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Abstract. In this paper, the notions of fuzzy P-sets and fuzzy $P^{'}$-sets in fuzzy topological spaces are introduced and several characterizations of these sets are established. A condition for the existence of fuzzy σ-nowhere dense sets in fuzzy hyperconnected spaces is obtained by means of fuzzy P-sets. The conditions under which fuzzy co-σ-boundary sets and fuzzy closed sets in fuzzy perfectly disconnected spaces become fuzzy $P^{'}$-sets are established. A condition for fuzzy closed F_σ-sets in fuzzy $F^{'}$-spaces to become fuzzy $P^{'}$-sets is also obtained in this paper.

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Keywords: Fuzzy G_δ-set, Fuzzy F_σ-set, Fuzzy somewhere dense set, Fuzzy P-space, Fuzzy σ-boundary set, Fuzzy $F^{'}$-space, Fuzzy weakly Baire space, Fuzzy perfectly disconnected space, Fuzzy hyperconnected space.

Corresponding Author: G.Thangaraj (g.thangaraj@rediffmail.com)

1. Introduction

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by Zadeh [1] in 1965. Any application of mathematical concepts depends firmly and closely how one introduces basic ideas that may yield various theories in various directions. If the basic idea is suitably introduced, then not only the existing theories stand but also the possibility of emerging new theories increases and on these lines, Chang [2] introduced the notion of fuzzy topological spaces by means of fuzzy sets in 1967 and his work paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In 1970, Veksler introduced P-sets [3] and P′-sets [4] in classical topology. Atalla [5] developed the concept of P-sets in F′-spaces. In recent years, the topological space theory has been embedding in the soft set theory to obtain some interesting applications [6, 7, 8, 9]. Recently, Şenel et al. [10] applied the concept of octahedron sets proposed by Lee et al. [11] to multi-criteria group decision making problems.

In this paper, the concepts of fuzzy P-sets and fuzzy $P^{'}$-sets in fuzzy topological spaces are introduced and several characterizations of these sets are established. A condition for the existence of fuzzy σ-nowhere dense sets in fuzzy hyperconnected spaces is obtained by means of fuzzy P-sets. The conditions for the existence of fuzzy $P^{'}$-sets in fuzzy perfectly disconnected spaces are also obtained. The conditions under which fuzzy P-sets become fuzzy $P^{'}$-sets in fuzzy topological spaces and the fuzzy co-σ-boundary sets and fuzzy closed sets in fuzzy perfectly disconnected spaces become fuzzy $P^{'}$-sets are established. It is also established that if the fuzzy co-σ-boundary sets are fuzzy dense sets in fuzzy perfectly disconnected spaces, then the fuzzy co-σ-boundary sets are not fuzzy $P^{'}$-sets. Finally a condition for fuzzy closed F_σ-sets in fuzzy $P^{'}$-spaces to become fuzzy $P^{'}$-sets is obtained in this paper. There is a need and scope of investigation considering different types of P-sets to study fuzzy $F^{'}$-spaces and fuzzy F-spaces.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel, are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval $[0, 1]$. A fuzzy set λ in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$ for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$ for all $x \in X$.

Definition 2.1 ([2])**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The *interior*, the *closure* and the *complemen*t of λ are defined respectively as follows:

(i) $int(\lambda) = \bigvee\{\mu/\mu \leq \lambda, \mu \in T\}$,
(ii) $cl(\lambda) = \bigwedge\{\mu/\lambda \leq \mu, 1-\mu \in T\}$,
(iii) $\lambda'(x) = 1 - \lambda(x)$, for all $x \in X$.

For a family $\{\lambda_i/i \in I\}$ of fuzzy sets in (X, T), the *union* $\psi = \bigvee_i \lambda_i$ and the *intersection* $\delta = \bigwedge_i \lambda_i$, are defined respectively as

(iv) $\psi(x) = sup_i\{\lambda_i(x)/x \in X\}$.
(v) $\delta(x) = inf_i\{\lambda_i(x)/x \in X\}$.

Lemma 2.2 ([12])**.** *For a fuzzy set λ of a fuzzy topological space X,*

(1) $1 - int(\lambda) = cl(1 - \lambda)$,
(2) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.3. A fuzzy set λ in a fuzzy topological space (X, T) is called a

(i) *fuzzy dense set*, if there exist no fuzzy closed set μ in X such that $\lambda < \mu < 1$, i.e., $cl(\lambda) = 1$ in X [13],
(ii) *fuzzy nowhere dense set*, if there exist no non-zero fuzzy open set μ in X such that $\mu < cl(\lambda)$, i.e., $intcl(\lambda) = 0$ in X [13],
(iii) *fuzzy somewhere dense set*, if there exists a non-zero fuzzy open set μ in X such that $\mu < cl(\lambda)$, i.e., $intcl(\lambda) \neq 0$ in X [14],
(iv) *fuzzy pre-open set*, if $\lambda \leq intcl(\lambda)$ and *fuzzy pre-closed set*, if $clint(\lambda) \leq \lambda$

in X [15],

(v) *fuzzy regular open set*, if $\lambda = intcl(\lambda)$ and *fuzzy regular closed set*, if $\lambda = clint(\lambda)$ in X [12],

(vi) *fuzzy G_δ-set*, if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ [17],

(vii) *fuzzy F_σ-set*, if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in T$ [17],

(viii) *fuzzy σ-boundary set*, if $\lambda = \vee_{i=1}^{\infty}(\mu_i)$, where $\mu_i = cl(\lambda_i) \wedge (1 - \lambda_i)$ and $(\lambda_i)'s$ are fuzzy regular open sets in X [17],

(ix) *fuzzy σ-nowhere dense set*, if λ is a fuzzy F_σ-set in X such that $int(\lambda) = 0$ [18].

Definition 2.4. A fuzzy topological space (X, T) is called a

(i) *fuzzy hyperconnected space*, if every non-null fuzzy open subset of X is fuzzy dense in X [19],

(ii) *fuzzy perfectly disconnected space*, if for any two non-zero fuzzy sets λ and μ defined on X with $\lambda \le 1 - \mu$, $cl(\lambda) \le 1 - cl(\mu)$ in X [20],

(iii) *fuzzy F'-space*, if $\lambda \le 1 - \mu$, where λ and μ are fuzzy F_σ-sets in X, then $cl(\lambda) \le 1 - cl(\mu)$ in X) [21],

(iv) f*uzzy weakly Baire space*, if $int(\bigvee_{i=1}^{\infty} \mu_i) = 0$, where $\mu_i = cl(\lambda_i) \wedge (1 - \lambda_i)$ and $(\lambda_i)'s$ are fuzzy regular open sets in X [17],

(v) *fuzzy globally disconnected space*, if each fuzzy semi-open set in X is fuzzy open, i.e., if $\lambda \le clint(\lambda)$ for a fuzzy set λ defined on X, then $\lambda \in T$ [22],

(vi) *fuzzy P-space*, if each fuzzy G_δ-set in X is fuzzy open in X [23],

(vii) *fuzzy open hereditarily irresolvable space*, if $intcl(\lambda) \ne 0$, then $int(\lambda) \ne 0$ for any non-zero fuzzy set λ in X [24].

Theorem 2.5 ([17])**.** *If λ is a fuzzy σ-boundary set in a topological space (X, T), then λ is a fuzzy F_σ-set in X.*

Theorem 2.6 ([20])**.** *If $\lambda \le 1 - \mu$, where μ is a fuzzy closed set in a fuzzy perfectly disconnected space (X, T), then there exists a fuzzy regular open set δ in X) such that $int(\lambda) \le \delta \le 1 - \mu$.*

Theorem 2.7 ([20])**.** *If λ is a fuzzy set in a fuzzy perfectly disconnected space (X, T), then $int(\lambda)$ is a fuzzy closed set in X.*

Theorem 2.8 ([14])**.** *If λ is a fuzzy somewhere dense set in a topological space (X, T), then $cl(\lambda)$ is a fuzzy somewhere dense set in X.*

Theorem 2.9 ([17])**.** *If λ is a fuzzy co-σ-boundary set in a fuzzy perfectly disconnected space (X, T), then there exists a fuzzy G_δ-set δ in X such that $\lambda \le \delta$.*

Theorem 2.10 ([17])**.** *Let (X, T) be the fuzzy topological space. Then the following are equivalent:*

(1) *X is a fuzzy weakly Baire space,*

(2) *$int(\lambda) = 0$ for each fuzzy σ-boundary set λ in X,*

(3) *$cl(\gamma) = 1$ for each fuzzy co-σ-boundary set γ in X.*

Theorem 2.11 ([18])**.** *In a fuzzy topological space (X, T), a fuzzy set λ is a fuzzy σ-nowhere dense set in X if and only if $1 - \lambda$ is a fuzzy dense and fuzzy G_δ-set in X.*

Theorem 2.12 ([25]). *If λ is a fuzzy somewhere dense set in a fuzzy hyperconnected and fuzzy open hereditarily irresolvable space (X, T), then $1 - \lambda$ is a fuzzy nowhere dense set in X.*

Theorem 2.13 ([25]). *If δ is a fuzzy somewhere dense set in a fuzzy hyperconnected, fuzzy open hereditarily irresolvable and fuzzy globally disconnected space (X, T), then δ is a fuzzy open set in X.*

3. Fuzzy P-sets

Motivated by the works of Veksler [3] and Atalla [5], the notion of fuzzy P-set in a fuzzy topological space is defined as follows:

Definition 3.1. A fuzzy closed set λ in a fuzzy topological space (X, T) is called a *fuzzy P-set*, if $\lambda \leq 1 - \mu$, where μ is a fuzzy F_σ- set in X, implies that $\lambda \leq 1 - cl(\mu)$ in X.

Example 3.2. Let $X = \{a, b, c\}$ and let α, β and γ be fuzzy sets in X defined as follows:

$$\alpha(a) = 0.6,\ \alpha(b) = 0.7,\ \alpha(c) = 0.8,$$
$$\beta(a) = 0.8,\ \beta(b) = 0.7,\ \beta(c) = 0.6,$$
$$\gamma(a) = 0.8,\ \gamma(b) = 0.6,\ \gamma(c) = 0.6.$$

Then $T = \{0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\wedge\beta, \alpha\wedge\gamma, 1\}$ is a fuzzy topology on X. By computation, one can find that $[1-(\alpha\wedge\gamma)] = (1-\alpha)\vee(1-\beta)\vee(1-\gamma)\vee[1-(\alpha\vee\beta)]\vee[1-(\alpha\wedge\beta)]$ is a fuzzy F_σ-set in (X, T). Also $[1 - (\alpha \wedge \gamma)]$ is a fuzzy closed set in (X, T). Thus $cl[1 - (\alpha \wedge \gamma)] = [1 - (\alpha \wedge \gamma)]$. Also $(1 - \alpha) \leq 1 - [1 - (\alpha \wedge \gamma)] = \alpha \wedge \gamma$ and $(1 - \alpha) \leq (\alpha \wedge \gamma) = 1 - [1 - (\alpha \wedge \gamma)] = 1 - cl[1 - (\alpha \wedge \gamma)]$. So $1 - \alpha$ is a fuzzy P-set in (X, T). Also one can find that $1 - \beta, 1 - \gamma, 1 - (\alpha \vee \beta), 1 - (\alpha \wedge \beta)$ are fuzzy P-sets in (X, T).

Example 3.3. Let $X = \{a, b, c\}$ and let α, β and γ be fuzzy sets in X defined as follows:

$$\alpha(a) = 0.5,\ \alpha(b) = 0.3,\ \alpha(c) = 0.5,$$
$$\beta(a) = 0.6,\ \beta(b) = 0.5,\ \beta(c) = 0.7,$$
$$\gamma(a) = 0.5,\ \gamma(b) = 0.4,\ \gamma(c) = 0.6.$$

Then $T = \{0, \alpha, \beta, \gamma, 1\}$ is a fuzzy topology on X. By computation, one can find that $1 - \alpha = (1 - \alpha) \vee (1 - \beta) \vee (1 - \gamma)$ is a fuzzy F_σ-set in (X, T). By computation, one can find that there is no fuzzy closed set λ in (X, T) such that $\lambda \leq 1 - (1 - \alpha)$, where $1 - \alpha$ is a fuzzy F_σ-set in (X, T). Thus no fuzzy closed set in (X, T) is a fuzzy P-set in (X, T).

Proposition 3.4. *If λ is a fuzzy P-set in a fuzzy topological space (X, T) such that $\lambda + \mu \leq 1$, where μ is a fuzzy F_σ-set in X, then $\lambda + cl(\mu) \leq 1$.*

Proof. Let λ be a fuzzy P-set in (X, T). Suppose that $\lambda + \mu \leq 1$, where μ is a fuzzy F_σ- set in X. Then $\lambda \leq 1 - \mu$. Since λ is a fuzzy P-set in X, $\lambda \leq 1 - cl(\mu)$. Thus $\lambda + cl(\mu) \leq 1$. □

Proposition 3.5. *If λ is a fuzzy P-set in a fuzzy topological space (X,T) such that $\lambda \leq 1-\mu_i$ $(i = 1$ to $\infty)$, where $(\mu_i)'s$ are fuzzy closed sets in X, then $\lambda \leq 1 - cl[\bigvee_{i=1}^{\infty} \mu_i]$ in X.*

Proof. Let λ be a fuzzy P-set in a fuzzy topological space X. Suppose that $\lambda \leq 1-\mu_i$, where $\mu_i's$ are fuzzy closed sets in the topological space X. Then $\bigwedge_{i=1}^{\infty} \lambda \leq \bigwedge_{i=1}^{\infty}(1-\mu_i)$. Thus $\lambda \leq 1-\bigvee_{i=1}^{\infty} \mu_i$. Since $(\mu_i)'s$ are fuzzy closed sets in X, $\bigvee_{i=1}^{\infty} \mu_i$ is a fuzzy F_σ-set in X. Let $\mu = \bigvee_{i=1}^{\infty} \mu_i$. Then $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ- set in X. Since λ is a fuzzy P-set in X, $\lambda \leq 1 - cl(\mu)$ in X. Thus $\lambda \leq 1 - cl[\bigvee_{i=1}^{\infty} \mu_i]$ in X. □

Corollary 3.6. *If λ is a fuzzy P-set in a fuzzy topological space (X,T) such that $\lambda \leq 1-\mu_i$ $(i = 1$ to $\infty)$, where $(\mu_i)'s$ are fuzzy closed sets in X, then $\lambda \leq 1 - intcl[\bigvee_{i=1}^{\infty} \mu_i]$ in X.*

Proof. Let λ be a fuzzy P-set in (X,T). Then λ is a fuzzy closed set in X. Thus $cl(\lambda) = \lambda$. By hypothesis, $\lambda \leq 1-\mu_i$, where $(\mu_i)'s$ are fuzzy closed sets in X. By the Proposition 3.5, $\lambda \leq 1 - cl[\vee_{i=1}^{\infty}(\mu_i)]$. So $cl(\lambda) \leq cl\{1 - cl[\vee_{i=1}^{\infty}(\mu_i)]\}$. Hence $\lambda \leq 1 - intcl[\bigvee_{i=1}^{\infty} \mu_i]$ in X. □

Proposition 3.7. *If $int(\lambda) \neq 0$ for a fuzzy P-set in a fuzzy topological space (X,T) such that $\lambda + \mu \leq 1$, where μ is a fuzzy F_σ-set in X, then μ is not a fuzzy dense set in X.*

Proof. Let λ be a fuzzy P-set in (X,T) such that $\lambda+\mu \leq 1$, where μ is a fuzzy F_σ-set in X. By Proposition 3.4, $\lambda \leq 1-cl(\mu)$ in (X,T). Then $int(\lambda) \leq int[1-cl(\mu)]$. Thus $int(\lambda) \leq 1 - cl[cl(\mu)]$. So $int(\lambda) \leq 1 - cl(\mu)$. Hence $cl(\mu) \leq 1 - int(\lambda)$.

Now let $\delta = int(\lambda)$. Then δ is a fuzzy open in X. Thus $cl(\mu) \leq 1-\delta$. So $cl(\mu) \neq 1$, in (X,T). Hence μ is not a fuzzy dense set in X. □

Proposition 3.8. *If λ is a fuzzy P-set in a fuzzy topological space (X,T) such that $\lambda \leq \mu$, where μ is a fuzzy G_δ-set in (X,T), then $\lambda \leq int(\mu)$ in X.*

Proof. Let λ be a fuzzy P-set in (X,T) such that $\lambda \leq \mu$, where μ is a fuzzy G_δ-set in X. Now $\lambda \leq 1-(1-\mu)$, where $1-\mu$ is a fuzzy F_σ-set in X. Since λ is a fuzzy P-set in X, $\lambda \leq 1 - cl(1-\mu)$ in X. Then $\lambda \leq 1 - [1 - int(\mu)]$. Thus $\lambda \leq int(\mu)$ in X. □

Proposition 3.9. *If λ is a fuzzy P-set in a fuzzy topological space (X,T) such that $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in (X,T), then δ is not a fuzzy nowhere dense set in X.*

Proof. Let λ be a fuzzy P-set in (X,T) such that $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in X. Then $\lambda \leq 1-(1-\delta)$, where $1-\delta$ is a fuzzy F_σ-set in X. Thus $\lambda \leq 1 - cl(1-\delta)$ in X. Now $clint(1-\delta) \leq cl(1-\delta)$, implies that $1 - cl(1-\delta) \leq 1 - clint(1-\delta)$. So $\lambda \leq 1 - clint(1-\delta) = intcl(\delta)$, in X. If $intcl(\delta) = 0$, then $\lambda = 0$, contradition to λ being a non-zero fuzzy set in X. Hence $intcl(\delta) \neq 0$. Therefore δ is not a fuzzy nowhere dense set in X. □

Remark 3.10. In view of the above proposition one will have the following result: "If λ is a fuzzy P-set in a fuzzy topological space (X,T) then there is no fuzzy nowhere dense and fuzzy G_δ-set μ in (X,T) such that $\lambda \leq \mu$".

Proposition 3.11. *If $\delta \leq \lambda$, where δ is a fuzzy P-set and λ is a fuzzy co-σ-boundary set in a fuzzy topological space (X, T), then $\delta \leq int(\lambda)$ in X.*

Proof. Let λ be a fuzzy co-σ-boundary set in (X, T). Then $1 - \lambda$ is a fuzzy σ-boundary set in X. Thus by Theorem 2.5, $1 - \lambda$ is a fuzzy F_σ-set in X. Now $\delta \leq \lambda$ implies that $\delta \leq 1 - (1 - \lambda)$. Since δ is a fuzzy P-set in X, $\delta \leq 1 - cl(1 - \lambda)$. Hence Lemma 2.2, $\delta \leq int(\lambda)$ in X. □

Proposition 3.12. *If $\delta \leq 1 - \lambda$, where δ is a fuzzy P-set in a fuzzy topological space (X, T) and λ is a fuzzy σ-boundary set in X, then $\delta \leq int(1 - \lambda)$ in X.*

Proof. Let λ be a fuzzy σ-boundary set in (X, T). Then by Theorem 2.5, λ is a fuzzy F_σ-set in X. Since δ is a fuzzy P-set in X and $\delta \leq 1 - \lambda$, $\delta \leq 1 - cl(\lambda)$. Thus $\delta \leq int(1 - \lambda)$ in X. □

Proposition 3.13. *If λ is a fuzzy P-set in a fuzzy topological space (X, T) such that $\lambda \leq 1 - \mu$, where μ is a fuzzy F_σ-set in X, then $\lambda \leq 1 - int(\mu)$ and $\lambda \leq 1 - cl(\mu)$ in X.*

Proof. Let λ be a fuzzy P-set in (X, T) such that $\lambda \leq 1 - \mu$, where μ is a fuzzy F_σ-set in X. Since λ is a fuzzy P-set, $\lambda \leq 1 - cl(\mu)$. Also $cl(\lambda) \leq cl(1 - \mu)$. Since λ is a fuzzy P- set, λ is a fuzzy closed set in X. Then $\lambda = cl(\lambda) \leq cl(1 - \mu) = 1 - int(\mu)$ in X. Thus $\lambda \leq 1 - cl(\mu)$ and $\lambda \leq 1 - int(\mu)$ in X. □

Corollary 3.14. *If λ is a fuzzy P-set in a fuzzy topological space (X, T), such that $\lambda \leq 1 - \mu$, where μ is a fuzzy F_σ-set in (X, T), then there exists a fuzzy open set δ in X such that $\lambda \leq \delta \leq 1 - int(\mu)$.*

Proof. Let λ be a fuzzy P-set in (X, T) such that $\lambda \leq 1 - \mu$, where μ is a fuzzy F_σ-set in (X, T). Then by Proposition 3.13, $\lambda \leq 1 - cl(\mu)$ and $\lambda \leq 1 - int(\mu)$ in X. Thus $\lambda \leq 1 - cl(\mu) \leq 1 - int(\mu)$. Now let $\delta = 1 - cl(\mu)$. Then δ is a fuzzy open set in X. Thus, $\lambda \leq \delta \leq 1 - int(\mu)$ in X. □

Proposition 3.15. *If λ is a fuzzy P-set in a fuzzy topological space (X, T) such that $\lambda \leq \mu$, where μ is a fuzzy G_δ-set in (X, T), then there exists a fuzzy open set η in X such that $\lambda \leq \eta \leq \mu$.*

Proof. Let λ be a fuzzy P-set in (X, T) such that $\lambda \leq \mu$, where μ is a fuzzy G_δ-set in X. Then by Proposition 3.8, $\lambda \leq int(\mu)$ in (X, T). Let $\eta = int(\mu)$. Then η is a fuzzy open set in X. Thus there exists a fuzzy open set η in X such that $\lambda \leq \eta \leq \mu$. □

4. Fuzzy P-sets and fuzzy F′-spaces

Proposition 4.1. *If λ is a fuzzy closed F_σ-set in a fuzzy F′-space such that $\lambda + \mu \leq 1$, where μ is a fuzzy F_σ-set in (X, T), then λ is a fuzzy P-set in X.*

Proof. Suppose that $\lambda + \mu \leq 1$, where λ is a fuzzy closed F_σ-set and μ is a fuzzy F_σ-set in X. Then $\lambda \leq 1 - \mu$, where λ and μ are fuzzy F_σ-sets in X. Since (X, T) is a fuzzy F′-space for $\lambda \leq 1 - \mu$, $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T). Also since λ is a fuzzy closed set, $\lambda = cl(\lambda) \leq 1 - cl(\mu)$. Thus $\lambda \leq 1 - cl(\mu)$ in X. So for the fuzzy closed set λ with $\lambda \leq 1 - \mu, \lambda \leq 1 - cl(\mu)$ in X implies that λ is a fuzzy P-set in X. □

Proposition 4.2. *If λ is a fuzzy F_σ-set in the fuzzy* F$'$*-space and fuzzy P-space (X, T) such that $\lambda \leq 1 - \mu$, where μ is a fuzzy F_σ-set in (X, T), then λ is a fuzzy P-set in X.*

Proof. Let λ be a fuzzy F_σ-set in (X, T) such that $\lambda \leq 1 - \mu$, where μ is a fuzzy F_σ-set in X. Since (X, T) is a fuzzy P-space, the fuzzy F_σ-set λ is a fuzzy closed set in X. Then by Proposition 4.1, the fuzzy closed F_σ-set λ in the fuzzy F$'$-space (X, T) is a fuzzy P-set in X. □

Proposition 4.3. *If λ is a fuzzy closed F_σ-set in a fuzzy* F$'$*-space and fuzzy P-space (X, T) such that $\lambda + \mu \leq 1$, where μ is a fuzzy F_σ-set in (X, T), then*
(1) *$1 - \mu$ is a fuzzy somewhere dense set in X,*
(2) *$int(\mu)$ is not a fuzzy dense set in X,*
(3) *$cl(1 - \mu)$ is a fuzzy somewhere dense set in X.*

Proof. (1) Let λ be a fuzzy closed F_σ-set in X such that $\lambda + \mu \leq 1$, where μ is a fuzzy F_σ-set in X. Since (X, T) is a fuzzy F$'$-space and fuzzy P-space by Proposition 4.2, λ is a fuzzy P-set in X. Now $\lambda + \mu \leq 1$, implies that $\lambda \leq 1 - \mu$ and $1 - \mu$ is a fuzzy G_δ-set in X. Then by Proposition 3.9, $1 - \mu$ is not a fuzzy nowhere dense set in X. That is, $intcl(1 - \mu) \neq 0$. Thus $1 - \mu$ is a fuzzy somewhere dense set in X.

(2) By (1), $intcl(1 - \mu) \neq 0$ in X. Then $1 - clint(\mu) \neq 0$. Thus $clint(\mu) \neq 1$ in X. So $int(\mu)$ is not a fuzzy dense set in X.

(3) By (1), $1 - \mu$ is a fuzzy somewhere dense set in X. Then by Theorem 2.8, $cl(1 - \mu)$ is a fuzzy somewhere dense set in X. □

Proposition 4.4. *If $\lambda \leq \gamma$, where λ is a fuzzy closed F_σ-set and γ is a fuzzy G_δ-set in a fuzzy* F$'$*-space (X, T), then λ is a fuzzy P-set in X.*

Proof. Let λ be a fuzzy closed F_σ-set in (X, T) such that $\lambda \leq \gamma$, where γ is a fuzzy G_δ-set. Then $\lambda \leq 1 - (1 - \gamma)$. Let $\mu = 1 - \gamma$. Then μ is a fuzzy F_σ-set in the fuzzy F$'$-space X. Since X is a fuzzy F$'$-space and $\lambda \leq 1 - \mu$ implies that $cl(\lambda) \leq 1 - cl(\mu)$ in X. Since λ is a fuzzy closed in X, $cl(\lambda) = \lambda$ in X. Thus $\lambda \leq 1 - cl(\mu)$ in X. So λ is a fuzzy P-set in X. □

The following proposition gives a condition for the existence of fuzzy σ-nowhere dense sets in fuzzy hyperconnected spaces by means of fuzzy P-sets.

Proposition 4.5. *If λ is a fuzzy P-set in a fuzzy hyperconnected space (X, T) such that $\lambda \leq 1 - \mu$, where μ is a fuzzy F_σ set in X, then μ is a fuzzy σ-nowhere dense set in X.*

Proof. Let λ be a fuzzy P-set in (X, T) such that $\lambda \leq 1 - \mu$, where μ is a fuzzy F_σ-set in X. By Corollary 3.14, there exists a fuzzy open set δ in X such that $\lambda \leq \delta \leq 1 - int(\mu)$. Then we have

$$cl(\lambda) \leq cl(\delta) \leq cl[1 - int(\mu)] \text{ and } cl(\lambda) \leq cl(\delta) \leq 1 - int[int(\mu)] = 1 - int(\mu) \text{ in } X.$$

Since X is a fuzzy hyperconnected space, the fuzzy open set δ is a fuzzy dense set in X. Thus $cl(\delta) = 1$, i.e., $1 \leq 1 - int(\mu)$ in X. This implies that $int(\mu) \leq 1 - 1 = 0$. So $int(\mu) = 0$ in X. Hence μ is a fuzzy F_σ-set such that $\mathrm{int}(\mu) = 0$. Therefore μ is a fuzzy σ-nowhere dense set in X. □

Proposition 4.6. *If* $\lambda \leq 1-\mu$, *where* λ *is a fuzzy closed set and* μ *is a fuzzy* F_σ*-set in a fuzzy perfectly disconnected space* (X,T), *then* λ *is a fuzzy P-set in* X.

Proof. Let λ be a fuzzy closed set in (X,T). Then $cl(\lambda) = \lambda$ in X. By hypothesis, $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ-set in X. Since (X,T) is the fuzzy perfectly disconnected space, $cl(\lambda) \leq 1 - cl(\mu)$. Thus $\lambda \leq 1 - cl(\mu)$, in (X,T). So λ is a fuzzy P-set in X. □

Proposition 4.7. *If* λ *is a fuzzy P-set such that* $\lambda \leq 1-\mu$, *where* μ *is a fuzzy* F_σ*-set in a fuzzy perfectly disconnected space* (X,T), *then there exists a fuzzy regular open set* δ *in* X *such that* $int(\lambda) \leq \delta \leq 1 - cl(\mu)$.

Proof. Let λ be a fuzzy P-set in (X,T) such that $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ-set in X. Since λ is a fuzzy P-set in X, $\lambda \leq 1 - cl(\mu)$. Also since (X,T) is the fuzzy perfectly disconnected space, by Theorem 2.6, there exists a fuzzy regular open set δ in X such that $int(\lambda) \leq \delta \leq 1 - cl(\mu)$. □

Proposition 4.8. *If* λ *is a fuzzy set defined on* X *such that* $int(\lambda) \leq 1-\mu$, *where* μ *is a fuzzy* F_σ*-set in a fuzzy perfectly disconnected space* (X,T), *then* $int(\lambda)$ *is a fuzzy P-set in* X.

Proof. Let λ be a fuzzy set defined on X in (X,T). Since (X,T) is the fuzzy perfectly disconnected space, by Theorem 2.7, $int(\lambda)$ is a fuzzy closed set in X. By hypothesis, $int(\lambda) \leq 1-\mu$, where μ is a fuzzy F_σ-set in X. Then $int[int(\lambda)] \leq int(1-\mu)$. Thus $int(\lambda) \leq 1 - cl(\mu)$ in X. So for a fuzzy closed set $int(\lambda)$ in (X,T), $int(\lambda) \leq 1-\mu$ implies that $int(\lambda) \leq 1 - cl(\mu)$. Hence $int(\lambda)$ is a fuzzy P-set in X. □

Proposition 4.9. *If* $\lambda \leq 1-\mu$, *where* μ *is a fuzzy* F_σ*-set and* λ *is a fuzzy closed set in a fuzzy P-space* (X,T), *then* λ *is a fuzzy P-set in* (X,T).

Proof. Let λ be a fuzzy closed set in X such that $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ-set in X. Since X is a fuzzy P-space, the fuzzy $F_\sigma-$set μ is a fuzzy closed set in X and $cl(\mu) = \mu$ in X. Then $\lambda \leq 1 - cl(\mu)$ in X. Thus λ is a fuzzy P-set in X. □

Remark 4.10. In view of the above proposition one will have the following result: "If $\lambda \leq \delta$, where δ is a fuzzy G_δ-set and λ is a fuzzy closed set in a fuzzy P-space (X,T), then λ is a fuzzy P-set in X".

5. Fuzzy P′-sets and fuzzy perfectly disconnected spaces

Definition 5.1. A fuzzy closed set λ in a fuzzy topological space (X,T) is called a *fuzzy* P′*-set* in X, if whenever $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ-set in X, then $\lambda \leq 1 - clint(\mu)$ in X.

Example 5.2. Let $X = \{a,b,c\}$. Consider the fuzzy sets α, β and γ are defined on X, as follows:

$$\alpha(a) = 0.5,\ \alpha(b) = 0.5\ \alpha(c) = 0.7,$$

$$\beta(a) = 0.6,\ \beta(b) = 0.7,\ \beta(c) = 0.5,$$

$$\gamma(a) = 0.6,\ \gamma(b) = 0.5\ \gamma(c) = 0.5.$$

Then $T = \{0, \alpha, \beta, \gamma, \alpha\vee\beta, \alpha\vee\gamma, \alpha\wedge\beta, 1\}$ is a fuzzy topology on X. By computation, one can find that

$$cl(\alpha) = 1,\ cl(\beta) = 1,\ cl(\gamma) = 1,\ cl(\alpha \vee \beta) = 1,\ cl(\alpha \vee \gamma) = 1,\ cl(\alpha \wedge \beta) = 1 - \alpha \wedge \beta,$$

$$int(1 - \alpha) = 0,\ int(1 - \beta) = 0,\ int(1 - \gamma) = 0,\ int[1 - (\alpha \vee \beta)] = 0,$$

$$int[1 - (\alpha \vee \gamma)] = 0,\ int[1 - (\alpha \wedge \beta)] = \alpha \wedge \beta.$$

Now $1 - (\alpha \wedge \beta) = (1 - \alpha) \vee (1 - \beta) \vee (1 - \gamma) \vee [1 - (\alpha \vee \beta)] \vee [1 - (\alpha \vee \gamma)]$ and $1-\gamma = (1-\beta) \vee [1-(\alpha\vee\beta)] \vee [1-(\alpha\vee\gamma)]$ implies that $1-(\alpha\wedge\beta)$ and $1-\gamma$ are fuzzy F_σ-set in (X, T). Now $1-\alpha \leq 1-(1-\gamma)$, where $(1-\alpha)$ is a fuzzy closed set and $(1-\gamma)$ is a fuzzy F_σ-set in X and $clint(1-\gamma) = 1-intcl(\gamma) = 1-int(1) = 1-1 = 0$. Then $1-clint(1-\gamma) = 1-0 = 1$ and clearly $1-\alpha \leq 1-clint(1-\gamma)$. Thus $1-\alpha$ is a fuzzy P'-set in X. By computation, one can find that $1 - \beta, 1 - \gamma, 1 - (\alpha \vee \beta), 1 - (\alpha \vee \gamma)$ and $1 - (\alpha \wedge \beta)$ are fuzzy P'-sets in X.

The following propositions provide conditions for the existence of fuzzy P'-sets in fuzzy perfectly disconnected spaces.

Proposition 5.3. *If $\lambda \leq 1 - \mu$, where λ is a fuzzy closed set and μ is a fuzzy F_σ-set in a fuzzy perfectly disconnected space (X, T), then λ is a fuzzy P'-set in X.*

Proof. Let λ be a fuzzy closed set in (X, T) such that $\lambda \leq 1-\mu$, where μ is a fuzzy F_σ-set in X. Now $\lambda \leq 1-\mu$ implies that $cl(\lambda) \leq cl(1-\mu)$. Then $cl(\lambda) \leq 1-int(\mu)$. Since λ is a fuzzy closed set in X, $cl(\lambda) = \lambda$. Thus $\lambda \leq 1 - int(\mu)$ in X. Since (X, T) is the fuzzy perfectly disconnected space, $\lambda \leq 1 - int(\mu)$ implies that $cl(\lambda) \leq 1 - clint(\mu)$. So $\lambda \leq 1 - clint(\mu)$. Hence λ is a fuzzy P'-set in X. □

Proposition 5.4. *If $\beta \leq \alpha$, where β is a fuzzy F_σ-set and α be a fuzzy open set in a fuzzy perfectly disconnected space (X, T), then $1 - \alpha$ is a fuzzy P'-set in X.*

Proof. Let α be a fuzzy open set in (X, T). Then $1 - \alpha$ is a fuzzy closed set in X. By hypothesis, $\beta \leq \alpha$, where β is a fuzzy F_σ-set. Then $1 - \alpha \leq 1 - \beta$ in X. Since X is the fuzzy perfectly disconnected space, by Proposition 5.3, $1 - \alpha$ is a fuzzy P'-set in X. □

Corollary 5.5. *If $\delta \leq \lambda$, where δ is a fuzzy σ-nowhere dense set and λ is a fuzzy open set in a fuzzy perfectly disconnected space (X, T), then $1 - \lambda$ is a fuzzy P'-set in X.*

Proof. Let δ be a fuzzy σ-nowhere dense set in X such that $\delta \leq \lambda$, where $\lambda \in T$. Since δ-is a fuzzy σ-nowhere dense set in X, δ is a fuzzy F_σ-set with $int(\delta) = 0$. Then $\delta \leq \lambda$, where δ is a fuzzy F_σ-set and λ is a fuzzy open set in the fuzzy perfectly disconnected space X. Then by Proposition 5.4, $1 - \lambda$ is a fuzzy P'-set in X. □

The following proposition shows that fuzzy P-sets in fuzzy topological spaces are fuzzy P'-sets.

Proposition 5.6. *If λ is a fuzzy P-set in a fuzzy topological space (X, T) then λ is a fuzzy P'-set in X.*

Proof. Let λ be a fuzzy closed set in (X, T) such that $\lambda \leq 1 - \mu$ in X, where μ is a fuzzy F_σ-set in X. Since λ is a fuzzy P-set in X, $\lambda \leq 1 - cl(\mu)$ in X. Now $clint(\mu) \leq cl(\mu)$ implies that $1 - cl(\mu) \leq 1 - clint(\mu)$. Then $\lambda \leq 1 - clint(\mu)$. Thus for $\lambda \leq 1 - \mu$ in X, $\lambda \leq 1 - clint(\mu)$. So λ is a fuzzy P′-set in X. □

Remark 5.7. The converse of the above proposition need not be true. That is, a fuzzy P′-set in a fuzzy topological space need not be a fuzzy P-set, since it need not be that $cl(\mu) \leq clint(\mu)$ in a fuzzy topological space.

The following proposition gives a condition for fuzzy P′-sets to become fuzzy P-sets in fuzzy topological spaces.

Proposition 5.8. *If λ is a fuzzy* P′*-set in a fuzzy topological space (X, T) in which fuzzy $F_\sigma$sets are fuzzy open, then λ is a fuzzy P-set in X.*

Proof. Let λ be a fuzzy P′-set in (X, T) and then λ is a fuzzy closed set in X. Suppose that $\lambda \leq 1 - \mu$, in X, where μ is a fuzzy F_σ-set in X. Since λ is a fuzzy P′-set in X, $\lambda \leq 1 - clint(\mu)$ in X. By hypothesis, the fuzzy F_σ-set μ is a fuzzy open set in X. Then $int(\mu) = \mu$. Thus $clint(\mu) = cl(\mu)$. So $\lambda \leq 1 - clint(\mu) = cl(\mu)$ in X. Hence for $\lambda \leq 1 - \mu$, in (X, T), $\lambda \leq 1 - cl(\mu)$. Therefore λ is a fuzzy P-set in X. □

Proposition 5.9. *If λ is a fuzzy* P′*-set in a fuzzy topological space (X, T) such that $\lambda + \mu \leq 1$, where μ is a fuzzy F_σ-set in X, then $\lambda + clint(\mu) \leq 1$ in X.*

Proof. Let λ be a fuzzy P′-set in (X, T) such that $\lambda + \mu \leq 1$, where μ is a fuzzy F_σ-set in X. Then $\lambda \leq 1 - \mu$. Since λ is a fuzzy P′-set in X, $\lambda \leq 1 - clint(\mu)$. Thus $\lambda + clint(\mu) \leq 1$ in X. □

Proposition 5.10. *If λ is a fuzzy* P′*-set in a fuzzy topological space (X, T) and $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in X, then δ is a fuzzy somewhere dense set in X.*

Proof. Let λ be a fuzzy P′-set in (X, T). Suppose that $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in X. Then $\lambda \leq 1 - (1 - \delta)$ and $1 - \delta$ is a fuzzy F_σ-set in X. Since λ is a fuzzy P′-set, $\lambda \leq 1 - (1 - \delta)$ implies that $\lambda \leq 1 - clint(1 - \delta)$ in X. Thus $\lambda \leq 1 - [1 - intcl(\delta)]$. So $\lambda \leq intcl(\delta)$. If $intcl(\delta) = 0$, then $\lambda = 0$ a contradiction to λ being a non-zero fuzzy set in X. Hence $intcl(\delta) \neq 0$ in X. Therefore δ is a fuzzy somewhere dense set in X. □

Corollary 5.11. *If λ is a fuzzy* P′*-set in a fuzzy topological space (X, T), then there is no fuzzy nowhere dense fuzzy G_δ-set δ in X such that $\lambda \leq \delta$.*

Proof. Let λ be a fuzzy P′-set in (X, T) such that $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in X. Then by Proposition 5.10, δ is a fuzzy somewhere dense set in X. Thus $intcl(\delta) \neq 0$ in (X, T). So δ is not a fuzzy nowhere dense set in X. Hence there is no fuzzy nowhere dense fuzzy G_δ-set δ in X such that $\lambda \leq \delta$. □

Remark 5.12. From the above proposition one will have the following result: "If λ is a fuzzy P′-set in a fuzzy topological space (X, T) and $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in X, then δ is not a fuzzy nowhere dense set in X."

The following proposition gives a condition for the fuzzy co-σ-boundary sets in fuzzy perfectly disconnected spaces to become fuzzy P′-sets.

Proposition 5.13. *If a fuzzy co-σ-boundary set α is a fuzzy closed set in a fuzzy perfectly disconnected space (X, T), then α is a fuzzy* P'*-set in X.*

Proof. Let α be a fuzzy co-σ-boundary set in (X, T) such that $cl(\alpha) = \alpha$. Since X is the fuzzy perfectly disconnected space, by Theorem 2.9, there exists a fuzzy G_δ-set δ in X such that $\alpha \leq \delta$. Then $1 - \delta \leq 1 - \alpha$, where $1 - \delta$ is a fuzzy F_σ-set and $1 - \alpha$ is a fuzzy open set in X. Thus by Proposition 5.4, $1 - (1 - \alpha)$ is a fuzzy P'-set in X. So α is a fuzzy P'-set in X. □

The following proposition gives a condition under which fuzzy co-σ-boundary sets in fuzzy perfectly disconnected spaces are not fuzzy P'-sets.

Proposition 5.14. *If α is a fuzzy co-σ-boundary set in a fuzzy perfectly disconnected and fuzzy weakly Baire space (X, T), then α is not a fuzzy* P'*-set in X.*

Proof. Let $\alpha(\neq 1)$ be a fuzzy co-σ-boundary set in (X, T). Since (X, T) is the fuzzy weakly Baire space, by Theorem 2.10, $cl(\alpha) = 1$ for the fuzzy co-σ-boundary set α in X. Then $cl(\alpha) \neq \alpha$. Thus α is not a fuzzy closed set in X. So by Proposition 5.13, α is not a fuzzy P'-set in X. □

Remark 5.15. In view of the above proposition, one will have the following result: "If the fuzzy co-σ-boundary sets are fuzzy dense sets in fuzzy perfectly disconnected spaces, then the fuzzy co-σ-boundary sets are not fuzzy P'-sets".

The following proposition gives a condition for fuzzy closed sets in fuzzy perfectly disconnected spaces to become fuzzy P'-sets by means of fuzzy co-σ-boundary sets.

Proposition 5.16. *If $\alpha \leq \beta$, where β is a fuzzy co-σ-boundary set and α is a fuzzy closed set in a fuzzy perfectly disconnected space (X, T), then α is a fuzzy* P'*-set in X.*

Proof. Let β be a fuzzy co-σ-boundary set in (X, T). Since (X, T) is the fuzzy perfectly disconnected space, by Theorem 2.9, there exists a fuzzy G_δ-set δ in X such that $\beta \leq \delta$. Then $\beta \leq 1 - (1 - \delta)$. Let $\mu = 1 - \delta$ and then μ is a fuzzy F_σ-set in X. Then by hypothesis, $\alpha \leq \beta$. Thus $\alpha \leq 1 - \mu$ in X. Since (X, T) is the fuzzy perfectly disconnected space, by Proposition 5.3, α is a fuzzy P'-set in X. □

Proposition 5.17. *If $\lambda \leq 1 - \mu$, where λ is a fuzzy closed set and μ is a fuzzy σ-nowhere dense set in a fuzzy perfectly disconnected space (X, T), then λ is a fuzzy* P'*-set in X.*

Proof. Suppose that $\lambda \leq 1 - \mu$, where μ is a fuzzy σ-nowhere dense set and λ is a fuzzy closed set in (X, T). Since μ is a fuzzy σ-nowhere dense set in X, μ is a fuzzy F_σ-set in X with $int(\mu) = 0$. Since (X, T) is the fuzzy perfectly disconnected space, by Proposition 5.3, λ is a fuzzy P'-set in X. □

Proposition 5.18. *If $\mu \leq \lambda$, where μ is a fuzzy F_σ-set and λ is a fuzzy open set in a fuzzy perfectly disconnected space (X, T), then $clint(\mu) \neq 1$ in X.*

Proof. Suppose that $\mu \leq \lambda$, where μ is a fuzzy F_σ-set and $\lambda \in T$ in X. Since X is the fuzzy perfectly disconnected space, by Proposition 5.4, $1 - \lambda$ is a fuzzy P'-set in X. Now $\mu \leq \lambda$ implies that $1 - \lambda \leq 1 - \mu$ in X. Since μ is a fuzzy F_σ-set, $1 - \mu$ is a fuzzy G_δ-set in X. Then, by Proposition 5.10, $1-\mu$ is a fuzzy somewhere dense set in X. Thus $intcl(1-\mu) \neq 0$ in X. So $1 - clint(\mu) \neq 0$. Hence $clint(\mu) \neq 1$ in X. □

Proposition 5.19. *If λ is a fuzzy closed set in a fuzzy topological space (X, T) such that $\lambda \leq 1-\mu$ in X, where μ is a fuzzy F_σ-set and fuzzy pre-closed set in a fuzzy topological space X, then λ is a fuzzy* P′*-set in X.*

Proof. Suppose that $\lambda \leq 1-\mu$. Then $\mu \leq 1-\lambda$. Since μ is a fuzzy pre-closed set, $clint(\mu) \leq \mu$. Now $clint(\mu) \leq \mu \leq 1-\lambda$ implies that $clint(\mu) \leq 1-\lambda$. Then $\lambda \leq 1 - clint(\mu)$. Thus for $\lambda \leq 1-\mu$, $\lambda \leq 1 - clint(\mu)$. So λ is a fuzzy P′-set in X. □

Proposition 5.20. *If $clint(\mu) = cl(\mu)$ for the fuzzy F_σ-set μ in a fuzzy topological space (X, T) and λ is a fuzzy* P′*-set in X such that $\lambda \leq 1-\mu$, then λ is a fuzzy P-set in X.*

Proof. Let λ be a fuzzy P′-set in (X, T) such that $\lambda \leq 1-\mu$ in X, where μ is a fuzzy F_σ-set in X. Since λ is a fuzzy P′-set in X, $\lambda \leq 1 - clint(\mu)$ in X. Then by hypothesis, $clint(\mu) = cl(\mu)$ for the fuzzy F_σ-set μ. Thus $\lambda \leq 1 - cl(\mu)$ in X. So λ is a fuzzy P-set in X. □

6. Fuzzy P′-sets and other fuzzy topological spaces

The following proposition gives a condition for fuzzy closed F_σ-sets in fuzzy F′-spaces to become fuzzy P′-sets.

Proposition 6.1. *If λ is a fuzzy closed F_σ-set in a fuzzy* F′*-space (X, T) such that $\lambda + \mu \leq 1$, where μ is a fuzzy F_σ-set in X, then λ is a fuzzy* P′*-set in X.*

Proof. Suppose that $\lambda + \mu \leq 1$, where λ is a fuzzy closed F_σ-set and μ is a fuzzy F_σ-set. Then $\lambda \leq 1-\mu$. Since X is a fuzzy F′-space, $\lambda \leq 1-\mu$ implies that $cl(\lambda) \leq 1 - cl(\mu)$. Since λ is a fuzzy closed set, $\lambda = cl(\lambda)$. Then $\lambda \leq 1 - cl(\mu)$. Now $int(\mu) \leq \mu$ implies that $clint(\mu) \leq cl(\mu)$ and $1 - cl(\mu) \leq 1 - clint(\mu)$ and $\lambda \leq 1 - clint(\mu)$. Thus $\lambda \leq 1-\mu$ implies that $\lambda \leq 1 - clint(\mu)$. So λ is a fuzzy P′-set in X. □

Proposition 6.2. *If λ is a fuzzy* P′*-set in a fuzzy hyperconnected and fuzzy open hereditarily irresolvable space (X, T) such that $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in X, then δ is a fuzzy dense set in X.*

Proof. Let λ be a fuzzy P′-set in X such that $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in X. Then by Proposition 5.10, δ is a fuzzy somewhere dense set in X. Since X is a fuzzy hyperconnected and fuzzy open hereditarily irresolvable space, by Theorem 2.12, $1-\delta$ is a fuzzy nowhere dense set in X. Thus $intcl(1-\delta) = 0$ and $1 - clint(\delta) = 0$, in X. So $clint(\delta) = 1$. Since $clint(\delta) \leq cl(\delta)$, $cl(\delta) = 1$. Hence δ is a fuzzy dense set in X. □

Proposition 6.3. *If λ is a fuzzy* P′*-set in a fuzzy hyperconnected and fuzzy open hereditarily irresolvable and fuzzy globally disconnected space (X, T) such that $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in X, then δ is a fuzzy open set in X.*

Proof. Let λ be a fuzzy P$'$-set in (X, T) such that $\lambda \leq \delta$, where δ is a fuzzy G_δ-set in X. The, by Proposition 5.10, δ is a fuzzy somewhere dense set in X. Since (X, T) is the fuzzy hyperconnected and fuzzy open hereditarily irresolvable and fuzzy globally disconnected space, by Theorem 2.13, δ is a fuzzy open set in X. □

Proposition 6.4. *If λ is a fuzzy closed F_σ-set and μ is a fuzzy F_σ-set such that $\lambda \leq 1 - \mu$ in a fuzzy* F$'$*-space (X, T), then*
(1) *$1 - \mu$ is a fuzzy somewhere dense set in X,*
(2) *$clint(\mu) \neq 1$ in X.*

Proof. (1) Let λ be a fuzzy closed F_σ-set and μ is a fuzzy F_σ-set in X such that $\lambda \leq 1 - \mu$. Then $\lambda + \mu \leq 1$ in X. Since (X, T) is a fuzzy F$'$-space, by Proposition 6.1, λ is a fuzzy P$'$-set in X. Since μ is a fuzzy F_σ-set, $1 - \mu$ is a fuzzy G_δ-set in X. Thus $\lambda \leq 1 - \mu$, where λ is a fuzzy P$'$-set and $1 - \mu$ is a fuzzy G_δ-set in X. So by Proposition 5.10, $1 - \mu$ is a fuzzy somewhere dense set in X.

(2) From (1), $1 - \mu$ is a fuzzy somewhere dense set in X. Then $intcl(1 - \mu) \neq 0$ and $1 - clint(\mu) \neq 0$ in X. Thus $clint(\mu) \neq 1$ in X. □

Proposition 6.5. *If λ is a fuzzy closed F_σ-set and μ is a fuzzy F_σ-set such that $\lambda + \mu \leq 1$ in a fuzzy open hereditarily irresolvable and fuzzy* F$'$*-space (X, T), then $cl(\mu) \neq 1$ in X.*

Proof. Let λ be a fuzzy closed F_σ-set and μ is a fuzzy F_σ-set in (X, T) such that $\lambda + \mu \leq 1$. Since X is a fuzzy F$'$-space, by Proposition 6.4(1), $1 - \mu$ is a fuzzy somewhere dense set in X. Then $intcl(1 - \mu) \neq 0$ in X. Since X is a fuzzy open hereditarily irresolvable space, $intcl(1-\mu) \neq 0$ implies that $int(1-\mu) \neq 0$ in (X, T). Thus $1 - cl(\mu) \neq 0$. So $cl(\mu) \neq 1$ in X. Hence μ is not a fuzzy dense set in X. □

7. Conclusions

In this paper, the notions of fuzzy P-sets and fuzzy $P^{'}$-sets in fuzzy topological spaces are introduced and studied. It is shown that if fuzzy P-sets are subsets of fuzzy G_δ sets in a fuzzy topological space, then fuzzy P-sets are not fuzzy nowhere dense sets and there are fuzzy open sets lying between those fuzzy P-sets and those fuzzy G_δ sets. It is obtained that whenever fuzzy P-sets are subsets of fuzzy co-σ-boundary sets, then those fuzzy P-sets are subsets of interiors of those fuzzy co-σ-boundary sets in a fuzzy topological space. A condition for the existence of fuzzy σ-nowhere dense sets in fuzzy hyperconnected spaces is obtained by means of fuzzy P-sets. The conditions for the existence of fuzzy $P^{'}$-sets in fuzzy perfectly disconnected spaces are obtained. The conditions under which fuzzy P-sets become fuzzy $P^{'}$-sets in fuzzy topological spaces and the fuzzy co-σ-boundary sets and fuzzy closed sets in fuzzy perfectly disconnected spaces become fuzzy $P^{'}$-sets are established. It is also established that if the fuzzy co-σ-boundary sets are fuzzy dense sets in fuzzy perfectly disconnected spaces, then the fuzzy co-σ-boundary sets are not fuzzy $P^{'}$-sets. Finally a condition for fuzzy closed F_σ-sets in fuzzy $P^{'}$-spaces to become fuzzy $P^{'}$-sets is obtained in this paper.

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G. Thangaraj (g.thangaraj@rediffmail.com)
Department of Mathematics, Thirualluvar University, Serkkadu, Vellore - 632 115, Tamilnadu, India

S. Muruganantham (murugananth2016@gmail.com)
Department of Mathematics, Thirualluvar University, Serkkadu, Vellore - 632 115, Tamilnadu, India