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Soft multiset in a new perspective

A. I. Isah

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Abstract. In this paper, the Concept of Soft multisets using multisets of equivalence relation approach is introduced. Soft multiset operations such as union, intersection, complement and so on are also demonstrated. It is further established that both Demorgan's laws and complementation properties hold. Some related results are also presented.

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1. Introduction

As in [7], soft set was initiated with the goal of addressing the inherent difficulties of theories like, fuzzy set, rough set, interval mathematics and vague set which are all suitable approaches to handling uncertainty. The theory of Soft Set which is a mathematical concept for dealing with uncertainty has applications in mathematics, decision making etc., as shown in [4, 5, 11, 12, 13, 14, 15, 16].

As in [17], multiset (mset, for short) which is an unordered collection of objects (called the elements) where unlike a standard (Cantorian) set, elements are allowed to repeat is initiated with the aim of addressing repetition. As emphasized in [8, 18, 19], the concept of multisets, could be viewed in two different ways: Sets with distinguishable repeated elements (e.g., houses or vehicles sharing a common property) and sets with indistinguishable repeated elements (e.g., sand or soup of elementary particles). In [18], the two notions are respectively, formalized as a *set equipped with an equivalence relation* or *with a function* and are referred to as multisets of equivalence relation approach and multisets of functional approach, respectively.

The theory of soft multisets (*soft msets, for short*) was first introduced in [1], there after, other scholars such as [2, 6, 9, 10, 20, 21] defined it in various ways which immensely contributed to its development. Moreover, as both ideas are excellent contribution to the theory of soft multiset, and as multisets are generalizations of Sets [3], the idea of soft multisets in [10] and [21] are the one that are more closer to the basic concept of multisets of functional approach.

However, as none of the scholars studied the theory on the basis of multiset of equivalence relation approach, this work aimed to introduced the idea of soft multisets using multisets of equivalence relation approach.

2. Preliminaries

Definition 2.1 ([7])**.** A pair (F,E) is called a soft set (over U), if F is a mapping of E into the set of all subsets of the set U.

Definition 2.2. A multiset X as in [8] is defined as a pair (X_0, ρ), where X_0 is a set and ρ an equivalence relation on X_0. The set X_0 is called the field of the multiset. Elements of X_0 in the same equivalence class will be said to be of the same sort and elements in different equivalence classes will be said to be of different sorts. The pair (X_0, ρ), where X_0 is a set and ρ the empty equivalence relation on X_0, is an ordinary set; as in [18], it is an empty multiset, if X_0 is an empty set and ρ an empty equivalence relation on X_0.

Definition 2.3 ([18])**.** Let the sort of an mset $X = (X_0, \rho)$ be denoted by $s_1, s_2, \ldots, s_n$. Then an mset (X_0, ρ) can be equivalently represented as a set of all its sorts, i.e.,

$$(X_0, \rho) = \{s_1, s_2, \ldots, s_n\}.$$

Clearly, any two sorts of an mset are either the same or disjoint.

A sort S' is said to be a subsort of a sort S, denoted $S' \subseteq S$, if every element of S' is an element of S.

Two sorts S and S' are said to be equal, denoted by $S = S'$, if $S' \subseteq S$ and $S \subseteq S'$.

Definition 2.4 ([8])**.** Let $X = (X_0, \rho)$ be a multiset. A submultiset $Y \subseteq X$ is a multiset of the form (Y_0, σ), where Y_0 is a subset of X_0 and σ is ρ-restricted to Y_0.

Definition 2.5 ([18])**.** Let $G = (G_0, \gamma), A = (A_0, \rho)$, and $B = (B_0, \sigma)$ be msets such that $A_0, B_0, \subseteq G_0$, and ρ, σ are γ-restricted to A_0, and B_0, respectively.

(i) The union of A and B, denoted $A \cup B$, is an mset $D = (D_0, \tau)$ with field $A_0 \cup B_0$ and the equivalence relation τ as γ-restricted to D_0.

(ii) The intersection of A and B, denoted $A \cap B$, is an mset $D = (D_0, \tau)$ with field $A_0 \cap B_0$ and the equivalence relation τ as γ-restricted to D_0.

Definition 2.6. Let $G = (G_0, \gamma)$, $X = (X_0, \rho)$ be msets such that $X_0 \subseteq G_0$ and ρ is $\gamma-$ restricted to X_0. Then the complement of X in Z, denoted $\overline{X}$, is an mset $\overline{X} = (\overline{X}_0, \sigma)$ such that $\overline{X}_0 = \{G_0 \backslash X_0\}$ and σ is $\gamma-$ restricted to $\overline{X}_0$, i.e., $x \in X \Longrightarrow x \notin \overline{X}$.

Remark 2.7. If N is a submultiset of M, then $\overline{N} \cap N = \varnothing$ and $\overline{N} \cup N = M$.

Definition 2.8 ([8])**.** Let X be a multiset. The power multiset of X, denoted $P(X)$ is defined as the multiset with field all the submultisets of X, and with equivalence relation such that every pair of elements in the field are related (i e., $P(X)$ has only one sort).

In the following section, the notion of soft multiset using multiset of equivalence relation approach is introduced.

3. Soft multiset

Definition 3.1. Let U be a multiset, E be a set of parameters and $A \subseteq E$. Then a pair (F, A) or F_A is called a soft multiset over U, with $F : A \longrightarrow P(U)$.

Example 3.2. Let the universal multiset

$$U = (U_0, \beta) = \left\{a, a', b, \ldots, b^{vii}, c, c', c^{''}, c^{'''}, d, d', \ldots, d^{ix}\right\} = \{s_1, s_2, s_3, s_4\},$$

where $s_i's$ are the respective sorts of U, and let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$, $A = \{e_1, e_4, e_7\}$ and $F : A \longrightarrow P(U)$ be defined as:

$$F(e_1) = \left\{a, b, b', b^{''}\right\},\ F(e_4) = \{c, c', d, d', d''\} \text{ and } F(e_7) = \{b, b', c, d, \ldots, d^v\}.$$

Then $(F, A) = \left\{\left(e_1, \left\{a, b, b', b^{''}\right\}\right), \left(e_4, \left\{c, c', d, d', d^{''}\right\}\right), (e_7, \{b, b', c, d, \ldots, d^v\})\right\}$.

Observe that $F(e_1)$, for instance, has two sorts, i.e., a is of its own sort while $b,\ b',\ b''$ are of the same sort different from a.

Moreover in (F, A), $F(e_2) = F(e_3) = F(e_5) = F(e_6) = F(e_8) = F(e_9) = \varnothing$.

Let $F(e) = \{s_1, s_2, \ldots, s_m\}$ and $G(e) = \{t_1, t_2, \ldots, t_k\}$. Then,

(i) $F(e) \subseteq G(e)$, if $\forall s_m \in F(e)$, $\exists\ t_k \in G(e)$ such that $s_m \subseteq t_k$.
That is, if $G(e) = (G(e)_0, \rho)$ and $F(e) = (F(e)_0,\ \sigma)$, then,
$F(e) \subseteq G(e)$ if $F(e)_0$ is a subset of $G(e)_0$ and σ is $\rho-$restricted to $F(e)_0$.

(ii) $F(e) \cup G(e) = H(e) = (H(e)_0, \tau)$, where $H(e)_0 = F(e)_0 \cup G(e)_0$ and τ is $\beta-$restricted to $H(e)_0$.

(iii) $F(e) \cap G(e) = H(e) = (H(e)_0, \gamma)$, where $H(e)_0 = F(e)_0 \cap G(e)_0$ and γ is $\beta-$restricted to $H(e)_0$.

Definition 3.3. Let (F, A) and (G, B) be soft multisets over U. Then (F, A) is called a soft submultiset of (G, B), written $(F, A) \sqsubset (G, B)$, if

(i) $A \subseteq B$,
(ii) $F(e) \subseteq G(e)$, $\forall\ e \in A$.

Definition 3.4. Two soft multisets (F, A) and (G, B) over U are said to be equal, written $(F, A) = (G, B)$, if $(F, A) \sqsubset (G, B)$ and $(G, B) \sqsubset (F, A)$.

Also, if $(F, A) \sqsubset (G, B)$ and $(F, A) \neq (G, B)$, then (F, A) is called a proper soft submultiset of (G, B). (F, A) is a whole soft submultiset of (G, B), if $\forall s_m \in F(e)\ \exists\ t_k \in G(e)$ such that $s_m = t_k$, $\forall\ e \in A$. It is a full soft submset, if $\forall t_k \in G(e)\ \exists\ s_m \in F(e)$ such that $s_m \subseteq t_k$, $\forall\ e \in B$.

Example 3.5. Consider the soft multisets given by:

$$(K, D) = \{(e_1, \{a, b, b', b^{''}\}), (e_4, \{c, c', d, d', d^{''}\}), (e_5, \{b, b', c, d, \ldots, d^v\})\},$$

$$(F, A) = \{e_1, \{a, b, b'\}), (e_5, \{b, b', c, d\})\},$$

$$(G, B) = \{(e_1, \{a, b, b', b^{''}\}), (e_4, \{c, c', d, d', d^{''}\})\},$$

$$(H, C) = \{e_1, \{a, b\}), (e_4, \{c, c', d\}), (e_5, \{b, b', c, d, d'\})\}.$$

Then we can easily see that (F, A) is a proper soft submultiset of (K, D), (G, B) is a whole soft submultiset of (K, D) and (H, C) is a full soft submultiset of (K, D).

Definition 3.6. Let (F, A) and (G, B) be two soft multisets over a universe U and parameter set E. Then

(i) the union of (F, A) and (G, B), denoted by $(F, A) \sqcup (G, B) = (H, C)$, where $C = A \cup B$ and $H(e) = \{F(e) \cup G(e),\ \forall e \in C\}$.

(ii) the intersection of (F, A) and (G, B), denoted by $(F, A) \sqcap (G, B) = (H, C)$, where $C = A \cap B$ and $H(e) = \{F(e) \cap G(e),\ \forall e \in C\}$.

Let us denote Φ_A, as the relative null soft multiset, such that $F(e) = \varnothing$, $\forall e \in A$ and U_A the relative absolute soft multiset, such that $F(e) = U$, $\forall e \in A$.

Definition 3.7. Let (F, A) and (G, B) be two soft multisets over U such that $(G, B) \sqsubset (F, A)$. Then their difference, denoted by $(F, A) \setminus (G, B) = (H, A)$, where $H(e) = \{F(e) \backslash G(e),\ \forall e \in A\}$.

Theorem 3.8. Let (F, A) and (G, B) be two soft multisets over U. Then

(1) $((F, A) \setminus (G, B)) \sqcup (G, B) = (F, A)$,

(2) $((F, A) \setminus (G, B)) \sqcap (G, B) = \Phi_A$.

Proof. (1) Let $((F, A) \setminus (G, B)) = (H, A)$ where $H(e) = \{F(e) \backslash G(e), \forall e \in A\}$.
Then clearly $((F, A) \setminus (G, B)) \sqcup (G, B) = (H, A) \sqcup (G, B)$.
Suppose $(H, A) \sqcup (G, B) = (K, A)$, where $K(e) = \{H(e) \cup G(e), \forall e \in A\}$.
Since $(G, B) \sqsubset (F, A)$, $K(e) = F(e), \forall e \in A$, i.e., $(K, A) = (F, A)$.
Thus $((F, A) \setminus (G, B)) \sqcup (G, B) = (F, A)$.

(2) Let $((F, A) \setminus (G, B)) = (H, A)$, where $H(e) = \{F(e) \backslash G(e), \forall e \in A\}$.
Then clearly, $((F, A) \setminus (G, B)) \sqcap (G, B) = (H, A) \sqcap (G, B)$.
Suppose $(H, A) \sqcap (G, B) = (L, A)$, where $L(e) = \{H(e) \cap G(e), \forall e \in A\}$.
Then $H(e) \cap G(e) = \varnothing, \forall e \in A$, i.e., $L(e) = \varnothing, \forall e \in A$.
Thus $(H, A) \sqcap (G, B) = \Phi_A$.
So $((F, A) \setminus (G, B)) \sqcap (G, B) = \Phi_A$. □

Definition 3.9. The complement of a soft multiset (F, A), is $(F, A)^c = (F^c, A)$, where $F^c : A \longrightarrow P(U)$ is defined as $F^c(e) = U \backslash F(e)$, $\forall e \in A$.

Example 3.10. Let (F, A) be a soft multiset over a universal multiset given by

$$U = \left\{a_1, a_1^{'}, a_1^{''}, a_1^{'''}, a_2, a_2^{'}, a_2^{''}, \dots, a_2{}^{vi}, a_3, a_3^{'}, a_3^{''}, \dots, a_3{}^{ix}, a_4, a_4^{'}, a_4^{''}, \dots a_4{}^{iix}\right\},$$

$$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\},\ A = \{e_1, e_2, e_3\},$$

$$F(e_1) = \{a_2, a_2^{'}, a_3, a_3^{'}, a_3^{''}\},\ F(e_2) = \{a_2, a_4, a_4^{'}\},$$
$$F(e_3) = \{a_1, a_1^{'}, a_2, a_2^{'}, a_2^{''}, a_2^{'''}, a_2{}^{iv}, a_3, a_3^{'}, a_3^{''}\}.$$

Then $(F, A)^c = (F^c, A)$, where

$$F^c(e_1) = \left\{a_1, a_1^{'}, a_1^{''}, a_1^{'''}, a_2^{''}, \dots, a_2{}^{vi}, a_3^{'''}, \dots, a_3{}^{ix}, a_4, a_4^{'}, a_4^{''}, \dots a_4{}^{iix}\right\},$$
$$F^c(e_2) = \left\{a_1, a_1^{'}, a_1^{''}, a_1^{'''}, a_2^{'}, a_2^{''}, \dots, a_2{}^{vi}, a_3, a_3^{'}, a_3^{''}, \dots, a_3{}^{ix}, a_4^{''}, \dots a_4{}^{iix}\right\},$$
$$F^c(e_3) = \{a_1^{''}, a_1^{'''}, a_2^{v}, a_2{}^{vi}, a_3^{'''}, \dots, a_3{}^{ix}, a_4, a_4^{'}, a_4^{''}, \dots a_4{}^{iix}\}.$$

Theorem 3.11. Let U be a multiset, E the set of parameters, $A \subseteq E$ and (F, A) a soft multisets over U. Then
(1) $(F, A) \sqcap (F, A)^c = \Phi_A$,
(2) $(F, A) \sqcup (F, A)^c = \mathrm{U}_A$.

Proof. (1) Let (F, A) be a soft multiset over $U = (U_0, \beta)$,
where $U_0 = \left\{a_1, a_1^{'}, a_1^{''}, \ldots, {a_1}^m, a_2, a_2^{'}, a_2^{''}, \ldots, {a_2}^k, \ldots, a_n, a_n^{'}, a_n^{''}, \ldots, {a_n}^r\right\}$ and β an equivalence relation on U_0.

Suppose $F(e) = U$. Then $\forall e \in A$, we have $F(e) = [F(e)_0, \alpha]$, where α is $\beta-$ restricted to $F(e)_0$. Since $(F, A)^c = (F^c, A)$, $\forall e \in A$, $F^c(e) = [F^c(e)_0, \gamma]$, where $F^c(e)_0 = U_0 \backslash F(e)_0$ and γ is $\beta-$ restricted to $F^c(e)_0$.

Now, suppose $(F, A) \sqcap (F, A)^c = (G, A)$. Then for each $e \in A$,

$$G(e) = F(e) \cap F^c(e) = [F(e)_0 \cap F^c(e)_0, \rho],$$

where ρ is $\beta-$ restricted to $F(e)_0 \cap F^c(e)_0$. But $F(e)_0 \cap F^c(e)_0$ is empty, i.e., $\forall e \in A, G(e) = \varnothing$. Thus $(F, A) \sqcap (F, A)^c = \Phi_A$.

(2) Let $(F, A) \sqcup (F, A)^c = (H, A)$. Then for each $e \in A$,

$$H(e) = F(e) \cup F^c(e) = [F(e)_0 \cup F^c(e)_0, \sigma],$$

where σ is $\beta-$ restricted to $F(e)_0 \cup F^c(e)_0$.
But $F(e)_0 \cup F^c(e)_0 = U_0$. Thus for all $e \in A$, $H(e) = U$.
So $(F, A) \sqcup (F, A)^c = \mathrm{U}_A$. □

Theorem 3.12. Let (F, A) and (G, B) be two soft sets over U. Then
(1) $((F, A) \sqcup (G, B))^c = (F, A)^c \sqcap (G, B)^c$,
(2) $((F, A) \sqcap (G, B))^c = (F, A)^c \sqcup (G, B)^c$.

Proof. Let $(F, A) \sqcup (G, B) = (H, C)$ where $C = A \cup B$, $(F, A) \sqcap (G, B) = (J, D)$ and $D = A \cap B$.

(1) Clearly, $H(e) = F(e) \cup G(e)$, for each $e \in C$. But $((F, A) \sqcup (G, B))^c = (H, C)^c = (H^c, C)$. Then for each $e \in C$, we have

$$\begin{aligned} H^c(e) &= U \backslash H(e) = U \backslash [F(e) \cup G(e)] \\ &= [U \backslash F(e)] \cap [U \backslash G(e)] \\ &= F^c(e) \cap G^c(e) \\ &= (F, A)^c \sqcap (G, B)^c, \text{ i.e.,} \end{aligned}$$

$$((F, A) \sqcup (G, B))^c \sqsubset (F, A)^c \sqcap (G, B)^c. \tag{3.1}$$

On the other hand, $(F, A)^c \sqcap (G, B)^c = (F^c, A) \sqcap (G^c, B) = (J^c, D)$. Then by definition, for each $e \in D$, we have

$$\begin{aligned} J^c(e) &= F^c(e) \cap G^c(e) = [U \backslash F(e)] \cap [U \backslash G(e)] \\ &= U \backslash [F(e) \cup G(e)] \\ &= U \backslash H(e) = H^c(e) \\ &= ((F, A) \sqcup (G, B))^c, \text{ i.e.,} \end{aligned}$$

$$(F, A)^c \sqcap (G, B)^c \sqsubset ((F, A) \sqcup (G, B))^c. \tag{3.2}$$

Thus from (3.1) and (3.2), $((F, A) \sqcup (G, B))^c = (F, A)^c \sqcap (G, B)^c$.

(2) It is obvious that $e \in D, J(e) = F(e) \cap G(e)$, for each $e \in D$. Since $((F, A) \sqcap (G, B))^c = (J, D)^c = (J^c, D)$, for each $e \in D$, we have

$$\begin{aligned} J^c(e) &= U \backslash J(e) = U \backslash [F(e) \cap G(e)] \\ &= [U \backslash F(e)] \cup [U \backslash G(e)] \\ &= (F^c, A) \sqcup (G^c, B) \\ &= (F, A)^c \sqcup (G, B)^c, \text{ i.e.,} \end{aligned}$$

$$((F, A) \sqcap (G, B))^c \sqsubset (F, A)^c \sqcup (G, B)^c. \tag{3.3}$$

On the other hand, $(F, A)^c \sqcup (G, B)^c = (F^c, A) \sqcup (G^c, B) = (H^c, C)$. Then by definition, for each $e \in C$, we have

$$\begin{aligned} H^c(e) &= F^c(e) \cup G^c(e) = [U \backslash F(e)] \cup [U \backslash G(e)] \\ &= U \backslash [F(e) \cap G(e)] = ((F, A) \sqcap (G, B))^c, \text{ i.e.,} \end{aligned}$$

$$(F, A)^c \sqcup (G, B)^c \sqsubset ((F, A) \sqcap (G, B))^c. \tag{3.4}$$

Thus from (3.3) and (3.4), $((F, A) \sqcap (G, B))^c = (F, A)^c \sqcup (G, B)^c$. □

Definition 3.13. The power soft multiset of a given soft multiset (F, A), denoted by $P(F_A)$ or $P(F, A)$, is the set of all soft submultisets of (F, A).

Example 3.14. Let $(F, A) = \left\{(e_1, \{a_1\}), \left(e_2, \left\{a_1, a_1^{'}, a_2\right\}\right)\right\}$. Then

$$\begin{aligned} P(F_A) = \{&\Phi_A, \{(e_1, \{a_1\})\}, \{(e_2, \{a_1\})\}, \{(e_2, \{a_1^{'}\})\}, \{(e_2, \{a_2\})\}, \\ &\{(e_2, \{a_1, a_1^{'}\})\}, \{(e_2, \{a_1, a_2\})\}, \{(e_2, \{a_1^{'}, a_2\})\}, \\ &\{(e_2, \{a_1, a_1^{'}, a_2\})\}, \{(e_1, \{a_1\}), (e_2, \{a_1, a_1^{'}, a_2\})\}\}. \end{aligned}$$

Definition 3.15. Let $(H, B) \in P(F, A)$. Then the relative complement of (H, B) in $P(F, A)$ is $\overline{(H, B)} = (\overline{H}, B)$, where $\overline{H} : B \longrightarrow P(U)$ is defined by $\overline{H}(e) = F(e) \backslash H(e)$, $\forall e \in A$.

Theorem 3.16. (1) $(H, B) \cup \overline{\ (H, B)} = (F, A)$.
(2) $(H, B) \cap \overline{\ (H, B)} = \Phi_A$.

Proof. (1) Let $(R, B) = \overline{\ (H, B)} = (\overline{H}, B)$, where $\overline{H}(e) = F(e) \backslash H(e), \forall e \in A$.
Let $(H, B) \sqcup (R, B) = (T, A)$, where $T(e) = H(e) \cup R(e) = F(e), \forall e \in A$.
Then clearly, $T(e) = F(e), \forall e \in A$. Thus $(H, B) \cup \overline{\ (H, B)} = (F, A)$.

(2) Let $(H, B) \sqcap (R, B) = (Z, A)$, where $Z(e) = H(e) \cap R(e), \forall e \in A$.
Then, it is obvious that $H(e) \cap R(e) = \varnothing, \forall e \in A$.
Thus $(H, B) \cap \overline{\ (H, B)} = \Phi_A$. □

4. Conclusion

The paper introduces the idea of soft multiset using multiset of equivalence relation approach in order to addresses the inherent difficulties of complementation and others, encountered in the theory of Soft multisets. This seems to attract a wider application for the theory of soft multisets.

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