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Abstract. M.Murali Krishna Rao [14] introduced the notion of tripolar fuzzy set as a generalization of fuzzy set, bipolar fuzzy set and intuitionistic fuzzy set.The tripolar fuzzy set representation is very useful in discriminating relevant elements, irrelevent elements and contrary elements. In this paper,we introduce the notion of tripolar fuzzy interior ideal,tripolar fuzzy soft ideal, tripolar fuzzy soft interior ideal over semigroup and study some of their algebraic properties.

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1. Introduction

The theory of fuzzy sets is the most appropriate theory for dealing with uncertainty was introduced by Zadeh [18] in 1965. After the introduction of the concept of fuzzy sets by Zadeh, several researchers conducted the researches on the generalizations of the notions of fuzzy sets with huge applications in computer science, artifcial intelligence,control engineering, robotics, automata theory, decision theory, finite state machine, graph theory, logic, operations research and many branches of pure and applied mathematics. There are many extensions of fuzzy sets, for example, intuitionistic fuzzy sets, hesitant fuzzy sets, interval valued fuzzy sets, vague sets, type-2 fuzzy sets, fuzzy multi sets, bipolar fuzzy sets and cubic sets. The fuzzification of algebraic structure was introduced by Rosenfeld [17] and he introduced the notion of fuzzy subgroups in 1971.

Many problems in different disciplines such as economics, engineering science, social sciences, medical and artificial intelligence are not precise. There are always many types of uncertainties involved in the data. In dealing with uncertainties, many theories have been recently developed. In 1999, Molodtsov [12] developed the theory of soft sets involving enough parameters so that many difficulties that we are facing become easier by applying soft sets. It has been shown by some authors that soft sets also have applications in decision making problems and in information science. Bipolar fuzzy set is an extension of fuzzy set whose membership degree range is $[-1,1]$. In 1994, Zhang [19] initiated the concept of bipolar fuzzy set as a generalization of fuzzy set. In 2000, Lee [9, 10] used the term bipolar valued fuzzy sets and applied it to algebraic structure. Kim and Jun [7] studied intutionistic fuzzy interior ideals in semigroups. Jun et al. [6] introduced the notion of bipolar fuzzy ideals and bipolar fuzzy filters in BCK/BCI-algebras. K. J. Lee [8] studied bipolar fuzzy subalgebras and bipolar fuzzy ideals of $BCK/BCI-$algebras. In 2001, Maji et al. [11] combined the concept of fuzzy set which was introduced by Zadeh in 1965 and the notion of soft set which was introduced by Molodstov in 1999. Fuzzy soft set theory has been investigated by many researchers and proved to be useful in many different fields such as decision making, data analysis forecasting and so on. The notion of an intuitionistic fuzzy set was introduced by Atanassov [3] as a generalization of notion of fuzzy set. Acar et al. [1] gave the basic concept of soft ring. Aktas and Cagman [2] introduced the concept of soft group which was extended to fuzzy soft group by Aygnnogln and Aygun [4]. Ghosh et al. [5] initiated the study of fuzzy soft rings and fuzzy soft ideals.

Rao [14] introduced the notion of tripolar fuzzy set as a generalization of fuzzy set, bipolar fuzzy set and intuitionistic fuzzy set, and studied tripolar fuzzy interior ideals of $\Gamma-$semigroup. Rao and Venkateswarlu [15, 16] investigated tripolar fuzzy interior ideal, tripolar fuzzy soft ideal and tripolar fuzzy soft interior ideal of $\Gamma-$semigroup and $\Gamma-$semiring. In 1999, Shum et al. [13] established that the regularity of a semigroup M does not imply the regularity of a soft semigroup over M. Also, the regularity of a soft semigroup over semigroup does not imply the regularity of the semigroup. In this paper, we introduce the notion of tripolar fuzzy interior ideal of semigroup, tripolar fuzzy soft ideals and tripolar fuzzy soft interior ideals over semigroup. We study some of their algebraic properties and relations between them.

2. Preliminaries

In this section, we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. A non-empty subset A of a semigroup M is called

(i) a subsemigroup of M, if $AA \subseteq A$,
(ii) an interior ideal of M, if $MAM \subseteq A$ and $AA \subseteq A$,
(iii) a left (right) ideal of M, if $MA \subseteq A(AM \subseteq A)$,
(iv) an ideal, if $AM \subseteq A$ and $MA \subseteq A$.

Definition 2.2 ([14])**.** A fuzzy subset μ of a semigroup M is called
(i) a fuzzy subsemigroup of M, if $\mu(xy) \geq min\ \{\mu(x), \mu(y)\}$, for all $x, y \in M$,
(ii) a fuzzy left (right) ideal of M, if $\mu(xy) \geq \mu(y)\ (\mu(x))$, for all $x, y \in M$,
(iii) a fuzzy ideal of M, if $\mu(xy) \geq max\ \{\mu(x), \mu(y)\}$, for all $x, y \in M$.

Definition 2.3 ([9]). A bipolar fuzzy set A of a non-empty set X is an object having the form $A = \{(x, \mu_A(x), \delta_A(x)) \mid x \in X\}$, where $\mu_A : X \to [0,1]; \delta_A : X \to [-1,0]$. $\mu_A(x)$ represents degree of satisfaction of an element x to the property corresponding to fuzzy set A and $\delta_A(x)$ represents degree of satisfaction of an element x to the implicit counter property of fuzzy set A.

Definition 2.4 ([11]). Let $(f, A), (g, B)$ be fuzzy soft sets over a semigroup M. The intersection of fuzzy soft sets (f, A) and (g, B) is denoted by $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$ is defined as: for all $c \in A \cup B$,

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B \\ g_c, & \text{if } c \in B \setminus A \\ f_c \cap g_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.5 ([11]). Let $(f, A), (g, B)$ be fuzzy soft sets over a semigroup M. The union of fuzzy soft sets (f, A) and (g, B) is denoted by $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$ is defined as: for all $c \in A \cup B$.

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B \\ g_c, & \text{if } c \in B \setminus A \\ f_c \cup g_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.6 ([11]). Let $(f, A), (g, B)$ be fuzzy soft sets over a semigroup M. Then the meet of (f, A) and (g, B), denoted by $(f, A) \wedge (g, B)$, is defined by:

$$(f, A) \wedge (g, B) = (f \cap g, C) = (h, C),$$

where $C = A \times B,\ h_c(x) = \min\{f_a(x), g_b(x)\}$, for all $c = (a, b) \in A \times B$ and $x \in M$.

Definition 2.7 ([11]). Let $(f, A), (g, B)$ be fuzzy soft sets over a semigroup M. Then the join of (f, A) or (g, B), denoted by $(f, A) \vee (g, B)$, is defined by:

$$(f, A) \vee (g, B) = (h, C),$$

where $C = A \times B$ and $h_c(x) = \max\{f_a(x), g_b(x)\}$, for all $c = (a, b) \in A \times B, x \in M$.

Definition 2.8 ([14]). A fuzzy set A of a universe set X is said to be tripolar fuzzy set, if

$$A = \{(x, \mu_A(x), \lambda_A(x), \delta_A(x)) \mid x \in X \text{ and } 0 \leq \mu_A(x) + \lambda_A(x) \leq 1\},$$

where $\mu_A : X \to [0,1],\ \lambda_A : X \to [0,1],\ \delta_A : X \to [-1,0]$.
The membership degree $\mu_A(x)$ characterizes the extent that the element x satisfies to the property corresponding to tripolar fuzzy set A,$\lambda_A(x)$ characterizes the extent that the element x satisfies to the not property(irrelevant) corresponding to tripolar fuzzy set A and $\delta_A(x)$ characterizes the extent that the element x satisfies to the implicit counter property of tripolar fuzzy set A. For simplicity $A = (\mu_A, \lambda_A, \delta_A)$ has been used for $A = \{(x, \mu_A(x), \lambda_A(x), \delta_A(x)) \mid x \in X, 0 \leq \mu_A(x) + \lambda_A(x) \leq 1\}$

Example 2.9 ([14]). Assuming the sweet taste of food stuff as a positive membership value $\mu_A(x)$ i.e. the element x is satisfying the sweet property. Then bitter taste of food stuff as a negative membership value $\delta_A(x)$ i.e. the element x is satisfying the bitter property, and the remaining tastes of food stuffs like acidic, chilly etc., as a non membership value $\lambda_A(x)$ i.e., the element is satisfying irrelevent to the sweet property.Then the tastes of food stuffs can be considered as a tripolar fuzzy set and

$$A = \{(x, \mu_A(x), \lambda_A(x), \delta_A(x)) \mid x \in X\}.$$

3. Tripolar fuzzy soft interior ideals over semigroups

In this section, we introduce the notion of tripolar fuzzy subsemigroup, tripolar fuzzy ideal, tripolar fuzzy interior ideal, tripolar fuzzy soft ideal and tripolar fuzzy soft interior ideal over semigroup. We study some of their algebraic properties and relations between them.

Definition 3.1. A tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ of a semigroup M is called a tripolar fuzzy subsemigroup of M, if A satisfies the following conditions: for all $x, y, z \in M$,

(i) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(x)\}$,
(ii) $\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\}$,
(iii) $\delta_A(xy) \leq \max\{\delta_A(x), \delta_A(y)\}$.

Definition 3.2. A tripolar fuzzy subsemigroup $A = (\mu_A, \lambda_A, \delta_A)$ of a semigroup M is called a tripolar fuzzy ideal of M, if A satisfies the following conditions: for all $x, y, z \in M$,

(i) $\mu_A(xy) \geq \max\{\mu_A(x), \mu_{A)}(x)\}$,
(ii) $\lambda_A(xy) \leq \min\{\lambda_A(x), \lambda_A(y)\}$,
(iii) $\delta_A(xy) \leq \min\{\delta_A(x), \delta_A(y)\}$.

Definition 3.3. A tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ of a semigroup M is called a tripolar fuzzy interior ideal of M, if A satisfies the following conditions: for all $x, y, z \in M$,

(i) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(x)\}$,
(ii) $\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\}$,
(iii) $\delta_A(xy) \leq \max\{\delta_A(x), \delta_A(y)\}$,
(iv) $\mu_A(xzy) \geq \mu_A(z)$,
(v) $\lambda_A(xzy) \leq \lambda_A(z)$,
(vi) $\delta_A(xzy) \leq \delta_A(z)$.

Example 3.4. Let $M = \{a, b, c, d\}$ Then M is a semigroup if the binary operation $\cdot$ on M is defined as follows:

$\cdot$	a	b	c	d
a	a	a	a	a
b	b	b	b	b
c	a	a	a	a
d	a	a	a	d

Consider the tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ defined as follows:

$\mu_A = \{(a, 0.9), (b, 0.7), (c, 0.1), (d, 0.1)\}, \lambda_A = \{(a, 0.1), (b, 0.2), (c, 0.3), (d, 0.8)\}$,
$\delta_A = \{(a, -0.7), (b, -0.8), (c, -0.3), (d, -0.1)\}$.

Then we can easily check that A is a tripolar fuzzy interior ideal of the semigroup M.

Definition 3.5. A tripolar fuzzy soft set (f, A) over a semigroup M is called a tripolar fuzzy soft semigroup over M, if $f(a) = \{(\mu_{f(a)}(x), \lambda_{f(a)}(x), \delta_{f(a)}(x)) \mid x \in M, a \in A\}$, where $\mu_{f(a)}(x) : M \to [0, 1]; \lambda_{f(a)}(x) : M \to [0, 1]; \delta_{f(a)}(x) : M \to [-1, 0]$ such that $0 \leq \mu_{f(a)}(x) + \lambda_{f(a)}(x) \leq 1$ satisfying the following conditions: for all $x, y \in M$,

(i) $\mu_{f(a)}(xy) \geq \min\{\mu_{f(a)}(x), \mu_{f(a)}(x)\}$,
(ii) $\lambda_{f(a)}(xy) \leq \max\{\lambda_{f(a)}(x), \lambda_{f(a)}(y)\}$,
(iii) $\delta_{f(a)}(xy) \leq \max\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}$.

Definition 3.6. A tripolar fuzzy soft set (f, A) over a semigroup M is called a tripolar fuzzy soft ideal over M, if for all $x, y \in M$ and $a \in A$,

(i) $\mu_{f(a)}(xy) \geq \max\{\mu_{f(a)}(x), \mu_{f(a)}(x)\}$,
(ii) $\lambda_{f(a)}(xy) \leq \min\{\lambda_{f(a)}(x), \lambda_{f(a)}(y)\}$,
(iii) $\delta_{f(a)}(xy) \leq \min\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}$.

Example 3.7. Let $M = \{x_1, x_2, x_3\}$. Then We define binary operation with the following table:

$\cdot$	x_1	x_2	x_3
x_1	x_1	x_3	x_3
x_2	x_3	x_2	x_3
x_3	x_3	x_3	x_3

Let $E = \{a, b, c\}$ and $B = \{a, b\}$. Then (ϕ, B) is a tripolar fuzzy soft set defined as $(\phi, B) = \{\phi(a), \phi(b)\}$, where

$\phi(a) = \{(x_1, 0.2, 0.7, -0.2), (x_2, 0.3, 0.6, -0.3), (x_3, 0.6, 0.3, -0.3)\}$
$\phi(b) = \{(x_1, 0.4, 0.5, -0.3), (x_2, 0.6, 0.3, -0.5), (x_3, 0.5, 0.4, -0.2)\}$.

Then we can see that (ϕ, B) is a tripolar fuzzy soft semigroup over M, (ϕ, B) is not a tripolar fuzzy soft ideal over M and (ϕ, B) is a tripolar fuzzy soft interior ideal over M.

Definition 3.8. A tripolar fuzzy soft set(f, A) over semigroup M is called a tripolar fuzzy soft interior ideal of M, if for all $x, y, z \in M$ and $a \in A$,

(i) $\mu_{f(a)}(xzy) \geq \mu_{f(a)}(z)$,
(ii) $\lambda_{f(a)}(xzy) \leq \lambda_{f(a)}(z)$,
(iii) $\delta_{f(a)}(xzy) \leq \delta_{f(a)}(z)$.

The proofs of the following theorems are trivial, so we omit the proofs

Theorem 3.9. *Every tripolar fuzzy ideal of a semigroup M is a tripolar fuzzy interior ideal of a semigroup M.*

Theorem 3.10. *If a tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ of a semigroup M is an interior ideal of a semigroup M then $(\mu_A, \overline{\mu}_A, \delta_A)$ where $\overline{\mu}_A = 1 - \mu_A$, is a tripolar fuzzy interior ideal of a semigroup M.*

Definition 3.11. Let $A = (\mu_A, \lambda_A, \delta_A)$ be a tripolar fuzzy set of a semigroup M and $\alpha \in [0, 1]$. Then the sets $\mu_{A,} = \{x \in M \mid \mu_A(x) \geq \alpha\}$, $\lambda_{A,} = \{x \in M \mid \lambda_A(x) \leq \alpha\}$ and $\delta_{A,-\alpha} = \{x \in M \mid \delta_A(x) \leq -\alpha\}$ are called $\mu-$level α cut, $\lambda-$level α cut and $\delta-$level $-\alpha$ cut of A, respectively.

Theorem 3.12. *If $A = (\mu_A, \lambda_A, \delta_A)$ is a tripolar fuzzy interior ideal of a semigroup M then $\mu-$level $t$cut, $\lambda-$level $t$cut and $\delta-$level $t$cut of A are interior ideals of the semigroup M, for all $t \in Im(\mu_A) \cap Im(\lambda_A) \subseteq [0,1]$ and $-t \in Im(\delta_A)$.*

Proof. Let $t \in Im(\mu_A) \cap Im(\lambda_A) \subseteq [0,1]$ and $-t \in Im(\delta_A)$ and $x, y \in \mu_{A,t}$. Then $\mu_A(x) \geq t$ and $\mu_A(y) \geq t$. Thus $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \geq t$. So $xy \in \mu_{A,t}$. Hence $\mu_{A,t}$ is a subsemigroup of M.

Let $x,\ y \in M,\ z \in \mu_{A,t}$. Then $\mu_A(xzy) \geq \mu_A(z) \geq t$. Thus $xzy \in \mu_{A,t}$. So $\mu_{A,t}$ is an interior ideal of the semigroup M.

Suppose $x,\ y \in \lambda_{A,t}$. Then $\lambda_A(x) \leq t$ and $\lambda_A(y) \leq t$. Thus

$$\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\} \leq t.$$

So $xy \in \lambda_{A,t}$. Hence $\lambda_{A,t}$ is a subsemigroup of M.

Let $x,\ y \in M,\ z \in \lambda_{A,t}$ Then $\lambda_A(xzy) \leq \lambda_A(z) \leq t$. Thus $xzy \in \lambda_{A,t}$.

Suppose $x, y \in \delta_{A,-t}$.Then $\delta_A(x) \leq -t, \delta_A(y) \leq -t$. Thus

$$\delta_A(xy) \leq \max\{\delta_A(x), \delta_A(y)\} \leq -t.$$

So $x\alpha y \in \delta_{A,-t}$.

Let $x, y \in M, z \in \delta_{A,-t}$. Then $\delta_A(xzy) \leq \delta_a(z) \leq -t$. Thus $xzy \in \delta_{A,-t}$. So $\delta_{A,-t}$ is an interior ideal of the semigroup M. Hence the theorem holds. □

Theorem 3.13. *If $A = (\mu_A, \lambda_A, \delta_A)$ is a tripolar fuzzy interior ideal of a semigroup M then*

(1) $\mu_A(x) = \sup\{\alpha \in [0,1] \mid x \in \mu_{A,\alpha}\}$,
(2) $\lambda_A(x) = \inf\{\alpha \in [0,1] \mid x \in \lambda_{A,\alpha}\}$,
(3) $\delta_A(x) = \inf\{\alpha \in [-1,0] \mid x \in \delta_{A,\alpha}\}$, *for all* $x \in M$.

Proof. Proofs of (1) *and* (2) are similar to proof of Theorem 3.12 in [3], so we omit the proof.

Let $\eta = \inf\{\alpha \in [-1,0] \mid x \in \delta_{A,\alpha}\}$. Then we have

$$\inf\{\alpha \in [-1,0] \mid x \in \delta_{A,\alpha}\} < \eta + \epsilon, \text{ for any } \epsilon > 0.$$

Thus $\alpha < \eta + \epsilon$, for some $\alpha < 0, \alpha \in [-1,0],\ x \in \delta_{A,\alpha}$. since $\alpha < 0.\delta_A(x) \leq \alpha$, $\delta_A(x) \leq \eta$.

Let $\delta_A(x) = \beta, \beta \in [-1,0]$. Then $x \in \delta_{A,\beta}$

$$\begin{aligned}
&\Rightarrow \beta \in \{\alpha \in [-1,0] \mid x \in \delta_{A,\alpha}\}\\
&\Rightarrow \inf\{\alpha \in [-1,0] \mid x \in \delta_{A,\alpha}\} \leq \beta\\
&\Rightarrow \eta \leq \beta = \delta_A(x).\\
&\Rightarrow \delta_A(x) = \eta.
\end{aligned}$$

Thus $\delta_A(x) = \inf\{\alpha \in [-1,0] \mid x \in \delta_{A,\alpha}\}$. So the theorem holds. □

Theorem 3.14. *Let $A = (\mu_A, \lambda_A, \delta_A)$ be a tripolar fuzzy set in a semigroup M such that non-empty sets $\mu_{A,\alpha}, \lambda_{A,\alpha}, \delta_{A,-\alpha}$ are interior ideals of M, for all $\alpha \in [0,1]$. Then A is a tripolar fuzzy interior ideal of M.*

Theorem 3.15. *A tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ is a fuzzy interior ideal of a semigroup M if and only if fuzzy subsets $\mu_A, \overline{\lambda}_A, \delta_A$ are fuzzy interior ideals of M.*

Proof. Suppose $A = (\mu_A, \lambda_A, \delta_A)$ is a tripolar fuzzy interior ideal of semigroup M. Then obviously, μ_A, δ_A are fuzzy interior ideals of M.

Let $x,\ y \in M$. Then

$$\begin{aligned}\overline{\lambda}_A(xy) &= 1 - \lambda_A(xy) \geq 1 - \max\{\lambda_A(x), \lambda_A(y)\}\\ &= \min\{1 - \lambda_A(x), 1 - \lambda_A(y)\}\\ &= \min\{\overline{\lambda}_A(x), \overline{\lambda}_A(y)\}.\end{aligned}$$

Suppose $x,\ y,\ z \in M$. Then $\overline{\lambda}_A(xzy) = 1 - \lambda_A(xzy) \geq 1 - \lambda_A(z) = \overline{\lambda}_A(z)$. Thus $\overline{\lambda}$ is an fuzzy interior ideal of M.

Conversely, suppose that $\mu_A, \overline{\lambda}_A, \delta_A$ are fuzzy interior ideals of a semigroup M. Let $x,\ y,\ z \in M$. Then

$$\begin{aligned}\lambda_A(xy) &= 1 - \overline{\lambda}_A(xy)\\ &\geq \max\{1 - \overline{\lambda}_A(x), 1 - \overline{\lambda}_A(y)\}\\ &= \max\{\lambda_A(x), \lambda_A(y)\}.\end{aligned}$$

Thus $\overline{\lambda}_A(xzy) \geq \overline{\lambda}_A(z)$. So $1 - \lambda_A(xzy) \geq 1 - \lambda_A(z)$. Hence $\lambda_A(xzy) \leq \lambda_A(z)$. Therefore this completes the proof. □

Corollary 3.16. *A tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ is a tripolar fuzzy interior ideal of a semigroup M if and only if the fuzzy subsets $(\mu_A, \overline{\mu}_A, \delta_A)$ and $(\overline{\lambda}_A, \lambda_A, \delta_A)$ are tripolar fuzzy interior ideals of M.*

Definition 3.17. Let $f : X \to Y$ be a map, $A = (\mu_A, \lambda_A, \delta_A)$ and $B = (\mu_B, \lambda_B, \delta_B)$ be tripolar fuzzy sets in X and Y respectively. Then pre-image of B under f, denoted by $f^{-1}(B)$ is a tripolar fuzzy set in X defined by:

$$f^{-1} = \big(f^{-1}(\mu_B), f^{-1}(\lambda_B), f^{-1}(\delta_B)\big),$$

where $f^{-1}(\mu_B) = \mu_B(f), f^{-1}(\lambda_B) = \lambda_B(f)$ and $f^{-1}(\delta_B) = \delta_B(f)$.

Theorem 3.18. *Let $f : M \to N$ be a homomorphism of semigroups. If $B = (\mu_b, \lambda_B, \delta_B)$ is a tripolar fuzzy interior ideal of a semigroup N, then $f^{-1}(B) = \big(f^{-1}(\mu_B), f^{-1}(\lambda_B), f^{-1}(\delta_B)\big)$ is a tripolar fuzzy interior ideal of M.*

Proof. Suppose $B = (\mu_B, \lambda_B, \delta_B)$ is a tripolar fuzzy interior ideal of the semigroup N and $x, y \in M$. Then

$$\begin{aligned}f^{-1}\big(\mu_B(xy)\big) &= \mu_B(f(xy)) = \mu_B\big(f(x)f(y)\big)\\ &\geq \min\{\mu_B(f(x)), \mu_B(f(y))\}\\ &= \min\{f^{-1}(\mu_B(x)), f^{-1}(\mu_B(y))\}.\end{aligned}$$

Suppose $x, y, z \in M$. Then we have

$$\begin{aligned}f^{-1}\big(\mu_B(xzy)\big) &= \mu_B(f(xzy)) = \mu_B\big(f(x)f(z)f(y)\big)\\ &\geq \mu_B f(z) = f^{-1}(\mu_B f(z)),\\ f^{-1}\big(\lambda_B(xy)\big) &= \lambda_B(f(xy)) = \lambda_B\big(f(x)f(y)\big)\\ &\leq \max\{\lambda_B(f(x)), \lambda_B(f(y))\}\\ &= \max\{f^{-1}(\lambda_B(x)), f^{-1}(\lambda_B(y))\},\\ f^{-1}\big(\lambda_B(xzy)\big) &= \lambda_B(f(xzy))\\ &= \lambda_B\big(f(x)f(z)f(y)\big)\\ &\leq \lambda_B(f(z)) = f^{-1}(\lambda_B(f(z))),\\ f^{-1}\big(\delta_B(xy)\big) &= \delta_B(f(xy)) = \delta_B\big(f(x)f(y)\big)\\ &\leq \max\{\delta_B(f(x)), \delta_B(f(y))\}\\ &= \max\{f^{-1}(\delta_B(x)), f^{-1}(\delta_B(y))\},\end{aligned}$$

$$f^{-1}\big(\delta_B(xzy)\big) = \delta_B(f(xzy)) = \delta_B\big(f(x)f(z)f(y)\big)$$
$$\leq \delta_B(f(z)) = f^{-1}(\delta_B f(z)).$$

Thus $f^{-1}(B) = \big(f^{-1}(\mu_B), f^{-1}(\lambda_B), f^{-1}(\delta_B)\big)$ is a tripolar fuzzy interior ideal of the semigroup M. □

Theorem 3.19. *Let M and N be semigroups and $\phi : M \to N$ be an onto homomorphism. If A is a homomorphism ϕ invariant tripolar interior ideal of a semigroup M then the image of A under homomorphism ϕ is a tripolar fuzzy interior ideal of a semigroup M.*

Proof. Let $A = (\mu_A, \lambda_A, \delta_A)$ bs e a tripolar interior ideal of the semigroup M and $x,\ y \in N$. Then there exist $a, b \in M$ such that $\phi(a) = x, \phi(b) = y$ and

$\phi(\mu_A(xy)) = \mu_A(ab) \geq \min\{\mu_A(x), \mu_A(b)\} = \min\{\phi(\mu_A)(x), \phi(\mu_A)(y)\}$.

Suppose $x,\ y,\ z \in N$. Then there exist $a, b, c \in N$ such that $\phi(a) = x, \phi(b) = y$ and $\phi(c) = z$. Thus $\phi(\mu_A(xzy)) = \mu_A(acb) \geq \mu_A(c) = \phi(\mu_A(z))$. So $\phi(\mu_A)$ is a fuzzy interior ideal of the semigroup M.

On one hand,

$\phi(\lambda_A(xy)) = \lambda_A(ab) \leq \min\{\lambda_A(x), \lambda_A(b)\} = \min\{\phi(\lambda_A)(x), \phi(\lambda_A)(y)\}$

and

$\phi(\lambda_A(xzy) = \lambda_A(acb) \leq \lambda_A(c) = \phi(\lambda_A(z))$.

Then $\phi(\lambda_A)$ is a fuzzy interior ideal of the semigroup M.

On the other hand,

$\phi(\delta_A(xy)) = \delta_A(ab) \leq \min\{\delta_A(x), \delta_A(b)\} = \min\{\phi(\delta_A)(x), \phi(\delta_A)(y)\}$

and

$\phi(\delta_A(xzy) = \delta_A(acb) \leq \delta_A(c) = \phi(\delta_A(z))$.

Thus $\phi(\delta_A)$ is a fuzzy interior ideal of the semigroup M. So $\phi(A)$ is a tripolar fuzzy interior ideal of the semigroup M. □

Theorem 3.20. *Every tripolar fuzzy soft ideal over a semigroup M is a tripolar fuzzy soft interior ideal over a semigroup M.*

Proof. Let (f, A) be a tripolar fuzzy soft ideal over a semigroup M. Then $f(a) = \{\mu(a), \lambda(a), f(a)\}$ is a tripolar fuzzy ideal of $M,\ a \in A$. Thus

(i) $\mu_{f(a)}(xzy) \geq \mu_{f(a)}(xz) \geq \mu_{f(a)}(z)$,
(ii) $\lambda_{f(a)}(xzy) \leq \lambda_{f(a)}(xz) \leq \lambda_{f(a)}(z)$,
(iii) $\delta_{f(a)}(xzy) \leq \delta_{f(a)}(xz) \leq \delta_{f(a)}(z)$ for all $x, y, z \in M$.

So (f, A) is a tripolar fuzzy soft interior ideal over M. □

Theorem 3.21. *Every tripolar fuzzy soft interior ideal over a regular semigroup M is a tripolar fuzzy soft ideal over M.*

Proof. Let (f, A) be tripolar fuzzy soft interior ideal over the regular semigroup M. Then $f(a) = \{\mu_{f(a)}, \lambda_{f(a)}, \delta_{f(a)}\}$ is a tripolar fuzzy ideal of $M,\ a \in A$.

Suppose $x, y \in M$. Then $xy \in M$ and there exists $z \in M$ such that $xy = xyzxy$. Thus we have

$\mu_{f(a)}(xy) = \mu_{f(a)}(xyzxy) = \mu_{f(a)}(xy(zxy) \geq \mu_{f(a)}(y)$,
$\mu_{f(a)}(xy) = \mu_{f(a)}((xyz)xy) \geq \mu_{f(a)}(x)$.

So $\mu_{f(a)}$ is a fuzzy ideal of M.

Also, we have

$\lambda_{f(a)}(xy) = \lambda_{f(a)}(xyzxy) \leq \lambda_{f(a)}(y)$,
$\lambda_{f(a)}(xy) = \lambda_{f(a)}((xyz)xy) \leq \lambda_{f(a)}(x)$.
Then $\lambda_{f(a)}$ is a fuzzy ideal of M.

Moreover, we have
$\delta_{f(a)}(xy) = \delta_{f(a)}(xyzxy) \leq \delta_{f(a)}(y)$,
$\delta_{f(a)}(xy) \leq \delta_{f(a)}(x)$.
Thus $\delta_{f(a)}$ is a fuzzy ideal of M. So $f(a)$ is a tripolar fuzzy ideal of the regular semigroup M. Hence (f, A) is a tripolar fuzzy soft ideal over the regular semigroup M. □

Definition 3.22. A soft set $(f\ A)$ over a semigroup M is called a soft set with cover, if $\bigcup\limits_{a \in A} f(a) = M$.

Definition 3.23. A soft semigroup $(f,\ A)$ over a semigroup M is called a soft regular semigroup, if for each $a \in A,\ f(a)$ is regular subsemigroup.

Theorem 3.24. *Let $(f,\ A)$ be a soft regular semigroup over a semigroup M with cover. If (f, A) is a tripolar fuzzy soft interior ideal over semigroup M, then (f, A) is a tripolar fuzzy soft ideal over a semigroup M.*

Proof. Suppose $(f,\ A)$ is a soft regular semigroup with cover, (g, A) is a tripolar fuzzy soft interior ideal over semigroup M and $x \in M$. Then $f(a)$ is a regular subsemigroup of M for all $a \in A$ and $M = \bigcup\limits_{a \in A} f(a)$. Thus there exists $b \in A$ such that $x \in f(b)$. Since $f(b)$ is regular, there exists $y \in f(b)$ such that $x = xyx$. That implies $y \in f(b) \subseteq \bigcup\limits_{a \in A} f(a) = M$. So M is a regular semigroup. Hence by Theorem 3.21, (f, A) is a tripolar fuzzy soft ideal over the semigroup M. □

Theorem 3.25. *If (f, A) and (g, B) are two tripolar fuzzy soft ideals over semigroup M then $(f, A) \cup (g, B)$ is a tripolar fuzzy soft ideal over M.*

Proof. Suppose (f, A) and (g, B) are two tripolar fuzzy soft ideals over semigroup M. Then by Definition 2.5, we have $(f, A) \cup (g, B) = (h, C)$, where $C = A \cup B$, and

$$h(c) = f \cup g(c) = \begin{cases} f(c) & \text{if } c \in A \setminus B \\ g(c) & \text{if } c \in B \setminus A \\ f(c) \cup g(c) & \text{if } c \in A \cap B \quad \text{for all} \quad c \in A \cup B. \end{cases}$$

Case(i): If $c \in A \setminus B$, then $f \cup g(c) = f(c)$. Thus we have

$$\begin{aligned}
\mu_{f\cup g(c)}(xy) &= \mu_{f(c)}(xy)\\
&\geq \max\{\mu_{f(c)}(xy), \mu_{f(c)}(xy)\}\\
&= \max\{\mu_{f\cup g(c)}(x), \mu_{f\cup g(c)}(y)\}.\\
\lambda_{f\cup g(c)}(xy) &= \lambda_{f(c)}(xy)\\
&\leq \min\{\lambda_{f(c)}(x), \lambda_{f(c)}(y)\}\\
&= \min\{\lambda_{f\cup g(c)}(x), \lambda_{f\cup g(c)}(y)\}.\\
\delta_{f\cup g(c)}(xy) &= \delta_{f(c)}(xy)\\
&\leq \min\{\delta_{f(c)}(x), \delta_{f(c)}(y)\}\\
&= \min\{\delta_{f\cup g(c)}(x), \delta_{f\cup g(c)}(y)\}.
\end{aligned}$$

Case(ii): If $c \in B \setminus A$, then $f \cup g(c) = g(c)$. Since $g(c)$ is a tripolar fuzzy ideal of a semigroup $M, f \cup g(c)$ is a tripolar fuzzy ideal of a semigroup M.

Case(iii): If $c \in A \cap B$, then $f \cup g(c) = f(c) \cup g(c)$. Thus we have

$$\begin{aligned}
\mu_{f\cup g(c)}(xy) &= \mu_{f(c)\cup g(c)}(xy)\\
&= \max\{\mu_{f(c)}(xy), \mu_{g(c)}(xy)\}\\
&\geq \max\{\max\{\mu_{f(c)}(x), \mu_{f(c)}(y)\}, \max\{\mu_{g(c)}(x), \mu_{g(c)}(y)\}\}\\
&= \max\{\max\{\mu_{f(c)}(x), \mu_{g(c)}(x)\}, \max\{\mu_{f(c)}(y), \mu_{g(c)}(y)\}\}\\
&= \max\{\mu_{f(c)\cup g(c)}(x), \mu_{f(c)\cup g(c)}(y)\}\\
&= \max\{\mu_{f\cup g(c)}(x), \mu_{f\cup g(c)}(y)\}.
\end{aligned}$$

Similarly, we can prove

$$\lambda_{f\cup g(c)}(xy) \leq \min\{\lambda_{f\cup g(c)}(x), \lambda_{f\cup g(c)}(y)\},$$

$$\delta_{f\cup g(c)}(xy) \leq \min\{\delta_{f\cup g(c)}(x), \delta_{f\cup g(c)}(y)\}.$$

So $f \cup g(c)$ is a tripolar fuzzy ideal of M. Hence $(f, A) \cup (g, B)$ is a tripolar fuzzy soft ideal over M. □

Corollary 3.26. *If (f, A) and (g, B) are two tripolar fuzzy soft interior ideals over a semigroup M, then $(f, A) \cup (g, B)$ is a tripolar fuzzy soft interior ideal of a semigroup M.*

Theorem 3.27. *If (f, A) and (g, B) are two tripolar fuzzy soft interior ideals over a semigroup M, then $(f, A) \wedge (g, B)$ is a tripolar fuzzy interior ideal over a semigroup M.*

Proof. Suppose (f, A) and (g, B) are two tripolar fuzzy soft interior ideals over the semigroup M. Then obviously, $(f, A) \wedge (g, B)$ is a soft tripolar fuzzy subsemigroup of M. By Definition 2.6, $(f, A) \wedge (g, B) = (f \cap g, C)$, where $C = A \times B$.

Suppose $(a, b) \in C, x, y \in M$. Then

$$\begin{aligned}
\mu_{f\wedge g(a,b)}(xyz) &= \mu_{f(a)\cap g(b)}(xyz) \\
&\geq \min\{\mu_{f(a)}(y), \mu_{g(b)}(y)\} \\
&= \mu_{f(a)\cap g(b)}(y) \\
&= \mu_{f\wedge g(a,b)}(y), \\
\lambda_{f\wedge g(a,b)}(xyz) &= \lambda_{f(a)\cap g(b)}(xyz) \\
&= \min\{\lambda_{f(a)}(xyz), \lambda_{g(b)}(xyz)\} \\
&\leq \min\{\lambda_{f(a)}(y), \lambda_{g(b)}(y)\} \\
&= \lambda_{f(a)\cap g(b)}(y) \\
&= \lambda_{f\wedge g(a,b)}(y), \\
\delta_{f\wedge g(a,b)}(xyz) &= \delta_{f(a)\cap g(b)}(xyz) \\
&= \min\{\delta_{f(a)}(xyz), \delta_{g(b)}(xyz)\} \\
&\leq \min\{\delta_{f(a)}(y), \delta_{g(b)}(y)\} \\
&= \delta_{f(a)\cap g(b)}(y) \\
&= \delta_{f\wedge g(a,b)}(y).
\end{aligned}$$

Thus $(f, A) \wedge (g, B)$ is a soft tripolar fuzzy soft interior ideal of the semigroup M. □

Corollary 3.28. *If (f, A) and (g, B) are two tripolar fuzzy soft ideals over a semigroup M then $(f, A) \wedge (g, B)$ is a tripolar fuzzy soft ideal over a semigroup M.*

Corollary 3.29. *If (f, A) and (g, B) are two tripolar fuzzy soft ideals over semigroup M then $(f, A) \cap (g, B)$ is a tripolar fuzzy soft ideal over M.*

Corollary 3.30. *If (f, A) and (g, B) are two tripolar fuzzy soft interior ideals over a semigroup M then $(f, A) \cap (g, B)$ is a tripolar fuzzy soft interior ideal of a semigroup M.*

The proof of following theorem follows from Theorem 3.21 and Corollary 3.30.

Theorem 3.31. *If (f, A) and (g, B) are two tripolar fuzzy soft interior ideals over a regular semigroup M then $(f, A) \cap (g, B)$ is a tripolar fuzzy soft ideal of a semigroup M.*

Definition 3.32. Let (f, A) and (g, B) be two tripolar fuzzy soft sets over a semigroup M. The the product (f, A) and (g, B) is defined as $(f, A) \circ (g, B) = (f \circ g, C)$, where $C = A \circ B$.

$$\mu_{(f\circ g)(c)}(x) = \begin{cases} \mu_{f(c)}(x) & \text{if } c \in A \setminus B \\ \mu_{g(c)}(x) & \text{if } c \in B \setminus A \\ \sup\limits_{x=yz} \left\{ \min\{\mu_{f(c)}(y), \mu_{g(c)}(z)\} \right\} & \text{if } c \in A \cap B, \end{cases}$$

$$\lambda_{(f\circ g)(c)}(x) = \begin{cases} \lambda_{f(c)}(x) & \text{if } c \in A \setminus B \\ \lambda_{g(c)}(x) & \text{if } c \in B \setminus A \\ \inf\limits_{x=yz} \big\{ \max\{\lambda_{f(c)}(y), \lambda_{g(c)}(z)\}\big\} & \text{if } c \in A \cap B, \end{cases}$$

$$\delta_{(f\circ g)(c)}(x) = \begin{cases} \delta_{f(c)}(x) & \text{if } c \in A \setminus B \\ \delta_{g(c)}(x) & \text{if } c \in B \setminus A \\ \inf\limits_{x=yz} \big\{ \max\{\delta_{f(c)}(y), \delta_{g(c)}(z)\}\big\} & \text{if } c \in A \cap B. \end{cases}$$

Theorem 3.33. *If (f, A) and (g, B) are tripolar fuzzy soft interior ideals over semigroup M then $(f, A) \circ (g, B)$ is a tripolar fuzzy soft interior ideal over semigroup M.*

Proof. Obviously $(f, A) \circ (g, B)$ is tripolar fuzzy soft subsemigroup over M. Let $x,\ y,\ z \in M$. By Definition 3.32, $(f, A) \circ (g, B) = (f \circ g, C)$, where $C = A \cup B$ and $c \in C,\ x \in M$.

case(i): If $c \in A \setminus B$, then we have

$$\mu_{f\circ g(c)} = \mu_{f(c)},\ \lambda_{f\circ g(c)} = \lambda_{f(c)},\ \delta_{f\circ g(c)} = \delta_{f(c)}.$$

Since (f, A) is a tripolar fuzzy soft interior ideal over M, $f \circ g(c)$ is a tripolar fuzzy soft interior ideal of M.

case(ii): If $c \in B \setminus A$, then we have

$$\mu_{f\circ g(c)} = \mu_{g(c)},\ \lambda_{f\circ g(c)} = \lambda_{g(c)},\ \delta_{f\circ g(c)} = \delta_{g(c)}.$$

Since (g, B) is a tripolar fuzzy soft interior ideal over M, $f \circ g(c)$ is a tripolar fuzzy soft interior ideal of M.

case(iii): If $c \in A \cap B$, then we have

$$\begin{aligned} \mu_{f\circ g(c)}(x) &= \sup_{x=ab} \big\{ \min\{\mu_{f(a)}(x), \mu_{g(b)}(x)\}\big\}, \\ \mu_{f\circ g(c)}(xyz) &= \sup_{x=ab} \big\{ \min\{\mu_{f(a)}(xyz), \mu_{g(b)}(xyz)\}\big\} \\ &\geq \sup_{x=ab} \big\{ \min\{\mu_{f(a)}(y), \mu_{g(b)}(y)\}\big\} \\ &= \mu_{f\circ g(c)}(y), \\ \lambda_{f\circ g(c)}(xyz) &= \inf_{x=ab} \big\{ \max\{\lambda_{f(a)}(xyz), \lambda_{g(b)}(xyz)\}\big\} \\ &\leq \sup_{x=ab} \big\{ \max\{\lambda_{f(a)}(y), \lambda_{g(b)}(y)\}\big\} \\ &= \lambda_{f\circ g(c)}(y), \\ \delta_{f\circ g(c)}(xyz) &= \inf_{x=ab} \big\{ \max\{\delta_{f(a)}(xyz), \delta_{g(b)}(xyz)\big\} \\ &\leq \inf_{x=ab} \big\{ \max\{\delta_{f(a)}(y), \delta_{g(b)}(y)\}\big\} \\ &= \delta_{f\circ g(c)}(y). \end{aligned}$$

Thus $f \circ g(c)$ is a tripolar fuzzy interior ideal of M. So $(f, A) \circ (g, B)$ is a tripolar fuzzy interior ideal over M. □

Theorem 3.34. *Let E be a parameter set and $\sum\limits_{E}(M)$ be the set of all tripolar fuzzy soft interior ideals over semigroup M. Then $(\sum\limits_{E}(M), \cup, \cap)$ forms a complete distributive lattices along with the relation $\subseteq$.*

Proof. Suppose (f, A) and (g, B) are soft interior ideals over M such that $A \subseteq E, B \subseteq E$. Then by Corollaries 3.26 and 3.30, $(f, A) \cup (g, B)$ and $(f, A) \cap (g, B)$ are tripolar fuzzy soft interior ideals over M. It is obvious that $(f, A) \cap (g, B)$ is lub of $\{(f, A), (g, B)\}$ and $(f, A) \cup (g, B)$ is glb of $\{(f, A), (g, B)\}$. Thus every sub collection of $\sum\limits_{E}(M)$ has lub and glb. So $\sum\limits_{E}(M)$ is a complete lattice. Moreover, we can prove that

$$(f, A) \cap \big((g, B) \cup (h, C)\big) = \big((f, A) \cap (g, B)\big) \cup \big((f, A) \cap (h, C)\big).$$

Hence $(\sum\limits_{E}(M), \cup, \cap)$ forms a complete distributive lattice. □

4. Conclusion

In this paper, we introduced the notion of tripolar fuzzy interior ideals of semigroup. We proved that for any homomorphism ϕ from a semigroup M to a semigroup N, if A is a tripolar fuzzy interior ideal of M then homomorphic image $\phi(A)$ is a tripolar fuzzy interior ideal of N and B is a tripolar fuzzy interior ideal of N then the pre image $\phi^{-1}(B)$ is a tripolar fuzzy interior ideal of M. We also introduced the notion of tripolar fuzzy soft subsemigroup, tripolar fuzzy soft ideal, tripolar fuzzy soft interior ideals over semigroup and studied some of their algebraic properties and relations between them. We proved that the$\sum\limits_{E}(M)$ be the set of all tripolar fuzzy soft interior ideals over semigroup M. Then $(\sum\limits_{E}(M), \cup, \cap)$ forms complete distributive lattices along with the relation $\subseteq$. where E is a parameter set. In continuity of this paper,we study topology on tripolar fuzzy soft interior ideals over semigroup.

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References

[1] U. Acar, F. Koyuncu and B. Tanay, Soft sets and Soft rings, Comput. Math. Appli. 59 (2010) 3458–3463.
[2] H. Aktas and N. Cagman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2726–2735.
[3] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems 20 (1986) 87–96.
[4] A. Aygunoglu and H. Aygun, Introduction to fuzzy soft group, Comput. Math. Appli. 58 (2009) 1279–1286.
[5] J. Ghosh, B. Dinda and T. K. Samanta, Fuzzy soft rings and Fuzzy soft ideals, Int. J. P. App. Sc. Tech. 2 (2) (2011) 66–74.
[6] Y. B. Jun, H. S. Kim and K. J. Lee, Bipolar fuzzy translation in BCK/BCI algebra, J. of the Chungcheong Math. Soc. (22) (3) (2009) 399–408.
[7] K. H. Kim and Y. B. Jun, Intuitionistic fuzzy interior ideals of semigroups, Int. J. of Math. and Math. Sci. 27 (5) (2001) 261–267.
[8] K. J. Lee, Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI algebras, Bull. Malays. Math. Soc. (32) (2009) 361–373.

[9] K. M. Lee, Bipolar valued fuzzy sets and their applications, Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand (2000) 307–312.

[10] K. M. Lee, Comparison of interval valued fuzzy sets, intuitionistic fuzzy sets and bipolar valued fuzzy sets, J. Fuzzy Logic intelligent Systems (14) (2) (2004) 125–129.

[11] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, The J. of Fuzzy Math. 9 (3) (2001) 589–602.

[12] D. A. Molodtsov, Soft set theory first results, Computers and math. with appli. 37 (1999) 19–31.

[13] Muhammad Irfan Ali, Muhammad Shabir and K. P. Shum, On Soft Ideals over Semigroups, Southeast Asian Bulletin of Mathematics 34 (2010) 595–610.

[14] M. Murali Krishna Rao, Tripolar fuzzy interior ideals of $\Gamma-$semigroup, Ann. Fuzzy Math. Inform. 15 (2) (2018) 199–206.

[15] M. Murali Krishna Rao and B. Venkateswarlu, Tripolar fuzzy ideals of $\Gamma-$semiring, Asia Pacific J. of Math. 5 (2) (2018) 192–207.

[16] M. Murali Krishna Rao and B. Venkateswarlu, Tripolar fuzzy soft interior ideals and tripolar fuzzy soft interior ideals over $\Gamma-$semiring, FACTA UNIVERSITATIS (NI S) Ser. Math. Inform. 35 (1) (2020) 29–42.

[17] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512–517.

[18] L. A. Zadeh, Fuzzy sets, Information and control 8 (1965) 338–353.

[19] W. R. Zhang, Bipolar fuzzy sets, Proc. of FUZZ-IEEE (1998) 835–840.

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