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Logistic entropy of bone mineral density at diabetes and nondiabetes and new type visualizations

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Abstract. In the medicine, the reference intervals are obtained help of statistical methods. These intervals are evaluated by physicians according to their vocational knowledge and thus disease is identified. The reference intervals must be obtained accurately as they are the basis of the analysis of situation and treatment. Are your reference intervals correct for your patient or is it suitable for the detection of diseases or do medical chart records and reference intervals provide complete and accurate information about the diseases they represent? In this paper, the entropy values in the reference intervals were calculated for normal, partially degraded and full degraded bone mineral density of diabetic and nondiabetics patients and were given entropy images using by fuzzy sets and its entropy concept. Thus, we have measured entropies of reference intervals and given entropy images. Also, we have calculated entropy values for T and Z scores and it has been noticed that entropy values very high in some bone mineral intervals.

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Keywords: Entropy, Fuzzy set, BMD, TBS-score, T-score, Z-score, Fuzzy set in medicine.

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1. Introduction

Let's start with this famous sentence: "Everything in medicine is fuzzy" (See Kazem Sadegh-Zadeh, [1], p.17). In the diagnosis of diseases, physical examination findings of the patient, preliminary information as well as a series of laboratory tests and imaging methods are used. Then the results are interpreted. The treatment method is determined. In these processes, there are many fuzzy situations

and we know that fuzzy sets are widely used in medicine. In medicine, the mathematical models were constructed using by fuzzy logic, fuzzy sets and its membership functions. These mathematical models have made an important methodological contribution to medical research (See [2, 3, 4, 5, 6]). One of the earliest mathematical models was created by Sanchez [7, 8].

Some therapy methods, for example endocrine therapy or chemotherapy, can have harmful effects on bone cells and can lead to rapid bone loss. As a result, the risk of bone fracture increases. Therefore, the images or graphs and various measurement ranges are important in bone density measurement as well as in many areas of medicine. Medical graphs contains medically data for patients. A good medical graph, chart or numeric values will represent a clear picture of the patient related to illness.

These images or graphs also provides vital information to allow healthcare practitioners to make sound decisions based on the information contained in the records. For example the ECG and EEG devices display a graphical image of heart rates and brain waves, respectively and the DXA devices gives numerical values. In medicine, paramedics must be able to interpret graphs very well in order to make the right decision. For instance, knowledge of ventilator waveforms for lungs is important for clinicians and sometimes graphs can be difficult to interpret and it can take long time (See [9, 10] and [11]). Even an infinitesimal amount of mistake can cause colossal difference in the diagnosis process, or some significant data can stay unnoticed, especially in the case of junior doctors. Some times, instead of just observing wave graphics, numerical data obtained from these diagrams are more reliable and easy to understand. In this respect, both graphical representation and numerical values have an important place in diagnosis as they complement each other. Recently, Şengönül et al. [12, 13] have made investigations in the same direction by using fuzzy sets and the entropy concept.

In this paper, the reference intervals of bone mineral density were transformed into fuzzy sets and calculated the entropy values to find the uncertainty contained in the reference interval of diabetes and nondiabetic, so the doctors can keep in view patient treatment using entropy values. Finally, for better observations of patients, with the help of the entropy functions, medical images were given for some bone diseases at diabetes. These images were obtained by using Wolfram Mathematica 7.0.

Now, to explain the main topic of this article, we will summarize the entropy concept for any fuzzy set and related concepts . After we will make some computations but these computations are completely different than Czogala and Leski's computations [14].

Let $\mathscr{X}$ be nonempty crisp set, $\mathbb{R}$ and $\mathbb{N}$ for the set of all real and natural numbers, respectively. According to Zadeh, a *fuzzy subset* of $\mathscr{X}$ is

$$\varnothing \neq \{(x, m(x)) : x \in \mathscr{X}\} \subseteq \mathscr{X} \times [0,1]$$

for some function $m : \mathscr{X} \to [0,1]$ [15]. Let us consider a function $m : \mathbb{R} \to [0,1]$ as a subset of a nonempty base space $\mathbb{R}$. In this case, the function $m(x)$ is called the *membership function* of the fuzzy set F. Let suppose that $m(x)$ be membership

function of the fuzzy set F and the function $h : [0,1] \to [0,1]$ satisfies the following properties:

(1) Monotonically increasing at $[0, \frac{1}{2}]$ and decreasing at $[\frac{1}{2}, 1]$,
(2) $h(x) = 0$ if $x = 0$,
(3) $h(x) = 1$ if $x = \frac{1}{2}$.

Then the function h is called an *entropy function* and the equality $H(m(x)) = h(m(x))$ holds for $x \in \mathbb{R}$.

Some well known entropy functions are given as follows:

$$h_1(x) = 4x(1-x),\ h_2(x) = -x\ln x - (1-x)\ln(1-x),\ h_3(x) = \min\{2x, 2-2x\}$$

and

$$h_4(x) = \begin{cases} 2x, & x \in [0, \frac{1}{2}] \\ 2(1-x), & x \in [\frac{1}{2}, 1]. \end{cases}$$

Note that the function h_1 is the logistic function, h_2 is called *Shannon function* and h_3 is the tent function (See [16, 17, 18, 20, 19, 21]).

The similarity of the membership functions does not reflect the conception of itself, but it will be used for examining the context of the membership functions. Whether a particular shape is suitable or not can be determined only in the context of a particular application. However, that many applications are not overly sensitive to variations in the shape. In such cases, it is convenient to use a simple shape, such as the triangular shape or Gaussian shape of membership function. Let us define fuzzy set A on the set $\mathbb{R}$ with membership function as follows:

$$A(x) = \begin{cases} \frac{h_A}{u_1 - u_0}(x - u_0), & x \in [u_0, u_1) \\ \frac{-h_A}{u_2 - u_1}(x - u_1) + h_A, & x \in [u_1, u_2] \\ 0, & \text{others}, \end{cases} \tag{1.1}$$

where the notations h_A denotes height of the fuzzy sets A, $u_0, u_1, u_2 \in \mathbb{R}$ and $u_0 \le u_1 \le u_2$. For brief, we write triple $(u_0, u_1 : h_A, u_2)$ for fuzzy set A. The notation $\mathscr{F}$ denotes the set of the all fuzzy sets in the form $u = (u_0, u_1 : h_A, u_2)$ on the $\mathbb{R}$.

In the fuzzy set theory, it is known that the fuzziness of a fuzzy set is a important matter and there are many method to measure the fuzziness of a fuzzy set. At first, the fuzziness was thought to be the distance between fuzzy set and its nearest nonfuzzy set. Later, the entropy was used instead of fuzziness [20, 21]. Well, then what is the entropy? These definitions are given in [12] and [13] but here it will be summarized.

Let $F \in \mathscr{F}$ and $m(x)$ be the membership function of the fuzzy set F and consider the function $H : \mathscr{F} \to \mathbb{R}^+$.

(1) $H(F) = 0$ iff F is crisp set,
(2) $H(F)$ has a unique maximum, if $m(x) = \frac{1}{2}$ for all $x \in \mathbb{R}$
(3) For $F, G \in \mathscr{F}$, if $G(x) \le F(x)$ for $F(x) \le \frac{1}{2}$ and $F(x) \le G(x)$ for $F(x) \ge \frac{1}{2}$ then $H(F) \ge H(G)$,
(4) $H(F^c) = H(F)$, where F^c is the complement of the fuzzy set F.

If the conditions (1)–(4) hold, then $H(F)$ is called an *entropy function* of the fuzzy set F (See [22]).

Let $\mathscr{X}$ be a continuous universal set, F is fuzzy set on $\mathscr{X}$, $m(x)$ be membership function of F, and $h \in \{h_1, h_2, h_3\}$. Then the total entropy of the fuzzy set F on the $\mathscr{X}$ is defined as follows:

$$\eta_F = \int_{x\in\mathscr{X}} h(m(x))p(x)dx, \tag{1.2}$$

where $p(x)$ is the probability density function of the available data in $\mathscr{X}$ [23]. If we take $p(x) = 1$ and $h = h_1$ in the (1.2), then the η_F is called a *logistic entropy* of the fuzzy set F.

It is known that the value of η_F is depend on support of the fuzzy set F. If the F is a fuzzy set on the set $\mathbb{R}$ with membership function (1.1), then we see that the logistic entropy of fuzzy set F is equal to

$$\eta_F = c(2h_F - \frac{4}{3}h_F^2)\ell(F) \tag{1.3}$$

for $p(x) = c$ and $h = h_1$, where $\ell(F) = \max\{x - y : x, y \in \overline{\{x \in \mathbb{R} : m(x) > 0\}}\}$ [12].

It is known that the T score is an expression of how much the patient's BMD (bone mineral density) measurements are above or below the average of the BMD measurements of young adults of the same sex. Similarly, the Z-score is a comparison of the bone mineral density of people of the same age, weight, and type as yours. As we said before, the medical images or graphs and various measurement ranges are very important in determining the disease especially medical graphs contains medically data for patients and good medical graph, chart or numeric values will give a clear picture of the patient related to illness. Clearly, these tools provides vital information to allow healthcare practitioners to make decisions based on the information contained in the record. For example, let's take EEG or ECG graphs. The ECG and EEG devices display a graphical image of heart rates and brain waves, respectively and the DXA devices gives numerical values. Likewise, although bone density is given numerically, it can be displayed graphically.

A high trabecular bone score indicates dense and well-connected bone microarchitecture; Conversely, a low trabecular bone score means that the bone microarchitecture is incomplete and weak.

The bone mineral density defines a measure of the amount of minerals (mostly calcium and phosphorous) contained in a certain volume of bone. Bone mineral density measurements are used to diagnose osteoporosis (a condition marked by decreased bone mass), to see how well osteoporosis treatments are working, and to predict how likely the bones are to break. Low bone mineral density can occur in patients treated for different illness. Also BMD is called, bone density and bone mass (For more, [24, 25, 26, 27, 28, 29]).

2. Some entropy computations and new type visualizations for BMD in diabetes and nondiabets

Recently, Siddapur et all. [27] have studied and compared on bone mineral density values. They have observed 30 patients for T scores at type 2 diabetic and nondiabetic postmenopausal women with osteoporosis. They have determined some intervals about 30 Type 2 diabetic and 30 nondiabetic postmenopausal women and these intervals have given as a table in [27]. According to the tables of Siddapur

et all., if Age $\in [59.44 \pm 7.42]$, Weight $\in [62.6 \pm 9.04]$ and Height $\in [152.95 \pm 5.29]$ then the BMI $\in [26.79 \pm 3.88]$, BMD $\in [0.83 \pm 0.06]$, T score $\in [-2.84 \pm 0.42]$, Fasting serum glucose $\in [164.5 \pm 62.19]$ and Serum zinc $\in [62.4 \pm 13.35]$ for diabetics. Also they have obtained that if Age $\in [59.36 \pm 7.44]$, Height $\in [151.03 \pm 6.45]$ and Weight $\in [53.73 \pm 9.28]$ then the BMI $\in [23.56 \pm 7.44]$, BMD $\in [0.79 \pm 0.09]$, T score $\in [-3.22 \pm 0.74]$, Fasting serum glucose $\in [87.6 \pm 7.29]$ and Serum zinc $\in [68.2 \pm 13.86]$ for nondiabetics, where $[\beta \pm \alpha]$ is equal the interval $[\beta - \alpha, \beta + \alpha]$.

Using to the data which it was given by Siddapur et all. [27], the entropies of BMD of the diabetic and nondiabetic are calculated as follows, with respect to $p(x) = 1$:

(2.1)

$$\eta_{diabetic}(BMD) = \int_{x\in[0.77,0.83]} h_1(s_{diabet}(BMD)(x))p(x)dx + \int_{x\in[0.83,0.88]} h_1(s_{diabet}(BMD)(x))p(x)dx = 0.05 \text{ (See Figure 1.)},$$

(2.2)

$$\eta_{nondiabetic}(BMD) = \int_{x\in[0.7,0.79]} h_1(s_{nondiabet}(BMD)(x))p(x)dx + \int_{x\in[0.79,0.88]} h_1(s_{nondiabet}(BMD)(x))p(x)dx = 0.075$$

(See Figure 2.) respectively. Where the notations $\eta_{diabet}(BMD)$, $\eta_{nondiabet}(BMD)$ are denotes membership functions of BMD of diabetic patients and nondiabetic patients, respectively and membership functions are determined with (1.1), given in section 4. It is seen that in (2.1) and (2.2) the entropy of BMD of nondiabetic is hight than diabetic person. That is

$$0.05 = \eta_{diabetic}(BMD) < \eta_{nondiabetic}(BMD) = 0.075.$$

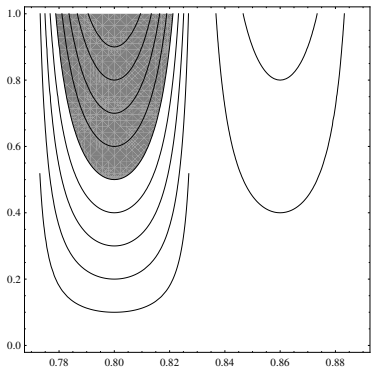

FIGURE 1.
New type visualizations of BMD of diabetes according to data of [27].

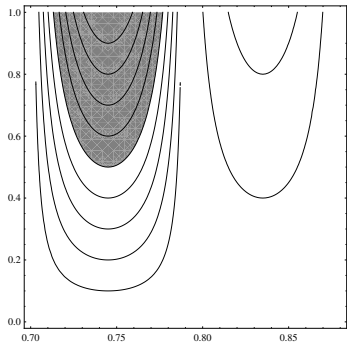

FIGURE 2.
New type visualizations of BMD of nondiabetes according to data of [27].

Similarly to above, we can calculate the entropies of Fasting serum glucose, serum zinc and T-scores for diabetics and non diabetics as follows:

(2.3)

$$\eta_{diabetic}(FSG) = \int_{x\in[102.31,164.5]} h_1(s_{diabet}(FSG)(x))p(x)dx + \int_{x\in[164.5,226.69]} h_1(s_{diabet}(FSG)(x))p(x)dx = 51.825 \text{ (See Figure 3.)},$$

(2.4)

$$\eta_{nondiabetic}(FSG) = \int_{x\in[80.31,87.6]} h_1(s_{nondiabet}(FSG)(x))p(x)dx + \int_{x\in[87.6,94.89]} h_1(s_{nondiabet}(FSG)(x))p(x)dx = 6.075 \text{ (See Figure 4.)}.$$

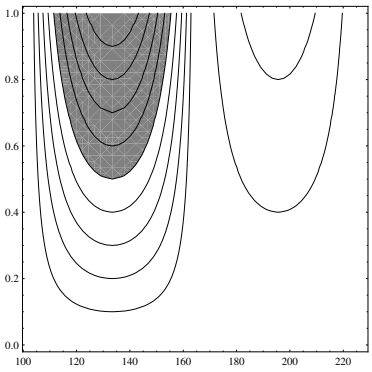

FIGURE 3.
New type visualizations of Fasting Serum Glucose of diabetes according to data of [27].

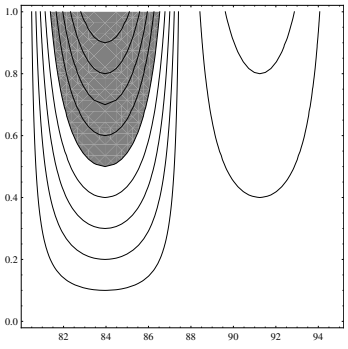

FIGURE 4.
New type visualizations of Fasting Serum Glucose of nondiabetic according to data of [27].

(2.5)

$$\eta_{diabetic}(SZ) = \int_{x\in[49.05,62.4]} h_1(s_{diabet}(SZ)(x))p(x)dx + \int_{x\in[62.4,75.75]} h_1(s_{diabet}(SZ)(x))p(x)dx = 11.125 \text{ (See Figure 5.)},$$

(2.6)

$$\eta_{nondiabetic}(SZ) = \int_{x\in[54.34,68.2]} h_1(s_{nondiabet}(SZ)(x))p(x)dx + \int_{x\in[68.2,82.06]} h_1(s_{nondiabet}(SZ)(x))p(x)dx = 11.55 \text{ (See Figure 6.)}.$$

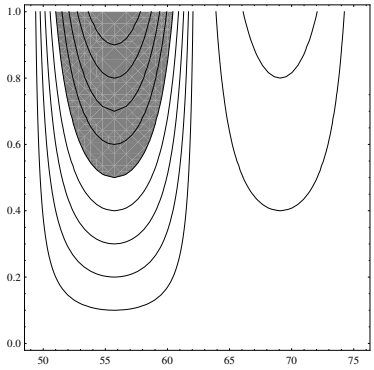

FIGURE 5.
New type visualizations of Serum Zinc of diabetes according to data of [27].

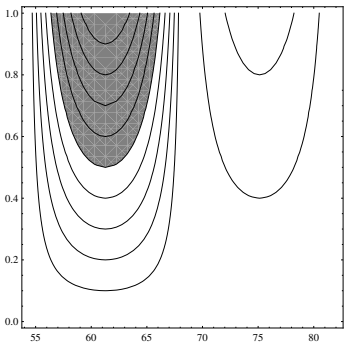

FIGURE 6.
New type visualizations of Serum Zinc of nondiabetes according to data of [27].

It is seen that in (2.5) and (2.6) the entropy of serum zinc of nondiabetic is hight than diabetic person. That is

$$11.25 = \eta_{diabetic}(SZ) < \eta_{nondiabetic}(SZ) = 11.55.$$

Similarly, we can write that a comparison between $\eta_{diabetic}(FSG)$ with $\eta_{nondiabetic}(FSG)$ as follows:

$$51.825 = \eta_{diabetic}(FSG) > \eta_{nondiabetic}(FSG) = 6.075.$$

3. Comments and Results

In generally, according to our computations:

(1) The entropy of the bone mineral density in diabetes is low than non diabetes person.
(2) The entropy of the serum zinc in diabetes is low than non diabetes person.
(3) The entropy of the fasting serum glucose in diabetes is biggest than non diabetes person.

4. Entropy functions list of bone mineral density

$$h_1(s_{diabet}(BMD)(x)) = \begin{cases} \frac{4(x-0.77)}{0.06}\frac{(1-(x-0.77))}{0.06}, & 0.77 \le x < 0.83 \\ \frac{(0.89-x)}{0.06}\frac{(1-(0.89-x))}{0.06}, & 0.83 \le x \le 0.89 \\ 0, & \text{other wise} \end{cases}$$

$$h_1(s_{nondiabet}(BMD)(x)) = \begin{cases} \frac{4(x-0.7)}{0.09}\frac{(1-(x-0.7))}{0.09}, & 0.7 \le x < 0.79 \\ \frac{(0.88-x)}{0.09}\frac{(1-(0.88-x))}{0.09}, & 0.79 \le x \le 0.88 \\ 0, & \text{other wise} \end{cases}$$

$$h_1(s_{diabet}(FSG)(x)) = \begin{cases} \frac{4(x-102.31)}{62.19}\frac{(1-(x-102.31))}{62.19}, & 102.31 \le x < 164.5 \\ \frac{(226.69-x)}{62.19}\frac{(1-(226.69-x))}{62.19}, & 164.5 \le x \le 226.69 \\ 0, & \text{other wise} \end{cases}$$

$$h_1(s_{nondiabet}(FSG)(x)) = \begin{cases} \frac{4(x-80.31)}{7.29}\frac{(1-(x-80.31))}{7.29}, & 80.31 \le x < 87.6 \\ \frac{(94.89-x)}{7.29}\frac{(1-((94.89-x))}{7.29}, & 87.6 \le x \le 94.89 \\ 0, & \text{other wise} \end{cases}$$

$$h_1(s_{nondiabet}(SZ)(x)) = \begin{cases} \frac{4(x-49.05)}{13.35}\frac{(1-(x-49.05))}{13.35}, & 49.05 \le x < 62.4 \\ \frac{(75.75-x)}{13.35}\frac{(1-(75.75-x))}{13.35}, & 62.4 \le x \le 75.75 \\ 0, & \text{other wise} \end{cases}$$

$$h_1(s_{nondiabet}(SZ)(x)) = \begin{cases} \frac{4(x-54.34)}{13.86}\frac{(1-(x-54.34))}{13.86}, & 54.34 \le x < 68.2 \\ \frac{(82.06-x)}{13.86}\frac{(1-(82.06-x))}{3.86}, & 68.2 \le x \le 82.06 \\ 0, & \text{other wise.} \end{cases}$$

5. Conflict of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

6. Author Contributions

This study was equally contributed by each authors.

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