \vee \beta$ and $\beta \vee [\alpha \wedge \gamma]$, where

$$\beta = [int(1 - \alpha)] \wedge [int(1 - \beta)] \wedge int[1 - (\beta \vee [\alpha \wedge \gamma])],$$

$$\alpha \vee \beta = [int(1 - \beta)] \wedge [int[1 - (\alpha \wedge \beta)]],$$

$$\beta \vee [\alpha \wedge \gamma] = [int(1 - \beta)] \wedge [int[1 - (\alpha \wedge \beta)]] \wedge [int[1 - (\beta \vee [\alpha \wedge \gamma])]].$$

Thus the fuzzy regular closed sets $1 - \beta$, $1 - (\alpha \vee \beta)$ and $1 - (\beta \vee [\alpha \wedge \gamma])$ are the fuzzy regular G_δ-sets $\alpha \vee \beta$, β and $\beta \vee [\alpha \wedge \gamma]$ respectively in X. So (X, T) is a fuzzy regular Oz-space.

Proposition 3.4. *If λ is a fuzzy open set in a fuzzy regular Oz-space(X, T), then $cl(\lambda)$ is a fuzzy regular G_δ-set in X.*

Proof. Let λ be a fuzzy open set in (X, T). Then by Theorem 2.7, $cl(\lambda)$ is a fuzzy regular closed set in X. Since (X, T) is a fuzzy regular Oz-space, $cl(\lambda)$ is a fuzzy regular G_δ-set in X. □

Corollary 3.5. *If μ is a fuzzy closed set in a fuzzy regular Oz-space (X, T), then $int(\mu)$ is a fuzzy regular F_σ-set in X.*

Proof. Let μ be a fuzzy closed set in X. Then $1 - \mu$ is a fuzzy open set in X. By Proposition 3.4, $cl(1 - \mu)$ is a fuzzy regular G_δ-set in X. Thus by Lemma 2.3, $cl(1 - \mu) = 1 - int(\mu)$. So then $int(\mu)$ is a fuzzy regular F_σ-set in X. □

Corollary 3.6. *If μ is a fuzzy closed set in a fuzzy regular Oz-space(X, T), then $int(\mu)$ is a fuzzy F_σ set in X.*

Proof. The proof follows from Corollary 3.5 and Theorem 2.10. □

It should be noted that in a fuzzy topological space (X, T), the fuzzy set 0_X is not a fuzzy regular F_σ-set in X. [For, if 0_X is a fuzzy regular F_σ-set in X, then $0_X = \bigvee_{i=1}^{\infty} cl(\mu_i)$, where $\mu_i \in T$. This will imply that $cl(\mu_i) = 0$ and then $\mu_i = 0$, a contradiction to $\mu_i \in T$].

Proposition 3.7. *If λ is a fuzzy set in a fuzzy regular Oz-space (X, T), then $intcl(\lambda)(\neq 0)$ is a fuzzy regular F_σ-set in X.*

Proof. Let λ be a fuzzy set in X. Then $cl(\lambda)$ is a fuzzy closed set in X. Thus $1-cl(\lambda)$ is a fuzzy open set in X. Since X is a fuzzy regular Oz-space, by Proposition 3.4, $cl[1 - cl(\lambda)]$ is a fuzzy regular G_δ-set in X. So by Lemma 2.3, $cl[1 - cl(\lambda)] = 1 - intcl(\lambda)$. Hence $1 - intcl(\lambda)$ is a fuzzy regular G_δ-set in X. This implies that $intcl(\lambda)$ is a fuzzy regular F_σ-set in X. □

Corollary 3.8. *If λ is a fuzzy set in a fuzzy regular Oz-space (X, T), then $intcl(\lambda)$ is a fuzzy F_σ-set in X.*

Proof. The proof follows from Proposition 3.7 and Theoem 2.10. □

Proposition 3.9. *If λ is a fuzzy set in a fuzzy regular Oz-space (X, T), then $clint(\lambda)$ is a fuzzy regular G_δ-set in X.*

Proof. Let λ be a fuzzy set in X. Then $int(\lambda)$ is a fuzzy open set in X. Since X is a fuzzy regular Oz-space, by Proposition 3.4, $cl[int(\lambda)]$ is a fuzzy regular G_δ-set in X. □

Corollary 3.10. *If λ is a fuzzy set in a fuzzy regular Oz-space (X, T), then $clint(\lambda)$ is a fuzzy G_δ-set in X.*

Proof. The proof follows from Proposition 3.9 and Theorem 2.9. □

Proposition 3.11. *If λ is a fuzzy pre-open set in a fuzzy regular Oz-space(X, T), then there exists a fuzzy regular F_σ-set μ in X such that $\lambda \leq \mu$.*

Proof. Let λ be a fuzzy pre-open set in X. Then $\lambda \leq intcl(\lambda)$. Since X is a fuzzy regular Oz-space, for the fuzzy set λ, by Proposition 3.7, $intcl(\lambda)$ is a fuzzy regular F_σ-set in X. Let $\mu = intcl(\lambda)$. Then for the fuzzy pre-open set λ in X, there exists a fuzzy regular F_σ-set μ in X such that $\lambda \leq \mu$. □

Corollary 3.12. *If δ is a fuzzy pre-closed set in a fuzzy regular Oz-space (X, T), then there exists a fuzzy regular G_δ-set θ in X such that $\theta \leq \delta$.*

Proof. Let δ be a fuzzy pre-closed set in X. Then $1 - \delta$ is a fuzzy pre-open set in X. Since X is a fuzzy regular Oz-space, by Proposition 3.11, there exists a fuzzy regular F_σ-set μ in X such that $1-\delta \leq \mu$. This implies that $1-\mu \leq \delta$. Let $\theta = 1-\mu$. Then θ is a fuzzy regular G_δ-set in X. Thus for the fuzzy pre-closed set δ in X, there exists a fuzzy regular G_δ-set θ in X such that $\theta \leq \delta$. □

Proposition 3.13. *If λ is a fuzzy somewhere dense set in a fuzzy regular Oz -space (X, T), then there exists a fuzzy regular G_δ-set η in X such that $\eta \leq cl[\lambda]$.*

Proof. Let λ be a fuzzy somewhere dense set in X. Then by Theorem 2.11, there exists a fuzzy regular closed set η in X such that $\eta \leq cl[\lambda]$. Since X is a fuzzy regular Oz-space, the fuzzy regular closed set η is a fuzzy regular G_δ-set in X. Thus for the fuzzy somewhere dense set λ in X, there exists a fuzzy regular G_δ-set η in X such that $\eta \leq cl[\lambda]$. □

Proposition 3.14. *If λ is a fuzzy somewhere dense set in a fuzzy regular Oz-space (X, T), then there exists a fuzzy G_δ-set η in (X, T) such that $\eta \leq cl(\lambda)$.*

Proof. The proof follows from Proposition 3.13 and Theorem 2.9. □

Proposition 3.15. *If λ is a fuzzy open set in a fuzzy regular Oz-space (X, T), then there exists a fuzzy regular open set δ in X such that $\delta \leq cl[\lambda]$.*

Proof. Let λ be a fuzzy open set in X. Then by Theorem 2.7, $cl(\lambda)$ is a fuzzy regular closed set in X. Since X is a fuzzy regular Oz-space, the fuzzy regular closed set $cl(\lambda)$ is a fuzzy regular G_δ-set in X and thus $cl(\lambda) = \bigwedge_{i=1}^{\infty} int(\lambda_i)$, where $1-\lambda_i \in T$. Now $int[\bigwedge_{i=1}^{\infty}(\lambda_i)] \leq \bigwedge_{i=1}^{\infty} int(\lambda_i)$ implies that $int[\bigwedge_{i=1}^{\infty}(\lambda_i)] \leq cl(\lambda)$. Since $\lambda_i's$ are fuzzy closed sets in X, $\bigwedge_{i=1}^{\infty}(\lambda_i)$ is a fuzzy closed set in X and thus $int(\bigwedge_{i=1}^{\infty} \lambda_i)$ is a fuzzy regular open set in X. Let $\delta = int(\bigwedge_{i=1}^{\infty} \lambda_i)$. Then for the fuzzy open set λ in X, there exists a fuzzy regular open set δ in X such that $\delta \leq cl[\lambda]$. □

Proposition 3.16. *If μ is a fuzzy closed set in a fuzzy regular Oz-space (X, T), then there exists a fuzzy regular closed set η in (X, T) such that $int(\mu) \leq \eta$.*

Proof. Let μ be a fuzzy closed set in X. Then $1-\mu$ is a fuzzy open set in X. Since X is a fuzzy regular Oz-space, by Proposition 3.15, for the fuzzy open set $1-\mu$, there exists a fuzzy regular open set δ in X such that $\delta \leq cl(1-\mu)$, and thus $\delta \leq 1-int(\mu)$. This implies that $int(\mu) \leq 1-\delta$. Let $\eta = 1-\delta$. Then η is a fuzzy regular closed set in X. Thus for the fuzzy closed set μ in X, there exists a fuzzy regular closed set η in X such that $int(\mu) \leq \eta$. □

Proposition 3.17. *If δ is a fuzzy G_δ-set in a fuzzy regular Oz-space (X, T), then $clint(\delta)$ is a fuzzy G_δ-set in X.*

Proof. Let δ be a fuzzy G_δ-set in X. Since X is a fuzzy regular Oz-space, by Proposition 3.9, $clint(\delta)$ is a fuzzy regular G_δ-set in X. Then by Theorem 2.9, $clint(\delta)$ is a fuzzy G_δ-set in X. □

Remark 3.18. It should be noted that if λ is a fuzzy G_δ-set in a fuzzy almost P-space (X, T), $int(\lambda)$ is a fuzzy G_δ-set in X.

Corollary 3.19. *If η is a fuzzy F_σ-set in a fuzzy regular Oz-space (X,T), then $intcl(\eta)$ is a fuzzy F_σ-set in X.*

Proof. Let η be a fuzzy F_σ-set in X. Then $1-\eta$ is a fuzzy G_δ-set in X. Then by Proposition 3.17, $clint(1-\eta)$ is a fuzzy G_δ-set in X. This implies that $1 - intcl(\eta)$ is a fuzzy G_δ-set. Thus $intcl(\eta)$ is a fuzzy F_σ-set in X. □

Proposition 3.20. *If δ is a fuzzy G_δ-set in a fuzzy regular Oz-space (X,T), then $\delta \leq \wedge_{i=1}^{\infty}(\eta_i)$, where η_i's are fuzzy regular G_δ-sets in X.*

Proof. Let δ be a fuzzy G_δ-set in X. Then $\delta = \bigwedge_{i=1}^{\infty}(\delta_i)$, where δ_i's are fuzzy open sets in X and $\delta = \bigwedge_{i=1}^{\infty}\delta_i \leq \bigwedge_{i=1}^{\infty} cl(\delta_i)$. Thus by Proposition 3.4, $cl(\delta_i)$'s are fuzzy regular G_δ-sets in X. So $\delta \leq \bigwedge_{i=1}^{\infty}\eta_i$, where η_i's are fuzzy regular G_δ-sets in X. □

Proposition 3.21. *If λ is a fuzzy semi-open set in a fuzzy regular Oz -space (X,T), then $cl(\lambda)$ is a fuzzy regular G_δ-set in X.*

Proof. Let λ be a fuzzy semi-open set in X. Then $\lambda \leq clint(\lambda)$. Now $cl(\lambda) \leq cl[clint(\lambda)] = clint(\lambda) \leq clintcl(\lambda) \leq clcl(\lambda) = cl(\lambda)$. This implies that $cl(\lambda) \leq clintcl(\lambda) \leq cl(\lambda)$. Thus $clintcl(\lambda) = cl(\lambda)$. So $cl(\lambda)$ is a fuzzy regular closed set in X. Since (X,T) is a fuzzy regular Oz-space, the fuzzy regular closed set $cl(\lambda)$ is a fuzzy regular G_δ-set in X. □

4. Fuzzy regular Oz-spaces and other fuzzy topological spaces

In this section, a study between fuzzy regular Oz-spaces, and other fuzzy topological spaces such as fuzzy Oz-spaces, fuzzy quasi-Oz-spaces, fuzzy extremally disconnected spaces, fuzzy almost P-spaces, weak fuzzy Oz-spaces, fuzzy Moscow spaces is undertaken.

Proposition 4.1. *If a fuzzy topological space (X,T) is a fuzzy regular Oz-space, then X is a fuzzy Oz-space.*

Proof. Let λ be a fuzzy regular closed set in X. Since X is a fuzzy regular Oz-space, λ is a fuzzy regular G_δ-set in X. Then by Theorem 2.9, λ is a fuzzy $G_\delta -$set in X. Thus X is a fuzzy Oz-space. □

Proposition 4.2. *If a fuzzy topological space (X,T) is a fuzzy regular Oz-space, then (X,T) is a fuzzy quasi-Oz-space.*

Proof. Let λ be a fuzzy regular closed set in X. Since X is a fuzzy regular Oz-space, λ is a fuzzy regular G_δ-set in X. Then by Theorem 2.9, λ is a fuzzy G_δ-set in X. Since λ is a fuzzy regular closed set in X, $\lambda = clint(\lambda)$. Thus for the fuzzy regular closed set λ in X, $\lambda = clint(\lambda)$, where λ is a fuzzy G_δ-set in X. So X is a fuzzy quasi-Oz-space. □

In view of Theorem 2.12, Propositions 4.1 and 4.2, the interrelations between fuzzy regular Oz-spaces, fuzzy Oz-spaces and fuzzy quasi-Oz-spaces can be stated as below:

Fuzzy regular Oz-spaces $\Rightarrow$ Fuzzy Oz-spaces $\Rightarrow$ Fuzzy quasi-Oz-spaces.

Remark 4.3. The converses of the above need not be true. That is, fuzzy quasi-Oz-spaces need not be fuzzy Oz-spaces and fuzzy Oz- Spaces need not be fuzzy regular Oz-spaces. For, consider the following examples.

Example 4.4. Let $X = \{a, b, c\}$. Let $I = [0, 1]$. Consider the fuzzy sets α, β and γ in X defined as follows:
$\alpha : X \to I$ is defined by $\alpha(a) = 0.4;\ \alpha(b) = 0.6;\ \alpha(c) = 0.5$,
$\beta : X \to I$ is defined by $\beta(a) = 0.6;\ \beta(b) = 0.7;\ \beta(c) = 0.8$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.5;\ \gamma(b) = 0.4;\ \gamma(c) = 0.7$.
Then $T = \{0, \alpha,\ \beta,\ \gamma,\ \alpha \vee \gamma,\ \alpha \wedge \gamma,\ 1\}$ is clearly the fuzzy topology on X. By computation, one can find that $cl(\alpha) = 1 - (\alpha \wedge \gamma)$; $cl(\beta) = 1$; $cl(\gamma) = 1$; $cl(\alpha \vee \gamma) = 1$; $cl(\alpha \wedge \gamma) = 1 - \alpha$ and $int(1 - \alpha) = \alpha \wedge \gamma$; $int(1 - \beta) = 0$; $int(1 - \gamma) = 0$; $int(1 - [\alpha \vee \gamma]) = 0$; $int(1 - [\alpha \wedge \gamma]) = \alpha$. Thus the fuzzy regular closed sets in (X, T) are $1 - \alpha$ and $1 - [\alpha \wedge \gamma]$. Also the fuzzy G_δ-sets in (X, T) are α and $\alpha \wedge \gamma$, where $\alpha = \alpha \wedge \beta \wedge (\alpha \vee \gamma)$ and $\alpha \wedge \gamma = \alpha \wedge \beta \wedge \gamma$. By computation, one can find that $1 - [\alpha \wedge \gamma] = clint(\alpha)$ and $1 - \alpha = clint(\alpha \wedge \gamma)$, in (X, T). So (X, T) is a fuzzy quasi-Oz-space. Since the fuzzy regular closed sets $1 - \alpha$ and $1 - [\alpha \wedge \gamma]$ are not fuzzy G_δ-sets in (X, T), (X, T) is not a fuzzy Oz-space.

Example 4.5. Let $X = \{a, b, c\}$. Let $I = [0, 1]$. Consider the fuzzy sets α, β and γ in X defined as follows:
$\alpha : X \to I$ is defined by $\alpha(a) = 0.5;\ \alpha(b) = 0.5;\ \alpha(c) = 0.6$,
$\beta : X \to I$ is defined by $\beta(a) = 0.5;\ \beta(b) = 0.6;\ \beta(c) = 0.5$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.6;\ \gamma(b) = 0.4;\ \gamma(c) = 0.5$.
Then $T = \{0, \alpha,\ \beta,\ \gamma,\ \alpha \vee \beta,\ \alpha \vee \gamma,\ \beta \vee \gamma,\ \alpha \wedge \beta,\ \alpha \wedge \gamma,\ \gamma \vee [\alpha \wedge \beta],\ \alpha \vee \beta \vee \gamma, 1\}$ is a fuzzy topology on X. Then by computation, one can find that

$cl(\alpha) = 1;$	$int(1 - \alpha) = 0;$
$cl(\beta) = 1 - (\alpha \wedge \gamma) = \beta;$	$int(1 - \beta) = \alpha \wedge \gamma;$
$cl(\gamma) = 1;$	$int(1 - \gamma) = 0;$
$cl(\alpha \vee \beta) = 1;$	$int(1 - [\alpha \vee \beta]) = 0;$
$cl(\alpha \vee \gamma) = 1;$	$int(1 - [\alpha \vee \gamma]) = 0;$
$cl(\beta \vee \gamma) = 1;$	$int(1 - [\beta \vee \gamma]) = 0;$
$cl(\alpha \wedge \beta) = 1 - (\alpha \wedge \beta) = \alpha \wedge \beta;$	$int(1 - [\alpha \wedge \beta]) = \alpha \wedge \beta;$
$cl(\alpha \wedge \gamma) = 1 - \beta = \alpha \wedge \gamma;$	$int(1 - [\alpha \wedge \gamma]) = \beta;$
$cl(\gamma \vee [\alpha \wedge \beta]) = 1;$	$int(1 - (\gamma \vee [\alpha \wedge \beta])) = 0;$
$cl(\alpha \vee \beta \vee \gamma) = 1;$	$int(1 - [\alpha \vee \beta \vee \gamma]) = 0.$

Thus the fuzzy regular closed sets in (X, T) are $1 - \beta$, $1 - (\alpha \wedge \beta)$ and $1 - (\alpha \wedge \gamma)$.
Now $1 - \beta = \beta \wedge \gamma \wedge (\alpha \wedge \beta) = \alpha \wedge \gamma$;
$1 - (\alpha \wedge \beta) = \alpha \wedge (\alpha \vee \gamma) \wedge [\gamma \vee (\alpha \wedge \beta)] = \alpha \wedge \beta$;
$1 - (\alpha \wedge \gamma) = (\alpha \vee \beta) \wedge (\beta \vee \gamma) \wedge (\alpha \vee \beta \vee \gamma) = \beta$.
Then $1 - \beta$, $1 - (\alpha \wedge \beta)$ and $1 - (\alpha \wedge \gamma)$ are fuzzy G_δ-sets in (X, T). Thus the fuzzy regular closed sets $1 - \beta$, $1 - (\alpha \wedge \beta)$ and $1 - (\alpha \wedge \gamma)$ are fuzzy G_δ-sets in (X, T). So (X, T) is a fuzzy Oz-space. By computation, the fuzzy regular G_δ-sets in (X, T) are $1 - \beta$, $1 - (\alpha \wedge \beta)$. Since the fuzzy regular closed set $1 - (\alpha \wedge \gamma)$ is not a fuzzy regular G_δ-set in (X, T), (X, T) is not a fuzzy regular Oz-space.

The following proposition gives a condition under which fuzzy regular Oz-spaces become fuzzy extremally disconnected spaces.

Proposition 4.6. *If a fuzzy topological space (X, T) is a fuzzy regular Oz and fuzzy P-space, then X is a fuzzy extremally disconnected space.*

Proof. Let λ be a fuzzy open set in X. Since X is a fuzzy regular Oz-space, by Proposition 3.4, $cl(\lambda)$ is a fuzzy regular G_δ-set in X. Then by Theorem 2.9, $cl(\lambda)$ is a fuzzy G_δ-set in X. Also since X is a fuzzy P-space, the fuzzy G_δ-set $cl(\lambda)$ is a fuzzy open set in X. So X is a fuzzy extremally disconnected space. □

The following proposition gives a condition under which fuzzy regular Oz-spaces become weak fuzzy Oz-spaces.

Proposition 4.7. *If each fuzzy F_σ-set is a fuzzy open set in a fuzzy regular O_z-space (X, T), then (X, T) is a weak fuzzy O_z-space.*

Proof. Let λ be a fuzzy F_σ-set in X. Then, by the hypothesis, λ is a fuzzy open set in X. Since X is a fuzzy regular Oz-space, by Proposition 3.4, $cl(\lambda)$ is a fuzzy regular G_δ-set in X and thus by Theorem 2.9, $cl(\lambda)$ is a fuzzy G_δ-set in X. So for the fuzzy F_σ-set λ, $cl(\lambda)$ is a fuzzy G_δ-set in X. Hence (X, T) is a weak fuzzy Oz-space. □

Proposition 4.8. *If (X, T) is a fuzzy regular Oz and weak fuzzy P-space, then (X, T) is a fuzzy extremally disconnected space.*

Proof. Let λ be a fuzzy open set in X. Since X is a fuzzy regular Oz-space, by Proposition 3.4, $cl(\lambda)$ is a fuzzy regular G_δ-set in X. Also since (X, T) is a weak fuzzy P-space, by Theorem 2.13, $cl(\lambda)$ is a fuzzy regular open set in X and then $intcl[cl(\lambda)] = cl(\lambda)$. Thus $intcl(\lambda) = cl(\lambda)$. So $cl(\lambda)$ is a fuzzy open set in X. Hence it follows that (X, T) is a fuzzy extremally disconnected space. □

Proposition 4.9. *If (X, T) is a fuzzy regular Oz and weak fuzzy P-space, then (X, T) is a fuzzy Moscow space.*

Proof. The proof follows from Proposition 4.8 and Theorem 2.14. □

Proposition 4.10. *If a fuzzy topological space (X, T) is a fuzzy regular Oz and fuzzy P-space, then X is a fuzzy Moscow space.*

Proof. The proof follows from Proposition 4.6 and Theorem 2.14. □

Fuzzy regular Oz-spaces in which fuzzy G_δ-sets are not fuzzy dense, are not fuzzy hyperconnected spaces. For, consider the following proposition.

Proposition 4.11. *If a fuzzy topological space (X, T) is a fuzzy regular Oz-space in which fuzzy G_δ-sets are not fuzzy dense, then X is not a fuzzy hyperconnected space.*

Proof. Let λ be a fuzzy open set in (X, T). Then by Theorem 2.7, $cl(\lambda)$ is a fuzzy regular closed set in X. Since X is a fuzzy regular Oz-space, $cl(\lambda)$ is a fuzzy regular G_δ-set in X. Thus by Theorem 2.9, $cl(\lambda)$ is a fuzzy G_δ- set in X. So by hypothesis, $cl(\lambda) \neq 1$. Hence (X, T) is not a fuzzy hyperconnected space. □

The following proposition gives a condition under which fuzzy semi-open sets become fuzzy pre-open sets in fuzzy regular Oz- spaces.

Proposition 4.12. *If λ is a fuzzy semi-open set in fuzzy regular Oz and weak fuzzy P-space (X, T), then λ is a fuzzy pre-open set in X.*

Proof. Let λ be a fuzzy semi-open set in (X, T). Then $\lambda \leq clint(\lambda)$, in X. Since X is a fuzzy regular Oz-space, by Proposition 3.9, for the fuzzy set λ, $clint(\lambda)$ is a fuzzy regular G_δ-set in X. Also since (X, T) is a weak fuzzy P-space, by Theorem 2.13, $clint(\lambda)$ is a fuzzy regular open set in X and then $intcl[clint(\lambda)] = clint(\lambda)$. This implies that $intclint(\lambda) = clint(\lambda)$ and thus $intcl(\lambda) \geq intclint(\lambda) = clint(\lambda) \geq \lambda$. So $intcl(\lambda) \geq \lambda$. Hence λ is a fuzzy pre-open set in X. □

Proposition 4.13. *If λ is a fuzzy open set in a fuzzy regular Oz and weak fuzzy P-space (X, T), then λ is not a fuzzy dense set in X.*

Proof. Let λ be a fuzzy open set in X. Since (X, T) is a fuzzy regular Oz-space, by Proposition 3.4, $cl(\lambda)$ is a fuzzy G_δ set in X. Also since (X, T) is a weak fuzzy P-space, by Theorem 2.19, $cl(\lambda)$ is not a fuzzy dense set in X. Then $cl[cl(\lambda)] \neq 1$, in X. Thus $cl(\lambda) \neq 1$. So λ is not a fuzzy dense set in (X, T). □

Corollary 4.14. *If (X, T) is a fuzzy regular Oz and weak fuzzy P-space, then X is not a fuzzy hyperconnected space.*

Proof. Let λ be a fuzzy open set in X. Since X is a fuzzy regular Oz and weak fuzzy P-space, by Proposition 4.13, λ is not a fuzzy dense set in X. Then X is not a fuzzy hyperconnected space. □

The following proposition gives a condition under which fuzzy regular Oz-spaces become fuzzy almost P-spaces.

Proposition 4.15. *If each fuzzy regular G_δ-set is a non-zero fuzzy set in a fuzzy regular Oz-space (X, T), then X is a fuzzy almost P-space.*

Proof. Let δ be a fuzzy G_δ-set in X. Since X is a fuzzy regular Oz-space, by Proposition 3.9, for the fuzzy set δ in X, $clint(\delta)$ is a fuzzy regular G_δ-set in X. Then by the hypothesis, $clint(\delta) \neq 0$. Thus $int(\delta) \neq 0$ [for, if $int(\delta) = 0$, then $clint(\delta) = 0$, a contradiction]. Thus for the fuzzy G_δ-set δ, $int(\delta) \neq 0$. So (X, T) is a fuzzy almost P-space. □

Proposition 4.16. *If each fuzzy regular G_δ-set is a non-zero fuzzy set in a fuzzy regular Oz and weak fuzzy P-space (X, T), then X is a fuzzy pseudo P-space.*

Proof. Let(X, T) be a fuzzy regular Oz and weak fuzzy P-space in which each fuzzy regular G_δ-set is a non-zero fuzzy set in X. Then by Proposition 4.15, X is a fuzzy almost P-space. Thus X is a fuzzy almost P-space and weak fuzzy P-space. So X is a fuzzy pseudo P-space. □

Proposition 4.17. *If there exists a fuzzy closed set μ in a fuzzy regular Oz and weak fuzzy P-space (X, T) in which each fuzzy regular G_δ set is a non-zero fuzzy set and if $int(\mu) < \delta$, for a fuzzy open set δ in X, then X is a fuzzy Baire space.*

Proof. Let μ be a fuzzy closed set in X. Since X is a fuzzy regular Oz-space, by Corollary 3.5, $int(\mu)$ is a fuzzy regular F_σ-set in X. Then by the hypothesis $int(\mu) < \delta$ for a fuzzy open set δ in X. Thus by Proposition 4.16, (X, T) is a fuzzy pseudo P-space. So by Theorem 2.16, X is a fuzzy Baire space. □

Proposition 4.18. *If μ is a fuzzy closed set in a fuzzy regular Oz and weak fuzzy P-space (X, T) in which each fuzzy regular G_δ-set is a non-zero fuzzy set, then*

(1) *$int(\mu)$ is a fuzzy Baire dense set in X,*
(2) *there exist a fuzzy second category set δ such that $\delta \leq int(\mu)$ in X,*
(3) *$clint(\mu)$ is a fuzzy second category and fuzzy regular G_δ-set in X.*

Proof. (1) Let μ be a fuzzy closed set in X in which each fuzzy regular G_δ-set is a non-zero fuzzy set in X. Since X is a fuzzy regular Oz-space, by Corollary 3.5, $int(\mu)$ is a fuzzy regular F_σ-set in X. Then by Proposition 4.16, the fuzzy regular Oz and weak fuzzy P-space X is a fuzzy pseudo P-space. Thus by Theorem 2.15, $int(\mu)$ is a fuzzy Baire dense set in X.

(2) For the fuzzy closed set μ in X, by (1), $int(\mu)$ is a fuzzy Baire dense set in X. Then by Theorem 2.17, there exist a fuzzy second category set δ in X such that $\delta \leq int(\mu)$.

(3) For the fuzzy closed set μ in X, by (1), $int(\mu)$ is a fuzzy Baire dense set in X. Then by Theorem 2.18, $clint(\mu)$ is a fuzzy second category set in X. Since X is a fuzzy regular Oz-space, by Proposition 3.9, for the fuzzy set μ, $clint(\mu)$ is a fuzzy regular G_δ-set in X. Thus for the fuzzy closed set μ in X, $clint(\mu)$ is a fuzzy second category and fuzzy regular G_δ-set in X. □

Proposition 4.19. *If λ_i's are fuzzy semi-open sets in a fuzzy regular Oz and weak fuzzy P-space (X, T) in which each fuzzy regular G_δ-set is a non-zero fuzzy set, $\bigvee_{i=1}^{\infty} cl(\lambda_i)$, is a fuzzy F_σ-set and a fuzzy regular closed set in X.*

Proof. Let λ_i's ($i = 1$ *to* ∞) be fuzzy semi-open sets in X in which each fuzzy regular G_δ-set is a non-zero fuzzy set in X. Since X is a fuzzy regular Oz and weak fuzzy P-space, by Proposition 4.16, X is a fuzzy pseudo P-space. Then by Theorem 2.20, $\bigvee_{i=1}^{\infty} cl(\lambda_i)$ is a fuzzy regular closed set in X. Since $[cl(\lambda_i)]'$s are fuzzy closed sets in X, $\bigvee_{i=1}^{\infty} cl(\lambda_i)$ is a fuzzy F_σ-set and fuzzy regular closed set in X. □

Remark 4.20. In view of Propositions 3.21 and 4.19, one will have the following result: "If λ_i's are fuzzy semi-open sets in a fuzzy regular Oz and weak fuzzy P-space (X, T) in which each fuzzy regular G_δ-set is a non-zero fuzzy set, then $[cl(\lambda_i)]'$s are fuzzy regular closed sets and $\bigvee_{i=1}^{\infty} cl(\lambda_i)$ is a fuzzy regular closed set in X."

5. Conclusion

In this paper, the concept of fuzzy regular Oz-space is introduced by means of fuzzy regular closed sets and fuzzy regular G_δ-sets. It is obtained that the fuzzy closure of a fuzzy open set is a fuzzy regular G_δ-set and the fuzzy interior of a fuzzy closed set is fuzzy regular F_σ-set in fuzzy regular Oz-spaces. Also fuzzy pre-open sets are having fuzzy regular F_σ-sets as their fuzzy super sets in fuzzy regular Oz-spaces. It is established that the fuzzy closure of fuzzy interior of any fuzzy set is a fuzzy regular G_δ-set in fuzzy regular Oz-spaces and the fuzzy closure of a fuzzy somewhere dense set is a fuzzy super set of a fuzzy regular G_δ-set and the fuzzy closure of a fuzzy open set is a fuzzy super set of a fuzzy regular open set in fuzzy regular Oz-spaces.

It is obtained that fuzzy regular Oz-spaces are fuzzy Oz-spaces and fuzzy regular Oz and fuzzy P-spaces are fuzzy extremally disconnected spaces. A condition under which fuzzy regular Oz-spaces become weak fuzzy Oz-spaces is obtained. It is established that fuzzy regular Oz and weak fuzzy P-spaces are fuzzy Moscow spaces. A condition under which fuzzy semi-open sets become fuzzy pre-open sets in fuzzy regular Oz-spaces is also obtained. The conditions, for fuzzy regular Oz and weak fuzzy P-spaces to become fuzzy Baire spaces and for fuzzy regular Oz-spaces to become fuzzy almost P-spaces, are also obtained in this paper. The existence of fuzzy regular G_δ-sets which are non-zero in fuzzy regular Oz and weak fuzzy P-spaces shows that such fuzzy regular Oz and weak fuzzy P-spaces are fuzzy pseudo P-spaces.

References

[1] L. A. Zadeh , Fuzzy Sets, Information and Control 8 (1965) 338–353.
[2] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl. 24 (1968) 182–190.
[3] R. L. Blair, Spaces in which special sets are z-embedded, Canad.J. Math. 28 (4) (1976) 673–690.
[4] V. E. Scepin, On topological products, groups and a new class of spaces that are more general than metric spaces, Dokl. Akad. Nauk SSSR 226 (1976) 527–529 ; Soviet Math. Dokl. 17 (1976) 152–155.
[5] T. Terada, Note on Z-, $C*$- and C-embedding, Sci. Rep. Tokyo Kyoiku Daigaku Sect. A 13 (1975) 129–132.
[6] A. Chigogidze, On perfectly κ-normal spaces, Soviet Math. Dokl. 21 (1980) 95–98.
[7] A. V. Arhangel'skii, Functional tightness, Q-spaces and τ-embeddings, Comment. Math. Univ. Carolinae 24 (1) (1983) 105–120.
[8] C. I. Kim, Quasi-Oz-spaces, Bull. Korean Math. Soc. 30 (1) (1993) 117–124.
[9] Ernest P. Lane, PM-normality and the insertion of a continuous function, Pacific J. Math. 82 (1) (1979) 155–162.
[10] G. Şenel, Soft topology generated by L-soft sets, J. New Theory 4 (24) (2018) 88–100.
[11] G. Şenel, A new approach to Hausdorff space theory via soft sets, Mathematical problems in Engineering 2016 (9) (2016) 1–6.
[12] G. Şenel and N. Çağman, Soft closed sets on soft bitopological spaces, Journal of New Results in Science 3 (5) (2014) 57–66.
[13] G. Şenel and N. Çağman, Soft Topological Subspaces, Ann. Fuzzy Math. Inform. 10 (4) (2015) 525–535.
[14] G. Şenel, J. G. Lee and K. Hur, Distance and similarity measures for Octahedron sets and their application to MCGDM problems, Mathematics 20208 (10) (2020) 1690.
[15] J. Kim, G. Şenel, P. K. Lim, J. G. Lee, K. Hur, Octahedron sets, Ann. Fuzzy Math. Inform. 19 (3) (2019) 211–238.
[16] Tareq M. Al-shami, H. Z. Ibrahim, A. A. Azzam and A. I. EL-Maghrabi, SR-fuzzy sets and their weighted aggregated operators in application to decision-making, Journal of Function Spaces 2022 (2022) Article ID 3653225.
[17] K. K. Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl. 82 (1981) 14–32.
[18] G. Balasubramanian, Maximal Fuzzy Topologies, Kybernetika 31 (5) (1995) 459–464.
[19] A. S. Bin Shahna, On fuzzy strong semi-continuity and fuzzy pre-continuity, Fuzzy Sets and Systems 44 (1991) 303–308.
[20] S. Soundara Rajan, A study on fuzzy Volterra spaces, Ph.D Thesis 2015, Thiruvalluvar University, Vellore, Tamilnadu State, India.
[21] G. Thangaraj and G. Balasubramanian, On somewhat fuzzy continuous functions, J. Fuzzy Math 11 (2) (2003) 725–736.
[22] G. Thangaraj and S. Senthil, On somewhere fuzzy continuous functions, Annl. Fuzzy Math. Inform. 15 (2) (2018) 181–198.
[23] G. Thangaraj and S. Lokeshwari, On fuzzy Baire dense sets and fuzzy Baire resolvable spaces, Adv. Fuzzy Sets and Sys. 23 (2-3) (2018) 93–116.
[24] G. Thangaraj and G. Balasubramanian, On fuzzy basically disconnected spaces, J. Fuzzy Math. 9 (1) (2001) 103–110.

[25] G. Thangaraj and C. Anbazhagan, On fuzzy almost P-spaces, Inter. J. Innov. Sci. Engin. and Tech. 2 (4) (2015) 389–407.

[26] G. Thangaraj and A. Vinothkumar, SomerRemarks on weak fuzzy P-spaces, Malaya Journal of Matematik 9 (1) (2021) 855–860.

[27] Miguel Caldas, Govindappa Navalagi and Ratnesh Saraf, On fuzzy weakly semi-open functions, Proyecciones 21 (1) (2002) 51–63.

[28] B. Ghosh, Fuzzy extremally disconnected spaces, Fuzzy Sets and System 46 (2) (1992) 245–250.

[29] G. Thangaraj and M. Ponnusamy, On fuzzy Oz-spaces, (Communicated to Journal of Advances and Applications in Mathematical Sciences).

[30] G. Thangaraj and M. Ponnusamy, On fuzzy quasi-Oz-spaces, (Communicated to Journal of Advances and Applications in Mathematical Sciences).

[31] G. Thangaraj and A. Vinothkumar, On fuzzy Moscow spaces, Annl. Fuzzy Math. and Inform. 23 (3) (2022) 225–242.

[32] G. Thangaraj and A. Vinothkumar, On fuzzy pseudo P-spaces, Adv. Fuzzy Sets and Sys. 25 (2) (2020) 85–108.

[33] G. Thangaraj and S. Anjalmose, On fuzzy Baire spaces, J. Fuzzy Math. 21 (3) (2013) 667–676.

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