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Abstract. In this paper, the notion of fuzzy Moscow space is introduced and studied. It is established that fuzzy extremally disconnected spaces are fuzzy Moscow spaces and fuzzy Moscow and fuzzy P-spaces are fuzzy extremally disconnected spaces. It is obtained that fuzzy Moscow spaces are not fuzzy hyperconnected spaces and not fuzzy superconnected spaces. A condition under which fuzzy Moscow spaces become fuzzy hyperconnected spaces is obtained. It is also established that fuzzy semi-open sets are fuzzy pre-open sets in fuzzy Moscow and fuzzy P-spaces.

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Keywords: Fuzzy G_δ-set, Fuzzy F_σ-set, Fuzzy semi-open set, Fuzzy residual set, Fuzzy P-space, Fuzzy Baire space, Fuzzy globally disconnected space, Fuzzy extremally disconnected space, Fuzzy hyperconnected space.

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1. Introduction

The concept of fuzzy set as a new approach for modelling uncertainties was introduced by Zadeh [1] in the year 1965. The concept of fuzzy topological spaces was introduced by Chang [2] in 1968. Chang's works paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts . The concept of Moscow space in classical topology was introduced by Arhangel'skii [3] and the notion of Moscow space can be considered as a straightforward generalization of extremal disconnectedness. The notion of a Moscow space generalizes the notion of a perfectly κ-normal space introduced independently by Blair [4]. The notion of a Moscow space plays a crucial role in the theory of C-embeddings.

The concept of fuzzy extremally disconnected spaces was introduced and studied by Ghosh [5]. The notion of fuzzy P-spaces was introduced and studied by Thangaraj and Balasubramanian [17]. In this paper, the notion of fuzzy Moscow space is introduced by means of fuzzy G_δ-sets. Several characterizations of fuzzy Moscow spaces are established. It is established that fuzzy extremally disconnected spaces are fuzzy Moscow spaces and fuzzy Moscow and fuzzy P-spaces are fuzzy extremally disconnected spaces and fuzzy basically disconnected spaces. It is obtained that fuzzy Moscow spaces, are not fuzzy hyperconnected spaces and not fuzzy superconnected spaces. The condition under which fuzzy Moscow spaces become fuzzy hyperconnected spaces is obtained. It is also established that fuzzy regular closed sets are fuzzy open sets and fuzzy semi-open sets are fuzzy pre-open sets in fuzzy Moscow and fuzzy P-spaces. Also it is established that fuzzy closure of fuzzy β-open sets are fuzzy open sets in fuzzy Moscow and fuzzy P-spaces. A condition under which fuzzy basically disconnected spaces becomes fuzzy Moscow spaces, is also obtained in this paper. In recent years, the topological space theory has been embedding in the soft set theory to obtain some interesting applications [6, 7, 8, 9]. Recently, Şenel et al. [10] defined an octahedron set composed of an interval-valued fuzzy set, an intuitionistic set and a fuzzy set that will provide nice information about uncertainty and vagueness and applied the concept of octahedron sets to multi-criteria group decision-making problems.

2. Preliminaries

Some basic notions and results used in the sequel, are given in order to make the exposition self-contained. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I the unit interval $[0, 1]$. A fuzzy set in X is a mapping from X into I. The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X is defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 ([2])**.** Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The *interior*, the *closure* and the *complement* of λ are defined respectively as follows:

(i) $int(\lambda) = \vee\{\mu \mid \mu \leq \lambda, \mu \in T\}$,
(ii) $cl(\lambda) = \wedge\{\mu \mid \lambda \leq \mu, 1 - \mu \in T\}$,
(iii) $\lambda'(x) = 1 - \lambda(x)$ for all $x \in X$.

For a family $\{\lambda_i \mid i \in I\}$ of fuzzy sets in (X, T), the union $\psi = \vee_i(\lambda_i)$ and intersection $\delta = \wedge_i(\lambda_i)$ are defined respectively as:

(iv) $\psi(x) = \sup_i\{\lambda_i(x) \mid x \in X\}$,
(v) $\delta(x) = \inf_i\{\lambda_i(x) \mid x \in X\}$.

Lemma 2.2 ([11])**.** *For a fuzzy set λ of a fuzzy topological space X,*
(1) $1 - int(\lambda) = cl(1 - \lambda)$,
(2) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.3. A fuzzy set λ in a fuzzy topological space (X, T) is called a

(i) *fuzzy regular-open*, if $\lambda = intcl(\lambda)$ and *fuzzy regular-closed*, if $\lambda = clint(\lambda)$ [11],
(ii) *fuzzy pre-open*, if $\lambda \leq intcl(\lambda)$ and *fuzzy pre-closed*, if $clint(\lambda) \leq \lambda$ [12],
(iii) *fuzzy semi-open*, if $\lambda \leq clint(\lambda)$ and *fuzzy semi-closed*, if $intcl(\lambda) \leq \lambda$ [11],
(iv) *fuzzy β-open*, if $\lambda \leq clintcl(\lambda)$ and *fuzzy β-closed*, if $intclint(\lambda) \leq \lambda$ [13],
(v) *fuzzy G_δ-set*, if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in I$ [14],

(vi) *fuzzy F_σ-set* if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$ [14].

Definition 2.4. The fuzzy set λ in a fuzzy topological space (X, T), is called a

(i) *fuzzy dense set*, if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$, i.e., $cl(\lambda) = 1$ in (X, T) [16],
(ii) *fuzzy nowhere dense set*, if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$, i.e. , $intcl(\lambda) = 0$ in (X, T) [16],
(iii) *fuzzy first category set*, if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category [16],
(iv) *fuzzy somewhere dense set*, if there exists a non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$, i.e, $intcl(\lambda) \neq 0$, in (X, T) [19],
(v) *fuzzy residual set*, if $1 - \lambda$ is a fuzzy first category set in (X, T) [20],
(vi) *fuzzy σ-boundary set*, if $\lambda = \vee_{i=1}^{\infty}(\mu_i)$, where $\mu_i = cl(\lambda_i) \wedge (1 - \lambda_i)$ and (λ_i)'s are fuzzy regular open sets in (X, T) [21],
(vii) *fuzzy co-σ-boundary set*, if $1 - \lambda$ is a fuzzy σ-boundary set in (X, T) [21],
(viii) *fuzzy σ-nowhere dense set*, if is a fuzzy F_σ-set with $int(\lambda) = 0$ in (X, T) [23].

Definition 2.5. A fuzzy topological space (X, T) is called a

(i) *fuzzy P-space* if each fuzzy G_δ-set in (X, T) is fuzzy open in (X, T) [17],
(ii) *fuzzy almost P-space*, if for every non-zero fuzzy G_δ-set λ in (X, T), $int(\lambda) \neq 0$ in (X, T) [24],
(iii) *fuzzy globally disconnected space*, if each fuzzy semi-open set is fuzzy open in (X, T) [26],
(iv) *fuzzy hyperconnected space*, if every non-null fuzzy open subset of (X, T) is fuzzy dense in (X, T) [29],
(v) *fuzzy perfectly disconnected space*, if for any two non-zero fuzzy sets λ and μ defined on X with $\lambda \leq (1 - \mu)$, $cl(\lambda) \leq 1 - cl(\mu)$ in (X, T) [27],
(vi) *fuzzy submaximal space*, if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, $\lambda \in T$ [14],
(vii) *fuzzy Baire space*, if $int\left(\vee_{i=1}^{\infty}(\lambda_i)\right) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T) [20],
(viii) *fuzzy superconnected space*, if it has no proper fuzzy regular open set [30],
(ix) *fuzzy DG_δ-space*, if each fuzzy dense (but not fuzzy open) set in (X, T) is a fuzzy G_δ-set in (X, T) [28],
(x) *fuzzy extremally disconnected space*, if the closure of every fuzzy open set of (X, T) is fuzzy open in (X, T) [5],
(xi) *fuzzy basically disconnected space*, if the closure of every fuzzy open F_σ-set of (X, T) is fuzzy open in (X, T) [17],
(xii) *fuzzy nodef space*, if each fuzzy nowhere dense set is a fuzzy F_σ-set in (X, T) [28],
(xiii) *fuzzy open hereditarily irresolvable space*, if $intcl(\lambda) \neq 0$, for any non-zero fuzzy set λ defined on X, $int(\lambda) \neq 0$ in (X, T) [18].

Definition 2.6 ([31])**.** Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T). The fuzzy semi-closure and the fuzzy semi-interior of λ are defined as follows:

(i) $s\text{-}cl(\lambda) = \wedge\{\mu \mid \lambda \leq \mu,\ \mu$ is fuzzy semi-closed in $(X,T)\}$,
(ii) $s\text{-}int(\lambda) = \vee\{\mu \mid \mu \leq \lambda,\ \mu$ is fuzzy semi-open in $(X,T)\}$.

Theorem 2.7 ([11]). *In a fuzzy space X,*
(1) *the closure of a fuzzy open set is a fuzzy regular closed set,*
(2) *the interior of a fuzzy closed set is a fuzzy regular open set.*

Theorem 2.8 ([20]). *If λ is a fuzzy residual set in a fuzzy topological space (X,T), then there exists a fuzzy G_δ-set μ in (X,T) such that $\mu \leq \lambda$.*

Lemma 2.9 ([31]). *Let A be a fuzzy set of a fuzzy topological space X. Then $int(A) \leq s\text{-}int(A) \leq A \leq s\text{-}cl(A) \leq cl(A)$.*

Theorem 2.10 ([15]). *For any fuzzy topological space (X,T), the following are equivalent:*
(1) *X is fuzzy extremally disconnected space,*
(2) *for each fuzzy closed set λ, $int(\lambda)$ is fuzzy closed,*
(3) *for each fuzzy open set λ, $cl(\lambda) + cl[1 - cl(\lambda)] = 1$,*
(4) *for every pair of fuzzy open sets λ and μ in X with $cl(\lambda) + \mu = 1$,*

$$cl(\lambda) + cl(\mu) = 1.$$

Theorem 2.11 ([21]). *If λ is a fuzzy σ-boundary set in a fuzzy topological space (X,T), then λ is a fuzzy F_σ-set in (X,T).*

Theorem 2.12 ([26]). *If λ is a fuzzy residual set in the fuzzy globally disconnected space (X,T), then λ is a fuzzy G_δ-set in (X,T).*

Theorem 2.13 ([20]). *If γ is a fuzzy residual set in a fuzzy almost P-space (X,T), then there exists a fuzzy G_δ-set μ in (X,T) such that $\mu \leq \gamma$ in (X,T).*

Theorem 2.14 ([22]). *Let (X,T) be a fuzzy topological space. Then, the following are equivalent:*
(1) *(X,T) is a fuzzy Baire space,*
(2) *each non-zero fuzzy open set is a fuzzy second category set in (X,T).*

Theorem 2.15 ([17]). *For any fuzzy topological space (X,T), the following are equivalent :*
(1) *X is fuzzy basically disconnected,*
(2) *for each fuzzy closed G_δ-set λ, $int(\lambda)$ is fuzzy closed,*
(3) *for each fuzzy open F_σ-set λ, $cl(\lambda) + cl[1 - cl(\lambda)] = 1$.*

Theorem 2.16 ([25]). *If λ is a fuzzy residual set in the fuzzy submaximal space (X,T), then is a fuzzy G_δ-set in (X,T).*

Theorem 2.17 ([21]). *If λ is a fuzzy co-σ-boundary set in a fuzzy perfectly disconnected space (X,T), then there exists a fuzzy G_δ-set δ in (X,T) such that $\lambda \leq \delta$.*

Theorem 2.18 ([5]). *A fuzzy topological space (X,T) is fuzzy extremally disconnected if and only if $FSO(X) \subset FPO(X)$.*

Theorem 2.19 ([5]). *The following are equivalent for a fuzzy topological space (X,T):*

(1) (X, T) *is fuzzy extremally disconnected,*
(2) *the closure of every fuzzy semi-pre-open set in* X *is fuzzy open.*
(3) *the closure of every fuzzy pre-open set in* X *is fuzzy open.*

Theorem 2.20 ([23]). *If* λ *is a fuzzy* σ*-nowhere dense set in the fuzzy topological space* (X, T)*, then* λ *is a fuzzy first category set in* (X, T)*.*

Theorem 2.21 ([5]). *In a fuzzy extremally disconnected space* (X, T)*,* $cl(\mu) = s\text{-}cl(\mu)$ *for any fuzzy semi-open set* μ *in* (X, T)*.*

Theorem 2.22 ([28]). *If* λ *is a fuzzy dense (but not fuzzy open) set in a fuzzy* DG_δ*-space* (X, T)*, then* λ *is a fuzzy residual set in* (X, T)*.*

Theorem 2.23 ([28]). *If* λ *is a fuzzy dense (but not fuzzy open) set in a fuzzy* DG_δ*-space* (X, T)*, then there exists a fuzzy* G_δ*-set* η *in* (X, T) *such that* $\eta \leq \lambda$*.*

Theorem 2.24 ([28]). *If* γ *is a fuzzy dense set in the fuzzy open hereditarily and fuzzy nodef space* (X, T)*, then* λ *is a fuzzy* G_δ*-set such that* $int(\lambda) \neq 0$*, in* (X, T)*.*

Theorem 2.25 ([25]). *If a fuzzy topological space* (X, T) *is a fuzzy hyperconnected and fuzzy P-space, then* (X, T) *is a fuzzy Baire space.*

3. Fuzzy Moscow Spaces

In this section, we introduce the concept of fuzzy Moscow spaces and study various properties related to it.

Definition 3.1. A fuzzy topological space (X, T) is called a *fuzzy Moscow space*, if for each fuzzy open set λ in (X, T), $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T).

Example 3.2. Let $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets α, β and γ are defined on X as follows:
$\alpha : X \to I$ is defined by $\alpha(a) = 0.6$; $\alpha(b) = 0.4$; $\alpha(c) = 0.6$,
$\beta : X \to I$ is defined by $\beta(a) = 0.5$; $\beta(b) = 0.5$; $\beta(c) = 0.5$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.4$; $\gamma(b) = 0.6$; $\gamma(c) = 0.4$.
Then $T = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \wedge \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \vee \gamma, , 1\}$ is a fuzzy topology on X. Moreover, we can easily check that the following equalities hold:

$$\alpha = \alpha \wedge (\alpha \vee \beta) \wedge (\alpha \vee \gamma),\ \alpha \wedge \beta = \alpha \wedge (\beta \vee \gamma) \wedge (\alpha \vee \beta),$$

$$\alpha \wedge \gamma = \alpha \wedge \beta \wedge (\beta \wedge \gamma),\ \beta \wedge \gamma = \beta \wedge \gamma \wedge (\alpha \vee \beta),\ \gamma = (\alpha \vee \gamma) \wedge (\beta \vee \gamma) \wedge \gamma.$$

Thus α, γ, $\alpha \wedge \beta$, $\alpha \wedge \gamma$ and $\beta \wedge \gamma$ are fuzzy G_δ-sets in (X, T). By computation, one can find that
$cl(\alpha) = 1 - \gamma = \alpha \vee (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$,
$cl(\beta) = 1 - \beta = (\alpha \wedge \beta) \vee (\beta \wedge \gamma)$,
$cl(\gamma) = 1 - \alpha = (\beta \wedge \gamma) \vee \gamma \vee (\alpha \wedge \gamma)$,
$cl(\alpha \vee \beta) = 1 - (\beta \wedge \gamma) = \alpha \vee (\beta \wedge \gamma)$,
$cl(\alpha \vee \gamma) = 1 - (\alpha \wedge \gamma) = \alpha \vee \gamma$,
$cl(\beta \vee \gamma) = 1 - (\alpha \wedge \beta) = (\alpha \wedge \beta) \vee (\beta \wedge \gamma) \vee \gamma$,
$cl(\alpha \wedge \beta) = 1 - (\beta \vee \gamma) = (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$,
$cl(\alpha \wedge \gamma) = 1 - (\alpha \vee \gamma) = (\alpha \wedge \gamma)$,

$cl(\beta \wedge \gamma) = 1 - (\alpha \vee \beta) = (\beta \wedge \gamma) \vee (\alpha \wedge \gamma)$.
So for each fuzzy open set λ ($=\alpha$, β, γ, $\alpha \vee \beta$, $\alpha \vee \gamma$, $\beta \vee \gamma$, $\alpha \wedge \beta$, $\alpha \wedge \gamma$, $\beta \wedge \gamma$) in (X, T), $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$ where (δ_i)'s ($=\alpha$, γ, $\alpha \wedge \beta$, $\alpha \wedge \gamma$ and $\beta \wedge \gamma$) are fuzzy G_δ-sets in (X, T) shows that (X, T) is a fuzzy Moscow space.

Example 3.3. Let $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets λ, μ, γ and η are defined on X as follows:

$\lambda : X \to I$ is defined by $\lambda(a) = 0.8$; $\lambda(b) = 0.6$; $\lambda(c) = 0.7$,
$\mu : X \to I$ is defined by $\mu(a) = 0.6$; $\beta(b) = 0.9$; $\beta(c) = 0.8$,
$\gamma : X \to I$ is defined by $\gamma(a) = 0.7$; $\gamma(b) = 0.5$; $\gamma(c) = 0.9$,
$\eta : X \to I$ is defined by $\eta(a) = 0.7$; $\eta(b) = 0.6$; $\eta(c) = 0.8$.

Then $T = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \vee \gamma, \mu \vee \gamma, \lambda \wedge \mu, \lambda \wedge \gamma, \mu \wedge \gamma, \lambda \vee (\mu \wedge \gamma), \mu \vee (\lambda \wedge \gamma), \gamma \vee (\lambda \wedge \mu), \lambda \wedge (\mu \vee \gamma), \mu \wedge (\lambda \vee \gamma), \gamma \wedge (\lambda \vee \mu), \lambda \vee \mu \vee \gamma, \lambda \wedge \mu \wedge \gamma, 1\}$ is a fuzzy topology on X. By computation, one can find that

$$\lambda = \lambda \wedge (\lambda \vee \mu) \wedge (\lambda \vee \gamma) \wedge [\lambda \vee (\mu \wedge \gamma)] \wedge (\lambda \vee \mu \vee \gamma),$$
$$\lambda \wedge \gamma = \gamma \wedge (\mu \vee \gamma) \wedge [\lambda \wedge (\mu \vee \gamma)] \wedge [\gamma \wedge (\lambda \vee \mu)] \wedge [\mu \vee (\lambda \wedge \gamma)] \wedge [\gamma \vee (\lambda \wedge \mu)],$$
$$\lambda \wedge \mu \wedge \gamma = \mu \wedge (\lambda \wedge \mu) \wedge (\mu \wedge \gamma) \wedge [\mu \wedge (\lambda \vee \gamma)],$$

and

$$\eta = (\mu \vee \gamma) \wedge [\mu \vee (\lambda \wedge \gamma)] \wedge [\gamma \vee (\lambda \wedge \mu)].$$

Thus λ, $\lambda \wedge \gamma$, $\lambda \wedge \mu \wedge \gamma$ and η are fuzzy G_δ-sets in (X, T). By computation, one can find that $cl(\lambda) = 1 \neq \lambda \vee (\lambda \wedge \gamma) \vee [\lambda \wedge \mu \wedge \gamma] \vee \eta$. This shows that (X, T) is not a fuzzy Moscow space.

Proposition 3.4. *If λ is a fuzzy open set in the fuzzy Moscow space (X, T), then there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq cl(\lambda)$.*

Proof. Let λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy Moscow space, $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Clearly, $\delta_i \leq \vee_{i=1}^{\infty}(\delta_i)$. Let $\mu = \delta_i$. Then $\mu \leq \vee_{i=1}^{\infty}(\delta_i) = cl(\lambda)$. Thus for the fuzzy open set λ in (X, T), there exists a fuzzy G_δ-set μ in (X, T) such that $\mu \leq cl(\lambda)$. □

Proposition 3.5. *If η is a fuzzy closed set in the fuzzy Moscow space (X, T), then there exists a fuzzy F_σ-set δ in (X, T) such that $int(\eta) \leq \delta$.*

Proof. Let η be a fuzzy closed set in (X, T). Then $1 - \eta$ is the fuzzy open set in (X, T). Since (X, T) is the fuzzy Moscow space and by Proposition 3.4, there exists an fuzzy G_δ-set μ in (X, T) such that $\mu \leq cl(1 - \eta)$. Thus $\mu \leq 1 - int(\eta)$. This implies that $int(\lambda) \leq 1 - \mu$. Let $\delta = 1 - \mu$. Then δ is a fuzzy F_σ-set in (X, T). Thus for the fuzzy closed set η in (X, T), there exists a fuzzy F_σ-set δ in (X, T) such that $int(\eta) \leq \delta$. □

Proposition 3.6. *If λ is a fuzzy regular closed set in the fuzzy Moscow space (X, T), then $\lambda = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T).*

Proof. Let λ be a fuzzy regular closed set in (X, T). Then $clint(\lambda) = \lambda$ in (X, T). Now $int(\lambda) \neq 0$ [otherwise, $int(\lambda) = 0$, will imply that $clint(\lambda) = cl[0] = 0$, a contradiction]. Since the fuzzy topological space (X, T) is the fuzzy Moscow space, for the fuzzy open set $int(\lambda)$ in (X, T), $cl[int(\lambda)] = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Thus $\lambda = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). □

Proposition 3.7. *If μ is a fuzzy regular open set in the fuzzy Moscow space (X,T), then $\mu = \wedge_{i=1}^{\infty}(\eta_i)$, where (η_i)'s are fuzzy F_σ-sets in (X,T).*

Proof. Let μ be a fuzzy regular open set in (X,T). Then $1-\mu$ is a fuzzy regular closed set in (X,T). Since (X,T) is the fuzzy Moscow space, by Proposition 3.6, $1-\mu = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X,T). Thus $\mu = 1-\vee_{i=1}^{\infty}(\delta_i) = \wedge_{i=1}^{\infty}(1-\delta_i)$. Let $\eta_i = 1-\delta_i$. Then (η_i)'s are fuzzy F_σ-sets in (X,T). Thus $\mu = \wedge_{i=1}^{\infty}(\eta_i)$, where (η_i)'s are fuzzy F_σ-sets in (X,T). □

Proposition 3.8. *If each λ_i ($i = 1$ to ∞) is a fuzzy residual set in the fuzzy Moscow space (X,T), then there exists a fuzzy regular closed set γ in (X,T) such that $\gamma \leq \vee_{i=1}^{\infty}(\lambda_i)$.*

Proof. Let each λ_i ($i = 1$ to ∞) be a fuzzy residual set in (X,T). Then by Theorem 2.8, there exist fuzzy G_δ-sets η_i in (X,T) such that $\eta_i \leq \lambda_i$. This implies that $\vee_{i=1}^{\infty}(\eta_i) \leq \vee_{i=1}^{\infty}(\lambda_i)$. Since the fuzzy topological space (X,T) is a fuzzy Moscow space, for some fuzzy open set δ in (X,T), $cl(\delta) = \vee_{i=1}^{\infty}(\eta_i)$. Thus $cl(\delta) \leq \vee_{i=1}^{\infty}(\lambda_i)$ in (X,T). Since δ is the fuzzy open set in (X,T), by Theorem 2.7 (1), $cl(\delta)$ is the fuzzy regular closed set in (X,T). Let $\gamma = cl(\delta)$. Thus, if (λ_i)'s ($i = 1$ to ∞) are fuzzy residual sets in (X,T), then there exists a fuzzy regular closed set γ in (X,T) such that $\gamma \leq \vee_{i=1}^{\infty}(\lambda_i)$. □

Proposition 3.9. *If each μ_i ($i = 1$ to ∞) is a fuzzy first category set in the fuzzy Moscow space (X,T), then there exists a fuzzy regular open set δ in (X,T) such that $\wedge_{i=1}^{\infty}(\mu_i) \leq \delta$.*

Proof. Let each μ_i ($i = 1$ to ∞) be a fuzzy first category set in (X,T). Then $(1-\mu_i)$'s are fuzzy residual sets in (X,T). Since the fuzzy topological space (X,T) is a fuzzy Moscow space, by Proposition 3.8, there exists a fuzzy regular closed set γ in (X,T) such that $\gamma \leq \vee_{i=1}^{\infty}(1-\mu_i)$. This implies that $\gamma \leq [1-\wedge_{i=1}^{\infty}(\mu_i)]$. Thus $\wedge_{i=1}^{\infty}(\mu_i) \leq (1-\gamma)$. Let $\delta = 1-\gamma$. Then δ is the fuzzy regular open set in (X,T). Thus for the fuzzy first category sets (μ_i)'s in (X,T), there exists a fuzzy regular open set δ in (X,T) such that $\wedge_{i=1}^{\infty}(\mu_i) \leq \delta$. □

Proposition 3.10. *If each μ_i ($i = 1$ to ∞) is a fuzzy first category set in the fuzzy Moscow space (X,T), then there exists a fuzzy open and fuzzy semi-closed set δ in (X,T) such that $\wedge_{i=1}^{\infty}(\mu_i) \leq \delta$.*

Proof. The Proof follows from Proposition 3.9 and from the fact that fuzzy regular open sets in fuzzy topological spaces (X,T) are both fuzzy open and fuzzy semi-closed. □

Proposition 3.11. *If each μ_i ($i = 1$ to ∞) is a fuzzy first category set in the Moscow space (X,T), then $s\text{-}cl\,[\wedge_{i=1}^{\infty}(\mu_i)] \neq 1$ in (X,T).*

Proof. Let each μ_i ($i = 1$ to ∞) be a fuzzy first category set in (X,T). Then by Proposition 3.9, there exists a fuzzy regular open set δ in (X,T) such that $\wedge_{i=1}^{\infty}(\mu_i) \leq \delta$. Since fuzzy regular open sets in fuzzy topological spaces are both fuzzy open and fuzzy semi-closed, δ is a fuzzy open and fuzzy semi-closed set in (X,T). Thus $\wedge_{i=1}^{\infty}(\mu_i) \leq \delta$ implies that $s\text{-}cl\,[\wedge_{i=1}^{\infty}(\mu_i)] \neq 1$ in (X,T). □

Remark 3.12. From the above Proposition, one will have the following result: "If (μ_i)'s $(i = 1$ to $\infty)$ are fuzzy first category sets in the Moscow space (X, T), then $\wedge_{i=1}^{\infty}(\mu_i)$ is not fuzzy semi-dense in (X, T)".

Proposition 3.13. *If each δ_i $(i = 1$ to $\infty)$ is a fuzzy G_δ-set in the fuzzy Moscow space (X, T), then s-int $[\vee_{i=1}^{\infty}(\delta_i)] \neq 0$ in (X, T).*

Proof. Let λ be a fuzzy semi-open set in (X, T). Then $\lambda \leq clint(\lambda)$ in (X, T). Since the fuzzy topological space (X, T) is a fuzzy Moscow space, for the fuzzy open set $int(\lambda)$ in (X, T), $cl[int(\lambda)] = \vee_{i=1}^{\infty}(\eta_i)$, where (η_i)'s are fuzzy G_δ-sets in (X, T) [(η_i)'s are considered from fuzzy G_δ-sets (δ_i)'s]. Thus $\lambda \leq \vee_{i=1}^{\infty}(\eta_i)$ in (X, T). This implies that $\lambda \leq \vee_{i=1}^{\infty}(\eta_i) \leq \vee_{i=1}^{\infty}(\delta_i)$. So *s-int* $[\vee_{i=1}^{\infty}(\delta_i)] \neq 0$ in (X, T). □

Proposition 3.14. *If each γ_i $(i = 1$ to $\infty)$ is a fuzzy F_σ-set in the fuzzy Moscow space (X, T), then s-cl $[\wedge_{i=1}^{\infty}(\gamma_i)] \neq 1$ in (X, T).*

Proof. Let each γ_i $(i = 1$ to $\infty)$ be a fuzzy F_σ-set in (X, T). Then $(1-\gamma_i)$'s are fuzzy G_δ-sets in (X, T). Since the fuzzy topological space (X, T) is a fuzzy Moscow space, by Proposition 3.13, *s-int* $[\vee_{i=1}^{\infty}(1-\gamma_i)] \neq 0$ in (X, T). Now *s-int* $[\vee_{i=1}^{\infty}(1-\gamma_i)]$=*s-int* $[1 - \wedge_{i=1}^{\infty}(\gamma_i)]$=$1 -$ *s-cl* $[\wedge_{i=1}^{\infty}(\gamma_i)]$, implies that $1 -$ *s-cl* $[\wedge_{i=1}^{\infty}(\gamma_i)] \neq 0$ in (X, T). Thus *s-cl* $[\wedge_{i=1}^{\infty}(\gamma_i)] \neq 1$ in (X, T). □

Remark 3.15. From the above Proposition, one will have the following result: "If (μ_i)'s $(i = 1$ to $\infty)$ are fuzzy F_σ-sets in the Moscow space (X, T), then $\wedge_{i=1}^{\infty}(\mu_i)$ is not fuzzy semi-dense in (X, T)".

Proposition 3.16. *If each λ_i $(i = 1$ to $\infty)$ is a fuzzy σ-boundary set in the fuzzy Moscow space (X, T), then $\wedge_{i=1}^{\infty}(\lambda_i)$ is a fuzzy open set in (X, T).*

Proof. Let each λ_i $(i = 1$ to $\infty)$ be a fuzzy σ-boundary set in (X, T). Then by Theorem 2.11, the fuzzy σ-boundary sets (λ_i)'s are fuzzy F_σ-sets in (X, T). Thus $(1 - \lambda_i))$'s are fuzzy G_δ-sets in (X, T). Since the fuzzy topological space (X, T) is a fuzzy Moscow space, for some fuzzy open set λ in (X, T), $cl(\lambda) = \vee_{i=1}^{\infty}(1 - \lambda_i)$, where $(1-\lambda_i))$'s are fuzzy G_δ-sets in (X, T). This implies that $cl(\lambda) = 1 - \wedge_{i=1}^{\infty}(\lambda_i)$. So $\wedge_{i=1}^{\infty}(\lambda_i) = 1 - cl(\lambda)$. Since $1 - cl(\lambda)$ is a fuzzy open set in (X, T), $\wedge_{i=1}^{\infty}(\lambda_i)$ is a fuzzy open set in (X, T). □

Proposition 3.17. *If each μ_i $(i = 1$ to $\infty)$ is a fuzzy co-σ-boundary set in the fuzzy Moscow space (X, T), then $\vee_{i=1}^{\infty}(\mu_i)$ is a fuzzy closed set in (X, T).*

Proof. Let each μ_i $(i = 1$ to $\infty)$ be a fuzzy co-σ-boundary set in (X, T). Then $(1 - \mu_i)$'s are fuzzy σ-boundary in (X, T). Since the fuzzy topological space (X, T) is a fuzzy Moscow space, by Proposition 3.16, $\wedge_{i=1}^{\infty}(1 - \mu_i)$ is a fuzzy open set in (X, T). Now $\wedge_{i=1}^{\infty}(1-\mu_i) = 1-\vee_{i=1}^{\infty}(\mu_i)$, implies that $1-\vee_{i=1}^{\infty}(\mu_i)$ is the fuzzy open set. Thus $\vee_{i=1}^{\infty}(\mu_i)$ is a fuzzy closed set in (X, T). □

Proposition 3.18. *If λ is a fuzzy semi-open set in the fuzzy Moscow space (X, T), then $\lambda \leq \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T).*

Proof. Let λ be a fuzzy semi-open set in (X, T). Then $\lambda \leq clint(\lambda)$ in (X, T). Now $int(\lambda) \neq 0$ [otherwise, $int(\lambda) = 0$, will imply that $clint(\lambda) = cl[0] = 0$ and this will imply that $\lambda = 0$, a contradiction]. Since the fuzzy topological space (X, T) is a fuzzy Moscow space, for the fuzzy open set $int(\lambda)$ in (X, T), $cl[int(\lambda)] = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Thus $\lambda \leq \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). □

Proposition 3.19. *If λ is a fuzzy pre-closed set in the fuzzy Moscow space (X, T), then $\vee_{i=1}^{\infty}(\delta_i) \leq \lambda$, where (δ_i)'s are fuzzy G_δ-sets in (X, T).*

Proof. Let λ be a fuzzy pre-closed set in (X, T). Then $clint(\lambda) \leq \lambda$, in (X, T). Since the fuzzy topological space (X, T) is a fuzzy Moscow space, for the fuzzy open set $int(\lambda)$ in (X, T), $cl[int(\lambda)] = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Thus $\vee_{i=1}^{\infty}(\delta_i) \leq \lambda$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). □

Proposition 3.20. *If λ is a fuzzy β-open set in the fuzzy Moscow space (X, T), then $\lambda \leq \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T).*

Proof. Let λ be a fuzzy β-open set in (X, T). Then $\lambda \leq clintcl(\lambda)$, in (X, T). Now $intcl(\lambda) \neq 0$. Since the fuzzy topological space (X, T) is the fuzzy Moscow space, for the fuzzy open set $intcl(\lambda)$ in (X, T), $cl[intcl(\lambda)] = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Thus $\lambda \leq \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). □

Proposition 3.21. *If λ is a fuzzy somewhere dense set in the fuzzy Moscow space (X, T), then*

(1) *$clintcl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T),*
(2) *$int(\lambda) \leq \vee_{i=1}^{\infty}(\delta_i)$ in (X, T).*

Proof. (1) Let λ be a fuzzy somewhere dense set in (X, T). Then $intcl(\lambda) \neq 0$, in (X, T). Since (X, T) is a fuzzy Moscow space, for the fuzzy open set $intcl(\lambda)$ in (X, T), $cl[intcl(\lambda)] = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Thus $clintcl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T).

(2). By (1), $clintcl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Then $intcl(\lambda) \leq \vee_{i=1}^{\infty}(\delta_i)$ and $int(\lambda) \leq intcl(\lambda)$, implies that $int(\lambda) \leq \vee_{i=1}^{\infty}(\delta_i)$ in (X, T). □

Proposition 3.22. *If each λ_i $(i = 1$ to $\infty)$ is a fuzzy σ-nowhere dense set in the fuzzy Moscow space (X, T), then there exists a fuzzy regular open set δ in (X, T) such that $\wedge_{i=1}^{\infty}(\lambda_i) \leq \delta$.*

Proof. Let each λ_i $(i = 1$ to $\infty)$ be a fuzzy σ-nowhere dense set in (X, T). Then by Theorem 2.20, (λ_i)'s are fuzzy first category sets in (X, T). Thus $(1-\lambda_i)$'s are fuzzy residual sets in (X, T). So by Theorem 2.8, there exist fuzzy G_δ-sets η_i in (X, T) such that $\eta_i \leq (1 - \lambda_i)$. This implies that $\vee_{i=1}^{\infty}(\eta_i \leq \vee_{i=1}^{\infty}(1 - \lambda_i$. Since (X, T) is a fuzzy Moscow space, for some fuzzy open set μ in (X, T), $cl(\mu) = \vee_{i=1}^{\infty}(\eta_i$. Hence $cl(\mu) \leq \vee_{i=1}^{\infty}(1 - \lambda_i)$ and $cl(\mu) \leq 1 - \wedge_{i=1}^{\infty}(\lambda_i)$ in (X, T). This implies that $\wedge_{i=1}^{\infty}(\lambda_i) \leq 1 - cl(\mu)$. By Theorem 2.7 (1), $cl(\mu)$ is a fuzzy regular closed. Therefore $1 - cl(\mu)$ is a fuzzy regular open set in (X, T). Let $\delta = 1 - cl(\mu)$. Then there exists a fuzzy regular open set δ in (X, T) such that $\wedge_{i=1}^{\infty}(\lambda_i) \leq \delta$. □

4. Fuzzy Moscow spaces and other fuzzy topological spaces

In this section, we discuss some relationships among fuzzy Moscow spaces and other fuzzy spaces.

Proposition 4.1. *If the fuzzy topological space* (X, T) *is a fuzzy Moscow and fuzzy P-space, then* (X, T) *is a fuzzy extremally disconnected space.*

Proof. Let λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy Moscow space, $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Since the fuzzy topological space (X, T) is the fuzzy P-space, the fuzzy G_δ-sets (δ_i)'s are fuzzy open sets in (X, T). Then $\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy open set in (X, T). Thus $cl(\lambda)$ is a fuzzy open set in (X, T). So for the fuzzy open set λ, $cl(\lambda)$ is a fuzzy open set in (X, T) implies that (X, T) is a fuzzy extremally disconnected space. □

Proposition 4.2. *If* μ *is a fuzzy closed set in the fuzzy Moscow and fuzzy P-space, then there exists a fuzzy* F_σ*-set* δ *in* (X, T) *such that* $clint(\mu) \leq \delta$.

Proof. Let μ be a fuzzy closed set in (X, T). Since (X, T) is a fuzzy Moscow and fuzzy P-space, by Proposition 4.1, (X, T) is a fuzzy extremally disconnected space. Then by Theorem 2.10, for the fuzzy closed set μ, $int(\mu)$ is fuzzy closed in (X, T) and thus $cl[int(\mu)] = int(\mu)$. Thus by Proposition 3.5, there exists a fuzzy F_σ-set δ in (X, T) such that $int(\mu) \leq \lambda$. So for the fuzzy closed set μ in (X, T), there exists a fuzzy F_σ-set δ in (X, T) such that $clint(\mu) \leq \delta$. □

Proposition 4.3. *If the fuzzy topological space* (X, T) *is a fuzzy Moscow space, then* (X, T) *is not a fuzzy superconnected space.*

Proof. Let the fuzzy topological space (X, T) be a fuzzy Moscow space and (μ_i)'s $(i = 1 \text{ to } \infty)$ be fuzzy first category sets in (X, T). Then by Proposition 3.9, there exists a fuzzy regular open set δ in (X, T) such that $\wedge_{i=1}^{\infty}(\mu_i) \leq \delta$. Thus (X, T) is not a fuzzy superconnected space. □

Proposition 4.4. *If the fuzzy topological space* (X, T) *is a fuzzy Moscow space, then* (X, T) *is not a fuzzy hyperconnected space.*

Proof. Let λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy Moscow space, $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Then by Theorem 2.7 (1), $cl(\lambda)$ being the closure of a fuzzy open set λ is a fuzzy regular closed set and thus a closed set in (X, T). So $cl(\lambda) \neq 1$, in (X, T). Hence for the fuzzy open set λ, $cl(\lambda) \neq 1$, in (X, T) implies that (X, T) is not a fuzzy hyperconnected space. □

The following Proposition gives a condition for the fuzzy Moscow spaces to become fuzzy hyperconnected spaces.

Proposition 4.5. *If each fuzzy* G_δ*-set is a fuzzy dense set in the fuzzy Moscow space* (X, T)*, then* (X, T) *is a fuzzy hyperconnected space.*

Proof. Let λ be a fuzzy open set in (X, T). Since (X, T) is an fuzzy Moscow space, $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Then by the hypothesis, the fuzzy G_δ-sets (δ_i)'s are fuzzy dense sets in (X, T). Now $cl[cl(\lambda)] = cl\,[\vee_{i=1}^{\infty}(\delta_i)] \geq \vee_{i=1}^{\infty}\,[cl(\delta_i)] = \vee_{i=1}^{\infty}(1) = 1$, implies that $cl[cl(\lambda)] = 1$. Since $cl[cl(\lambda)] = cl(\lambda)$, $cl(\lambda) = 1$. Thus the fuzzy open set λ is a fuzzy dense set in (X, T). So (X, T) is the fuzzy hyperconnected space. □

The following Proposition shows that fuzzy extremally disconnected spaces are fuzzy Moscow spaces.

Proposition 4.6. *If the fuzzy topological space (X, T) is a fuzzy extremally disconnected space, then (X, T) is a fuzzy Moscow space.*

Proof. Let λ be the fuzzy open set in (X, T). Since (X, T) is the fuzzy extremally disconnected space, $cl(\lambda)$ is the fuzzy open set in (X, T). Since $\lambda \leq cl(\lambda)$, $\lambda = \lambda \wedge cl(\lambda)$. Now $\lambda = \lambda \wedge cl(\lambda)$, where $\lambda \in T$. Then $cl(\lambda) \in T$ implies that λ is a fuzzy G_δ-set in (X, T). Since $cl(\lambda)$ is the fuzzy open set in (X, T) and (X, T) is the fuzzy extremally disconnected space, $cl[cl(\lambda)]$ is a fuzzy open set in (X, T). Now $cl(\lambda) \leq cl[cl(\lambda)]$ and $cl(\lambda) = cl(\lambda) \wedge cl[cl(\lambda)]$. Thus $cl(\lambda)$ is the fuzzy G_δ-set in (X, T). So $cl(\lambda) = \lambda \vee cl(\lambda)$, where λ and $cl(\lambda)$ are fuzzy G_δ-sets in (X, T). Hence (X, T) is a fuzzy Moscow space. □

The following Proposition gives a condition for fuzzy basically disconnected spaces to become fuzzy Moscow spaces.

Proposition 4.7. *If a fuzzy topological space (X, T) is an fuzzy basically disconnected space in which fuzzy open sets are fuzzy F_σ-sets, then (X, T) is an fuzzy Moscow space.*

Proof. Let λ be an fuzzy open set in (X, T). Then by the hypothesis, λ is an fuzzy F_σ-set in (X, T). Since (X, T) is an fuzzy basically disconnected space, by Theorem 2.15, for the fuzzy open F_σ-set λ, $cl(\lambda) + cl[1 - cl(\lambda)] = 1$. Thus $cl(\lambda) + 1 - intcl(\lambda) = 1$. So $cl(\lambda) = intcl(\lambda)$. This implies that $cl(\lambda)$ is an fuzzy open set in (X, T). Hence (X, T) is a fuzzy extremally disconnected space. Therefore by Proposition 4.6, (X, T) is a fuzzy Moscow space. □

Proposition 4.8. *If each δ_i $(i = 1$ to $\infty)$ is a fuzzy residual set in the fuzzy Moscow and fuzzy globally disconnected space (X, T), then $\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy semi-open set in (X, T).*

Proof. Let each δ_i $(i = 1$ to $\infty)$ be a fuzzy residual set in (X, T). Since (X, T) is a fuzzy globally disconnected space, by Theorem 2.12, (δ_i)'s are fuzzy G_δ-sets in (X, T). Now consider $\vee_{i=1}^{\infty}(\delta_i)$ in (X, T) and (X, T) being the fuzzy Moscow space, for some fuzzy open set λ in (X, T), $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Since λ is a fuzzy open set in (X, T), $\lambda \leq intcl(\lambda)$. Then $cl(\lambda) \leq clintcl(\lambda)$. Thus $cl(\lambda)$ is a fuzzy semi-open set in (X, T). So $\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy semi-open set in (X, T). □

Proposition 4.9. *If each η_i $(i = 1$ to $\infty)$ is a fuzzy first category set in the fuzzy Moscow and fuzzy globally disconnected space (X, T), then $\wedge_{i=1}^{\infty}(\eta_i)$ is a fuzzy semi-closed set in (X, T).*

Proof. Let each η_i $(i = 1$ to $\infty)$ be a fuzzy first category set in (X, T). Then $(1 - \eta_i)$'s are fuzzy residual sets in (X, T). Since (X, T) is the fuzzy Moscow and fuzzy globally disconnected space, by Proposition 4.8, $\vee_{i=1}^{\infty}(1 - \eta_i)$ is a fuzzy semi-open set in (X, T). Now $\vee_{i=1}^{\infty}(1 - \eta_i) = 1 - \wedge_{i=1}^{\infty}(\eta_i)$ and this implies that $1 - \wedge_{i=1}^{\infty}(\eta_i)$ is a fuzzy semi-open set in (X, T). Thus $\wedge_{i=1}^{\infty}(\eta_i)$ is a fuzzy semi-closed set in (X, T). □

Proposition 4.10. *If the fuzzy topological space (X, T) is a fuzzy Moscow and fuzzy P-space, then (X, T) is a fuzzy basically disconnected space.*

Proof. Let (X, T) be a fuzzy Moscow and fuzzy P-space. Then by Proposition 4.1, (X, T) is a fuzzy extremally disconnected space. Since fuzzy extremally disconnected spaces are fuzzy basically disconnected spaces, (X, T) is a fuzzy basically disconnected space. □

Proposition 4.11. *If each γ_i ($i = 1$ to ∞) is a fuzzy residual set in the fuzzy Moscow and fuzzy almost P-space (X, T), then there exists a fuzzy open set in (X, T) such that $cl(\lambda) \leq \vee_{i=1}^{\infty}(\gamma_i)$.*

Proof. Let each γ_i ($i = 1$ to ∞) be a fuzzy residual set in (X, T). Since (X, T) is a fuzzy almost P-space, by Theorem 2.13, there exists fuzzy G_δ-sets (δ_i)'s in (X, T) such that $\delta_i \leq \gamma_i$, in (X, T). Then $\vee_{i=1}^{\infty}(\delta_i) \leq \vee_{i=1}^{\infty}(\gamma_i)$ in (X, T). Since (X, T) is a fuzzy Moscow space, for some fuzzy open set λ in (X, T), $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Thus $cl(\lambda) \leq \vee_{i=1}^{\infty}(\gamma_i)$ in (X, T). So, if (δ_i)'s ($i = 1$ to ∞) are fuzzy residual sets in the fuzzy Moscow and fuzzy almost P-space (X, T), then there exists a fuzzy open set λ in (X, T) such that $cl(\lambda) \leq \vee_{i=1}^{\infty}(\gamma_i)$. □

Proposition 4.12. *If each η_i ($i = 1$ to ∞) is a fuzzy first category set in the fuzzy Moscow and fuzzy almost P-space (X, T), then there exists a fuzzy open set α in (X, T) such that $\wedge_{i=1}^{\infty}(\eta_i) \leq \alpha$.*

Proof. Let each η_i ($i = 1$ to ∞) be a fuzzy first category set in (X, T). Then $(1 - \eta_i)$'s are fuzzy residual sets in (X, T). Since (X, T) is a fuzzy Moscow and fuzzy almost P-space, by Proposition 4.11, for the fuzzy residual sets $(1 - \eta_i)$'s in (X, T), there exists a fuzzy open set λ in (X, T) such that $cl(\lambda) \leq \vee_{i=1}^{\infty}(1 - \eta_i)$. Now $\vee_{i=1}^{\infty}(1 - \eta_i) = 1 - \wedge_{i=1}^{\infty}(\eta_i)$. Thus $cl(\lambda) = 1 - \wedge_{i=1}^{\infty}(\eta_i)$. This implies that $\wedge_{i=1}^{\infty}(\eta_i) = 1 - cl(\lambda)$. Let $\alpha = 1 - cl(\lambda)$. Then α is a fuzzy open set in (X, T). Thus, if (η_i)'s are fuzzy first category sets in the fuzzy Moscow and fuzzy almost P-space (X, T), then there exists a fuzzy open set α in (X, T) such that $\wedge_{i=1}^{\infty}(\eta_i) \leq \alpha$ in (X, T). □

Proposition 4.13. *If each γ_i ($i = 1$ to ∞) is a fuzzy residual set in the fuzzy Moscow and fuzzy P-space (X, T), then $int\,(\vee_{i=1}^{\infty}(\gamma_i)) \neq 0$, in (X, T).*

Proof. Let each γ_i ($i = 1$ to ∞) be a fuzzy residual set in (X, T). Since (X, T) is a fuzzy Moscow and fuzzy P-space, by Proposition 4.1, (X, T) is a fuzzy extremally disconnected space. Since (X, T) is a fuzzy P-space, (X, T) is a fuzzy almost P-space [since for every non-zero fuzzy G_δ-set λ in (X, T), $int(\lambda) = \lambda \neq 0$]. Then (X, T) is a fuzzy Moscow and fuzzy almost P-space. Then by Proposition 4.11, there exists a fuzzy open set λ in (X, T) such that $cl(\lambda) \leq \vee_{i=1}^{\infty}(\gamma_i)$. Since (X, T) is a fuzzy extremally disconnected space, $cl(\lambda)$ is a fuzzy open set in (X, T). Thus $cl(\lambda) \leq \vee_{i=1}^{\infty}(\gamma_i)$ and $cl(\lambda) \in T$ implies that $int\,(\vee_{i=1}^{\infty}(\gamma_i)) \neq 0$ in (X, T). □

Proposition 4.14. *If each η_i ($i = 1$ to ∞) is a fuzzy first category set in the fuzzy Moscow and fuzzy P-space (X, T), then $cl\,(\vee_{i=1}^{\infty}(\gamma_i)) \neq 1$, in (X, T).*

Proof. Let each η_i ($i = 1$ to ∞) be a fuzzy first category set in (X, T). Then $(1 - \eta_i)$'s are fuzzy residual sets in (X, T). Since (X, T) is a fuzzy Moscow and fuzzy P-space, for the fuzzy residual sets $(1 - \eta_i)$'s in (X, T), by Proposition 4.13, $int\,(\vee_{i=1}^{\infty}(1 - \eta_i)) \neq 0$ in (X, T). Thus $int\,(1 - \wedge_{i=1}^{\infty}(\eta_i)) \neq 0$ in (X, T). So by

Lemma 2.2, $int\,(1 - \wedge_{i=1}^{\infty}(\eta_i)) = 1 - cl\,(\wedge_{i=1}^{\infty}(\eta_i))$. Hence $1 - cl\,(\wedge_{i=1}^{\infty}(\eta_i)) \neq 0$, implies that $cl\,(\wedge_{i=1}^{\infty}(\eta_i)) \neq 1$ in (X, T). □

Remark 4.15. From the above Proposition, one will have the following result: "If (η_i)'s ($i = 1$ to ∞) are fuzzy first category sets in the fuzzy Moscow and fuzzy P-space (X, T), then $\wedge_{i=1}^{\infty}(\eta_i)$ is not fuzzy dense in (X, T)".

Proposition 4.16. *If λ is a fuzzy open set in the fuzzy Baire, fuzzy Moscow and fuzzy P-space (X, T), then $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy second category sets in (X, T).*

Proof. Let λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy Moscow space, $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T) and (X, T) being the fuzzy P-space, the fuzzy G_δ-sets (δ_i)'s are fuzzy open sets in (X, T). In the fuzzy Baire space (X, T), by Theorem 2.14, the fuzzy open sets are (δ_i)'s are fuzzy second category sets in (X, T). Then for the fuzzy open set in (X, T), $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy second category sets in (X, T). □

Corollary 4.17. *If the fuzzy topological space (X, T) is a fuzzy Baire, fuzzy Moscow and fuzzy P-space, then for the fuzzy second category sets (δ_i)'s, $\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy regular closed set in (X, T).*

Proof. The Proof follows from Proposition 4.15 and Theorem 2.7 (1). □

Proposition 4.18. *If each fuzzy G_δ-set is a fuzzy dense set in a fuzzy Moscow and fuzzy P-space (X, T), then for a fuzzy open set in (X, T), $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy second category sets in (X, T).*

Proof. Let λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy Moscow space, $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Since the fuzzy G_δ-sets are fuzzy dense sets in the fuzzy Moscow space (X, T), by Proposition 4.5, (X, T) is an fuzzy hyperconnected space. Then (X, T) is a fuzzy hyperconnected and fuzzy P-space. Thus by Theorem 2.25, (X, T) is a fuzzy Baire space. So by Proposition 4.16, for the fuzzy open set λ in the fuzzy Baire, fuzzy Moscow and fuzzy P-space (X, T), $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy second category sets in (X, T). □

Proposition 4.19. *If each δ_i ($i = 1$ to ∞) is a fuzzy G_δ-set in a fuzzy Moscow and fuzzy extremally disconnected space (X, T), then $\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy open set in (X, T).*

Proof. Let each δ_i ($i = 1$ to ∞) be a fuzzy G_δ-set in (X, T). Since the fuzzy topological space (X, T) is a fuzzy Moscow space, for some fuzzy open set λ in (X, T), $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Since (X, T) is a fuzzy extremally disconnected space, for the fuzzy open set λ in (X, T), $cl(\lambda)$ is a fuzzy open set in (X, T). Then $\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy open set in (X, T). □

Proposition 4.20. *If each δ_i ($i = 1$ to ∞) is a fuzzy residual set in the fuzzy submaximal and fuzzy Moscow space (X, T), then $\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy regular closed set in (X, T).*

Proof. Let each δ_i ($i = 1$ to ∞) be a fuzzy residual set in (X, T). Since (X, T) is a fuzzy submaximal space, by Theorem 2.16, the fuzzy residual sets (δ_i)'s are fuzzy G_δ-sets in (X, T) and (X, T) being the fuzzy Moscow space, for some fuzzy open set δ in (X, T), $cl(\delta) = \vee_{i=1}^{\infty}(\delta_i)$. Since δ is a fuzzy open set in (X, T). Then by Theorem 2.7 (1), $cl(\delta)$ is a fuzzy regular closed set in (X, T). Thus $\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy regular closed set in (X, T). □

Corollary 4.21. *If each η_i ($i = 1$ to ∞) is a fuzzy first category set in the fuzzy submaximal and fuzzy Moscow space (X, T), then $\wedge_{i=1}^{\infty}(\eta_i)$ is a fuzzy regular open set in (X, T).*

Proof. Let each η_i ($i = 1$ to ∞) be a fuzzy first category set in (X, T). Then $(1-\eta_i)$'s ($i = 1$ to ∞) are fuzzy residual sets in (X, T). Since (X, T) is a fuzzy submaximal and fuzzy Moscow space, by Proposition 4.20, $\vee_{i=1}^{\infty}(1-\eta_i)$ is an fuzzy regular closed set in (X, T). Now $\vee_{i=1}^{\infty}(1-\eta_i) = 1 - \wedge_{i=1}^{\infty}(\eta_i)$ implies that $1 - \wedge_{i=1}^{\infty}(\eta_i)$ is an fuzzy regular closed set in (X, T). Thus $\wedge_{i=1}^{\infty}(\eta_i)$ is an fuzzy regular open set in (X, T). □

Proposition 4.22. *If each λ_i ($i = 1$ to ∞) is a fuzzy co-σ-boundary set in the fuzzy Moscow and fuzzy perfectly disconnected space (X, T), then there exists a fuzzy regular closed set δ in (X, T) such that $\vee_{i=1}^{\infty}(\lambda_i) \leq \delta$.*

Proof. Let each λ_i ($i = 1$ to ∞) be a fuzzy co-σ-boundary set in (X, T). Since (X, T) is an fuzzy perfectly disconnected space, by Theorem 2.17, there exist fuzzy G_δ-sets (δ_i)'s in (X, T) such that $\lambda_i \leq \delta_i$. This implies that $\vee_{i=1}^{\infty}(\lambda_i) \leq \vee_{i=1}^{\infty}(\delta_i)$. Since (X, T) is a fuzzy Moscow space, for some fuzzy open set η in (X, T), $cl(\eta) = \vee_{i=1}^{\infty}(\delta_i)$. Then by Theorem 2.7 (1), $cl(\eta)$ is a fuzzy regular closed set in (X, T). Let $\delta = cl(\eta)$. Then there exists a fuzzy regular closed set δ in (X, T) such that $\vee_{i=1}^{\infty}(\lambda_i) \leq \delta$. □

Corollary 4.23. *If each μ_i ($i = 1$ to ∞) is a fuzzy σ-boundary set in the fuzzy Moscow and fuzzy perfectly disconnected space (X, T), then there exists a fuzzy regular open set η in (X, T) such that $\eta \leq \wedge_{i=1}^{\infty}(\mu_i)$.*

Proof. Let each μ_i ($i = 1$ to ∞) be a fuzzy σ-boundary set in (X, T). Then $(1 - \mu_i)$'s ($i = 1$ to ∞) are fuzzy co-σ-boundary sets in (X, T). Since (X, T) is a fuzzy Moscow and fuzzy perfectly disconnected space, by Proposition 4.22, there exists a fuzzy regular closed set δ in (X, T) such that $\vee_{i=1}^{\infty}(1 - \mu_i) \leq \delta$. This implies that $1 - \wedge_{i=1}^{\infty}(\mu_i) \leq \delta$ and $1 - \delta \leq \wedge_{i=1}^{\infty}(\mu_i)$. Let $\eta = 1 - \delta$. Then η is a fuzzy regular open set in (X, T). Thus there exists a fuzzy regular open set η in (X, T) such that $\eta \leq \wedge_{i=1}^{\infty}(\mu_i)$. □

Proposition 4.24. *If λ is a fuzzy regular closed set in the fuzzy Moscow and fuzzy P-space (X, T), then λ is a fuzzy open set in (X, T).*

Proof. Let λ be a fuzzy regular closed set in (X, T). Since (X, T) is a fuzzy Moscow space, by Proposition 3.6, $\lambda = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Since (X, T) is a fuzzy P-space, the fuzzy G_δ-sets (δ_i)'s are fuzzy open sets in (X, T). Then $\vee_{i=1}^{\infty}(\delta_i)$ is a fuzzy open set in (X, T). Thus λ is a fuzzy open set in (X, T). □

Proposition 4.25. *If the fuzzy topological space (X, T) is a fuzzy Moscow and fuzzy P-space, then each fuzzy semi-open set in (X, T) is a fuzzy pre-open set in (X, T).*

Proof. Let λ be a fuzzy semi-open set in (X, T). Since (X, T) is a fuzzy Moscow and fuzzy P-space, by Proposition 4.1, (X, T) is a fuzzy extremally disconnected space. Then by Theorem 2.18, λ is a fuzzy pre-open set in (X, T). □

Proposition 4.26. *Let (X, T) be a fuzzy Moscow and fuzzy P-space, then*

(1) *if λ is a fuzzy β-open (semi pre-open) set, then $cl(\lambda)$ is a fuzzy open set in (X, T),*
(2) *if λ is a fuzzy pre-open set, then $cl(\lambda)$ is a fuzzy open set in (X, T).*

Proof. (1) Suppose λ is a fuzzy β-open (semi pre-open) set in (X, T). Since (X, T) is a fuzzy Moscow and fuzzy P-space, by Proposition 4.1, (X, T) is a fuzzy extremally disconnected space. Then, by Theorem 2.19 (2), $cl(\lambda)$ is a fuzzy open set in (X, T).

(2) Suppose λ is a fuzzy pre-open set in (X, T). Since (X, T) is a fuzzy Moscow and fuzzy P-space, by Proposition 4.1, (X, T) is a fuzzy extremally disconnected space. Then by Theorem 2.19 (3), $cl(\lambda)$ is the fuzzy open set in (X, T). □

Proposition 4.27. *If λ is a fuzzy G_δ-set in the fuzzy Moscow and fuzzy P-space (X, T), then there exists a fuzzy G_δ-set η in (X, T) such that $cl(\lambda) \leq \eta$.*

Proof. Let λ be a fuzzy G_δ-set in (X, T). Then $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy open sets in (X, T). Now $cl(\lambda) = cl\left(\wedge_{i=1}^{\infty}(\lambda_i)\right) \leq \wedge_{i=1}^{\infty}\left(cl(\lambda_i)\right)$. Since (X, T) is the fuzzy Moscow and fuzzy P-space, by Proposition 4.1, (X, T) is the fuzzy extremally disconnected space. Thus $(cl(\lambda_i))$'s are fuzzy open sets in (X, T). So $\wedge_{i=1}^{\infty}\left(cl(\lambda_i)\right)$ is a fuzzy G_δ-set in (X, T). Let $\eta = \wedge_{i=1}^{\infty}\left(cl(\lambda_i)\right)$. Then η is a fuzzy G_δ-set in (X, T). Thus for the fuzzy G_δ-set λ, there exists a fuzzy G_δ-set η in (X, T) such that $cl(\lambda) \leq \eta$. □

Proposition 4.28. *If λ is a fuzzy open set in the fuzzy Moscow and fuzzy P-space (X, T), then there exists a fuzzy open set μ in (X, T) such that $\lambda \leq cl(\mu)$.*

Proof. Let λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy Moscow space, $cl(\lambda) = \vee_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy G_δ-sets in (X, T). Since (X, T) is a fuzzy Moscow and fuzzy P-space, by Proposition 4.27, for the fuzzy G_δ-sets (δ_i)'s in (X, T), there exist fuzzy G_δ-sets (η_i)'s in (X, T) such that $cl(\delta_i) \leq \eta_i$. Then we get

$$\vee_{i=1}^{\infty}(\delta_i) \leq \vee_{i=1}^{\infty}\left(cl(\delta_i)\right) \leq \vee_{i=1}^{\infty}(\eta_i).$$

This implies that $cl(\lambda) \leq \vee_{i=1}^{\infty}(\eta_i)$. Since (X, T) is a fuzzy Moscow space, for some fuzzy open set μ in (X, T), $cl(\mu) = \vee_{i=1}^{\infty}(\eta_i)$. Thus $\lambda \leq cl(\lambda) \leq cl(\mu)$. So there exists a fuzzy open set μ in (X, T) such that $\lambda \leq cl(\mu)$. □

Corollary 4.29. *If λ is the fuzzy open set in the fuzzy Moscow and fuzzy P-space (X, T), then there exists a fuzzy open set η in (X, T) such that $\lambda \leq \eta$.*

Proof. Let λ be a fuzzy open set in (X, T). Since (X, T) is a fuzzy Moscow and fuzzy P-space (X, T), by Proposition 4.28, there exists a fuzzy open set μ in (X, T) such that $\lambda \leq cl(\mu)$. Then by Theorem 2.7 (1), $cl(\mu)$ is a fuzzy regular closed set in (X, T). Thus by Proposition 4.24, the fuzzy regular closed set $cl(\mu)$ is a fuzzy open set in (X, T). Let $cl(\mu) = \eta$. Then for the fuzzy open set λ, there exists a fuzzy open set η in (X, T) such that $\lambda \leq \eta$. □

Proposition 4.30. *If λ is a fuzzy semi-open set in the fuzzy Moscow and fuzzy P-space (X, T), then $cl(\lambda) = s\text{-}cl(\lambda)$, in (X, T).*

Proof. The Proof follows from Theorem 2.21 and Proposition 4.1. □

Proposition 4.31. *If (λ_i)'s ($i = 1$ to ∞) are fuzzy dense (but not fuzzy open) sets in the fuzzy Moscow and fuzzy DG_δ-space (X, T), then there exists a fuzzy regular closed set γ in (X, T) such that $\gamma \le \vee_{i=1}^{\infty}(\lambda_i)$.*

Proof. The Proof follows from Theorem 2.22 and Proposition 3.8. □

Proposition 4.32. *If each λ_i ($i = 1$ to ∞) is a fuzzy dense (but not fuzzy open) set in the fuzzy Moscow and fuzzy DG_δ-space (X, T), then there exists a fuzzy open set η in (X, T) such that $cl(\eta) \le \vee_{i=1}^{\infty}(\lambda_i)$.*

Proof. Let each λ_i ($i = 1$ to ∞) be a fuzzy dense (but not fuzzy open) set in (X, T). Since (X, T) is the fuzzy DG_δ-space, by Theorem 2.23, there exist fuzzy G_δ-sets (η_i)'s in (X, T) such that $\eta_i \le \lambda_i$. Then $\vee_{i=1}^{\infty}(\eta_i) \le \vee_{i=1}^{\infty}(\lambda_i)$. Since (X, T) is the fuzzy Moscow space, for some fuzzy open set η in (X, T), $cl(\eta) \le \vee_{i=1}^{\infty}(\lambda_i)$. Thus $cl(\eta) \le \vee_{i=1}^{\infty}(\lambda_i)$ in (X, T). □

Proposition 4.33. *If each λ_i ($i = 1$ to ∞) is a fuzzy dense set in the fuzzy open hereditarily irresolvable, fuzzy Moscow and fuzzy nodef space (X, T), then there exists a fuzzy open set μ in (X, T) such that $cl(\mu) = \vee_{i=1}^{\infty}(\lambda_i)$, where $int(\lambda_i) \ne 0$ in (X, T).*

Proof. Let each λ_i ($i = 1$ to ∞) be a fuzzy dense set in (X, T). Since (X, T) is a fuzzy open hereditarily irresolvable and fuzzy nodef space, by Theorem 2.24, (λ_i)'s are fuzzy G_δ-sets such that $int(\lambda_i) \ne 0$ in (X, T). Now consider $\vee_{i=1}^{\infty}(\lambda_i)$ in (X, T). Since (X, T) is the fuzzy Moscow space, for some fuzzy open set μ in (X, T) such that $cl(\mu) = \vee_{i=1}^{\infty}(\lambda_i)$ in (X, T). Then $cl(\mu) = \vee_{i=1}^{\infty}(\lambda_i)$, where $int(\lambda_i) \ne 0$ in (X, T). □

Proposition 4.34. *If λ is a fuzzy open set in the fuzzy Moscow and fuzzy P-space (X, T), then there exists a fuzzy G_δ-set η in (X, T) such that $\eta \le intcl(\lambda)$.*

Proof. Let λ be a fuzzy open set in (X, T). Then $1 - \lambda$ is a fuzzy closed set in (X, T). Since (X, T) is the fuzzy Moscow and fuzzy P-space, by Proposition 4.2, there exists a fuzzy F_σ-set δ in (X, T) such that $clint(1 - \lambda) \le \delta$. Then $1 - intcl(\lambda) \le \delta$. Thus $1 - \delta \le intcl(\lambda)$. Let $\eta = 1 - \delta$. Then η is a fuzzy G_δ-set in (X, T). Thus there exists a fuzzy G_δ-set η in (X, T) such that $\eta \le intcl(\lambda)$. □

5. Conclusions

In this paper, the notion of fuzzy Moscow space is introduced by means of fuzzy G_δ-sets. It is established that fuzzy extremally disconnected spaces are fuzzy Moscow spaces and fuzzy Moscow and fuzzy P-spaces, are fuzzy extremally disconnected spaces and fuzzy basically disconnected spaces. It is obtained the fuzzy Moscow spaces, are not fuzzy hyperconnected spaces and not fuzzy superconnected spaces. The condition under which fuzzy Moscow spaces, become fuzzy hyperconnected spaces, is obtained. Fuzzy closure of an fuzzy open set in fuzzy Baire, fuzzy Moscow and fuzzy P-space, is obtained by means of fuzzy second category sets . It is also established that fuzzy regular closed sets are fuzzy open sets and fuzzy semi-open sets are fuzzy pre-open sets in fuzzy Moscow and fuzzy P-spaces. A condition under which fuzzy basically disconnected spaces becomes fuzzy Moscow spaces, is also obtained in this paper.

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