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Abstract. In this work, we define the fuzzy soft symmetric operator, which is a special type of fuzzy soft linear operators in fuzzy soft Hilbert spaces based on fuzzy soft inner product spaces, initiated by Faried et al. [6]. Moreover, an example of the fuzzy soft symmetric operator is introduced. Furthermore, related fuzzy soft point spectrum theorem is investigated.

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1. Introduction

In 1965, Zadeh [18] proposed an extension of the set theory which is the theory of fuzzy sets to deal with uncertainty. Just as a crisp set on a universal set X is defined by its characteristic function from X to $\{0, 1\}$, a fuzzy set on a domain X is defined by its membership function from X to $[0, 1]$. In 1999, Molodtsov [10] introduced an extension of the set theory namely soft set theory to overcome uncertainties and solve complicated problems which can't be dealt with by classical methods in many areas such as Riemann integration, decision making, game theory, computer science, medicine, economics and many other fields. The soft set is a mathematical tool for modeling uncertainty by associating a set with a set of parameters. After that, many researchers introduced new extended concepts based on soft sets (See [1, 4, 5, 12, 13, 14, 15, 16, 17]). But in real life problems and situations, we still have inexact information about our considered objects. So, to improve those two concepts; fuzzy set and soft set, Maji et al. [9] combined them together in one concept and called this new concept fuzzy soft set. This new concept widened the soft sets approach from ordinary cases to fuzzy cases which is more general than any other. In recent years, many researchers applied this notion and gave some concepts such as fuzzy soft point [11], fuzzy soft metric spaces [2] and fuzzy soft normed spaces

[3]. In present, Faried et al. [6] introduced the fuzzy soft inner product spaces with studying its properties and some related results. Also, they [7] gave the definition of the fuzzy soft Hilbert space with studying its properties and many more related results. In addition, they [8] continued by defining the fuzzy soft linear operators in fuzzy soft Hilbert spaces with their related theorems including spectral theory and proving fuzzy soft Hilbert space's fuzzy soft self-duality. In this paper, we progress on these stated previous studies by introducing a special type of fuzzy soft linear operators in fuzzy soft Hilbert spaces, which is the fuzzy soft symmetric operator, establishing an example of it and introducing its fuzzy soft spectral theory.

2. Preliminaries

The aim of this section is to list some definitions needed in the following discussion.

Definition 2.1 ([9])**.** Let U be a universal set, E be a set of parameters and $A \subseteq E$. A pair (G, A) is called a fuzzy soft set over U, where G is a mapping given by $G : A \to \mathcal{F}(U)$, $\mathcal{F}(U)$ is the family of all fuzzy subsets of U and the fuzzy subset of U is defined as a map f from U to $[0, 1]$. The family of all fuzzy soft sets (G, A) over a universal set U, in which all the parameter sets A are the same, is denoted by $FSS(U)_A = FSS(\tilde{U})$.

Definition 2.2 ([6, 11])**.** The fuzzy soft set $(G, A) \in FSS(\tilde{U})$ is called a fuzzy soft point over U, denoted by $(u_{f_{G(e)}}, A)$ (briefly denoted by $\tilde{u}_{f_{G(e)}}$), if for the element $e \in A$ and $u \in U$,

$$f_{G(e)}(u) = \begin{cases} \alpha \ , \ if \ u = u_0 \in U \ and \ e = e_0 \in A \\ 0 \ , \ if \ u \in U - \{u_0\} \ or \ e \in A - \{e_0\}, \end{cases}$$

where $\alpha \in (0, 1]$ is the value of the membership degree. The fuzzy soft point can be considered as the quadruple (u_0, e_0, G, α).

Note that $\tilde{\theta} = (\tilde{0}, \tilde{0}, \tilde{0}, \tilde{0})$ and $\tilde{j} = (\tilde{1}, \tilde{1}, \tilde{1}, \tilde{1})$.

Definition 2.3 ([6])**.** Let $\tilde{H}$ be a fuzzy soft inner product space and $\tilde{v}^1_{f_{1_{G(e_1)}}}, \tilde{v}^2_{f_{2_{G(e_2)}}} \tilde{\in} \tilde{H}$. Then $\tilde{v}^1_{f_{1_{G(e_1)}}}$ is said to be fuzzy soft orthogonal to $\tilde{v}^2_{f_{2_{G(e_2)}}}$ ($\tilde{v}^1_{f_{1_{G(e_1)}}} \tilde{\perp} \tilde{v}^2_{f_{2_{G(e_2)}}}$), if

$$< \widetilde{\tilde{v}^1_{f_{1_{G(e_1)}}}, \tilde{v}^2_{f_{2_{G(e_2)}}}} > \tilde{=} \tilde{0}.$$

Definition 2.4 ([7])**.** $\ell_2(A)$ the space of all fuzzy soft square-summable sequences is a fuzzy soft Hilbert space with the complex fuzzy soft inner product defined as for all $\tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} \tilde{\in} \ell_2(A)$,

$$< \widetilde{\tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}}} > \tilde{=} \sum_{i=1}^{\infty} \tilde{v}^i_{f_{i_{G(e_i)}}} \overline{\tilde{u}^i_{f_{i_{G(e_i)}}}}.$$

Definition 2.5 ([8])**.** Let $\mathbb{C}(A)$ be the set of all fuzzy soft complex numbers, $\tilde{H}$ be a fuzzy soft Hilbert space and $\tilde{T}$ be a fuzzy soft linear operator on $\tilde{H}$. Then

(i) $\tilde{\lambda} \tilde{\in} \mathbb{C}(A)$ is said to be a fuzzy soft eigenvalue of $\tilde{T}$, if there exists $\tilde{\theta} \tilde{\neq} \tilde{v}_{f_{G(e)}} \tilde{\in} \tilde{H}$ such that $\tilde{T}\tilde{v}_{f_{G(e)}} \tilde{=} \tilde{\lambda} \tilde{v}_{f_{G(e)}}$,

(ii) $\tilde{v}_{f_{G(e)}} \tilde{\neq} \tilde{\theta}$ is called the fuzzy soft eigenvector of $\tilde{T}$ corresponding to $\tilde{\lambda}$.

(iii) The set of all such $\tilde{\lambda}$ is called the fuzzy soft point spectrum of $\tilde{T}$ and denoted by $\tilde{\sigma}_p(\tilde{T})$.

3. Main Results

The aim of this section is to introduce the fuzzy soft symmetric operators in fuzzy soft Hilbert spaces and to study an example and theorem involving their fuzzy soft eigenvalues and fuzzy soft eigenvectors.

Definition 3.1. (Fuzzy soft symmetric operator)
Let $\tilde{H}$ be a fuzzy soft Hilbert space and $\tilde{\mathbb{B}}(\tilde{H})$ be the set of all fuzzy soft bounded linear operators on $\tilde{H}$. $\tilde{T}\tilde{\in}\tilde{\mathbb{B}}(\tilde{H})$ is called fuzzy soft symmetric operator if we have:

$$< \widetilde{\tilde{T}\tilde{v}^1_{f_{1_{G(e_1)}}}, \tilde{v}^2_{f_{2_{G(e_2)}}}} > \tilde{=} < \widetilde{\tilde{v}^1_{f_{1_{G(e_1)}}}, \tilde{T}\tilde{v}^2_{f_{2_{G(e_2)}}}} >, \tag{3.1}$$

for each $\tilde{v}^1_{f_{1_{G(e_1)}}}, \tilde{v}^2_{f_{2_{G(e_2)}}} \tilde{\in}\tilde{H}$.

Example 3.2. Let $\tilde{\mathbb{L}}(\ell_2(A))$ be the set of all fuzzy soft linear operators on $\ell_2(A)$. Define $\tilde{T}\tilde{\in}\tilde{\mathbb{L}}(\ell_2(A))$ by

$$\tilde{T}\{\tilde{v}^n_{f_{n_{G(e_n)}}}\}\tilde{=}\{\frac{\tilde{v}^n_{f_{n_{G(e_n)}}}}{\tilde{n}}\},\ for\ \tilde{v}_{f_{G(e)}}\tilde{=}\{\tilde{v}^n_{f_{n_{G(e_n)}}}\}^{\infty}_{n=1}. \tag{3.2}$$

Then, $\tilde{T}$ is fuzzy soft bounded, $\widetilde{\|\tilde{T}\|}\tilde{=}\tilde{1}$ and $\tilde{T}$ is fuzzy soft symmetric operator.

Solution. By using the given formula (3.2) of $\tilde{T}$, we have:

$$\widetilde{\|\tilde{T}\tilde{v}_{f_{G(e)}}\|}^2 \tilde{=} \sum_{n=1}^{\infty} |\frac{\tilde{v}^n_{f_{n_{G(e_n)}}}}{\tilde{n}}|^2 \tilde{=} \sum_{n=1}^{\infty} \frac{|\tilde{v}^n_{f_{n_{G(e_n)}}}|^2}{\tilde{n}^2} \tilde{\leq} \sum_{n=1}^{\infty} |\tilde{v}^n_{f_{n_{G(e_n)}}}|^2 \tilde{=} \widetilde{\|\tilde{v}_{f_{G(e)}}\|}^2.$$

Then $\widetilde{\|\tilde{T}\tilde{v}_{f_{G(e)}}\|}\tilde{\leq}\widetilde{\|\tilde{v}_{f_{G(e)}}\|}$. Thus $\tilde{T}$ is fuzzy soft bounded and $\widetilde{\|\tilde{T}\|}\tilde{\leq}\tilde{1}$.

Let $\tilde{v}_{f_{G(e)}}\tilde{=}\tilde{e}_1\tilde{=}(\tilde{j}, \tilde{\theta}, \tilde{\theta}, \tilde{\theta}, \cdots, \tilde{\theta}, \cdots)$. Then by using the given formula (3.2) of $\tilde{T}$, we have:

$$\tilde{T}\tilde{v}_{f_{G(e)}}\tilde{=}\tilde{T}\tilde{e}_1\tilde{=}(\frac{\tilde{j}}{\tilde{1}}, \frac{\tilde{\theta}}{\tilde{2}}, \frac{\tilde{\theta}}{\tilde{3}}, \frac{\tilde{\theta}}{\tilde{4}}, \cdots, \frac{\tilde{\theta}}{\tilde{n}}, \cdots)\tilde{=}(\tilde{j}, \tilde{\theta}, \tilde{\theta}, \tilde{\theta}, \cdots, \tilde{\theta}, \cdots)\tilde{=}\tilde{e}_1.$$

Thus $\widetilde{\|\tilde{T}\tilde{e}_1\|}\tilde{=}\widetilde{\|\tilde{e}_1\|}\tilde{=}\tilde{1}$. So $\widetilde{\|\tilde{T}\tilde{e}_1\|}\tilde{\geq}\tilde{1}$. Hence $\widetilde{\|\tilde{T}\|}\tilde{=}\tilde{1}$.

Now, we prove that $\tilde{T}$ is fuzzy soft symmetric operator. By using the given formula (3.2) of $\tilde{T}$ and using the fuzzy soft inner product of $\ell_2(A)$ defined in Definition (2.4), we obtain:

$$\begin{aligned} <\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}}}> &\tilde{=} \sum_{n=1}^{\infty} \frac{\tilde{v}^n_{f_{n_{G(e_n)}}}}{\tilde{n}} \cdot \overline{\tilde{u}}^n_{f_{n_{G(e_n)}}} \\ &\tilde{=} \sum_{n=1}^{\infty} \tilde{v}^n_{f_{n_{G(e_n)}}} \cdot \frac{\overline{\tilde{u}}^n_{f_{n_{G(e_n)}}}}{\tilde{n}} \\ &\tilde{=} \sum_{n=1}^{\infty} \tilde{v}^n_{f_{n_{G(e_n)}}} \cdot \overline{(\frac{\tilde{u}^n_{f_{n_{G(e_n)}}}}{\tilde{n}})} \\ &\tilde{=} <\widetilde{\tilde{v}_{f_{G(e)}}, \tilde{T}\tilde{u}_{f_{G(e)}}}>, \end{aligned}$$

for all $\tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} \tilde{\in} \ell_2(A)$. Then $\tilde{T}$ is fuzzy soft symmetric operator.

Lemma 3.3. *Let $\tilde{T}\tilde{\in}\tilde{\mathbb{B}}(\tilde{H})$ be fuzzy soft symmetric. Then for $\tilde{v}_{f_{G(e)}} \tilde{\in} \tilde{H}$, $<\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}>$ is fuzzy soft real and*

$$\tilde{\inf_{\|\widetilde{\tilde{v}_{f_{G(e)}}}\| \tilde{=} \tilde{1}}} <\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}> \tilde{\leq} <\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}> \tilde{\leq} \tilde{\sup_{\|\widetilde{\tilde{v}_{f_{G(e)}}}\| \tilde{=} \tilde{1}}} <\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}>.$$

Proof. By using (3.1) from Definition (3.1) of fuzzy soft symmetric operator, we have

$$<\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}> \tilde{=} \overline{<\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}>}.$$

Then $<\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}>$ is fuzzy soft real for $\tilde{v}_{f_{G(e)}} \tilde{\in} \tilde{H}$.

For the second part, we can define (write, shortly):

$$\tilde{m}(\tilde{T}) \tilde{=} \tilde{\inf_{\|\widetilde{\tilde{v}_{f_{G(e)}}}\| \tilde{=} \tilde{1}}} <\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}> \text{ and } \tilde{M}(\tilde{T}) \tilde{=} \tilde{\sup_{\|\widetilde{\tilde{v}_{f_{G(e)}}}\| \tilde{=} \tilde{1}}} <\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}>.$$

It is clear from the definition of $\tilde{\sup}$ and $\tilde{\inf}$ that $\tilde{m}(\tilde{T}) \tilde{\leq} <\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}> \tilde{\leq} \tilde{M}(\tilde{T})$. □

Theorem 3.4. *If $\tilde{T}\tilde{\in}\tilde{\mathbb{B}}(\tilde{H})$ is fuzzy soft symmetric and $\tilde{\lambda}$ is a fuzzy soft eigenvalue of $\tilde{T}$, then $\tilde{\lambda}$ is fuzzy soft real and $\tilde{m}(\tilde{T}) \tilde{\leq} \tilde{\lambda} \tilde{\leq} \tilde{M}(\tilde{T})$. Fuzzy soft eigenvectors corresponding to different fuzzy soft eigenvalues are fuzzy soft orthogonal.*

Proof. Let $\tilde{\lambda}$ be a fuzzy soft eigenvalue of $\tilde{T}$. Then from Definition (2.5), there exists $\tilde{\theta} \tilde{\neq} \tilde{v}_{f_{G(e)}} \tilde{\in} \tilde{H}$ such that $\tilde{T}\tilde{v}_{f_{G(e)}} \tilde{=} \tilde{\lambda}\tilde{v}_{f_{G(e)}}$. Suppose that $\|\widetilde{\tilde{v}_{f_{G(e)}}}\| \tilde{=} \tilde{1}$. Then

$$<\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}> \tilde{=} <\widetilde{\tilde{\lambda}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}> \tilde{=} \tilde{\lambda} <\widetilde{\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}> \tilde{=} \tilde{\lambda} \|\widetilde{\tilde{v}_{f_{G(e)}}}\|^2 \tilde{=} \tilde{\lambda}.$$

Now, we have $\tilde{\lambda} \tilde{=} <\widetilde{\tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{v}_{f_{G(e)}}}>$. Thus by using the above Lemma (3.3), we obtain that $\tilde{\lambda}$ is fuzzy soft real and $\tilde{m}(\tilde{T}) \tilde{\leq} \tilde{\lambda} \tilde{\leq} \tilde{M}(\tilde{T})$.

To prove the second part, let $\tilde{T}\tilde{v}_{f_{G(e)}} \tilde{=} \tilde{\lambda}\tilde{v}_{f_{G(e)}}$ and $\tilde{T}\tilde{u}_{f_{G(e)}} \tilde{=} \tilde{\mu}\tilde{u}_{f_{G(e)}}$; $\tilde{\lambda} \tilde{\neq} \tilde{\mu}$, $\tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} \tilde{\neq} \tilde{\theta}$. Then by using the first part of the Theorem and applying (3.1) from Definition (3.1), we get:

$$
\begin{aligned}
\tilde{\lambda} \widetilde{< \tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} >} &\tilde{=} \widetilde{< \tilde{\lambda}\tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} >} \\
&\tilde{=} \widetilde{< \tilde{T}\tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} >} \\
&\tilde{=} \widetilde{< \tilde{v}_{f_{G(e)}}, \tilde{T}\tilde{u}_{f_{G(e)}} >} \\
&\tilde{=} \widetilde{< \tilde{v}_{f_{G(e)}}, \tilde{\mu}\tilde{u}_{f_{G(e)}} >} \\
&\tilde{=} \tilde{\mu} \widetilde{< \tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} >}.
\end{aligned}
$$

Thus $(\tilde{\lambda} - \tilde{\mu}) \widetilde{< \tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} >} \tilde{=} \tilde{0}$. Since $\tilde{\lambda} \tilde{\neq} \tilde{\mu}$, $\tilde{\lambda} - \tilde{\mu} \tilde{\neq} \tilde{0}$. So $\widetilde{< \tilde{v}_{f_{G(e)}}, \tilde{u}_{f_{G(e)}} >} \tilde{=} \tilde{0}$. Hence $\tilde{v}_{f_{G(e)}}$ and $\tilde{u}_{f_{G(e)}}$ are fuzzy soft orthogonal. □

4. Conclusions

Introducing the fuzzy version or the soft version of topics like metric spaces, normed spaces and Hilbert spaces has been studied by many mathematicians. On the other hand, combining fuzzy and soft sets together gives us more extended, generalized and accurate results. Few researchers have studied some of those general extensions concepts. In our study, a special type of fuzzy soft linear operators, which is the fuzzy soft symmetric operator has been introduced. To make the picture complete, an example and a fuzzy soft point spectrum of it have been established.

Conflict of interest

The authors declare that they have no conflict of interest.

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