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Fuzzy tri-ideals and fuzzy soft tri-ideals over semirings

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Abstract. In this paper, we introduce the notion of fuzzy tri-ideal, fuzzy soft tri-ideal over semirings and study some of their properties. Every fuzzy soft left (right) ideal over semiring is a fuzzy soft right (left) tri- ideal over semiring. We characterize the regular semiring in terms of fuzzy left (right) tri ideals and fuzzy soft left (right) tri-ideals over semiring.

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1. Introduction

Semirings play an essential role in studying matrices and determinants. Semirings are helpful in theoretical computer science and the solution of graph theory and optimization theory, particularly for studying automata, coding theory, and formal languages. Ideals play an important role in advanced studies of algebraic structures. Many mathematicians proved significant results and characterization of algebraic structures by using the concept and the properties of a generalization of ideals in algebraic structures. During 1950-1980, the concepts of bi-ideals, quasi-ideals, interior ideals and the applications of these ideals were studied by many mathematicians. The author [1, 2, 3, 4, 5, 6, 7] introduced and studied weak interior ideals , bi-interior ideals, bi-quasi ideals, quasi interior ideals and bi quasi interior ideals of Γ-semirings, semirings, Γ-semigroups, semigroups as a generalization of bi-ideal, quasi ideal and interior ideal of algebraic structures and characterized regular algebraic structures as well as simple algebraic structures using these ideals.

The fuzzy set theory was developed by Zadeh [8] in 1965. Many papers on fuzzy sets appeared, showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory, etc.

The fuzzification of the algebraic structure was introduced by Rosenfeld [9], and he introduced the notion of fuzzy subgroups in 1971. Kuroki [10] studied fuzzy interior ideals in semigroups.

Molodtsov [11] introduced the concept of a soft set theory, a new mathematical tool for dealing with uncertainties, as a parameterical family of subsets of the universal set. Feng et al. [12] studied soft semirings using the soft set theory. Soft set theory has many applications in game theory, operations research, Riemann integration, computer science, economics, data analysis, medical science, decision-making, and measurement theory. Aktas and Çağman [13] defined the soft set and soft groups. Jayanth Ghosh et al. [14] initiated the study of fuzzy soft rings and fuzzy soft ideals.

Rao et al. introduced and studied fuzzy soft ideals, Fuzzy soft Γ-semirings homomorphism, Fuzzy soft Γ−semiring, fuzzy soft k ideal over Γ−semiring, fuzzy soft quasi-ideal, and fuzzy soft interior ideal over Γ−semirings [6, 15, 1]. Soft set theory and fuzzy soft set theories have applications in fields like forecasting, simulation, evaluation of sound quality, and smoothness of functions.

This paper aims to introduce the notion of fuzzy (soft) tri-ideals of a semiring and characterize regular semiring in terms of fuzzy (soft) tri-ideals.

2. Preliminaries

In this section, we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1 ([16])**.** A set M together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called a *semiring*, provided

(i) $+$ is a commutative operation,
(ii) $\cdot$ distributes over the addition both from the left and from the right,
(iii) there exists $0 \in M$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in M$.

Definition 2.2 ([17])**.** Let M be a semiring. If there exists $1 \in M$ such that $a \cdot 1 = 1 \cdot a = a$ for all $a \in M$, then 1 is called a *unity element* of M. In this case, M is said to be a *semiring with unity.*

Definition 2.3 ([17])**.** An element a of a semiring M is called a *regular element*, if there exists an element b of M such that $a = aba$.

Definition 2.4 ([17])**.** A semiring M is called a *regular semiring*, if every element of M is a regular element.

Definition 2.5 ([17])**.** An element a of a semiring M is called a *multiplicatively idempotent* (an *additively idempotent*) element, if $aa = a(a + a = a)$.

Definition 2.6 ([17])**.** An element b of a semiring M is called an *inverse element* of a of M, if $ab = ba = 1$.

Definition 2.7 ([17])**.** A semiring M is called a *division semiring*, if for each non-zero element of M has multiplication inverse.

Definition 2.8 ([6]). A function $f : R \to M$ is said to be a *semiring homomorphism*, if $f(a+b) = f(a) + f(b)$ and $f(ab) = f(a)f(b)$ for all $a, b \in R$, where R and M are semirings.

Definition 2.9 ([6]). Let M be a non-empty set. A mapping $f : M \to [0, 1]$ is called a *fuzzy subset* of M

Definition 2.10 ([6]). Let f be a fuzzy subset of a non-empty set M and let $t \in [0, 1]$. Then the set $f_t = \{x \in M \mid f(x) \geq t\}$ is called a *level subset* of M with respect to f.

Definition 2.11 ([6]). Let M be a semiring. A fuzzy subset μ of M is said to be a *fuzzy subsemiring* of M, if it satisfies the following conditions: for all $x, y \in M$,
(i) $\mu(x+y) \geq min\ \{\mu(x), \mu(y)\}$,
(ii) $\mu(xy) \geq min\ \{\mu(x), \mu(y)\}$.

Definition 2.12 ([6]). A fuzzy subset μ of a semiring M is called a *fuzzy left* (*right*) *ideal* of M, if it satisfies the following conditions: for all $x, y \in M$,
(i) $\mu(x+y) \geq min\{\mu(x), \mu(y)\}$,
(ii) $\mu(xy) \geq \mu(y)\ (\mu(x))$.

Definition 2.13 ([6]). A fuzzy subset μ of a semiring M is called a *fuzzy ideal* of M, if it satisfies the following conditions: for all $x, y \in M$,
(i) $\mu(x+y) \geq min\{\mu(x), \mu(y)\}$,
(ii) $\mu(xy) \geq max\ \{\mu(x), \mu(y)\}$.

Definition 2.14 ([6]). For any two fuzzy subsets λ and μ of M, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in M$.

Definition 2.15 ([6]). Let f and g be fuzzy subsets of semiring M. Then $f \circ g, f + g, f \cup g, f \cap g$, are defined by: for each $x \in M$,

$$f \circ g(z) = \begin{cases} \sup\limits_{z=xy} \{min\{f(x), g(y)\}\}, \\ 0, \quad \text{otherwise.} \end{cases} ; f + g(z) = \begin{cases} \sup\limits_{z=x+y} \{min\{f(x), g(y)\}\}, \\ 0, \quad \text{otherwise} \end{cases}$$

$$f \cup g(z) = max\{f(z), g(z)\} \ \ ; \ f \cap g(z) = min\{f(z), g(z)\} \text{ for all } x, y, z \in M.$$

Definition 2.16 ([6]). Let A be a non-empty subset of M. The *characteristic function* of A is a fuzzy subset of M, denoted by $\chi_{_A}$, defined as follows: for each $x \in X$,

$$\chi_{_A}(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A. \end{cases}$$

Definition 2.17 ([18]). Let U be an initial Universe set and E be the set of parameters. Let $P(U)$ denotes the power set of U. A pair (μ, E) is called a *soft set* over U, where μ is a mapping given by $\mu : E \to \ P(U)$.

Definition 2.18 ([18]). Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (μ, A) is called a *fuzzy soft set* over U, where μ is a mapping given by $\mu : A \to [0, 1]^U$ and $[0, 1]^U$ denotes the collection of all fuzzy subsets of U. $\mu(a), a \in A$, is a fuzzy subset and is denoted by μ_a

Definition 2.19 ([18]). Let $(\mu, A), (\lambda, B)$ be fuzzy soft sets over U. Then (μ, A) is said to be a *fuzzy soft subset* of (λ, B), denoted by $(\mu, A) \subseteq (\lambda, B)$, if $A \subseteq B$ and $\mu_a \subseteq \lambda_a$ for all $a \in A$.

Definition 2.20 ([18]). Let $(\mu, A),\ (\lambda, B)$ be fuzzy soft sets. The *intersection* of (μ, A) and (λ, B), denoted by $(\mu, A) \cap (\lambda, B) = (\gamma, C)$, where $C = A \cup B$ for all $c \in C, \gamma(c) = \gamma_c$ for any fuzzy subset of M is defined as:

$$\gamma_c = \begin{cases} \mu_c, & \text{if } c \in A \setminus B, \\ \lambda_c, & \text{if } c \in B \setminus A, \\ \mu_c \cap \lambda_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.21. [18]Let $(\mu, A),\ (\lambda, B)$ be fuzzy soft sets. The *union* of (μ, A) and (λ, B), denoted by $(\mu, A) \cup (\lambda, B) = (\gamma, C)$, where $C = A \cup B$ for all $c \in C, \gamma(c) = \gamma_c$ for any fuzzy subset of M is defined as:

$$\gamma_c = \begin{cases} \mu_c, & \text{if } c \in A \setminus B, \\ \lambda_c, & \text{if } c \in B \setminus A, \\ \mu_c \cup \lambda_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.22 ([18]). Let M be a semiring and E be a parameter set, let $A \subseteq E$ and let $\mu : A \to [0, 1]^M$ be a mapping. Then (μ, A) is called a *fuzzy soft semiring* over M, if for each $a \in A$, $\mu(a) = \mu_a$ is the fuzzy subsemiring of M, i.e., for all $x,\ y \in M$,
(i) $\mu_a(x + y) \geq \min\{\mu_a(x), \mu_a(y)\}$,
(ii) $\mu_a(xy) \geq \min\{\mu_a(x), \mu_a(y)\}$.

Definition 2.23 ([18]). Let M be a semiring and E be a parameter set, let $A \subseteq E$ and let $\mu : A \to [0, 1]^M$ be a mapping. Then (μ, A) is called a *fuzzy soft left*(right) *ideal* over M, if for each $a \in A$, the corresponding fuzzy subset $\mu_a : M \to [0, 1]$ is a fuzzy left(right) ideal of M, i.e., for all $x,\ y \in M$,
(i) $\mu_a(x + y) \geq \min\{\mu_a(x), \mu_a(y)\}$,
(ii) $\mu_a(xy) \geq \mu_a(y)(\mu_a(x))$.

3. Fuzzy tri-ideals

In this section, we introduce the notion of a fuzzy right(left) tri ideal of a semiring and study the properties of fuzzy right(left) tri ideals.

Definition 3.1. A fuzzy subset μ of a semiring M is called a *fuzzy left* (*right*) *tri-ideal*, if it satisfies the following conditions:
(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$ for all $x,\ y \in M$,
(ii) $\mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu\big(\mu \circ \mu \circ \chi_M \circ \mu \subseteq \mu\big)$.

A fuzzy subset μ of a semiring M is called a *fuzzy tri-ideal*, if it is a left tri-ideal and a right tri-ideal of M.

Example 3.2. Let $M = \{0, a, b, c\}$. The binary operations $+,\ \cdot$ abe defined by the following tables:

$+$	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	c
c	c	c	c	c

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	a	b	b
c	0	a	b	$c.$

Then $(M, +, \cdot)$ is a semiring. Let $B = \{0, a, c\}$. Then B is a left tri-ideal of M. Now let μ be a fuzzy set in M defined as follows: for each $x \in M$,

$$\mu(x) = \begin{cases} 1 & \text{if } x \in B, \\ 0, & \text{otherwise.} \end{cases}$$

Then μ is a fuzzy tri-ideal of M.

Theorem 3.3. *A non-empty fuzzy subset μ of a semiring M is a fuzzy subsemiring of M if and only if $\mu \circ \mu \subseteq \mu$.*

Proof. Suppose μ is a fuzzy subsemiring of M and let $x \in M$. Then

$$\begin{aligned} \mu \circ \mu(x) &= \sup_{x=ab} \min\{\mu(a), \mu(b)\} \\ &\leq \sup_{x=ab} \mu(ab), \text{ since } \mu(ab) \geq \min\{\mu(a), \mu(b)\} \\ &= \mu(x). \end{aligned}$$

If $a, b \in M$ does not exist such that $x = ab$, then $\mu \circ \mu(x) = 0 \leq \mu(x)$ for all $x \in M$. Thus $\mu \circ \mu \subseteq \mu$.

Conversely, suppose $\mu \circ \mu \subseteq \mu$ and let $x,\ y \in M$. Then

$$\begin{aligned} \mu(xy) &\geq \mu \circ \mu(xy) \\ &= \sup \min\{\mu(x), \mu(y)\} \\ &\geq \min\{\mu(x), \mu(y)\}. \end{aligned}$$

Thus μ is a fuzzy subsemiring of M. $\square$

Theorem 3.4. *Every fuzzy right ideal of a semiring M is a fuzzy left tri-ideal of M.*

Proof. Let μ be a fuzzy right ideal of the semiring M and let $x \in M$. Then

$$\begin{aligned} \mu \circ \chi_M(x) &= \sup_{x=ab} \min\{\mu(a), \chi_M(b)\},\ a,\ b \in M, \\ &= \sup_{x=ab} \mu(a) \\ &\leq \sup_{x=ab} \mu(ab) \\ &= \mu(x). \end{aligned}$$

Thus $\mu \circ \chi_M(x) \leq \mu(x)$.

Now $$\begin{aligned} \mu \circ \chi_M \circ \mu \circ \mu(x) &= \sup_{x=uvs} \min\{\mu \circ \chi_M(uv), \mu \circ \mu(s)\} \\ &\leq \sup_{x=uvs} \min\{\mu(uv), \mu(s)\} \end{aligned}$$

$$= \mu(x).$$

So μ is a fuzzy left tri-ideal of M. □

Corollary 3.5. *Every fuzzy left ideal of a semiring M is a fuzzy right tri-ideal of M.*

Corollary 3.6. *Every fuzzy ideal of a semiring M is a fuzzy tri-ideal of M.*

Theorem 3.7. *Let M be a semiring and μ be a non-empty fuzzy subset of M. A fuzzy subset μ is a fuzzy left tri-ideal of a semiring M if and only if the level subset μ_t of μ is a left tri-ideal of a semiring M for every $t \in [0, 1]$, where $\mu_t \neq \phi$.*

Proof. Suppose μ is a fuzzy left tri-ideal of the semiring M and let $\mu_t \neq \phi, t \in [0, 1]$ and $a,\ b \in \mu_t$. Then we get

$$\mu(a) \geq t, \mu(b) \geq t.$$

Thus we have

$$\mu(a + b) \geq \min\{\mu(a), \mu(b)\} \geq t, \mu(ab) \geq \min\{\mu(a), \mu(b)\} \geq t.$$

So $a + b,\ ab \in \mu_t$.

Let $x \in \mu_t M \mu_t \mu_t$. Then $x = badc$, where $a \in M, b, c, d \in \mu_t$. Thus we have

$$\mu \circ \chi_M \circ \mu \circ \mu(x) \geq t.$$

So $\mu(x) \geq \mu \circ \chi_M \circ \mu \circ \mu(x) \geq t$. Hence $x \in \mu_t$. Therefore μ_t is a left tri-ideal M.

Conversely, suppose μ_t is a left tri-ideal of the semiring M for all $t \in Im(\mu)$. Let $x, y \in M, \mu(x) = t_1, \mu(y) = t_2$ and $t_1 \geq t_2$. Then $x,\ y \in \mu_{t_2}$. Thus we get

$$\mu(x + y) \geq t_2 = \min\{t_1, t_2\} = \min\{\mu(x), \mu(y)\}$$

and

$$\mu(xy) \geq t_2 = \min\{t_1, t_2\} = \min\{\mu(x), \mu(y)\}.$$

So $x + y \in \mu_{t_2},\ xy \in \mu_{t_2}$.

We have $\mu_l M \mu_l \mu_l \subseteq \mu_l$ for all $l \in Im(\mu)$. Let $t = \min\{Im(\mu)\}$. Then $\mu_t M \mu_t \mu_t \subseteq \mu_t$. Thus $\mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$. So μ is a fuzzy left tri-ideal of M. □

Corollary 3.8. *Let M be a semiring and μ be a non-empty fuzzy subset of M. A fuzzy subset μ is a fuzzy right tri-ideal of a semiring if and only if the level subset μ_t of μ is a right tri-ideal of a semiring M for every $t \in [0, 1]$, where $\mu_t \neq \phi$.*

Theorem 3.9. *Let I be a non-empty subset of a semiring M and χ_I be the characteristic function of I. Then I is a right tri-ideal of a semiring M if and only if χ_I is a fuzzy right tri-ideal of a semiring M.*

Proof. Let I be a non-empty subset of the semiring M and let χ_I be the characteristic function of I. Suppose I is a right tri-ideal of the semiring M. Obviously χ_I is a fuzzy subsemiring of M. We have $IIMI \subseteq I$. Then

$$\chi_I \circ \chi_I \circ \chi_M \circ \chi_I = \chi_{IIMI} \subseteq \chi_I.$$

Thus χ_I is a fuzzy right tri-ideal of M.

Conversely, suppose χ_I is a fuzzy right tri-ideal of M. Then I is a subsemiring of M. Thus $\chi_I \circ \chi_I \circ \chi_M \circ \chi_I \subseteq \chi_I$. So $\chi_{IIMI} \subseteq \chi_I$. Hence $IIMI \subseteq I$. Therefore I is a right tri-ideal of M. □

Theorem 3.10. *Let M be a regular semiring. Then μ is a fuzzy left tri-ideal of M if and only if μ is a fuzzy right left ideal of M.*

Proof. Suppose μ is a fuzzy left tri-ideal of M and let $x \in M$. Then $\mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$. Since M is a regular, there exists $y \in M$ such that $x = xyx$. Thus we get

$$\begin{aligned}\text{Then } \mu \circ \chi_M \circ \mu \circ \mu(x) &= \sup_{x=xyx} \min\{\mu \circ \chi_M(x), \mu \circ \mu(yx)\}) \\ &\leq \sup_{x=xyx} \min\{\mu \circ \chi_M(x), \mu(yx)\}\}\} \\ &\leq \sup_{x=xyx} \min\{\mu(x), \mu(yx)\}\}\} \text{ (if } \mu \circ \mu(x) \leq \mu(x)).\end{aligned}$$

So $\mu \circ \chi_M(x) \leq \mu(x)$. Hence μ is a fuzzy right ideal of M. The converse is obvious. □

Corollary 3.11. *Let M be a regular semiring. Then μ is a fuzzy right tri-ideal of M if and only if μ is a fuzzy left ideal of M.*

Theorem 3.12. *If μ and λ are fuzzy right tri- ideals of a semiring M, then $\mu \cap \lambda$ is a fuzzy right tri-ideal of M.*

Proof. Suppose μ and λ are fuzzy right-tri ideals of M and let $x,\ y \in M$. Then

$$\begin{aligned}\mu \cap \lambda(x+y) &= \min\{\mu(x+y), \lambda(x+y)\} \\ &\geq \min\{\min\{\mu(x), \mu(y)\}, \min\{\lambda(x), \lambda(y)\}\} \\ &= \min\{\min\{\mu(x), \lambda(x)\}, \min\{\mu(y), \lambda(y)\}\} \\ &= \min\{\mu \cap \lambda(x), \mu \cap \lambda(y)\},\end{aligned}$$

$$\begin{aligned}\mu \cap \lambda(xy) &= \min\{\mu(xy), \lambda(xy)\} \\ &\geq \min\{\min\{\mu(x), \mu(y)\}, \min\{\lambda(x), \lambda(y)\}\} \\ &= \min\{\min\{\mu(x), \lambda(x)\}, \min\{\mu(y), \lambda(y)\}\} \\ &= \min\{\mu \cap \lambda(x), \mu \cap \lambda(y)\},\end{aligned}$$

$$\begin{aligned}\chi_M \circ \mu \cap \lambda(x) &= \sup_{x=ab} \min\{\chi_M(a), \mu \cap \lambda(b)\} \\ &= \sup_{x=ab} \min\{\chi_M(a), \min\{\mu(b), \lambda(b)\}\} \\ &= \sup_{x=ab} \min\{\min\{\chi_M(a), \mu(b)\}, \min\{\chi_M(a), \lambda(b)\}\} \\ &= \min\{\sup_{x=ab} \min\{\chi_M(a), \mu(b)\}, \sup_{x=ab} \min\{\chi_M(a), \lambda(b)\}\} \\ &= \min\{\chi_M \circ \mu(x), \chi_M \circ \lambda(x)\} \\ &= \chi_M \circ \mu \cap \chi_M \circ \lambda(x).\end{aligned}$$

Thus $\chi_M \circ \mu \cap \chi_M \circ \lambda = \chi_M \circ \mu \cap \lambda$. On the other hand, for each $x \in M$,

$$\begin{aligned}((\mu \cap \lambda) \circ (\mu \cap \lambda))(x) &= \sup_{x=ab} \min\{(\mu \cap \lambda)(a), (\mu \cap \lambda)(b)\} \\ &= \sup_{x=ab} \min\{\min\{\mu(a), \lambda(a)\}, \{\min\{\mu(b), \lambda(b)\} \\ &= \sup_{x=ab} \min\{\min\{\mu(a), \lambda(a)\}, \sup_{x=ab} \min\{\min\{\mu(b), \lambda(b)\} \\ &= \min\{\sup_{x=ab}\{\min\{\mu(a), \mu(b)\}, \sup_{x=ab}\{\min\{\lambda(a), \lambda(b)\}\} \\ &= \min\{\mu \circ \mu(x), \lambda \circ \lambda(x)\} \\ &= (\mu \circ \mu) \cap (\lambda \circ \lambda)(x).\end{aligned}$$

Then we get

$$\begin{aligned}&(\mu \cap \lambda \circ \mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda)(x) \\ =& \sup_{x=abc} \min\{\mu \circ \mu \cap \lambda \circ \lambda(a), \chi_M \circ \mu \cap \lambda(bc)\}\} \\ =& \sup_{x=abc} \min\{\mu \circ \mu(a), \lambda \circ \lambda(a)\}\}, \min\{\chi_M \circ \mu(bc), \chi_M \circ \lambda(bc)\}\} \\ =& \sup_{x=abc} \min\{\min\{\mu \circ \mu(a), \chi_M \circ \mu(bc)\}\}, \min\{\lambda \circ \lambda(a), \chi_M \circ \lambda(bc)\}\} \\ =& \min\{\sup_{x=abc} \min\{\mu \circ \mu(a), \chi_M \circ \mu(bc)\}, \sup_{x=abc} \min\{\lambda \circ \lambda(a), \chi_M \circ \lambda(bc)\}\} \\ =& \min\{\mu \circ \mu \circ \chi_M \circ \mu(x), \lambda \circ \lambda \circ \chi_M \circ \lambda(x)\} \\ =& \mu \circ \mu \circ \chi_M \circ \mu \cap \lambda \circ \lambda \circ \chi_M \circ \lambda(x).\end{aligned}$$

So $\mu \cap \lambda \circ \mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda = \mu \circ \mu \circ \chi_M \circ \mu \cap \lambda \circ \lambda \circ \chi_M \circ \lambda \subseteq \mu \cap \lambda$. Hence

$$\mu \cap \lambda \circ \mu \cap \lambda \circ \chi_M \circ \mu \cap \lambda = \mu \circ \mu \circ \chi_M \circ \mu \cap \lambda \circ \lambda \circ \chi_M \circ \lambda \subseteq \mu \cap \lambda.$$

Therefore $\mu \cap \lambda$ is a fuzzy right tri-ideal of M. □

Corollary 3.13. *If μ and λ are fuzzy left tri-ideals of a semiring M, then $\mu \cap \lambda$ is a fuzzy left tri-ideal of M.*

Corollary 3.14. *Let μ and λ be fuzzy tri-ideals of a semiring M. Then $\mu \cap \lambda$ is a fuzzy tri-ideal of M.*

Theorem 3.15. *If μ and λ are fuzzy right tri-ideals of a semiring M, then $\mu \cup \lambda$ is a fuzzy right tri-ideal of M.*

Proof. Suppose μ and λ are fuzzy right tri-ideals of M, and let $x,\ y \in M$. Then

$$\begin{aligned}\mu \cup \lambda(x+y) &= \max\{\mu(x+y), \lambda(x+y)\} \\ &\geq \max\{\min\{\mu(x), \mu(y)\}, \min\{\lambda(x), \lambda(y)\}\} \\ &= \min\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\} \\ &= \min\{\mu \cup \lambda(x), \mu \cup \lambda(y)\},\end{aligned}$$

$$\begin{aligned}\mu \cup \lambda(xy) &= \max\{\mu(xy), \lambda(xy)\} \\ &\geq \max\{\min\{\mu(x), \mu(y)\}, \min\{\lambda(x), \lambda(y)\}\} \\ &= \min\{\max\{\mu(x), \lambda(x)\}, \max\{\mu(y), \lambda(y)\}\} \\ &= \min\{\mu \cup \lambda(x), \mu \cup \lambda(y)\},\end{aligned}$$

$$\begin{aligned}\chi_M \circ \mu \cup \lambda(x) &= \sup_{x=ab} \min\{\chi_M(a), \mu \cup \lambda(b)\} \\ &= \sup_{x=ab} \min\{\chi_M(a), \max\{\mu(b), \lambda(b)\}\} \\ &= \sup_{x=ab} \max\{\min\{\chi_M(a), \mu(b)\}, \min\{\chi_M(a), \lambda(b)\}\} \\ &= \max\{\sup_{x=ab} \min\{\chi_M(a), \mu(b)\}, \sup_{x=ab} \min\{\chi_M(a), \lambda(b)\}\} \\ &= \max\{\chi_M \circ \mu(x), \chi_M \circ \lambda(x)\} \\ &= (\chi_M \circ \mu \cup \chi_M \circ \lambda)(x).\end{aligned}$$

Thus $\chi_M \circ \mu \cup \chi_M \circ \lambda = \chi_M \circ \mu \cup \lambda$. On the other hand, for each $x \in M$,

$$\begin{aligned}((\mu \cup \lambda) \circ (\mu \cup \lambda))(x) &= \sup_{x=ab} \min\{(\mu \cup \lambda)(a), (\mu \cup \lambda)(b)\} \\ &= \sup_{x=ab} \min\{\max\{\mu(a), \lambda(a)\}, \{\max\{\mu(b), \lambda(b)\}\} \\ &= \sup_{x=ab} \min\{\max\{\mu(a), \lambda(a)\}, \sup_{x=ab} \min\{\max\{\mu(b), \lambda(b)\}\} \\ &= \max\{\sup_{x=ab} \{\min\{\mu(a), \mu(b)\}, \sup_{x=ab} \{\min\{\lambda(a), \lambda(b)\}\} \\ &= \max\{\mu \circ \mu(x)\}, \{\lambda \circ \lambda(x)\} \\ &= ((\mu \circ \mu) \cup (\lambda \circ \lambda))(x).\end{aligned}$$

So we have

$$\begin{aligned}&\mu \cup \lambda \circ \mu \cup \lambda \circ \chi_M \circ \mu \cup \lambda(x) \\ =& \sup_{x=abc} \min\{\mu \circ \mu \cup \lambda \circ \lambda(a), \chi_M \circ \mu \cup \lambda(bc)\} \\ =& \sup_{x=abc} \max\{\min\{\mu \circ \mu(a), \chi_M \circ \mu(bc)\}, \min\{\lambda \circ \lambda(a), \chi_M \circ \lambda(bc)\}\} \\ =& \max\{\sup_{x=abc} \min\{\mu \circ \mu(a), \chi_M \circ \mu(bc)\}, \sup_{x=abc} \min\{\lambda \circ \lambda(a), \chi_M \circ \lambda(bc)\}\} \\ =& \max\{\mu \circ \mu \circ \chi_M \circ \mu(x), \lambda \circ \lambda \circ \chi_M \circ \lambda(x)\} \\ =& \mu \circ \mu \circ \chi_M \circ \mu \cup \lambda \circ \lambda \circ \chi_M \circ \lambda(x).\end{aligned}$$

Hence $\mu \cup \lambda \circ \mu \cup \lambda \circ \chi_M \circ \mu \cup \lambda = \mu \circ \mu \circ \chi_M \circ \mu \cup \lambda \circ \lambda \circ \chi_M \circ \lambda \subseteq \mu \cup \lambda$.
Therefore $\mu \cup \lambda$ is a fuzzy right tri-ideal of M. □

Corollary 3.16. *If μ and λ are fuzzy left tri-ideals of a semiring M then $\mu \cup \lambda$ is a fuzzy left tri-ideal of M.*

Corollary 3.17. *Let μ and λ be fuzzy tri-ideals of a semiring M. Then $\mu \cup \lambda$ is a fuzzy tri-ideal of M.*

Theorem 3.18. *M is a regular semiring if and only if $\mu = \mu \circ \chi_M \circ \mu \circ \mu$ for any fuzzy left tri-ideal μ of M.*

Proof. Let μ be a fuzzy left tri-ideal of the regular semiring M and let $x \in M$. Then there exists $y \in M$ such that $x = xyx$ and $\mu \circ \chi_M \circ \mu \circ \mu \subseteq \mu$. Thus

$$\begin{aligned}\mu \circ \chi_M \circ \mu \circ \mu(x) &= \sup_{x=xyx} \{\min\{\mu \circ \chi_M(x), \mu \circ \mu(yx)\}\} \\ &\geq \sup_{x=xyx} \{\min\{\mu(x), \mu(yx)\}\} \\ &= \mu(x).\end{aligned}$$

So $\mu \subseteq \mu \circ \chi_M \circ \mu \circ \mu$. Hence $\mu \circ \chi_M \circ \mu \circ \mu = \mu$.

Conversely, suppose $\mu = \mu \circ \chi_M \circ \mu \circ \mu$ for any fuzzy tri-ideal μ of M. Let B be a left tri-ideal of M. Then by Theorem 3.4, χ_B is a fuzzy left tri-ideal of M. Thus

$$\chi_B = \chi_B \circ \chi_M \circ \chi_B \circ \chi_B = \chi_{BMBB},\ B = BMBB.$$

So M is a regular semiring. □

4. Fuzzy soft tri-ideals over semirings

In this section, we introduce the notion of fuzzy soft tri-ideals over semirings and study their properties.

Definition 4.1. Let M be a semiring and E be a parameter set and $A \subseteq E$. Let $\mu : A \to [0,1]^M$ be a mapping. Then (μ, A) is called a *fuzzy soft left* (*right*) *tri-ideal* over M, if it satisfies the following conditions: for each $a \in A$, and for all $x,\ y \in M$

(i) $\mu_a(x + y) \geq \min\{\mu_a(x), \mu_a(y)\}$,

(ii) $\mu_a \circ \chi_M \circ \mu_a \circ \mu_a \subseteq \mu_a \big(\mu_a \circ \mu_a \circ \chi_M \circ \mu_a \subseteq \mu_a\big)$.

A fuzzy soft subset(μ, A) of M is called a *fuzzy soft tri-ideal*, if it is both fuzzy soft left tri-ideal and fuzzy soft right tri-ideal of M.

Example 4.2. Let $M = \{0, a, b, c\}$, define the binary operations " $+$ " and " $\cdot$ " on M with the following tables

$+$	0	a	b	c
0	0	a	b	c
a	a	a	b	c
b	b	b	b	c
c	b	b	b	c

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	a	b	b
c	0	a	b	c

Then $(M, +, \cdot)$ is a semiring and $B = \{0, a, c\}$ is a left tri-ideal of M.
Let $E = \{e_1, e_2, e_3\}$.

Define

	0	a	b	c
f_{e_1}	0.3	0.4	0.2	0.5
f_{e_2}	0.4	0.5	0.3	0.7
f_{e_3}	0.2	0.4	0.1	0.6

$\{f_{e_i}\}, i = 1, 2, 3$. (F, E) is a fuzzy soft tri-ideal over M.

Theorem 4.3. *Let M be a semiring, E be a parameter set and $A \subseteq E$. Let (μ, A) is a fuzzy soft set over M, and χ_M is the characteristic fuzzy set. If (μ, A) is a fuzzy soft right ideal over M, then $\mu_a \circ \chi_M \subseteq \mu_a$ for all $a \in A$.*

Proof. Suppose (μ, A) is a fuzzy soft right ideal over M and let $z \in M$. Then

$$\begin{aligned} \mu_a \circ \chi_M(z) &= \sup_{z=lm} \min\{\mu_a(l), \chi_M(m)\}\ l, m \in M. \\ &= \sup_{z=lm} \min\{\mu_a(l), 1\} \\ &= \sup_{z=lm} \{\mu_a(l)\} \\ &\leq \sup_{z=lm} \mu_a(lm) \\ &= \sup_{z=lm} \{\mu_a(z)\} \\ &= \mu_a(z). \end{aligned}$$

Thus $\mu_a \circ \chi_M \subseteq \mu_a$, for all $a \in A$. □

Theorem 4.4. *Let M be a semiring, E be a parameter set and $A \subseteq E$. If (μ, A) is a fuzzy soft right ideal over M, then (μ, A) is a fuzzy soft left tri-ideal over M.*

Proof. Suppose (μ, A) is a fuzzy soft right ideal of M. Then for each $a \in A$, μ_a is a fuzzy right ideal of the semiring. Let $z \in M$. Then we have

$$\begin{aligned}\mu_a \circ \chi_M(z) &= \sup_{z=lm} \min\{\mu_a(l), \chi_M(m)\} \; l, m \in M,\\ &= \sup_{z=lm} \min\{\mu_a(l), 1\}\\ &= \sup_{z=lm} \{\mu_a(l)\}\\ &\leq \sup_{z=lm} \mu_a(lm)\\ &= \mu_a(z).\end{aligned}$$

Thus $\mu_a \circ \chi_M \subseteq \mu_a$. On the other hand,

$$\begin{aligned}\mu_a \circ \chi_M \circ \mu_a \circ \mu_a(z) &= \sup_{z=lmq} \min\{\mu_a \circ \chi_M(lm), \mu_a \circ \mu_a(q)\}\\ &\leq \sup_{z=lmq} \min\{\mu_a(lm), \mu_a(q)\}\\ &= \mu_a(z).\end{aligned}$$

So (μ, A) is a fuzzy soft left tri-ideal over M. □

Corollary 4.5. *Every fuzzy soft left ideal of a semiring M is a fuzzy soft right tri-ideal over M.*

Corollary 4.6. *Every fuzzy soft ideal of a semiring M is a fuzzy soft tri-ideal over M.*

Theorem 4.7. *Let M be a semiring, E be a parameter set and $A \subseteq E$. Let I be a non-empty subset of a semiring M. Then (I, A) is a soft right tri-ideal of a semiring M if and only if (χ_I, A) is a fuzzy soft right tri-ideal of a semiring M.*

Proof. Suppose (I, A) is a soft right tri-ideal over M and let $a \in A$. Then clearly, I_a is a right tri-ideal of M and χ_I is a fuzzy subsemiring of M. Thus $I_a I_a M I_a \subseteq I_a$. So we have

$$\chi_{I_a} \circ \chi_{I_a} \circ \chi_M \circ \chi_{I_a} = \chi_{I_a I_a M I_a} \subseteq \chi_{I_a}.$$

Hence $\chi_{I_a}$is a fuzzy right tri-ideal of M. Therefore (χ_I, A) is a fuzzy soft right tri-ideal over M.

Conversely, suppose (χ_I, A) is a fuzzy soft right tri-ideal over M. Then I is a subsemiring of M. Thus for each $a \in A$, we have

$$\chi_{I_a} \circ \chi_{I_a} \circ \chi_M \circ \chi_{I_a} \subseteq \chi_{I_a} \Rightarrow \chi_{I_a I_a M I_a} \subseteq \chi_{I_a} \quad \Rightarrow I_a I_a M I_a \subseteq I_a.$$

So I_a is a right tri-ideal of M. Hence (I, A) is a soft right tri-ideal over M. □

Theorem 4.8. *Let M be a semiring, $A \subseteq E$ and (η, A) be a non-empty fuzzy soft over M. Then (μ, A) is a fuzzy soft right tri-ideal over M if and only if the level* *subset* $(\mu_a)_k$ *of* (μ, A) *is a right tri-ideal over* $M, a \in A$ *for every* $k \in [0,1]$, *where* $(\mu_a)_k \neq \phi$.

Proof. Let M be a semiring and (μ, A) be a non-empty fuzzy soft subset over M. Let μ_a be a fuzzy right tri-ideal over U, $(\mu_a)_k \neq \phi, k \in [0,1]$ and $l, m \in M$. Then

$$\begin{aligned} &\mu_a(l) \geq k, \mu_a(m) \geq k \\ \Rightarrow &\mu_a(l+m) \geq \min\{\mu_a(l), \mu_a(m)\} \geq k, \\ \Rightarrow &\mu_a(lm) \geq \min\{\mu_a(l), \mu_a(m)\} \geq k, \\ \Rightarrow &l+m \in (\mu_a)_k, lm \in (\mu_a)_k. \end{aligned}$$

Let $z \in (\mu_a)_k M (\mu_a)_k (\mu_a)_k$. Then $z = lmnp$, where $m \in M, l, n, p \in (\mu_a)_k$. Then

$$\begin{aligned} &\mu_a \circ \mu_a \circ \chi_M \circ \mu_a(z) \geq k \\ \Rightarrow &\mu_a(z) \geq \mu_a \circ \mu_a \circ \chi_M \circ \mu_a(z) \end{aligned}$$

Therefore $z \in (\mu_a)_k$.

Hence $(\mu_a)_k$ is a right tri-ideal over M.

Conversely, suppose that $(\mu_a)_k$ is a right tri-ideal of M and let $a \in A$ for all $k \in [0,1]$. Let $x, y \in M$, $\mu_a(x) = k_1, \mu_a(y) = k_2$ and $k_1 \geq k_2$. Then $x, y \in (\mu_a)_{k2}$. Thus we have

$$\mu_a(x+y) \geq k_2 = \min\{k_1, k_2\} = \min\{\mu_a(x), \mu_a(y)\}$$

and

$$\mu_a(xy) \geq k_2 = \min\{k_1, k_2\} = \min\{\mu_a(x), \mu_a(y)\}.$$

So $x+y \in (\mu_a)_{k_2}$, $xy \in (\mu_a)_{k_2}$. Hence $(\mu_a)_l (\mu_a)_l M (\mu_a)_l \subseteq (\mu_a)_k$ for all $l \in Im\{(\mu_a)\}$.

Now let $k = \min Im\{(\mu_a)\}$. Then $(\mu_a)_k (\mu_a)_k M (\mu_a)_k \subseteq (\mu_a)_k$. Thus $(\mu_a) \circ (\mu_a) \circ \chi_M \circ (\mu_a) \subseteq (\mu_a)$. So (μ_a) is a fuzzy right tri- ideal of M. Hence (μ, A) is a fuzzy soft right tri- ideal over M. □

Theorem 4.9. *Let* M *be a semiring,* E *be a parameter set and* $A,\ B \subseteq E$. *If* (μ, A) *and* (λ, B) *are fuzzy soft right tri-ideals over* M, *then* $(\mu, A) \cap (\lambda, B)$ *is a fuzzy soft right tri-ideal over* M.

Proof. Suppose (μ, A) and (λ, B) are fuzzy soft right tri-ideals of M. By Definition 2.16, we have $(\mu, A) \cap (\lambda, B) = (\gamma, C)$, where $C = A \cup B$.

Case (i): If $c \in A \setminus B$, then $\gamma_c = \mu_c$. Thus γ_c is a fuzzy right tri-ideal of M, since (μ, A) is a fuzzy soft right tri-ideal over M.

Case (ii): If $c \in B \setminus A$, then $\gamma_c = \lambda_c$. Thus γ_c is a fuzzy right tri-ideal of M, since (θ, B) is a fuzzy soft right tri-ideal over M.

Case (iii): If $c \in A \cap B$, and $x, y \in M$, then $\gamma_c = \mu_c \cap \lambda_c$. Thus by Theorem 3.12, γ_c is a fuzzy right tri-ideal of M. So $(\mu, A) \cap (\lambda, B)$ is a fuzzy soft right tri-ideal over M. □

Corollary 4.10. *If* (μ, A) *and* (λ, B) *are fuzzy soft left tri-ideals over semiring* M, *then* $(\mu, A) \cap (\lambda, B)$ *is a fuzzy soft left tri-ideal over* M.

Theorem 4.11. *Let M be a semiring, E be a parameter set and $A,\ B \subseteq E$. If (μ, A) and (λ, B) are fuzzy soft right tri-ideals over M, then $(\mu, A) \cup (\lambda, B)$ is a fuzzy soft right tri-ideal over M.*

Proof. Suppose (μ, A) and (λ, B) are fuzzy soft right tri-ideals over M. By Definition 2.17, we have that $(\mu, A) \cup (\lambda, B) = (\gamma, C)$, where $C = A \cup B$.

Case (i): If $c \in A \setminus B$, then $\gamma_c = \mu_c$. Thus γ_c is a fuzzy right tri-ideal of M, since (μ, A) is a fuzzy soft right tri-ideal over M.

Case (ii): If $c \in B \setminus A$, then $\gamma_c = \lambda_c$. Thus γ_c is a fuzzy right tri-ideal of M, since (λ, B) is a fuzzy soft right tri-ideal over M.

Case (iii): If $c \in A \cap B$, and $x, y \in M$, then $\gamma_c = \mu_c \cup \lambda_c$. Thus by Theorem 3.15, γ_c is a fuzzy right tri-ideal ideal of M. So $(\mu, A) \cup (\lambda, B)$ is a fuzzy soft right tri-ideal over M. □

Corollary 4.12. *If (μ, A) and (λ, B) are fuzzy soft left tri-ideals over a semiring M, then $(\mu, A) \cup (\lambda, B)$ is a fuzzy soft left tri-ideal over M.*

Theorem 4.13. *Let M be a semiring, E be a parameter set and $A \subseteq E$. Then M is a regular if and only if $\mu_a = \mu_a \circ \chi_M \circ \mu_a \circ \mu_a, a \in A$ and (μ, A) is a fuzzy soft left tri-ideal of M.*

Proof. Suppose M is a regular and et (μ, A) be a fuzzy soft left tri-ideal of M and let $x \in M$. Then there exists $y \in M$ such that for each $a \in A$, $\mu_a \circ \chi_M \circ \mu_a \circ \mu_a \subseteq \mu_a$. Thus we have

$$\begin{aligned}
\mu_a \circ \chi_M \circ \mu_a \circ \mu_a(z) &= \sup_{z=zpz} \{\min\{\mu_a(z), \chi_M \circ \mu_a \circ \mu_a(pz)\}\} \\
&\geq \sup_{z=zpz} \{\min\{\mu_a(z), \mu_a(pz)\}\} \\
&\geq \min\{\mu_a(z), \mu_a(pz)\} \\
&\geq \mu_a(z).
\end{aligned}$$

So $\mu_a \subseteq \mu_a \circ \chi_M \circ \mu_a \circ \mu_a$. Hence $\mu_a \circ \chi_M \circ \mu_a \circ \mu_a = \mu_a$. Therefore (μ, A) is a fuzzy soft left tri-ideal of M.

Conversely, suppose $\mu_a = \mu_a \circ \chi_M \circ \mu_a \circ \mu_a$ for any fuzzy soft left tri-ideal (μ, A) over M and $a \in A$. Let T be a tri-ideal of the semiring M. Then χ_T be a fuzzy left tri-ideal of M. Thus $\chi_T = \chi_T \circ \chi_M \circ \chi_T \circ \chi_T = \chi_{TMTT}$, $T = TMTT$. So M is a regular semiring . □

Corollary 4.14. *Let M be a semiring and let E be a parameter set and $A \subseteq E$. Then M is regular if and only if $\mu_a = \mu_a \circ \chi_M \circ \mu_a \circ \mu_a, a \in A$ and (μ, A) is a fuzzy soft right tri-ideal of M.*

Theorem 4.15. *Let M be a semiring, let E be a parameter set and let $A,\ B \subseteq E$. Then M is a regular if and only if $\mu_a \cap \lambda_b \subseteq \mu_a \circ \lambda_b \circ \mu_a \circ \mu_a$ for every fuzzy soft right tri-ideal (μ, A) and every fuzzy soft ideal (λ, B) over M, $a \in A, b \in B$.*

Proof. Suppose M is a regular semiring and let $z \in M$. Then there exists $p \in M$ such that $z = zpz$. Thus we have

$$\begin{aligned}
\mu_a \circ \lambda_b \circ \mu_a \circ \mu_a(z) &= \sup_{z=zpz} \{\min\{\mu_a \circ \lambda_b(zp), \mu_a \circ \mu_a(z)\}\} \\
&= \min\Big\{ \sup_{zp=zpzp} \{\min\{\mu_a(z), \lambda_b(pzp)\}, \\
&\qquad \sup_{z=zpz} \{\min\{\mu_a(z), \mu_a(pz)\}\}\Big\} \\
&\geq \min\{\min\{\mu_a(z), \lambda_b(z)\}, min\{\mu_a(z), \mu_a(pz)\}\} \\
&= \min\{\mu_a(z), \lambda_b(z)\} = \mu_a \cap \lambda_b(z).
\end{aligned}$$

So $\mu_a \cap \lambda_b \subseteq \mu_a \circ \lambda_b \circ \mu_a \circ \mu_a$.

Conversely, suppose $\mu_a \cap \lambda_b \subseteq \mu_a \circ \lambda_b \circ \mu_a \circ \mu_a$ for every fuzzy soft right tri-ideal (μ, A) and every fuzzy soft ideal (λ, B) over M, $a \in A, b \in B$. Let (μ, A) be a fuzzy soft left tri-ideal of M. Then for each $a \in A$ $\mu_a \cap \chi_M \subseteq \mu_a \circ \chi_M \circ \mu_a \circ \mu_a$ and $\mu_a \subseteq \mu_a \circ \chi_M \circ \mu_a \circ \mu_a$. Thus M is a regular semiring. □

Corollary 4.16. *Let M be a semiring, E be a parameter set and A, $B \subseteq E$. Then M is a regular if and only if $\mu_a \cap \lambda_b \subseteq \mu_a \circ \mu_a \circ \lambda_b \circ \mu_a$, for every fuzzy soft left tri-ideal (μ, A) and every fuzzy soft ideal (λ, B) over M, $a \in A, b \in B$.*

5. Conclusion

As a further generalization of ideals, we introduced the notion of a tri-ideal of semiring as a generalization of ideal, (left, right, bi, quasi, bi-quasi, bi-interior, bi-quasi interior, and interior) ideal of semirings [7]. We introduced the notion of fuzzy (soft)right (left) tri-ideal of a semiring and characterized the regular semiring in terms of fuzzy (soft) right(left) tri- ideals of a semiring and studied some of their algebraic properties. We prove that if (μ, A) and (λ, B) are fuzzy soft right(left) tri-ideals over M, then $(\mu, A) \cap (\lambda, B)((\mu, A) \cup (\lambda, B))$ is a fuzzy soft right(left) tri-ideal over semiring M. In continuity of this paper, we study prime tri-ideals, maximal and minimal tri-ideals and fuzzy soft prime tri-ideals over semirings.

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