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On (f, g)-derivations of lattice implication algebras

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Abstract. In this paper, we introduce the notion of (f, g)-derivation of lattice implication algebra and investigated some related properties. Also, we prove that if D is an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x \in L$, then we get $D(x \to y) = f(x) \to D(y)$ for all $x, y \in L$.

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Keywords: Lattice implication algebra, (f, g)-derivation, normal filter, D-normal filter, $Fix_D(L)$, $KerD$.

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1. Introduction

The concept of lattice implication algebra was proposed by Xu [1], in order to establish an alternative logic knowledge representation. Also, in [2], Xu and Qin discussed the properties lattice H implication algebras, and gave some equivalent conditions about lattice H implication algebras. Xu and Qin [3] introduced the notion of filters in a lattice implication, and investigated their properties. The present author [4, 5] introduced the notion of derivation and f-derivation in lattice implications algebras and obtained some related results. In this paper, we introduce the notion of (f, g)-derivation of lattice implication algebra and investigated some related properties. Also, we prove that if D is an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x \in L$, then we get $D(x \to y) = f(x) \to D(y)$ for all $x, y \in L$.

2. Preliminary

A *lattice implication algebra* is an algebra $(L; \wedge, \vee, ', \to, 0, 1)$ of type $(2, 2, 1, 2, 0, 0)$, where $(L; \wedge, \vee, 0, 1)$ is a bounded lattice, " $'$ " is an order-reversing involution and " $\to$ " is a binary operation satisfying the following axioms: for all $x, y, z \in L$,

(L1) $x \to (y \to z) = y \to (x \to z)$,
(L2) $x \to x = 1$,

(L3) $x \to y = y' \to x'$,
(L4) $x \to y = y \to x = 1 \Rightarrow x = y$,
(L5) $(x \to y) \to y = (y \to x) \to x$,
(L6) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$,
(L7) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$.

If L satisfies conditions (L1) – (L5), then we say that L is a *quasi lattice implication algebra*. A lattice implication algebra L is called a *lattice H implication algebra*, if it satisfies $x \vee y \vee ((x \wedge y) \to z) = 1$ for all $x, y, z \in L$ (See [1]).

In the sequel, the binary operation " $\to$ " will be denoted by juxtaposition. We can define a partial ordering " $\leq$ " on a lattice implication algebra L by $x \leq y$ if and only if $x \to y = 1$ for all $x, y \in L$.

Proposition 2.1 ([1])**.** *In a lattice implication algebra L, the following hold for all $x, y, z \in L$.*
(1) $0 \to x = 1$, $1 \to x = x$ *and* $x \to 1 = 1$,
(2) $x \to y \leq (y \to z) \to (x \to z)$,
(3) $x \leq y$ *implies* $y \to z \leq x \to z$ *and* $z \to x \leq z \to y$,
(4) $x' = x \to 0$.
(5) $x \vee y = (x \to y) \to y$,
(6) $((y \to x) \to y')' = x \wedge y = ((x \to y) \to x')'$,
(7) $x \leq (x \to y) \to y$.

Definition 2.2 ([1])**.** In a lattice H implication algebra L, the following hold: for all $x, y, z \in L$,
(8) $x \to (x \to y) = x \to y$,
(9) $x \to (y \to z) = (x \to y) \to (x \to z)$.

Definition 2.3 ([3])**.** A subset F of a lattice implication algebra L is called a *filter* of L, if it satisfies the following axioms: for all $x, y \in L$,
(F1) $1 \in F$,
(F2) $x \in F$ and $x \to y \in F$ imply $y \in F$.

Definition 2.4 ([4])**.** Let L_1 and L_2 be lattice implication algebras.
(i) A mapping $f : L_1 \to L_2$ is an *implication homomorphism*, if

$$f(x \to y) = f(x) \to f(y) \text{ for all } x,\ y \in L_1.$$

(ii) A mapping $f : L_1 \to L_2$ is an *lattice implication homomorphism*, if

$$f(x \vee y) = f(x) \vee f(y),\ f(x \wedge y) = f(x) \wedge f(y),\ f(x') = f(x)' \text{ for all } x,\ y \in L_1.$$

Definition 2.5 ([5])**.** Let L be a lattice implication algebra and let $f : L \to L$ be an implication homomorphism on L. A mapping $d : L \to L$ is called an *f-derivation* of L, if there exists an implication homomorphism f such that

$$d(x \to y) = (f(x) \to d(y)) \vee (d(x) \to f(y)) \text{ for all } x,\ y \in L.$$

Proposition 2.6 ([5])**.** *Let d be a f-derivation on L. Then the following conditions hold: for every $x,\ y \in L$,*
(1) $d(1) = 1$,
(2) $d(x) = d(x) \vee f(x)$,

(3) $f(x) \leq d(x)$,
(4) $f(x) \vee f(y) \leq d(x) \vee d(y)$,
(5) $d(x \to y) = f(x) \to d(y)$.

3. (f, g)-derivations of lattice implication algebras

In what follows, let L denote a lattice implication algebra unless otherwise specified.

Definition 3.1. Let L be a lattice implication algebra and let f, g be two self maps on L. A map $D : L \to L$ is an (f, g)-*derivation* of L, if

$$D(x \to y) = (f(x) \to D(y)) \vee (D(x) \to g(y)),$$

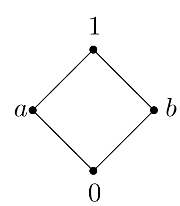
for all $x, y \in L$, where f and g are implication homomorphisms on L.

Let L be a lattice implication algebra and let f and g be implication homomorphisms on L. If $f = g$, then D is an f-derivation on L.

Example 3.2. Let $X = \{x, y\}$. Then

$$L = \mathcal{P}(X) = \{\varnothing, \{x\}, \{y\}, X\}.$$

Let $0 = \varnothing$, $a = \{x\}$, $b = \{y\}$, $1 = X$. Then $L = \{0, a, b, 1\}$ is a bounded lattice with above Hasse diagram.



We can make an implication $\to$ on L such as

$$a \to b = \{x\}^C \cup \{y\} = \{y\} \cup \{y\} = \{y\} = b.$$

Thus we have the operation table of the implication :

x	x'
0	1
a	b
b	a
1	0

$\to$	0	a	b	1
0	1	1	1	1
a	b	1	b	1
b	a	a	1	1
1	0	a	b	1

Define three maps $D : L \to L$, $f : L \to L$ and $g : L \to L$ by

$$D(x) = \begin{cases} b & \text{if } x = 0, a \\ 1 & \text{if } x = b, 1 \end{cases} \quad f(x) = \begin{cases} 0 & \text{if } x = 0, a \\ 1 & \text{if } x = b, 1 \end{cases} \quad g(x) = \begin{cases} b & \text{if } x = 0, a \\ 1 & \text{if } x = b, 1. \end{cases}$$

Then it is easy to check that D is an (f, g)-derivation on lattice implication algebra L.

Example 3.3. In Example 3.2, Define three maps $D : L \to L$, $f : L \to L$ and $g : L \to L$ by

$$D(x) = \begin{cases} a & \text{if } x = 0 \\ 1 & \text{if } x = a, b, 1 \end{cases} \qquad f(x) = \begin{cases} 0 & \text{if } x = 0, b \\ 1 & \text{if } x = a, 1 \end{cases} \qquad g(x) = \begin{cases} b & \text{if } x = 0, a \\ 1 & \text{if } x = 1 \\ 0 & \text{if } x = b. \end{cases}$$

Then it is easy to check that D is an (f, g)-derivation on lattice implication algebra L.

Proposition 3.4. *Let f and g be implication homomorphisms on L and let D be an (f, g)-derivation on L. Then the following conditions hold: for every $x,\ y \in L$,*

(1) $D(1) = 1$,
(2) $D(x) = D(x) \vee g(x)$,
(3) $g(x) \leq D(x)$,
(4) $g(x) \to y \leq D(x) \to y$.

Proof. (1) Let D be an (f, g)-derivation on L. Then

$$\begin{aligned} D(1) &= D(1 \to 1) = (f(1) \to D(1) \vee (D(1) \to g(1)) \\ &= (1 \to D(1)) \vee (D(1) \to 1) = D(1) \vee 1 = 1. \end{aligned}$$

(2) Let $x \in L$. Then we have

$$\begin{aligned} D(x) &= D(1 \to x) = (f(1) \to D(x)) \vee (D(1) \to g(x)) \\ &= (1 \to D(x)) \vee (1 \to g(x)) = D(x) \vee g(x). \end{aligned}$$

(3) Let $x \in L$. Then by part (2), we obtain

$$\begin{aligned} g(x) \to D(x) &= g(x) \to (D(x) \vee g(x)) = g(x) \to (D(x) \to g(x) \to g(x)) \\ &= (D(x) \to g(x)) \to (g(x) \to g(x)) = (D(x) \to g(x)) \to 1 \\ &= 1. \end{aligned}$$

Thus $g(x) \leq D(x)$.

(4) Let $x,\ y \in L$. Then by part (3), we have $g(x) \leq D(x)$. Thus by Proposition 2.1 (3), we get $g(x) \to y \leq D(x) \to y$. □

Proposition 3.5. *Let f, g be implication homomorphisms on L and let D be an (f, g)-derivation on L. If $f(x) \leq g(x)$ for every $x \in L$, then the following conditions hold: for all $x,\ y \in L$,*

(1) $D(x) \to D(y) \leq D(x \to y)$,
(2) $D(x) \to f(y) \leq f(x) \to D(y)$,
(3) $f(x) \to f(y) \leq D(x \to y)$.

Proof. (1) Let $x,\ y \in L$. Then from Definition 3.1 and Proposition 2.1 (7), we have

$$x \to D(y) \leq (f(x) \to D(y)) \vee (D(x) \to g(y)) = D(x \to y).$$

Now from $f(x) \leq D(x)$ and Proposition 2.1 (3), we get $D(x) \to D(y) \leq f(x) \to D(y)$. Thus $D(x) - D(y) \leq D(x \to y)$.

(2) Let $x,\ y \in L$. Then from $f(x) \leq D(x)$ and $f(y) \leq D(y)$, we get

$$D(x) \to f(y) \leq f(x) \to f(y) \text{ and } f(x) \to f(y) \leq f(x) \to D(y)$$

by using by Proposition 2.1 (3). Then we obtain $D(x) \to f(y) \leq f(x) \to D(y)$.
(3) Let $x,\ y \in L$. from Definition 3.1 and Proposition 2.1 (7), we have

$$f(x) \to D(y) \leq (f(x) \to D(y)) \vee (D(x) \to g(y)) = D(x \to y).$$

Since $f(y) \leq D(y)$, we get $f(x) \to f(y) \leq f(x) \to D(y)$. Thus $f(x) \to f(y) \leq D(x \to y)$. □

Theorem 3.6. *Let D be an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x \in L$. Then we get $D(x \to y) = f(x) \to D(y)$ for all $x,\ y \in L$.*

Proof. Suppose that D is an (f, g)-derivation on L and let $x, y \in L$. since $f(x) \leq D(x)$, $D(x) \to D(y) \leq f(x) \to D(y)$. Since $g(y) \leq D(y)$, $D(x) \to g(y) \leq D(x) \to D(y)$ Then we have $D(x) \to g(y) \leq f(x) \to D(y)$. Thus we get

$$\begin{aligned} D(x \to y) &= (f(x) \to D(y)) \vee (D(x) \to g(y)) \\ &= ((f(x) \to D(y)) \to (D(x) \to g(y))) \to (D(x) \to g(y)) \\ &= ((D(x) \to g(y)) \to (f(x) \to D(y))) \to (f(x) \to D(y)) \\ &= 1 \to (f(x) \to D(y)) = f(x) \to D(y) \end{aligned}$$

from (L5) and by Proposition 2.1 (3). This completes the proof. □

Theorem 3.7. *Let D be an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x \in L$. If it satisfies $D(x \to y) = D(x) \to f(y)$ for every $x,\ y \in L$, we have $D(x) = f(x)$.*

Proof. Let D be an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x \in L$. If it satisfies $D(x \to y) = D(x) \to f(y)$ for all $x,\ y \in L$, then by Theorem 3.6, we have

$$\begin{aligned} D(x) &= D(1 \to x) = D(1) \to f(x) \\ &= 1 \to f(x) = f(x) \end{aligned}$$

for every $x \in L$. This completes the proof.

□

Theorem 3.8. *Let D be an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x \in L$. If f is a lattice implication homomorphism on L, then we have $D(x \vee y) = D(x) \vee D(y)$ for every $x,\ y \in L$.*

Proof. Let $x,\ y \in L$. Then by Theorem 3.6, we obtain

$$\begin{aligned} D(x \vee y) &= D(x'' \vee y'') = D((x' \wedge y') \to 0) \\ &= (f(x') \wedge f(y')) \to D(0) = (f(x') \to D(0)) \vee (f(y') \to D(0)) \\ &= D(x' \to 0) \vee D(y' \to 0) = D(x) \vee D(y). \end{aligned}$$

□

Theorem 3.9. *Let D be an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x \in L$. Then D is an isotone generalized derivation on L.*

Proof. Let $x_1,\ x_2 \in L$ be such that $x_1 \leq x_2$. Then by Theorem 3.8, we get $D(x_1 \vee x_2) = D(x_1) \vee D(x_2)$. Since $x_1 \leq x_2$, we have $x_1 \vee x_2 = x_2$. Thus $D(x_1) \vee D(x_2) = D(x_2)$. So $D(x_1) \leq D(x_2)$. □

Definition 3.10. Let L be a lattice implication algebra L and let D be an (f, g)-derivation on L.

(i) D is called a *monomorphic (f, g)-derivation* of L, if D is one-to- one.
(ii) D is called an *epic (f, g)-derivation* of L, if D is onto.

Theorem 3.11. *Let L be a lattice implication algebra L and let D be an (f, g)-derivation on L. Then the following conditions are equivalent:*

(1) *$D(x) = x$ for all $x \in L$,*
(2) *D is a monomorphic (f, g)-derivation of L,*
(3) *D is an epic (f, g)-derivation of L.*

Proof. (1) $\Rightarrow$(2): The proof is clear.

(2) $\Rightarrow$(1): Let D be a monomorphic (f, g)-derivation of L and $x \in L$. By hypothesis, we have $D(D(x)) = D(x)$ for every $x \in L$. Since D is monomorphic, we get $D(x) = x$ for all $x \in L$.

(1) $\Rightarrow$(3): The proof is trivial.

(3) $\Rightarrow$(1): Let D be an epic (f, g)-derivation of L and $x \in L$. Then there exists $y \in L$ such that $D(y) = x$. Thus we have $D(x) = D(D(y)) = D^2(y) = D(y) = x$. □

Let L be a lattice implication algebra and let D be an (f, g)-derivation on L. Define a set $Fix_D(L)$ by

$$Fix_D(L) := \{x \in L \mid D(x) = f(x)\}$$

for all $x \in L$. Clearly, $1 \in Fix_D(L)$.

Proposition 3.12. *Let D be an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x \in L$. Then the following properties hold.*

(1) *If $x \in L$ and $y \in Fix_D(L)$, we have $x \to y \in Fix_D(L)$.*
(2) *If $x \in L$ and $y \in Fix_D(L)$, we have $x \vee y \in Fix_D(L)$.*

Proof. (1) Let $x \in L$ and $y \in Fix_D(L)$. Then we have $D(y) = f(y)$. Thus by Theorem 3.6, we get

$$D(x \to y) = f(x) \to D(y) = f(x) \to f(y) = f(x \to y).$$

This completes the proof.

(2) Let $x, y \in Fix_D(L)$. Then by Theorem 3.6, we get

$$\begin{aligned} D(x \vee y) &= D((x \to y) \to y) = f(x \to y) \to D(y) \\ &= f(x \to y) \to f(y) = f((x \to y) \to y) \\ &= f(x \vee y). \end{aligned}$$

This completes the proof. □

Proposition 3.13. *Let f be a lattice implication homomorphism on L and let D be an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x,\ y \in L$. L. If $x \leq y$ and $x \in Fix_D(L)$, we have $y \in Fix_D(L)$.*

Proof. Let $x \leq y$ and $x \in Fix_D(L)$. The by the hypothesis, $D(x) = x$ for every $x \in L$. Thus from Theorem 3.6, we get

$$\begin{aligned} D(y) &= D((1 \to y) = D((x \to y) \to y) \\ &= D((y \to x) \to x) = f(y \to x) \to D(x) \\ &= f(y \to x) \to f(x) = (f(y) \to f(x)) \to f(x) \\ &= (f(x) \to f(y)) \to f(y) = f(x) \vee f(y) = f(x \vee y) = f(y), \end{aligned}$$

So $y \in Fix_d(L)$. □

Definition 3.14. Let L be a lattice implication algebra. A non-empty set F of L is called a *normal filter*, if it satisfies the following conditions:

(i) $1 \in F$,
(ii) $x \in L$ and $y \in F$ imply $x \to y \in F$.

Example 3.15. In Example 3.3, let $F = \{1, a\}$. Then F is a normal filter of a lattice implication algebra L.

Proposition 3.16. *Let L be a lattice implication algebra L and let D be an (f, g)-derivation on L. Then $Fix_D(L)$ is a normal filter of L.*

Proof. Clearly, $1 \in Fix_D(L)$. By Proposition 3.12 (1), we know that $x \in L$ and $y \in F$ imply $x \to y \in F$. This completes the proof. □

Let L be a lattice implication algebra and let D be an (f, g)-derivation on L. Define a set $KerD$ by

$$KerD = \{x \in L \mid D(x) = 1\}.$$

Proposition 3.17. *Let L be a lattice implication algebra L and let D be an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x,\ y \in L$.*

(1) *If $y \in KerD$, then we have $x \vee y \in KerD$ for all $x \in L$.*
(2) *If $x \leq y$ and $x \in KerD$, then $y \in KerD$.*
(3) *If $y \in KerD$, we have $x \to y \in KerD$ for all $x \in L$.*

Proof. (1) Let D be an (f, g)-derivation on L and $y \in KerD$. Then we get $D(y) = 1$. Thus from Theorem 3.6,

$$D(x \vee y) = D((x \to y) \to y) = f(x \to y) \to D(y) = f(x \to y) \to 1 = 1.$$

So we have $x \vee y \in KerD$.

(2) Let $x \leq y$ and $x \in KerD$. Then we get $x \to y = 1$ and $D(x) = 1$. Thus from Theorem 3.6,

$$\begin{aligned} D(y) &= D(1 \to y) = D((x \to y) \to y) \\ &= D((y \to x) \to x) = f(y \to x) \to D(x) \\ &= f(y \to x) \to 1 = 1. \end{aligned}$$

So we have $y \in KerD$.

(3) Let $y \in KerD$. Then $D(y) = 1$. Thus from Theorem 3.6, we have

$$D(x \to y) = f(x) \to D(y) = f(x) \to 1 = 1$$

So we get $x \to y \in KerD$. □

Theorem 3.18. *Let D be an (f, g)-derivation on L and $f(x) \leq g(x)$ for every $x,\ y \in L$. Then $KerD$ is a normal filter of L.*

Proof. Clearly, $1 \in KerD$. Let $x \in L$ and $y \in KerD$. Then we have $d(y) = 1$. Thus

$$D(x \to y) = f(x) \to D(y) = x \to 1 = 1,$$

So from Theorem 3.6, $x \to y \in KerD$. Hence $KerD$ is a normal filter of L. □

Definition 3.19. Let D be an (f, g)-derivation of lattice implication algebra L. A normal filter F of L is called a *D-normal filter*, if $D(F) = F$.

Since $D(1) = 1$, it can be easily observed that the zero normal filter $\{1\}$ is a D-normal filter of L. If L is onto, then $D(L) = L$, which implies L is a D-normal filter of L.

Example 3.20. In Example 3.3, let $F = \{1, a, b\}$. Then F is a normal filter of D. It can be verified that $D(F) = F$. Thus F is an D-normal filter of L.

Lemma 3.21. *Let D be an (f, g)-derivation on L and let I, J be any two D-normal filters of L. Then we have $I \subseteq J$ implies $D(I) \subseteq D(J)$.*

Proof. Let $I \subseteq J$ and $x \in D(I)$. Then we have $x = D(y)$ for some $y \in I \subseteq J$. Thus we get $x = D(y) \in D(J)$. So $D(I) \subseteq D(J)$. □

Proposition 3.22. *Let D be an (f, g)-derivation on L. Then an intersection of any two D-normal filters is also a D-normal filter of L.*

Proof. Let $x \in D(I \cap J)$. Then $x = D(a)$ for some $a \in I$ and $a \in J$. Thus $x = D(a) \in D(I) = I$ and $x = D(a) \in D(J) = J$, which implies $x \in I \cap J$. Now let $x \in I \cap J$. Then $x \in I = D(I)$ and $x \in J = D(J)$. Thus we have $x \in D(I) \cap D(J)$. So $I \cap J$ is a D-normal filter of L. □

Definition 3.23. Let D be an (f, g)-derivation on L. A normal filter F of L is called an *injective normal filter* with respect to D, if for $x, y \in L$, $D(x) = D(y)$ and $x \in F$ implies $y \in F$.

Evidently, $KerD$ is an injective normal filter of L. Though the zero normal filter $\{1\}$ is a D-normal filter, there is no guarantee that it is injective normal filter.

Theorem 3.24. *Let D be an (f, g)-derivation on L. Then the following conditions are equivalent:*

(1) *$\{1\}$ is injective with respect to D,*
(2) *$KerD = \{1\}$,*
(3) *$D(x) = 1$ implies that $x = 1$ for all $x \in L$.*

Proof. (1) ⇒ (2): Suppose that $\{1\}$ is injective with respect to D. Let $x \in KerD$. Then $D(x) = D(1)$. Since $\{1\}$ is injective, we can get $x \in \{1\}$. Thus $KerD = \{1\}$.

(2) ⇒ (3): The proof is trivial.

(3) ⇒ (1): Let $D(x) = D(y)$ and $x \in \{1\}$. Then $D(y) = D(x) = D(1) = 1$. Thus $y = 1 \in \{1\}$.

□

Theorem 3.25. *Let D be an (f, g)-derivation on L and let D be idempotent. Then a D-normal filter F of L is injective with respect to D if and only if for any $x \in L, D(x) \in F$ implies $x \in F$.*

Proof. Let F be a D-normal filter of L and let F be injective with respect to D. Suppose that $D(x) \in F = D(F)$ and $x \in L$. Then $D(x) = D(a)$ for some $a \in F$. Since F is injective and $a \in F$, we get that $x \in F$.

Conversely, let $x, y \in L, D(x) = D(y)$ and $x \in F$. Since $x \in D(F)$, we get $x = D(a)$ for some $a \in F$. Then $D(y) = D(x) = D(D(a)) = D(a) \in D(F)$, which implies that $y \in F$. Thus F is an injective normal filter of L with respect to D.

□

4. Conclusion

We investigate the (f, g)-derivation, which is a generalization of f-derivation in lattice implication algebras. Also, we prove that if D is an (f, g)-derivation on lattice implication algebra L and $f(x) \leq g(x)$ for every $x \in L$, then we get $D(x \to y) = f(x) \to D(y)$ for all $x, y \in L$. In the future, we study (f, g)-derivation in other algebraic structure by using results obtained lattice implication algebras.

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References

[1] [2] Y. Xu, Lattice implication algebras, J. Southwest Jiaotong Univ. 1 (1993) 20–27.
[2] [3] Y. Xu and K. Y. Qin, Lattice H implication algebras and lattice implication algebra classes, J. Hebei Mining and Civil Engineering Institute 3 (1992) 139–143.
[3] [4] Y. Xu and K. Y. Qin, On filters of lattice implication algebras, J. Fuzzy Math. 1 (2) (1993), 251–260.
[4] [1] S. D. Lee and K. H. Kim, On derivations of lattice implication algebras, Ars Combinatoria 27 (4) (2019), 1119-1131.
[5] [5] Y. H. Yon and K. H. Kim, On f-derivations of lattice implication algebras, Ars Combinatoria 110 (2013) 205-215.

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