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Abstract. Information entropies have been widely applied in constructing heuristic attribute reduction. However, little attention has been paid to the information entropies for dynamic information systems with varying object set. In this paper, we present an incremental approach to update conditional complementary entropy for dynamic information systems with varying objects. Based on the new incremental formulas, we develop an incremental attribute reduction algorithm for decision table with varying object set. By a numerical experiment, we express the efficiency of the new method.

2010 AMS Classification: 03E75, 68T20, 68W25

Keywords: Information entropy, Conditional complementary entropy, Incremental approach, Dynamic information system; Attribute reduction.

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1. Introduction

Rough set theory [16, 18, 19, 20] pioneered by Pawlak in 1982 is aimed at data analysis problems involving uncertain, imprecise or incomplete information. As an objective soft computing tool, rough set theory has been widely applied to various fields such as medical diagnosis[15], investment decisions [2], feature selection [24, 25], knowledge discovery[11] and data mining [8]. Many techniques have been developed for attribute reductions in the past decades. In order to describe the uncertainty of information systems, some uncertainty measures have been studied [7, 12]. Several authors have acquired entropy reducts which can remain the entropy of target decision unchanged. The concept of entropy was introduced by Shannon in 1948, and its initial goal was to evaluate uncertainty of a system. In order to measure uncertainty in rough set theory, Beaubouef and Duentsch et al. [4, 3] utilized

Shannon's entropy and its variants. Slezak [22, 23] proposed Shannon's information entropy for searching reducts. In [10], Liang et al. acquired reducts by using a new information entropy and defined some new information entropies like rough entropy, combination entropy. The reference [21] brought forward combination entropy for measuring the uncertainty of information systems. Dai et al. [6, 5] investigated information entropies, including Shannon's entropy, conditional entropy and joint entropy for incomplete decision systems, and presented a new form of conditional entropy and constructed three attribute selection approaches, etc. In practice, information systems usually vary with time. Specially, the objects in business database are changing all the time. Some researchers have studied attribute reductions of dynamic information systems[1, 9, 13, 14, 26]. For example, Bang et al. [1] presented an incremental inductive learning algorithm to find a minimal set of rules for an information system when adding a new object. Hu et al. [9] gave an incremental attribute reduction algorithm when adding some new objects. Wang et al. [26] discussed how to update entropy for dynamic information system when changing data values. However, in their work, the object set and attribute set must remain unchanged, which limit its application. Moreover, the updating mechanisms in their article are only applicable when data are varied one by one. Their ideas are reasonable but their work needs further improvements.

Following the above work, we provide an incremental approach to updating conditional complementary entropy for dynamic information systems when the object set evolves with time. In order to study the dynamic changes of information system, we study from simple case to complex. Firstly, when only one object is removed, we update the conditional complementary entropy by definition and present an incremental approach to acquire new information entropy from the former information entropy. When only one object is added, we also establish recursive formula to construct new conditional complementary entropy by definition. Secondly, when only one object is changed, we update the information entropy by removing the old object and adding the new object. When several objects are removed, we use the recursive formula in the first case several times, then we construct an incremental algorithm. When several objects are added, we apply the recursive formula in the first case several times, then we acquire an incremental algorithm. Thirdly, for varying object set, by combining the first case and the second case, we can renew the information entropy.

The rest of this paper is organized as follows. Some preliminaries related to rough set theory are briefly reviewed in Section 2. Section 3 presents an incremental approach to updating conditional complementary entropy for dynamic information systems. Based on the above analysis, we construct a new algorithm for decision table with varying object set in Section 4. In order to demonstrate the practicality and effectiveness of our proposed method, we give a numerical experiment in Section 5. Section 6 provides the conclusion of this paper and some future works.

2. Preliminaries

In this section, we recall some basic concepts of rough sets and information entropy.

Definition 2.1 ([17]). An information system (or decision table) is defined as a pair $\langle U, A\rangle$ where U is a non-empty finite set of objects, $A = C \cup D$ is a non-empty finite set of attributes, C denotes the set of condition attributes and D denotes the set of decision attributes, $C \cap D = \varnothing$. Each attribute $a \in A$ is associated with a set V_a of its value, called the domain of a.

The concept of information system provides a convenient framework to represent the objects in the universe. Actually, every $B \subseteq A$ can determine an equivalence relation R_B as

$$\forall x, y, R_B(x, y) \Longleftrightarrow \forall a \in B : a(x) = a(y),$$

where $a(x)$ denotes the attribute value of x respect to a. Every R_B can partition U into some equivalence classes given by $U/R_B = \{[x]_B|x \in U\}$, or just U/B for short, where $[x]_B = \{y \in U|(x, y) \in R_B\}$ denotes the equivalence class including x with respect to B. Using these elementary sets in U/R, we can approximate arbitrary set $X \subseteq U$:

$$\underline{apr}(X) = \bigcup\{[x]_R|[x]_R \subseteq X\} = \{x \in U|[x]_R \subseteq X\},$$
$$\overline{apr}(X) = \bigcup\{[x]_R|[x]_R \cap X \neq \phi\} = \{x \in U|[x]_R \cap X \neq \phi\}.$$

The lower approximation $\underline{apr}(X)$ contains all the elementary sets which are the subsets of X, and upper approximation $\overline{apr}(X)$ contains all the elementary sets which have a non-empty intersection with X.

The complementary entropy is used to measure the uncertainty of an information system, and the conditional complementary entropy defined as below can measure the uncertainty of a decision table.

Definition 2.2 ([26]). Let $S = (U, C \cup D)$ be an information system, $B \subseteq C$, $U/B = \{X_1, X_2, \cdots X_m\}$ and $U/D = \{Y_1, Y_2, \cdots Y_n\}$. A conditional complementary entropy of B relative to D is defined as

$$E_U(D|B) = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{|X_i \cap Y_j|}{|U|} \cdot \frac{|Y_j^C - X_i^C|}{|U|} = \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{|X_i \cap Y_j|}{|U|} \cdot \frac{|X_i - Y_j|}{|U|}.$$

The entropy reduction was proposed by Skowron in 1993. Then, Slezak introduced Shannon's entropy to get reducts in classical rough set model [22, 23].

Definition 2.3 ([26]). Let $S = (U, C \cup D)$ be information system and $B \subseteq C$. Then, B is a relative reduct, if it satisfies:
(i) $E(D|B) = E(D|C)$,
(ii) $\forall a \in B, E(D|B - \{a\}) \neq E(D|B)$.

The first condition indicates that B keeps the same entropy as C, and the second condition indicates that each attribute in B is individually necessary.

3. An incremental approach to computing conditional complementary entropy for dynamic information systems

In this section, we establish a mathematical fundamental to compute the entropy for dynamic data sets.

Let $S = (U, A = C \cup D, V, f)$ with $C \cap D = \varnothing$, $B \subseteq C$. Then, one can obtain the partitions $U/B = \{X_1, X_2, \cdots X_m\}$ and $U/D = \{Y_1, Y_2, \cdots Y_n\}$. The complementary conditional entropy of D with respect to B is $E_U(D|B)$. By revising the object set, we can easily update conditional complementary entropy. When one object enter or get out of the system, we can ignore new condition equivalence classes and new decision equivalence classes by assuming that some $X_i \in U/B$, $Y_j \in U/D$ can be empty sets [26].

Lemma 3.1. *Suppose only one object x get out of the system S, and $x \in X_{p_1}, x \in Y_{q_1}$. Then we get a new information system $S^{'}$ and $U^{'} = U - \{x\}$, $U^{'}/B = \{X_1, X_2, \cdots X^{'}_{p_1}, \cdots X_m\}$, $U^{'}/D = \{Y_1, Y_2, \cdots Y^{'}_{q_1}, \cdots Y_n\}$, where $X^{'}_{p_1} = X_{p_1} - \{x\}$, $Y^{'}_{q_1} = Y_{q_1} - \{x\}$. New entropy $E_{U'}(D|B)$ can be got as the following.*

$$\frac{(|U|-1)^2}{|U|^2} \cdot E_{U'}(D|B) = E_U(D|B) - \frac{2|X_{p_1} - Y_{q_1}|}{|U|^2}.$$

Proof. By Definition 2.2, the new conditional complementary entropy is

$$\begin{aligned} E_{U'}(D|B) = &\sum_{i=1,i\neq p_1}^{m} \sum_{j=1,j\neq q_1}^{n} \frac{|X_i \cap Y_j|}{|U|-1} \cdot \frac{|X_i - Y_j|}{|U|-1} + \sum_{i=1,i\neq p_1}^{m} \frac{|X_i \bigcap Y^{'}_{q_1}|}{|U|-1} \cdot \frac{|X_i - Y'_{q_1}|}{|U|-1} \\ &+ \sum_{j=1,j\neq q_1}^{n} \frac{|X'_{p_1} \bigcap Y_j|}{|U|-1} \cdot \frac{|X'_{p_1} - Y_j|}{|U|-1} + \frac{|X'_{p_1} \bigcap Y^{'}_{q_1}|}{|U|-1} \cdot \frac{|X'_{p_1} - Y'_{q_1}|}{|U|-1}. \end{aligned}$$

Then, using $X^{'}_{p_1} = X_{p_1} - \{x\}$ and $Y^{'}_{q_1} = Y_{q_1} - \{x\}$, we have

$$\begin{aligned} &\frac{(|U|-1)^2}{|U|^2} \cdot E_{U-\{x\}}(D|B) \\ &= \sum_{i=1,i\neq p_1}^{m} \sum_{j=1,j\neq q_1}^{n} \frac{|X_i \cap Y_j|}{|U|} \cdot \frac{|X_i - Y_j|}{|U|} + \sum_{i=1,i\neq p_1}^{m} \frac{|X_i \cap Y_{q_1}|}{|U|} \cdot \frac{|X_i - Y_{q_1}|}{|U|} \\ &\quad + \sum_{j=1,j\neq q_1}^{n} \frac{|X_{p_1} \cap Y_j|}{|U|} \cdot \frac{|X_{p_1} - Y_j| - 1}{|U|} + \frac{|X_{p_1} \cap Y_{q_1}| - 1}{|U|} \cdot \frac{|X_{p_1} - Y_{q_1}|}{|U|} \\ &= E_U(D|B) - (\sum_{j=1,j\neq q_1}^{n} \frac{|X_{p_1} \bigcap Y_j|}{|U|^2} + \frac{|X_{p_1} - Y_{q_1}|}{|U|^2}) \\ &= E_U(D|B) - (\sum_{j=1}^{n} \frac{|X_{p_1} \cap Y_j|}{|U|^2} - \frac{|X_{p_1} \cap Y_{q_1}|}{|U|^2} + \frac{|X_{p_1} - Y_{q_1}|}{|U|^2}) \\ &= E_U(D|B) - (\frac{|X_{p_1}|}{|U|^2} - \frac{|X_{p_1} \cap Y_{q_1}|}{|U|^2} + \frac{|X_{p_1} - Y_{q_1}|}{|U|^2}) \\ &= E_U(D|B) - \frac{2|X_{p_1} - Y_{q_1}|}{|U|^2}. \end{aligned}$$

□

Lemma 3.2. *Suppose only one object x enter the system S, and $x \in X_{p_2}, x \in Y_{q_2}$. Then we get a new information system $S^{'}$ and $U^{'} = U \cup \{x\}$, $U^{'}/B =$*

$\{X_1, X_2, \cdots X'_{p_2}, \cdots X_m\}$, $U'/D = \{Y_1, Y_2, \cdots Y'_{q_2}, \cdots Y_n\}$, where $X'_{p_2} = X_{p_2} \bigcup \{x\}$ and $Y'_{q_2} = Y_{q_2} \bigcup \{x\}$. New entropy $E_{U'}(D|B)$ can be got as follows:

$$\frac{(|U|+1)^2}{|U|^2} \cdot E_{U'}(D|B) = E_U(D|B) + \frac{2|X_{p_2} - Y_{q_2}|}{|U|^2}.$$

Proof. By Definition 2.2, the new entropy is

$$\begin{aligned} E_{U'}(D|B) = & \sum_{i=1, i\neq p_2}^{m} \sum_{j=1, j\neq q_2}^{n} \frac{|X_i \cap Y_j|}{|U|+1} \cdot \frac{|X_i - Y_j|}{|U|+1} + \sum_{i=1, i\neq p_2}^{m} \frac{|X_i \bigcap Y'_{q_2}|}{|U|+1} \cdot \frac{|X_i - Y'_{q_2}|}{|U|+1} \\ & + \sum_{j=1, j\neq q_2}^{n} \frac{|X'_{p_2} \bigcap Y_j|}{|U|+1} \cdot \frac{|X'_{p_2} - Y_j|}{|U|+1} + \frac{|X'_{p_2} \bigcap Y'_{q_2}|}{|U|+1} \cdot \frac{|X'_{p_2} - Y'_{q_2}|}{|U|+1}. \end{aligned}$$

Then, using $X'_{p_2} = X_{p_2} \cup \{x\}$ and $Y'_{q_2} = Y_{q_2} \cup \{x\}$, we obtain

$$\begin{aligned} & \frac{(|U|+1)^2}{|U|^2} \cdot E_{U\cup\{x\}}(D|B) \\ =& [E_U(D|B) - \sum_{i=1, j\neq p_2}^{m} \frac{|X_i \cap Y_{q_2}||X_i - Y_{q_2}|}{|U|^2} \\ & - \sum_{j=1, j\neq q_2}^{n} \frac{|X_{p_2} \cap Y_j||X_{p_2} - Y_j|}{|U|^2} - \frac{|X_{p_2} \cap Y_{q_2}||X_{p_2} - Y_{q_2}|}{|U|^2}] \\ & + \sum_{i=1, j\neq p_2}^{m} \frac{|X_i \cap Y_{q_2}||X_i - Y_{q_2}|}{|U|^2} \\ & + \sum_{j=1, j\neq q_2}^{n} \frac{|X_{p_2} \cap Y_j|(|X_{p_2} - Y_j|+1)}{|U|^2} + \frac{(|X_{p_2} \cap Y_{q_2}|+1)|X_{p_2} - Y_{q_2}|}{|U|^2} \\ =& E_U(D|B) + \sum_{j=1, j\neq q_2}^{n} \frac{|X_{p_2} \cap Y_j|}{|U|^2} + \frac{|X_{p_2} - Y_{q_2}|}{|U|^2} \\ =& E_U(D|B) + \sum_{j=1}^{n} \frac{|X_{p_2} \cap Y_j|}{|U|^2} - \frac{|X_{p_2} \cap Y_{q_2}|}{|U|^2} + \frac{|X_{p_2} - Y_{q_2}|}{|U|^2} \\ =& E_U(D|B) + \frac{|X_{p_2}|}{|U|^2} - \frac{|X_{p_2} \cap Y_{q_2}|}{|U|^2} + \frac{|X_{p_2} - Y_{q_2}|}{|U|^2} \\ =& E_U(D|B) + \frac{2|X_{p_2} - Y_{q_2}|}{|U|^2}. \end{aligned}$$

□

Wang et al. [26] have shown the process of computing new entropy when attribute values of x are varied and x is changed to x'. We can replace this process by removing x from system first, and then add x' into the system. In other words, we can get Wang's Theorem as our inference.

Theorem 3.3 ([26]). *If one and only one object* $x \in U$ *is changed to* x', $x \in X_{p_1}$, $x \in Y_{q_1}$, $x' \in X'_{p_2}, x \in Y'_{q_2}$. *The new complementary conditional entropy becomes*

$$E_{U'}(D|B) = E_U(D|B) + \frac{2|X_{p_2} - Y_{q_2}| - 2|X_{p_1} - Y_{q_1}|}{|U|^2}.$$

Proof. For the change of x to x', we pull out in two stages as follows. First, we remove x from S, then the combination conditional entropy of $E_{U-\{x\}}(D|B)$ can be got By Lemma 3.1, that is

$$\frac{(|U|-1)^2}{|U|^2} \cdot E_{U-\{x\}}(D|B) = E_U(D|B) - \frac{2|X_{p_1} - Y_{q_1}|}{|U|^2}.$$

Second, by adding x', we get the new combination conditional entropy $E_{U'}(D|B) = E_{U-\{x\}\cup\{x'\}}(D|B)$ by lemma 3.2 as

$$\frac{|U|^2}{(|U|-1)^2} \cdot E_{U-\{x\}\cup\{x'\}}(D|B) = E_{U-\{x\}}(D|B) + \frac{2|X_{p_2} - Y_{q_2}|}{(|U|-1)^2},$$

thus

$$E_{U-\{x\}\cup\{x'\}}(D|B) = \frac{(|U|-1)^2}{|U|^2} \cdot E_{U-\{x\}}(D|B) + \frac{(|U|-1)^2}{|U|^2} \cdot \frac{2|X_{p_2} - Y_{q_2}|}{(|U|-1)^2},$$

by using

$$E_{U-\{x\}}(D|B) = \frac{(|U|^2}{|U|-1)^2} \cdot (E_U(D|B) - \frac{2|X_{p_1} - Y_{q_1}|}{|U|^2}),$$

we get

$$\begin{aligned} &E_{U-\{x\}\cup\{x'\}}(D|B) \\ &= \frac{(|U|-1)^2}{|U|^2} \cdot \frac{(|U|^2}{|U|-1)^2} \cdot (E_U(D|B) - \frac{2|X_{p_1} - Y_{q_1}|}{|U|^2}) + \frac{(|U|-1)^2}{|U|^2} \cdot \frac{2|X_{p_2} - Y_{q_2}|}{(|U|-1)^2} \\ &= E_U(D|B) + \frac{2|X_{p_2} - Y_{q_2}| - 2|X_{p_1} - Y_{q_1}|}{|U|^2}. \end{aligned}$$

□

When M objects get out of the system, we can iterate Theorem 3.1 for M times. Then we get the following theorem.

Lemma 3.4. *If M objects get out of the system, M_{ij} represents that there M_i objects that get out of the conditional class X_i and M_{ij} objects of them come from decision class Y_j, then the new combination conditional entropy denoted by $E_{U-M}(D|B)$ becomes*

$$\frac{(|U|-M)^2}{|U|^2} \cdot E_{U-M}(D|B) = E_U(D|B) - 2\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{M_{ij}|X_i - Y_j|}{|U|^2}.$$

Proof. We remove M objects from S one by one. When M_{11} objects get out, we can apply Theorem 3.1 for M_{11} times. So we get

$$\frac{(|U|-M_{11})^2}{|U|^2}\cdot E_{U-M_{11}}(D|B)=E_U(D|B)-\frac{2M_{11}|X_1-Y_1|}{|U|^2}.$$

Next, let M_{12} objects leave this new system. By using Lemma 3.1 for M_{12} times, we obtain

$$\frac{(|U|-M_{11}-M_{12})^2}{(|U|-M_{11})^2}\cdot E_{U-M_{11}-M_{12}}(D|B)=E_{U-M_{11}}(D|B)-\frac{2M_{12}|X_1-Y_2|}{(|U|-M_{11})^2}.$$

Since

$$E_{U-M_{11}}(D|B)=\frac{|U|^2}{(|U|-M_{11})^2}\cdot (E_U(D|B)-\frac{2M_{11}|X_1-Y_1|}{|U|^2}),$$

we get

$$\begin{aligned}\frac{(|U|-M_{11}-M_{12})^2}{(|U|-M_{11})^2}\cdot E_{U-M_{11}-M_{12}}(D|B)=&\frac{|U|^2}{(|U|-M_{11})^2}\cdot (E_U(D|B)\\&-\frac{2M_{11}|X_1-Y_1|}{|U|^2})-\frac{2M_{12}|X_1-Y_2|}{(|U|-M_{11})^2}.\end{aligned}$$

In other words,

$$\frac{(|U|-M_{11}-M_{12})^2}{|U|^2}\cdot E_{U-M_{11}-M_{12}}(D|B)=(E_U(D|B)-\frac{2M_{11}|X_1-Y_1|}{|U|^2})-\frac{2M_{12}|X_1-Y_2|}{|U|^2}.$$

Repeating this operation, we can complete the proof by induction axiom. □

When N objects enter the system, by using Lemma 3.2, we get the following lemma.

Lemma 3.5. *If N objects enter the system, N_{ij} represents that there N_i objects enter the conditional class X_i and N_{ij} objects of them enter decision class Y_j. The new combination conditional entropy $E_{U+N}(D|B)$ becomes*

$$\frac{(|U|+N)^2}{|U|^2}\cdot E_{U+N}(D|B)=E_U(D|B)+2\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{N_{ij}|X_i-Y_j|}{|U|^2}.$$

Proof. We add these N objects into S one by one. When N_{11} objects get in, by using Lemma 3.2 for N_{11} times, we get

$$\frac{(|U|+N_{11})^2}{|U|^2}\cdot E_{U+N_{11}}(D|B)=E_U(D|B)+\frac{2N_{11}|X_1-Y_1|}{|U|^2}.$$

Next, when adding N_{12} objects, we use Lemma 3.2 for N_{12} times, that is

$$\frac{(|U|+N_{11}+N_{12})^2}{(|U|+N_{11})^2}\cdot E_{U+N_{11}+N_{12}}(D|B)=E_{U+N_{11}}(D|B)+\frac{2N_{12}|X_1-Y_2|}{(|U|+N_{11})^2}.$$

Using

$$E_{U+N_{11}}(D|B)=\frac{|U|^2}{(|U|+N_{11})^2}\cdot (E_U(D|B)+\frac{2N_{11}|X_1-Y_1|}{|U|^2}),$$

we acquire

$$\frac{(|U|+N_{11}+N_{12})^2}{(|U|+N_{11})^2}\cdot E_{U+N_{11}+N_{12}}(D|B)=\frac{|U|^2}{(|U|+N_{11})^2}\cdot(E_U(D|B)+\frac{2N_{11}|X_1-Y_1|}{|U|^2})+\frac{2N_{12}|X_1-Y_2|}{(|U|+N_{11})^2}.$$

In other words,

$$\frac{(|U|+N_{11}+N_{12})^2}{|U|^2}\cdot E_{U+N_{11}+N_{12}}(D|B)=E_U(D|B)+\frac{2N_{11}|X_1-Y_1|}{|U|^2}+\frac{2N_{12}|X_1-Y_2|}{|U|^2}.$$

Repeating this operation, we can complete the proof by induction axiom. □

Suppose there are N objects enter the system and M objects get out of the system at the same time. N_{ij} represents that there N_i objects that enter the conditional class X_i and N_{ij} objects of them enter decision class Y_j. M_{ij} represents that there M_i objects that get out of the conditional class X_i and M_{ij} objects of them come from decision class Y_j. Combining Lemma 3.4 and Lemma 3.5, we acquire the conclusion below.

Theorem 3.6. *If N objects enter the system, and M objects get out of the system at the same time. The new combination conditional entropy which we denote by $E_{U+N-M}(D|B)$ becomes*

$$\frac{(|U|+N-M)^2}{|U|^2}\cdot E_{U+N-M}(D|B)=E_U(D|B)+2\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{(N_{ij}-M_{ij})|X_i-Y_j|}{|u|^2}.$$

Proof. After adding these N objects into S one by one, by Lemma 3.5 we get

$$\frac{(|U|+N)^2}{|U|^2}\cdot E_{U+N}(D|B)=E_U(D|B)+2\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{N_{ij}|X_i-Y_j|}{|U|^2}.$$

Then we remove M objects from the system above, by Lemma 3.4,

$$\frac{(|U|+N-M)^2}{(|U|+N)^2}\cdot E_{U+N-M}(D|B)=E_{U+N}(D|B)-2\sum_{i=1}^{m}\sum_{j=1}^{n}\frac{M_{ij}|X_i-Y_j|}{(|U|+N)^2}.$$

Combine these two equations and eliminate $E_{U+N}(D|B)$, then we complete our proof. □

4. An incremental attribute reduction algorithm for decision table with dynamically varying object set

Based on the incremental formula in Theorem 3.6, in this section, we will introduce an incremental attribute reduction algorithm for dynamically varying object set.

We first need to run the classic algorithm denoted by CAR [26] and store the result U/B, U/D, $E(D|B)$. Then we add or delete some objects and perform our incremental algorithm as follows.

Algorithm: An incremental attribute reduction algorithm for decision table with dynamically varying object set (IAR)

Input: A decision table $S = (U, C \cup D)$, N new objects enter S and M objects escape from S.

Output: Attribute reduction RED_{U+N-M} of new decision table $S' = (U + N - M, C \cup D)$.

Step 1: Run classical attribute reduction algorithm and store U/B, U/D, $E(D|B)$ for $B \subseteq C$.

Step 2: For each $x \in N$ or $x \in M$, check $x \in X_i$ and $x \in Y_i$, where $X_i \in U/B$,$Y_j \in U/D$. Then we get N_{ij} and $M_{ij}, i = 1, 2, \cdots, |U/B|, j = 1, 2, \cdots, |U/D|$.

Step 3: $reduct \leftarrow \phi$, $i \leftarrow 1$.

Step 4: According to Theorem 3.6, we compute $E_{U+N-M}(D|reduct)$.

Step 5: If $E_{U+N-M}(D|reduct) = E_{U+N-M}(D|C)$, then turn to Step 7; else turn to next Step 6.

Step 6: While $(E_{U+N-M}(D|reduct) \neq E_{U+N-M}(D|C))$ do

{

$i \leftarrow i + 1$;

$reduct \leftarrow reduct \cup c_i$, where c_i is the i-th important attribute in C.

}

Step 7: $RED_{U+N-M} \leftarrow reduct$, return RED_{U+N-M} and end.

By [26], in classic algorithm (CAR), we first need to acquire partition U/C with time complexity being $O(|U||C|)$; then we compute conditional entropy $E(D|B)$ by its definition, the corresponding time complexity is $\Theta = O(|U||C| + |U| + |U||U|) = O(|U|^2)$; finally, we add each $a \in C$ and check Definition 2.3, the time complexity is $O(|C|\Theta)$. Thus, the time complexity of CAR is $O(|U||C| + \Theta + |C|\Theta) = O(|C|^2|U| + |C||U|^2)$.

While, by IAR, in Step 1, we need to run CAR so as to get $U/B, U/D, E(D|B)$, the time complexity is $O(|C|^2|U|$; in Step 2, the time complexity of checking N M new objects $x \in X_i, x \in Y_i$ is $O(NM|U/B||U/D|)$; in Step 4, the time complexity of updating $E_{U+N-M}(D|B)$ is $\Theta' = O(|U/B||U/D||X_i||Y_j|)$; in Step 5–7, the time complexity of adding attributes is $O(|C|\Theta')$. Hence, the total time complexity of algorithm IAR is $O(|C|^2|U| + NM|U/B||U/D| + |U/B||U/D||X_i||Y_j| + |C||U/B||U/D||X_i||Y_j|) = O(|C|^2|U| + NM|U/B||U/D| + |C||U/B||U/D||X_i||Y_j|)$.

Generally speaking $N, M, |U/B|, |U/D|, |X_i|, |Y_j|$ are usually much smaller than $|U|$, then $NM|U/B||U/D| + |C||U/B||U/D||X_i||Y_j| \ll |C||U|^2$. As a result, IAR usually performs better than CAR.

5. Experimental analysis

Table 1. Time consumed by CAR and IAR

The size of the data set $\|U\|$	t (seconds)	Δt (seconds)
2000	74.943	0
2100	83.415	0.016
2200	92.166	0.031
2300	101.744	0.031
2400	110.698	0.016
2500	121.51	0.031
2600	136.189	0.031
2700	140.729	0.031
2800	156.36	0.031
2900	161.742	0.031
3000	173.114	0.031
3100	184.518	0.031
3196	201.975	0.031

Here, we give an example to express the effectiveness and efficiency of the proposed incremental reduction algorithm. The data set "Chess (King-Rook vs. King-Pawn).txt" is downloaded from UCI repository of machine learning databases. The experiments have been carried out on a personal computer with window 7 and Intel (R) Pentium (R) CPU G2030, 3.00GHz and 4 GB memory. The software used is Mathematica 4.0.

Firstly, we take out the first $|U|$ (for example $|U| = 2000$) objects from the data set named "Chess (King-Rook vs. King-Pawn).txt". Then, we run the classical reduction algorithm and list the consumed time denoted by "t" in table 1. During above calculation, we have stored each U/B, U/D, $E(D|B)$, $B \subseteq C$, so as to continue our incremental algorithm IAR. In what follows, we add 100 new objects step by step and perform our new algorithm. At the same time, we also record the consumed time denoted by "Δt" in Table 1.

According to the above analysis, we can clearly see that IAR works better than CAR for this experiment. The main reason is that we make full use of existing information including U/B, U/D, $E(D|B)(B \subseteq C)$ obtained by CAR. We only need to check each new object $x \in N$, confirm whether $x \in X_i$ or $x \in Y_j$. Then we update the new entropy $E_{U+N}(D|B)$ by Theorem 3.6. As $|U/B| \ll |U|, |U/D| \ll |U|$, the time consumed by IAR is obviously less than CAR. As it is extremely time-consuming to compute $E(D|B)$, the efficiency of IAR is improved.

6. Conclusions

Updating information entropies for dynamic data sets is a challenging issue. In this paper, we construct an incremental algorithm to renew conditional complementary entropy for dynamic information systems. We carefully divided new objects into the corresponding condition classes and decision classes and calculate new entropy incrementally. We consider the variation of data values as our special case by deleting some old objects from data sets and adding some new objects into data sets. By our incremental algorithm, we can get new entropies without computing repeatedly by their definition. It should be pointed out that updating mechanisms of the conditional complementary entropy introduced in our work are only applicable when attribute sets do not change. And it is worthy to point out that the variation of data values can be replaced by the variation of data sets. We think that the results which we give here can be widely used.

It is expected to construct incremental algorithms to update information entropies for dynamic data sets whose attribute sets can be changed. When attribute sets change, all equivalence classes can change, so it is more complex.

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