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Abstract. In this paper, we prove the existence and uniqueness of common fixed point for Ćirić-Riech-Rus contraction mapping in the setting of quasi-partial b-metric space. Some examples are given to verify the effectiveness of our results.

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1. Introduction

The French mathematician Maurice Frechet initiated the study of metric spaces [27] in 1905. The fixed-point theorem, generally known as the Banach contraction principle, appeared in explicit form in Banach′s thesis in 1922, where it was used to establish the existence of a solution to an integral equation. A number of generalizations of the well-known Banach contraction theorem are obtained in various directions(See [5, 6, 7, 8, 9, 22, 23, 25, 26]). In 1989, Bakhtin [4] introduced the concept of b-metric space and gave the contraction mapping which was the generalization of the Banach Contraction Principle. In 1993, Czerwick [6] extended this concept of b-metric spaces, whereas Shukla [28] introduced partial b-metric in 2014. As a further generalization for quasi-metric spaces and partial-metric spaces, Karapinar [15] introduced the notion of quasi-partial metric space and discussed the existence of fixed points of self-mappings T on quasi-partial metric spaces. Gupta and Gautam [12] further, generalized quasi-partial metric spaces to the class of quasi-partial b-metric spaces and proved some fixed point results in the setting of quasi-partial metric space (See [2, 4, 10, 11]). Jungck [13] extended the Banach contraction mapping principle in different direction which stated as follows.

Let (X, d) be a complete metric space. Let S be a continuous self map on X and T be any self map on X that commutes with S. Further let S and T satisfy $T(X) \subset S(X)$ and there exists a constant $\alpha \in (0, 1)$ such that for every $x, y \in X$,

$$d(Tx, Ty) \leq \alpha d(Sx, Sy)$$

for all $x, y \in X$. Then S and T have a unique common fixed point.

Many results exist in literature on common fixed points given by Kannan, Chatterjea and Zamfirescu contractive conditions have been recently obtained in Refs. [1] and [5], respectively. Das and Naik [7] derived common fixed point result for ĆirićÔÇÖs fixed point theorem which is stated as below.

Let (X, d) be a complete metric space. Let S be a continuous self map on X and T be any self map on X that commutes with S. Further let S and T satisfy $T(X) \subset S(X)$. If there exists a constant $h \in (0, 1)$ such that for every $x, y \in X$,

$$d(Tx, Ty) \leq hM(x, y),$$

where, $M(x, y) = max\{(Sx, Sy), d(Sx, Tx), d(Sy, Ty), d(Sx, Ty), d(Sy, Tx)\}$ then S and T have a unique common fixed point.

More research work related to this was mentioned in ([3, 15, 16, 17, 18, 19, 20, 24]).

In this paper, we have discussed the existence and uniqueness of common fixed points for Ćirić-Reich-Rus mapping [21] when the underlying space is Complete. Throughout this paper, N, R and R^+ denote the set of all positive integers, set of real numbers and the set of all non-negative real numbers respectively.

2. Preliminaries and basic properties

Definition 2.1 ([12])**.** A quasi-partial b-metric on a non-empty set X is a mapping $qp_b : X \times X \to R^+$ such that for some real numbers $s \geq 1$ and for all $x, y, z \in X$,

(QP_{b_1}) $qp_b(x, x) = qp_b(x, y) = qp_b(y, y)$ implies $x = y$,

(QP_{b_2}) $qp_b(x, x) \leq qp_b(x, y)$,

(QP_{b_3}) $qp_b(x, x) \leq qp_b(y, x)$,

(QP_{b_4}) $qp_b(x, y) + qp_b(z, z) \leq s[qp_b(x, y) + qp_b(y, z)]$.

A quasi-partial b-metric space is a pair (X, qp_b), where X is a non-empty space and qp_b is quasi-partial b-metric on X. The number s is called coefficient of (X, qp_b).

Lemma 2.2 ([11])**.** *Let (X, qp_b) be a quasi-partial b-metric space. Then the following holds:*

(1) *if $qp_b(x, y) = 0$, then $x = y$,*

(1) *if $x \neq y$, then $qp_b(x, y) > 0$ and $qp_b(y, x) > 0$.*

Definition 2.3 ([11])**.** Let (X, qp_b) be a quasi-partial b-metric space.

(i) A sequence $\{x_n\} \subset X$ is said to converges to $x \in X$, if

$$qp_b(x, x) = \lim_{n\to\infty} qp_b(x_n, x) = \lim_{n\to\infty} qp_b(x, x_n).$$

(ii) A sequence $\{x_n\} \subset X$ is called a Cauchy sequence, if

$$\lim_{n,m\to\infty} qp_b(x_n, x_m) = \lim_{m,n\to\infty} qp_b(x_m, x_n) \text{ exist and finite.}$$

(iii) The quasi-partial b-metric space (X, qp_b) is said to be complete, if every sequence $\{x_n\} \subset X$ converges with respect to τ_{qp_b} to a point $x \in X$ such that

$$qp_b(x,x) = \lim_{n,m\to\infty} qp_b(x_n, x_m) = \lim_{m,n\to\infty} qp_b(x_m, x_n).$$

Lemma 2.4 ([8])**.** *Let (X, qp_b) be a quasi-partial b-metric space and $\{x_n\}_{n=1}^{\infty}$ be a sequence in X. If $x_n \to x$, $x_n \to y$ and $qp_b(x,x) = qp_b(y,y) = 0$, then $x = y$.*

Definition 2.5. Let (X, qp_b) be a quasi-partial b-metric space. A sequence $\{x_n\} \subset X$ is said to converges to $x \in X$, if for every $\epsilon > 0$, there exists $K(\epsilon) \in N$ such that $x_n \in B(x, \epsilon)$, for all $n \geq K(\epsilon)$.

The above definition is derived from Definition 2.3 (i) and by the definition of Balls in qp_b-metric space $B(x,\epsilon) = \{y \in X | qp_b(x,y) < \epsilon$ and $qp_b(y,x) < \epsilon\}$.

Definition 2.6 ([10])**.** A mapping $f : X \to X$ is said to be continuous at $x_0 \in X$, i, for every $\epsilon > 0$, there exists $\delta(\epsilon) > 0$ such that $f(B(x_0, \delta)) \subset B(f(x_0), \epsilon)$.

Lemma 2.7. *(Sequential Criterion for continuity in qp_b metric space) Let X be a Quasi-Partial b-metric and $A \subseteq X$, $f : A \to X$ is a continuous function at a point $x_0 \in A$ if and only if for every sequence (x_n) in A that converges to x_0, the sequence $(f(x_n))$ converges to $f(x_0)$.*

Proof. Suppose $f : A \to X$ is a continuous function at a point $x_0 \in A$ and let (x_n) any sequence in A converging to x_0. We need to prove that $(f(x_n))$ converges to $f(x_0)$. Let $\epsilon > 0$ be given. Then by Definition 2.5, there exists $\delta(\epsilon) > 0$ such that

$$(B(x_0, \delta)) \subset B(f(x_0), \epsilon). \tag{2.1}$$

Now, we apply the definition of convergent sequence corresponding to the given δ to obtain a natural number $k(\delta)$ such that $x_n \in B(x, \delta)$ for all $n \geq k(\delta)$. Then by (2.1), $f(x_n) \in B(f(x_0), \epsilon)$ for all $n \geq k(\epsilon)$. Thus the sequence $(f(x_n))$ converges to $f(x_0)$.

Conversely, suppose the necessary condition holds and assume that $f : A \to X$ is not continuous at x_0. Then there exist $\epsilon_0 > 0$, for every $\delta > 0$ such that $f(B(x_0, \delta)) \not\subset B(f(x_0), \epsilon)$. Thus there exist $\epsilon_0 > 0$, for every $\delta > 0$ there exist at least one $x \in B(x_0, \delta)$ such that $f(x) \notin B(f(x_0), \epsilon)$. Choose $\delta = \frac{1}{n} > 0$. Then clearly,we have a sequence $x_n \in B(x_0, \frac{1}{n})$ such that $f(x_n) \notin B(f(x_0), \epsilon)$. Since $x_n \in B(x_0, \frac{1}{n})$ equivalent to $qp_b(x_n, x_0) < \frac{1}{n}$ and $qp_b(x_0, x_n) < \frac{1}{n}$, x_n converges to x_0 and $f(x_n) \notin B(f(x_0), \epsilon)$ equivalent to either $qp_b(f(x_n), f(x_0)) \geq \epsilon_0$ or $qp_b(f(x_0), f(x_n)) \geq \epsilon_0$ for all $n \in N$. Thus, $f(x_n)$ does not converges to $f(x_0)$, which contradicts the fact that for every sequence (x_n) in A that converges to x_0, the sequence $(f(x_n))$ converges to $f(x_0)$. So f is continuous function. □

Definition 2.8 ([1])**.** Let S and T be self maps of a nonempty set X. If there exists $x \in X$ such that $Sx = Tx$, then x is called a coincidence point of S and T, while $y = Sx = Tx$ is called a point of coincidence (or coincidence value) of S and T. If $Sx = Tx = x$, then x is called a common fixed point of S and T.

Definition 2.9 ([14])**.** Let S and T be self maps of a nonempty set X. The pair of mappings S and T is said to be weakly compatible, if they commute at their coincidence points.

3. Main result

In this section, common fixed point theorems for cyclic Ćirić-Reich-Rus contraction mapping is proved in the setting of quasi-partial b-metric space.

Theorem 3.1. *Let (X, qp_b) be a vomplete quasi-partial b-metric space. Let A and T be continuous self-map on X that commutes and $T(X) \subset A(X)$, further, let A and T satisfy the following: for any $x,\ y \in X$,*

$$qp_b(Tx, Ty) \leq \alpha qp_b(Ax, Ay) + \beta qp_b(Ax, Tx) + \beta qp_b(Ay, Ty), \tag{3.1}$$

where $\alpha, \beta \in (0, 1)$ such that for $s \geq 1$ and $s(\alpha + 2\beta) < 1$, A and T have a unique fixed point.

Proof. Let $x_0 \in X$ and consider a sequence $x_n \in X$ satisfying the following:

$$T(x_0) = A(x_1), T(x_1) = A(x_2), \cdots, T(x_n) = A(x_{n+1}).$$

by considering condition (3.1), we have

$$qp_b(Ax_1, Ax_2) = qp_b(Tx_0, Tx_1) \leq \alpha qp_b(Ax_0, Ax_1) + \beta qp_b(Ax_0, Tx_0) + \beta qp_b(Ax_1, Tx_1),$$
$$qp_b(Ax_1, Ax_2) \leq \alpha qp_b(Ax_0, Ax_1) + \beta qp_b(Ax_0, Ax_1) + \beta qp_b(Ax_1, Ax_2),$$
$$(1-\beta) qp_b(Ax_1, Ax_2) \leq (\alpha + \beta) qp_b(Ax_0, Ax_1),$$

Then we get

$$qp_b(Ax_1, Ax_2) \leq \frac{(\alpha+\beta)}{(1-\beta)} qp_b((Ax_0, Ax_1). \tag{3.2}$$

Thus from (3.2), we have

$$qp_b(Ax_2, Ax_1) = qp_b(Tx_1, Tx_2) \leq \alpha qp_b(Ax_1, Ax_0) + \beta qp_b(Ax_1, Tx_1) + \beta qp_b(Ax_0, Tx_0),$$
$$qp_b(Ax_2, Ax_1) \leq \alpha qp_b(Ax_1, Ax_0) + \beta qp_b(Ax_1, Ax_2) + \beta qp_b(Ax_0, Ax_1),$$
$$qp_b(Ax_2, Ax_1) \leq \alpha qp_b(Ax_1, Ax_0) + \beta (\frac{(\alpha+\beta)}{1-\beta}) qp_b(Ax_0, Ax_1) + \beta qp_b(Ax_0, Ax_1).$$

Now Take $h = Max\{qp_b(Ax_1, Ax_0), qp_b(Ax_0, Ax_1)\}$. Then

$$qp_b(Ax_1, Ax_2) \leq \frac{(\alpha+\beta)}{(1-\beta)} h \text{ and } qp_b(Ax_2, Ax_1) \leq \frac{(\alpha+\beta)}{(1-\beta)} h.$$

Similarly,

$$qp_b(Ax_2, Ax_3) \leq (\frac{\alpha+\beta}{1-\beta})^2 h \text{ and } qp_b(Ax_3, Ax_2) \leq (\frac{\alpha+\beta}{1-\beta})^2 h.$$

Thus we have: for every $n \in N$,

$$qp_b(Ax_n, Ax_{n+1}) \leq (\frac{\alpha+\beta}{1-\beta})^n h \text{ and } qp_b(Ax_{n+1}, Ax_n) \leq (\frac{\alpha+\beta}{1-\beta})^n h. \tag{3.3}$$

Let $m, n \in N$ and $m < n$. Then by 3.3)and QP_{b_4}, we have

$$\begin{aligned} qp_b(Ax_n, Ax_m) &\leq s[qp_b(Ax_n, Ax_{m+1}) + qp_b(Ax_{m+1}, Ax_m)] \\ &\quad - qp_b(Ax_{m+1}, Ax_{m+1}), \\ qp_b(Ax_n, Ax_m) &\leq s[qp_b(Ax_n, Ax_{m+1}) + qp_b(Ax_{m+1}, Ax_m)], \\ qp_b(Ax_n, Ax_m) &\leq s^2 qp_b(Ax_n, Ax_{m+2}) + s^2 qp_b(Ax_{m+1}, Ax_{m+2}) \\ &\quad + s qp_b(Ax_{m+1}, Ax_m), \\ qp_b(Ax_n, Ax_m) &\leq s^{n-m} qp_b(Ax_n, Ax_{n-1}) + \cdots + s^2 qp_b(Ax_{m+2}, Ax_{m+1}) \end{aligned}$$

$+sqp_b(Ax_{m+1}, Ax_m),$
$qp_b(Ax_n, Ax_m) \leq s^{n-m}(\frac{\alpha+\beta}{1-\beta})^{n-1}h + \cdots + s(\frac{\alpha+\beta}{1-\beta})^m h.$
Thus we get

$$qp_b(Ax_n, Ax_m) \leq sh(\frac{\alpha+\beta}{1-\beta})^m(\frac{1-(\frac{s(\alpha+\beta)}{1-\beta})^{n-m}}{1-\frac{s(\alpha+\beta)}{1-\beta}}). \tag{3.4}$$

First, we will show $s(\frac{\alpha+\beta}{1-\beta}) < 1$. Since $s(\alpha+2\beta) < 1$, $s\alpha + s\beta + s\beta < 1$. Since $s \geq 1$, $s(\alpha+\beta) < 1 - s\beta < 1 - \beta$. As $n, m \to \infty$ and using above inequality in equation (3.4), we have the following:

$$\lim_{n,m\to\infty} qp_b(Ax_n, Ax_m) = 0.$$

Similarly, $\lim_{n,m\to\infty} qp_b(Ax_m, Ax_n) = 0$. Then $\{A(x_n)\}$is a Cauchy Sequence in X. Since X is a complete qp_b metric space, there exist $t \in X$ such that $A(x_n)$ converges to t in qp_b metric space. Thus
$qp_b(t,t) = \lim_{n\to\infty} qp_b(Ax_n, t) = \lim_{n\to\infty} qp_b(t, Ax_n) = 0$
$= \lim_{m,n\to\infty} qp_b(Ax_n, Ax_m) = \lim_{n,m\to\infty} qp_b(Ax_m, Ax_n).$
Since $T(x_n) = A(x_{n+1})$, $T(x_n)$ converges to t in qp_b metric space. So by Lemma 2.7, $A(T(x_n))$ converges to $A(t)$ and $T(A(x_n))$ converges to $T(t)$.

Next, we will show $A(t) = T(t)$. It is sufficient to show that

$$qp_b(A(t), T(t)) = 0.$$

Consider
$qp_b(A(T(x_n)), T(t))$
$= qp_b(T(A(x_n)), T(t))$
$\leq \alpha qp_b(A(A(x_n)), A(t)) + \beta qp_b(A(A(x_n)), T(A(x_n))) + \beta qp_b(A(t), T(t)).$
As $n \to \infty$, we have

$$qp_b(A(t), T(t)) \leq \alpha qp_b(A(t), A(t)) + 2\beta qp_b(A(t), T(t)),$$
$$qp_b(A(t), T(t)) \leq \frac{\alpha}{1-2\beta} qp_b(A(t), A(t)) \leq \frac{\alpha}{1-2\beta} qp_b(A(t), T(t)).$$

By the hypothesis, $(\alpha + 2\beta) < \frac{1}{s} \leq 1$, i.e., $\alpha + 2\beta < 1$, i.e., $\frac{\alpha}{1-2\beta} < 1$. By using this inequality, we have $qp_b(A(t), T(t)) \leq 0$ but $qp_b(A(t), T(t)) \geq 0$. Then $qp_b(A(t), T(t)) = 0$. Thus t is a coincidence point.

Now, we will show $A(t)$ is a fixed point. Since A and T commutes, $T(A(t)) = A(T(t))$ for all $x \in X$.
Since $A(t) = T(t)$, so $A^2(t) = T^2(t)$. Then it is sufficient to show $qp_b(A^2(t), A(t)) = 0$ for $A^2(t) = A(t)$.

On the other hand,
$qp_b(A^2(t), A(t)) = qp_b(T^2(t), T(t))$
$\leq \alpha qp_b(A(T(t)), A(t)) + \beta qp_b(A(T(t)), T^2(t))$
$+\beta qp_b(A(t), T(t)),$
$qp_b(A^2(t), A(t)) \leq \alpha qp_b(A^2(t), A(t)) + \beta qp_b(A^2(t), A^2(t)) + \beta qp_b(A(t), A(t)),$
$(1-\alpha) qp_b(A^2(t), A(t)) \leq \beta[qp_b(A^2(t), A^2(t)) + qp_b(A(t), A(t))],$
$(1-\alpha) qp_b(A^2(t), A(t)) \leq \beta[qp_b(A^2(t), A^2(t)) + qp_b(A^2(t), A(t))],$
$(1-\alpha-2\beta) qp_b(A^2(t), A(t)) \leq 0.$
Since $1 - \alpha - 2\beta > 0$, $qp_b(A^2(t), A(t)) = 0$. Thus $A(t) = T(t)$ is a fixed point.

At last we need to show $A(t)$ is a unique fixed point. Let w and v be two fixed points, i.e., $A(w) = T(w) = w$ and $A(v) = T(v) = v$. We have to show $v = w$.

Consider

$$\begin{aligned} qp_b(v,v) &= qp_b(T(v),T(v)) \\ &\leq \alpha qp_b(A(v),A(v)) + 2\beta qp_b(A(v),T(v)) \\ &= (\alpha+\beta)qp_b(v,v). \end{aligned}$$

Then $(1-\alpha-\beta)qp_b(v,v) \leq 0$. Thus $qp_b(v,v) = 0$. Similarly, we have $qp_b(w,w) = 0$.

On the other hand,

$$\begin{aligned} qp_b(v,w) &= qp_b(Tv,Tw) \\ &\leq \alpha qp_b(A(v),A(w)) + \beta qp_b(A(v),T(v)) + \beta qp_b(A(w),T(w)) \\ &= \alpha qp_b(v,w) + \beta qp_b(v,v) + \beta qp_b(w,w). \end{aligned}$$

So $(1-\alpha)qp_b(v,w) \leq [qp_b(v,v) + qp_b(w,w)]$. Since $\alpha < 1$, $qp_b(v,w) = 0$. Hence $v = w$. □

Example 3.2. Let $X = [0,1]$ and let $s \geq 1$. Define the quasi-partial b-metric as:

$$qp_b(x,y) = |x-y| + |x|.$$

Then we can easily see that (X, qp_b) is a complete quasi-partial b-metric space. Also $T(x) = \frac{x}{4}$ and $A(x) = \frac{x}{2}$ are continuous self maps on X that commutes. We will check A and T satisfy the hypothesis of Theorem 3.1.

$$\begin{aligned} &T(X) = [0,\tfrac{1}{4}] \subset [0,\tfrac{1}{2}] = A(X), \\ &qp_b(Tx,Ty) = qp_b(x/4,y/4) = |x/4-y/4| + |x/4|, \\ &qp_b(Ax,Ay) = qp_b(x/2,y/2) = |x/2-y/2| + |x/2|, \\ &qp_b(Ax,Tx) = qp_b(x/2,x/4) = |x/2-x/4| + |x/2| = 3x/4, \\ &qp_b(Ay,Ty) = qp_b(y/2,y/4) = |y/2-y/4| + |y/2| = 3y/4, \\ &qp_b(Tx,Ty) \leq \tfrac{1}{2}qp_b(Ax,Ay), \\ &qp_b(Tx,Ty) \leq \tfrac{1}{2}qp_b(Ax,Ay) + \beta qp_b(Ax,Tx) + \beta qp_b(Ay,Ty). \end{aligned}$$

Choose $\beta \geq 0$ such that $(\frac{1}{2}+2\beta) < 1/s$. Then A and T have a unique common fixed point, i.e., $t = 0$.

Corollary 3.3. *Let (X, qp_b) be a complete quasi-partial b-metric space. Let A and T are continuous self-map on X that commutes and $T(X) \subset A(X)$, further, let A and T satisfy the following Kannan contraction condition: for all $x,\ y \in X$,*

$$qp_b(Tx,Ty) \leq \beta qp_b(Ax,Tx) + \beta qp_b(Ay,Ty),$$

where $\beta \in (0,1/2)$ such that for $s \geq 1$ and $s\beta < \frac{1}{2}$.
Then A and T have a unique fixed point.

Proof. In Theorem 3.1, Taking $\alpha = 0$ and $A =$ identity map. Then we can easily show that A and T have a unique fixed point. □

Corollary 3.4. *Let (X, qp_b) be a complete quasi-partial b-metric space. Let A and T be continuous self-map on X that commutes and $T(X) \subset A(X)$, further, let A and T satisfy the following Banach contraction condition: for all $x,\ y \in X$,*

$$qp_b(Tx,Ty) \leq \alpha qp_b(Ax,Ay),$$

where $\alpha \in (0,1)$ such that for $s \geq 1$ and $s\alpha < 1$.

Then A and T have a unique fixed point.

Proof. In Theorem 3.1, Taking
beta $= 0$ and $A =$ identity map. Then we can easily prove that A and T have a unique fixed point. □

Corollary 3.5. *Let (X, qp_b) be a Complete quasi-partial b-metric space and $T : X \to X$ be a self-map satisfying,*

$$qp_b(Tx, Ty) \leq \alpha qp_b(x, y) + \beta qp_b(x, Tx) + \beta qp_b(y, Ty)$$

for all $x, y \in X$, $(\alpha + 2\beta) \in [0, 1)$. Then T has a unique fixed point in X.

Proof. In Theorem 3.1, Taking $A =$ the identity map. Then we can easily check that T has a unique fixed point in X. □

4. Conclusion

The main contribution of the paper is to ensure the existence of common fixed points for Ćirić-Riech-Rus type contraction mappings of the quasi-partial b-metric space concerned in its definition. The study of the uniqueness of fixed points for these maps and their applications in the solution of nonlinear integral equations would be interesting topics for future work. It will also pave way for study of fixed points in Menger and fuzzy metric space.

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