Annals of Fuzzy Mathematics and Informatics
Volume 18, No. 3, (December 2019) pp. 245–271
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr
https://doi.org/10.30948/afmi.2019.18.3.245

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Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 18, No. 3, December 2019

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Neutrosophic soft set and its application in multicriteria decision making problems

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Received 20 February 2019; Revised 15 March 2019; Accepted 31 May 2019

Abstract. In this paper, neutrosophic soft set was studied and an observation made of the potential of its application in real life problems, multicriteria decision making problems in particular. To achieve some of the underlying goals, there is a need to define certain algebraic operations, namely, restricted intersection, extended intersection and restricted union. Some basic properties emerging from the definitions are presented and they include union, AND-product and OR-product operations. Some De Morgan's laws and the concept of inclusions are also established in neutrosophic soft set context. Some examples of the application of neutrosophic soft set in decision making problems using level soft sets of neutrosophic soft sets were presented. Furthermore, the concept of weighted neutrosophic soft set were discussed and applied to multicriteria decision making problems.

2010 AMS Classification: 03E20, 03E75

Keywords: Neutrosophic set, Soft set, Universe set, Parameter, Threshold.

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1. Introduction

The notion of fuzzy set was introduced by Zadeh in 1965 [19]. Since then fuzzy set was considered as the most appropriate mathematical tool for dealing with uncertainty and ambiguity in real life situations. The classical fuzzy set is characterized by membership value. One major setback in fuzzy set is the difficulty in assigning the membership values. As a result, the concept of interval valued fuzzy set [4] was introduced so as to solve the difficulty in assigning the membership values (to capture the uncertainty in the grade of membership values).

In real life applications, such as expert system, belief system, information fusion, medicine, social sciences and so on, one needs to consider the membership and non-membership for proper description of an object in uncertain and imprecise environment. None of the mentioned mathematical tools is appropriate for such situation. Only intuitionistic fuzzy set introduced by Atanassov [3] is appropriate for such situation. The intuitionistic fuzzy sets can deal only with incomplete information considering membership and non-membership values. But it does not handle the indeterminate and inconsistent information which exists in belief system. The concept neutrosophic set was introduced by Smarandache [18] is a mathematical tool for dealing with problems involving imprecise, indeterminate and inconsistent data.

The introduction of soft set theory by Molodtsov [16] has enriched its potentiality in dealing with aforementioned problems. Based on the several operations on soft sets introduced in [2, 13, 14] some more properties and algebra may be found in [1]. Various neutrosophic theory and applications are found in [5, 6, 7, 8, 9].

The aim of this research paper, is to extend the work of Maji in [15] by defining more algebraic operations such as restricted intersection, extended intersection, restricted union and present their various algebraic properties including various De Morgan's laws and Inclusion in neutrosophic soft set context which has indeed enriched the work of Maji on neutrosophic soft set. We also present the application of neutrosophic soft set in multicriteria decision making problems. The method of multicriteria approach adopted is easy to arrive at the optimum decision within a shortest time possible compared to Roy-Maji [14] approach that requires a lot of computations.

2. Preliminary Concepts

2.1. **Neutrosophic set.**

Definition 2.1.1 [18]**.** A Neutrosophic set A on the universe set of discourse X is defined as:

$$A = \left\{ \langle x,\ T_A(x),\ I_A(x),\ F_A(x), \rangle : x \in X \right\},$$

where $T,\ I, F\ : X\ \to]^-0,\ 1^+ [$ and $^-0 \leq T_A(x) +\ I_A(x) +\ F_A(x) \leq 3^+$.

From the philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets $]^-0,\ 1^+ [$. However, in real life applications in scientific and engineering problems, it is difficult to use neutrosophic set with value from real standard or non-standard subset $]^-0,\ 1^+ [$. As a result, we consider the neutrosophic set which takes the value from the subset of $[0,\ 1]$.

Definition 2.1.2 [18]**.** A Neutrosophic set A is contained in another neutrosophic set B, denoted by $A\ \subseteq B$, if $\forall\, x \in X, T_A(x)\ \leq\ T_B(x),\ I_A(x)\ \leq\ I_B(x),\ F_A(x)\ \geq F_B(x)$.

2.2. **Soft Set.**

We first recall some basic notions in soft set theory. Let U be an initial universe set, E be a set of parameters or attributes with respect to U, $P(U)$ be the power set of U and A⊆E.

Definition 2.2.1 [16]**.** A pair $(F,\ A)$ is called a soft set over U, where F is a mapping given by $F: A\ \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $x\in A$, $F(x)$ may be considered as the set of x-elements or as the set of x-approximate elements of the soft set $(F,\ A)$. The soft set $(F,\ A)$ can be represented as a set of ordered pairs as follows:

$$(F,\ A) =\ \{(\ x,\ F(x)),\ x \in A,\ F(x) \in P(U)\}\ .$$

Definition 2.2.2 [14]**.** Let $(F,\ A)$ and (G, B) be two soft sets over U. Then

(i) $(F,\ A)$ is said to be a soft subset of (G, B), denoted by $(F,\ A)\,\widetilde{\subseteq}\,(\mathrm{G},\ \mathrm{B})$, if $A\subseteq \mathrm{B}$ and $F(x) \subseteq \mathrm{G}(x)$, $\forall x\in \mathrm{A}$

(ii) $(F,\ A)$ and $(G,\ B)$ are said to be soft equal, denoted by $(F,\ A) = (\mathrm{G},\ \mathrm{B})$, if $(F,\ A)\,\widetilde{\subseteq}\,(\mathrm{G},\ \mathrm{B})$ and $(\mathrm{G},\ \mathrm{B})\,\widetilde{\subseteq}\,(F,\ A)$.

Definition 2.2.3 [2]**.** Let $(F,\ A)$ be a soft set over U. Then the support of $(F,\ A)$ written supp$(F,\ A)$ is defined by supp$(F,\ A) = \{x\in A : F(x)\neq\varnothing\}$.

(i) $(F,\ A)$ is called a non-null soft set if supp $(F,\ A)\neq\varnothing$.

(ii) $(F,\ A)$ is called a relative null soft set denoted by $\varnothing_A\ if\ F(x) = \varnothing,\ \forall x\in A$

(iii) $(F,\ A)$ is called a relative whole soft set, denoted by $U_A\ if\ F(x) = U,\ \forall x\in A$.

Definition 2.2.4 [11]**.** Let $(F,\ A)$ be a soft set over U. If $F(x)\neq\varnothing$ for all $x\in A$, then $(F,\ A)$ is called a non-empty soft set.

Definition 2.2.5 [2]**.** Let $(F,\ A)$ and $(G,\ B)$ be two soft sets over U. Then the union of $(F,\ A)$ and $(G,\ B)$, denoted by $(F, A)\,\widetilde{\cup}(G,\ B)$ is a soft set defined as: $(F, A)\,\widetilde{\cup}\,(G,\ B) = (H,\ C)$, $where\ C = A\cup B\ and\ \forall x\in C$,

$$H(x) = \begin{cases} F(x), & if\ x\in A-B \\ G(x), & if x\in B-A \\ F(x)\cup\ G(x), & if\ x\in A\cap B. \end{cases}$$

Definition 2.2.6 [11]**.** Let $(F,\ A)$ and $(G,\ B)$ be two soft sets over U. Then the restricted union of $(F,\ A)$ and $(G,\ B)$, denoted by $(\Gamma,\ A)\ \widetilde{\cup}_R\ (G,\ B)$ is a soft set defined as:

$$(F,\ A)\widetilde{\cup}_R\ (G,\ B) = (H,\ C),$$

where $C = A\cap B\ \neq\varnothing$ and $\forall\ x\in C,\ H(x) = F(x)\cup G(x)$.

Definition 2.2.7 [2]**.** Let $(F,\ A)$ and $(G,\ B)$ be two soft sets over U. Then the extended intersection of $(F,\ A)$ and $(G,\ B)$, denoted by $(F,\ A)\widetilde{\cap}_E\ (G,\ B)$, is a soft set defined as: $(F,\ A)\widetilde{\cap}_E\ (G,\ B) = (H,\ C)$, where $C = A\cup B$ and $\forall\ x\in C$,

$$H(x) = \begin{cases} F(x), & if\ x\in A-B \\ G(x), & if x\in B-A \\ F(x)\cap\ G(x), & if\ x\in A\cap B. \end{cases}$$

Definition 2.2.8 [2]**.** Let (F, A) and $(G,\ B)$ be two soft sets over U. Then the restricted intersection of (F, A) and $(G,\ B)$ denoted by $(F, A)\Cap(G,\ B)$, is a soft set defined as: $(F, A) \Cap (G,\ B) = (H,\ C)$, where $C = A \cap B$ and $\forall\ x \in C$, $H(x) = F(x) \cap G\ (x)$.

Definition 2.2.9 [14]**.** Let $(F,\ A)$ and $(G,\ B)$ be two soft sets over U. Then the AND-product or AND-intersection of $(F,\ A)$ and $(G,\ B)$ denoted by $(F,\ A)\widetilde{\bigwedge}$ $(G,\ B)$ is a soft set defined as:

$$(F,\ A)\widetilde{\bigwedge}\ (G,\ B) = (H,\ C)\,,$$

where $C = A \times B$ *and* $\forall\ (x,\ y) \in A \times B$, $H(x,\ y) = F(x) \cap G(y)$.

Definition 2.2.10 [14]**.** Let $(F,\ A)$ and $(G,\ B)$ be two soft sets over U. Then the OR-product or OR-union of $(F,\ A)$ and $(G,\ B)$, denoted by $(F,\ A)\widetilde{\bigvee}\ (G,\ B)$ is a soft set defined as:

$$(F,\ A)\widetilde{\bigvee}\ (G,\ B) = (H, C)\,,$$

where $C = A \times B$ *and* $\forall\ (x, y) \in A \times B$, $H(x,\ y) = F(x) \cup G(y)$.

Definition 2.2.11 [17]**.** Let $(F,\ A)$ and $(G,\ B)$ be two nonempty soft sets over U. The sum $(F,\ A)\,\tilde{+}(G,\ B)$ is define as the soft set $(H,\ C) = (F,\ A)\,\tilde{+}(G,\ B)$, where $C = A \times B$ and $H\,(x, y) =\ F\,(x) + G\,(y)\,,\ \forall\ (x,\ y) \in C$.

3. Neutrosophic soft set and its properties

Definition 3.1 [15]**.** Let U be an initial Universe set and E be a set of parameters. Let $A \subseteq E$ and $P(U)$ denotes the set of all neutrosophic sets of U. The pair (F, A) is termed to be the neutrosophic soft set over U, where F is a mapping given by $F : A \to P(U)$.

For illustration we consider Example 3.1 below.

Example 3.1. Let U be a set of cars Mr X is considering for transportation purchase during his wedding celebration and E is a set of decision parameters. Each parameter is a neutrosophic word or a sentence involving neutrosophic words. Consider

$$E = \{beautiful, costly, very\ costly, cheap, expensive, model\ 2010, black, made\ in\ Japan\}.$$

In this case, to define a neutrosophic soft set means to point out beautiful cars, costly cars, very costly cars, cheap cars and so on. Suppose that there are five cars in the universe U given by $U = \{C_1, C_2, C_3, C_4, C_5\}$ and set of parameters $A = \{a_1, a_2, a_3, a_4\}$, where a_1 stands for the parameter beautiful, a_2 stands for the parameter costly, a_3 stands for the parameter model 2010 and a_4 stands for the parameter made in Japan. Suppose that

$$F\,(beautiful) = \left\{ \begin{array}{c} \langle C_1, 0.6, 0.7, 0.3\rangle, \langle C_2, 0.4, 0.8, 0.5\rangle, \langle C_3, 0.6, 0.3, 0.3\rangle, \\ \langle C_4, 0.8, 0.4, 0.3\rangle, \langle C_5, 0.8, 0.3, 0.4\rangle \end{array} \right\},$$
$$F\,(costly) = \left\{ \begin{array}{c} \langle C_1, 0.7, 0.4, 0.5\rangle, \langle C_2, 0.7, 0.5, 0.4\rangle, \langle C_3, 0.8, 0.2, 0.3\rangle, \\ \langle C_4, 0.7, 0.2, 0.3\rangle, \langle C_5, 0.9, 0.4, 0.6\rangle \end{array} \right\},$$
$$F\,(model\ 2010) = \left\{ \begin{array}{c} \langle C_1, 0.8, 0.5, 0.4\rangle, \langle C_2, 0.6, 0.7, 0.3\rangle, \langle C_3, 0.7, 0.3, 0.5\rangle, \\ \langle C_4, 0.5, 0.3, 0.6\rangle, \langle C_5, 0.7, 0.4, 0.4\rangle \end{array} \right\},$$
$$F\,(made\ in\ Japan) = \left\{ \begin{array}{c} \langle C_1, 0.8, 0.7, 0.5\rangle, \langle C_2, 0.6, 0.8, 0.5\rangle, \langle C_3, 0.7, 0.6, 0.3\rangle, \\ \langle C_4, 0.8, 0.7, 0.6\rangle, \langle C_5, 0.8, 0.6, 0.5\rangle \end{array} \right\}.$$

Then the neutrosophic soft set $(F,\ A)$ is a parameterized family

$$\{F\,(a_i)\,,\ i = 1, 2, 3, \ldots, n\}$$

of all neutrosophic sets of U and describes a collection of approximation of an object. The mapping F here is 'car', where dot (.) is to be filled up by a parameter $a \in E$. Thus $F\,(a_1)$ means 'cars (beautiful)' whose functional-value is the neutrosophic set:

$$\{\langle C_1, 0.6, 0.7, 0.3\rangle, \langle C_2, 0.4, 0.8, 0.5\rangle, \langle C_3, 0.6, 0.3, 0.3\rangle, \langle C_4, 0.8, 0.4, 0.3\rangle, \langle C_5, 0.8, 0.3, 0.4\rangle\}.$$

So we can view the neutrosophic soft set $(F,\ A)$ as a collection of approximation below:

$$(F,\ A) = \left\{ \begin{array}{c} beautiful = \left\{ \begin{array}{c} \langle C_1, 0.6, 0.7, 0.3\rangle, \langle C_2, 0.4, 0.8, 0.5\rangle, \langle C_3, 0.6, 0.3, 0.3\rangle, \\ \langle C_4, 0.8, 0.4, 0.3\rangle, \langle C_5, 0.8, 0.3, 0.4\rangle \end{array} \right\}, \\ costly = \left\{ \begin{array}{c} \langle C_1, 0.7, 0.4, 0.5\rangle, \langle C_2, 0.7, 0.5, 0.4\rangle, \langle C_3, 0.8, 0.2, 0.3\rangle, \\ \langle C_4, 0.7, 0.2, 0.3\rangle, \langle C_5, 0.9, 0.4, 0.6\rangle \end{array} \right\}, \\ Model\ 2010 = \left\{ \begin{array}{c} \langle C_1, 0.8, 0.5, 0.4\rangle, \langle C_2, 0.6, 0.7, 0.3\rangle, \langle C_3, 0.7, 0.3, 0.5\rangle, \\ \langle C_4, 0.5, 0.3, 0.6\rangle, \langle C_5, 0.7, 0.4, 0.4\rangle \end{array} \right\}, \\ made\ in\ Japan = \left\{ \begin{array}{c} \langle C_1, 0.8, 0.7, 0.5\rangle, \langle C_2, 0.6, 0.8, 0.5\rangle, \langle C_3, 0.7, 0.6, 0.3\rangle, \\ \langle C_4, 0.8, 0.7, 0.6\rangle, \langle C_5, 0.8, 0.6, 0.5\rangle \end{array} \right\} \end{array} \right\},$$

where each approximation has two parts:

(1) A predicate p and
(2) An approximate value-set v (or simply referred to as value-set v).

For instance, for the approximation

$beautiful\ cars =$
$\{\langle C_1,\ 0.6,\ 0.7,\ 0.3\rangle, \langle C_2,\ 0.4,\ 0.8,\ 0.5\rangle, \langle C_3,\ 0.6,\ 0.3,\ 0.3\rangle, \langle C_4,\ 0.8,\ 0.4,\ 0.3\rangle,$
$\langle C_5,\ 0.8,\ 0.3,\ 0.4\rangle\}.$

We have (i) predicate name beautiful cars and (ii) the approximate value-set is

$\{\langle C_1,\ 0.6,\ 0.7,\ 0.3\rangle, \langle C_2,\ 0.4,\ 0.8,\ 0.5\rangle, \langle C_3,\ 0.6,\ 0.3,\ 0.3\rangle, \langle C_4,\ 0.8,\ 0.4,\ 0.3\rangle$
$\langle C_5,\ 0.8,\ 0.3,\ 0.4\rangle\}.$

Hence, a neutrosophic soft set $(F,\ A)$ can be viewed as a collection of approximation like $(F,\ A) = \ \{p_1 = v_1,\ p_2 = v_2, \ldots, p_n = v_n\}$.
For the purpose of storing a neutrosophic soft set in a computer, we could represent it in the form of table as shown below (corresponding to the neutrosophic soft set in

Table 3.1. Tabular representation of the neutrosophic soft set $(F,\ A)$ in Example 3.1.

U/A	beautiful	costly	model 2010	made in Japan
C_1	$\langle 0.6, 0.7, 0.3\rangle$	$\langle 0.7, 0.4, 0.5\rangle$	$\langle 0.8, 0.5, 0.4\rangle$	$\langle 0.8, 0.7, 0.5\rangle$
C_2	$\langle 0.4, 0.8, 0.5\rangle$	$\langle 0.7, 0.5, 0.4\rangle$	$\langle 0.6, 0.7, 0.3\rangle$	$\langle 0.6, 0.8, 0.5\rangle$
C_3	$\langle 0.6, 0.3, 0.3\rangle$	$\langle 0.8, 0.2, 0.3\rangle$	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.7, 0.6, 0.3\rangle$
C_4	$\langle 0.8, 0.4, 0.3\rangle$	$\langle 0.7, 0.2, 0.3\rangle$	$\langle 0.5, 0.3, 0.6\rangle$	$\langle 0.8, 0.7, 0.6\rangle$
C_5	$\langle 0.8, 0.3, 0.4\rangle$	$\langle 0.9, 0.4, 0.6\rangle$	$\langle 0.7, 0.4, 0.4\rangle$	$\langle 0.8, 0.6, 0.5\rangle$

Example 3.1). In this table, the entries are c_{ij} corresponding to the car c_i and the parameter a_j, where
c_{ij} = (true-membership value of c_i, indeterminacy-membership value c_i, falsity-membership value of c_i) in $F(a_j)$. The tabular representation of the neutrosophic soft set $(F,\ A)$ is as follows:

Definition 3.2. The class of all value-set of a neutrosophic soft set $(F,\ E)$ is called **value-class** of the neutrosophic soft set and it is denoted by $C_{(F,E)}$. For the Example 3.1, $C_{(F,E)} \subset P(U)$.

Definition 3.3. Let $(F,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over a common universe set U. $(F,\ A)$ is said to be neutrosophic soft subset of $(G,\ B)$, if $A \subset B$ and $T_{F(e)}(x) = T_{G(e)}(x)$,

$$I_{F(e)}(x) = G_{G(e)}(x), F_{F(e)}\left(x\right) = F_{G(e)}\left(x\right),\ \forall\ e \in A,\ x \in U.$$

We denote it mathematically by $(F,\ A)\tilde{\subseteq}(G,\ B)$.

$(F,\ A)$ is said to be neutrosophic soft super set of $(G,\ B)$, if $(G,\ B)$ is a neutrosophic soft subset of $(F,\ A)$. We denote it by $(F,\ A)\tilde{\supseteq}(G,\ B)$.

Example 3.2. Consider the two neutrosophic soft sets $(F,\ A)$ and $(G,\ B)$ over a common universe set $U = \{h_1,\ h_2,\ h_3,\ h_4,\ h_5\}$. The neutrosophic soft set $(F,\ A)$ describes the sizes of the objects and the neutrosophic soft set $(G,\ B)$ describes its surface textures. Let

$$(F,\ A) = \left\{ \begin{array}{c} \textit{small size} = \left\{ \begin{array}{c} \langle h_1, 0.4, 0.5, 0.7\rangle, \langle h_2, 0.5, 0.2, 0.5\rangle, \langle h_3, 0.6, 0.4, 0.8\rangle, \\ \langle h_4, 0.6, 0.3, 0.6\rangle, \langle h_5, 0.4, 0.3, 0.6\rangle \end{array} \right\}, \\ \textit{large size} = \left\{ \begin{array}{c} \langle h_1, 0.5, 0.2, 0.8\rangle, \langle h_2, 0.6, 0.5, 0.8\rangle, \langle h_3, 0.4, 0.3, 0.7\rangle, \\ \langle h_4, 0.2, 0.6, 0.8\rangle, \langle h_5, 0.5, 0.2, 0.7\rangle \end{array} \right\}, \\ \textit{moderate size} = \left\{ \begin{array}{c} \langle h_1, 0.4, 0.6, 0.6\rangle, \langle h_2, 0.6, 0.4, 0.7\rangle, \langle h_3, 0.4, 0.4, 0.9\rangle, \\ \langle h_4, 0.7, 0.5, 0.8\rangle, \langle h_5, 0.4, 0.3, 0.4\rangle \end{array} \right\}, \end{array} \right\}$$

and

$(G,\ B)$

$$= \left\{ \begin{array}{c} small\ texture = \left\{ \begin{array}{c} \langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.6, 0.4, 0.3\rangle, \langle h_3, 0.7, 0.5, 0.4\rangle, \\ \langle h_4, 0.5, 0.6, 0.2\rangle, \langle h_5, 0.6, 0.3, 0.2\rangle \end{array} \right\}, \\ large\ texture = \left\{ \begin{array}{c} \langle h_1, 0.5, 0.4, 0.5\rangle, \langle h_2, 0.6, 0.7, 0.3\rangle, \langle h_3, 0.4, 0.4, 0.5\rangle, \\ \langle h_4, 0.5, 0.3, 0.1\rangle, \langle h_5, 0.6, 0.3, 0.2\rangle \end{array} \right\}, \\ moderate\ texture = \left\{ \begin{array}{c} \langle h_1, 0.6, 0.7, 0.3\rangle, \langle h_2, 0.7, 0.4, 0.5\rangle, \langle h_3, 0.5, 0.6, 0.3\rangle, \\ \langle h_4, 0.5, 0.4, 0.2\rangle, \langle h_5, 0.4, 0.5, 0.4\rangle \end{array} \right\}, \\ very\ smoot\ texture = \left\{ \begin{array}{c} \langle h_1, 0.8, 0.4, 0.3\rangle, \langle h_2, 0.5, 0.6, 0.8\rangle, \langle h_3, 0.8, 0.2, 0.5\rangle, \\ \langle h_4, 0.5, 0.7, 0.2\rangle, \langle h_5, 0.4, 0.1, 0.8\rangle \end{array} \right\} \end{array} \right\}.$$

Obviously, $A \subset B$ and $(F,\ A)\widetilde{\subseteq}(G,\ B)$.

Definition 3.4. (Equality of two neutrosophic soft sets) Let $(F,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over a common universe set U. Then $(F,\ A)$ is said to be equal to $(G,\ B)$, if $(F,\ A)$ is a neutrosophic soft subset of $(G,\ B)$ and $(G,\ B)$ is a neutrosophic soft subset of $(F,\ A)$. We denote it by $(F,\ A) = (G,\ B)$.

Definition 3.5. (NOT set of a set of parameters) Let $E = \{e_1,\ e_2,\ \ldots,\ e_n\}$ be a set of parameters. The NOT set of E is denoted by $\neg E$ is define by $\neg E = \{\neg e_1,\ \neg e_2,\ \ldots,\ \neg e_n\}$, where $\neg e_i = not\ e_i,\ \forall\ i$.

Example 3.3. Consider Example 3.1. Here

$$\neg A = \{not\ beautiful, not\ costly, not\ model\ 2010, not\ made\ in\ Japan\}.$$

Definition 3.6. (Complement of a neutrosophic soft set) The complement of a neutrosophic soft set $(F,\ A)$ denoted by $(F,\ A)^C$ and is defined as $(F,\ A)^C = \left(F^C,\ \neg A\right)$, where $F^C : \neg A \rightarrow P(U)$ is a mapping given by $F^C(\alpha) =$ neutrosophic soft complement with $T_{F^C(x)} = F_{F(x)}$, $I_{F^C(x)} = I_{F(x)}$ and $F_{F^C(x)} = T_{F(x)}$.

Example 3.4. Consider Example 3.1. The $(F,\ A)^C$ describes the not attractiveness of the cars. We have

$$F\,(not\ beautiful) = \left\{ \begin{array}{c} \langle C_1, 0.3, 0.7, 0.6\rangle, \langle C_2, 0.5, 0.8, 0.4\rangle, \langle C_3, 0.3, 0.3, 0.6\rangle, \\ \langle C_4, 0.3, 0.4, 0.8\rangle, \langle C_5, 0.4, 0.3, 0.8\rangle \end{array} \right\},$$

$$F\,(not\ costly) = \left\{ \begin{array}{c} \langle C_1, 0.5, 0.4, 0.7\rangle, \langle C_2, 0.4, 0.5, 0.7\rangle, \langle C_3, 0.3, 0.2, 0.8\rangle, \\ \langle C_4, 03, 0.2, 0.7\rangle, \langle C_5, 0.6, 0.4, 0.9\rangle \end{array} \right\},$$

$$F\,(\ not\ model\ 2010) = \left\{ \begin{array}{c} \langle C_1, 0.4, 0.5, 0.8\rangle, \langle C_2, 0.3, 0.7, 0.6\rangle, \langle C_3, 0.5, 0.3, 0.7\rangle, \\ \langle C_4, 0.6, 0.3, 0.5\rangle, \langle C_5, 0.4, 0.4, 0.7\rangle \end{array} \right\},$$

$$F\,(not\ made\ in\ Japan) = \left\{ \begin{array}{c} \langle C_1, 0.5, 0.7, 0.8\rangle, \langle C_2, 0.5, 0.8, 0.6\rangle, \langle C_3, 0.3, 0.6,\ 0.7\rangle, \\ \langle C_4, 0.6, 0.7, 0.8\rangle, \langle C_5, 0.5, 0.6, 0.8\rangle \end{array} \right\}.$$

Definition 3.7. (Empty or Null neutrosophic soft set with respect to a parameter) A neutrosophic soft set $(F,\ A)$ over the universe U is said to be empty or null neutrosophic soft set with respect to the parameter A, if $T_{F(e)}(x) = 0,\ F_{F(e)}(x) = 0$

and $I_{F(e)}(x) = 0,\ \forall e \in A,\ \forall x \in U$. In this case, the null neutrosophic soft set is denoted by $(F,\ A)_{\varnothing}$.

Example 3.5. Let $U = \{C_1, C_2, C_3, C_4, C_5\}$, the set of five cars be considered as the universe set and $A = \{costly,\ cheap,\ expensive\}$ be the set of parameters that characterizes the cars. Consider the neutrosophic soft set $(F,\ A)$ which describes the attractiveness of the cars and

$$F\,(very\ costly) = \left\{ \begin{array}{c} \langle C_1, 0, 0, 0\rangle, \langle C_2, 0, 0, 0\rangle, \langle C_3, 0, 0, 0\rangle, \\ \langle C_4, 0, 0, 0\rangle, \langle C_5, 0, 0, 0\rangle \end{array} \right\},$$

$$F\,(cheap) = \left\{ \begin{array}{c} \langle C_1, 0, 0, 0\rangle, \langle C_2, 0, 0, 0\rangle, \langle C_3, 0, 0, 0\rangle, \\ \langle C_4, 0, 0, 0\rangle, \langle C_5, 0, 0, 0\rangle \end{array} \right\},$$

$$F\,(\ expensive) = \left\{ \begin{array}{c} \langle C_1, 0, 0, 0\rangle, \langle C_2, 0, 0, 0\rangle, \langle C_3, 0, 0, 0\rangle, \\ \langle C_4, 0, 0, 0\rangle, \langle C_5, 0, 0, 0\rangle \end{array} \right\}.$$

Then the neutrosophic soft set $(F,\ A)$ is a null neutrosophic soft set.

Definition 3.8. (Union of two neutrosophic soft set) Let $(F,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over a common universe set U. The union of $(F,\ A)$ and $(G,\ B)$ denoted by $(F,\ A)\widetilde{\cup}(G,\ B)$ is defined as $(F,\ A)\,\widetilde{\cup}\,(G,\ B) = (H,\ C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and false-membership of $(H,\ C)$ are as follows:

$$\begin{aligned} T_{H(e)}(x) &= T_{F(e)}(x), && if\ e \in A/B\ , \\ &= T_{G(e)}(x) && if\ e \in B/A\ , \\ &= max\left(T_{F(e)}(x), T_{G(e)}(x)\ \right) && if\ e \in A \cap B\ , \end{aligned}$$

$$\begin{aligned} I_{H(e)}(x) &= I_{F(e)}(x), && if\ e \in A/B\ , \\ &= I_{G(e)}(x) && if\ e \in B/A\ , \\ &= \tfrac{I_{F(e)}(x) + I_{G(e)}(x)}{2} && if\ e \in A \cap B\ , \end{aligned}$$

$$\begin{aligned} F_{H(e)}(x) &= F_{F(e)}(x), && if\ e \in A/B\ , \\ &= F_{G(e)}(x) && if\ e \in B/A\ , \\ &= min\left(F_{F(e)}(x), F_{G(e)}(x)\ \right) && if\ e \in A \cap B\ . \end{aligned}$$

Example 3.6. Let $(F,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over a common universe set U given as follows:

$(F,\ A)$

$$= \left\{ \begin{array}{c} beautiful = \left\{ \begin{array}{c} \langle c_1, 0.7, 0.4, 0.7\rangle, \langle c_2, 0.5, 0.6, 0.5\rangle, \langle c_3, 0.8, 0.4, 0.3\rangle, \\ \langle c_4, 0.9, 0.5, 0.4\rangle, \langle c_5, 0.7, 0.8, 0.1\rangle \end{array} \right\}, \\ costly = \left\{ \begin{array}{c} \langle c_1, 0.7, 0.4, 0.5\rangle, \langle c_2, 0.6, 0.8, 0.3\rangle, \langle c_3, 0.6, 0.4, 0.5\rangle, \\ \langle c_4, 0.7, 0.4, 0.6\rangle, \langle c_5, 0.6, 0.4, 0.5\rangle \end{array} \right\}, \\ model\ 2010 = \left\{ \begin{array}{c} \langle c_1, 0.6, 0.4, 0.6\rangle, \langle c_2, 0.7, 0.6, 0.4\rangle, \langle c_3, 0.8, 0.5, 0.6\rangle, \\ \langle c_4, 0.7, 0.6, 0.7\rangle, \langle c_5, 0.8, 0.6, 0.6\rangle \end{array} \right\}, \end{array} \right\},$$

$(G,\ B)$

$$= \left\{ \begin{array}{c} made\ in\ Japan = \left\{ \begin{array}{c} \langle c_1, 0.8, 0.2, 0.5\rangle, \langle c_2, 0.7, 0.4, 0.3\rangle, \langle c_3, 0.6, 0.5, 0.3\rangle, \\ \langle c_4, 0.7, 0.3, 0.1\rangle, \langle c_5, 0.5, 0.6, 0.4\rangle \end{array} \right\}, \\ model\ 2010 = \left\{ \begin{array}{c} \langle c_1, 0.8, 0.6, 0.7\rangle, \langle c_2, 0.6, 0.8, 0.4\rangle, \langle c_3, 0.5, 0.7, 0.5\rangle, \\ \langle c_4, 0.8, 0.4, 0.6\rangle, \langle c_5, 0.6, 0.4, 0.5\rangle \end{array} \right\}, \end{array} \right\}.$$

Then the union of $(F,\ A)$ and $(G,\ B)$ is $(H,\ C)$, where

$(H,\ C)$

$$= \left\{ \begin{array}{c} beautiful = \left\{ \begin{array}{c} \langle C_1, 0.7, 0.4, 0.7\rangle, \langle C_2, 0.5, 0.6, 0.5\rangle, \langle C_3, 0.8, 0.4, 0.3\rangle, \\ \langle C_4, 0.9, 0.5, 0.4\rangle, \langle C_5, 0.7, 0.8, 0.1\rangle \end{array} \right\}, \\ costly = \left\{ \begin{array}{c} \langle C_1, 0.7, 0.4, 0.5\rangle, \langle C_2, 0.6, 0.8, 0.3\rangle, \langle C_3, 0.6, 0.4, 0.5\rangle, \\ \langle C_4, 0.7, 0.4, 0.6\rangle, \langle C_5, 0.6, 0.4, 0.5\rangle \end{array} \right\}, \\ Model\ 2010 = \left\{ \begin{array}{c} \langle C_1, 0.8, 0.5, 0.6\rangle, \langle C_2, 0.7, 0.7, 0.4\rangle, \langle C_3, 0.8, 0.6, 0.5\rangle, \\ \langle C_4, 0.8, 0.5, 0.6\rangle, \langle C_5, 0.8, 0.5, 0.5\rangle \end{array} \right\}, \\ made\ in\ Japan = \left\{ \begin{array}{c} \langle C_1, 0.8, 0.2, 0.5\rangle, \langle C_2, 0.7, 0.4, 0.3\rangle, \langle C_3, 0.6, 0.5, 0.3\rangle, \\ \langle C_4, 0.7, 0.3, 0.1\rangle, \langle C_5, 0.5, 0.6, 0.4\rangle \end{array} \right\} \end{array} \right\}.$$

Definition 3.9. (Restricted intersection of two neutrosophic soft sets) Let $(F,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over a common universe set U. The intersection of $(F,\ A)$ and $(G,\ B)$ denoted by $(F,\ A) \Cap (G,\ B)$ is defined as $(F,\ A) \Cap (G,\ B) = (H,\ C)$, where $C = A \cap B \neq \varnothing$ and the truth-membership, indeterminacy-membership and false-membership of $(H,\ C)$ are as follows: for each $x \in C$,

$$T_{H(e)}(x) = min\left(T_{F(e)}(x), T_{G(e)}(x)\right),$$

$$I_{H(e)}(x) = \frac{I_{F(e)}(x) + I_{G(e)}(x)}{2}$$

and

$$F_{H(e)}(x) = max\left(F_{F(e)}(x), F_{G(e)}(x)\right).$$

Example 3.7. Consider the Example 3.6 above for the union. Then the neutrosophic soft set $(F,\ A) \Cap (G,\ B) = (H,\ C)$, where

$(H,\ C)$

$$= \left\{ Model\ 2010 = \left\{ \begin{array}{c} \langle C_1, 0.6, 0.5, 0.7\rangle, \langle C_2, 0.6, 0.7, 0.4\rangle, \langle C_3, 0.5, 0.6, 0.6\rangle, \\ \langle C_4, 0.7, 0.5, 0.7\rangle, \langle C_5, 0.6, 0.5, 0.6\rangle \end{array} \right\} \right\}.$$

Proposition 3.1. For any two neutrosophic soft sets $(F,\ A)$ and $(G,\ B)$ over the same universe set U, we have the following:

(1) $(F,\ A) \tilde{\cup} (F,\ A) = (F,\ A)$,
(2) $(F,\ A) \tilde{\cup} (G,\ B) = (G,\ B) \tilde{\cup} (F,\ A)$,
(3) $(F,\ A) \tilde{\cup} (F,\ A)_{\varnothing} = (F,\ A)$,
(4) $(F,\ A) \Cap (F,\ A) = (F,\ A)$,
(5) $(F,\ A) \Cap (G,\ B) = (G,\ B) \Cap (F,\ A)$,

(6) $(F,\ A)\Cap(F,\ A)_{\varnothing} = (F,\ A)_{\varnothing}$,

(7) $\left[(F,\ A)^{C}\right]^{C} = (F, A)$.

Proof. The proofs are obvious. □

Proposition 3.2. Let $(F,\ A)$, $(G,\ B)$ and $(H,\ C)$ be three neutrosophic soft sets over a common universe set U, then

(1) $(F,\ A)\,\widetilde{\cup}\,\big((G,\ B)\,\widetilde{\cup}\,(H,\ C)\big) = \big((F,\ A)\,\widetilde{\cup}\,(G,\ B)\big)\,\widetilde{\cup}\,(H,\ C)$,
(2) $(F,\ A)\Cap((G,\ B)\Cap(H,\ C)) = ((F,\ A)\Cap(G,\ B))\Cap(H,\ C)$,
(3) $(F,\ A)\,\widetilde{\cup}\,((G,\ B)\Cap(H,\ C)) = \big((F,\ A)\,\widetilde{\cup}\,(G,\ B)\big)\Cap\big((F,\ A)\,\widetilde{\cup}\,(H,\ C)\big)$,
(4) $(F,\ A)\Cap\big((G,\ B)\,\widetilde{\cup}\,(H,\ C)\big) = ((F,\ A)\Cap(G,\ B))\,\widetilde{\cup}\,((F,\ A)\Cap(H,\ C))$.

Proof. The proofs are obvious. □

Definition 3.10. (AND-product operation of two neutrosophic soft sets) Let $(F,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over a common universe set U. The AND-product of $(F,\ A)$ and $(G,\ B)$ denoted by $(F,\ A)\widetilde{\bigwedge}(G,\ B)$ is defined as $(F,\ A)\widetilde{\bigwedge}(G,\ B) = (H,\ C)$, where $C = A \times B$ and the truth-membership, indeterminacy-membership and false-membership of $(H,\ C)$ are as follows:

$$T_{H(e_1,e_2)}(x) = min\left(T_{F(e_1)}(x), T_{G(e_2)}(x)\right),$$
$$I_{H(e_1,e_2)}(x) = \frac{I_{F(e_1)}(x)+I_{G(e_2)}(x)}{2} \text{ and}$$
$$F_{H(e_1,e_2)}(x) = max\left(F_{F(e_1)}(x), F_{G(e_2)}(x)\right), \quad \forall\ e_1 \in A,\ \forall\ e_2 \in B.$$

Example 3.8. Consider Example 3.6. Then $(F,\ A)\widetilde{\bigwedge}(G,\ B) = (H,\ C)$. Let $a_1 =$ beautiful, $a_2 =$ *costly*, $a_3 =$ *model* 2010, and $b_1 =$ *made in Japan*, $b_2 =$ *model* 2010. Then

$$(H,\ C) = \left\{\begin{array}{l}
(a_1,\ b_1) = \left\{\begin{array}{c}\langle C_1, 0.7, 0.3, 0.7\rangle, \langle C_2, 0.5, 0.5, 0.5\rangle, \langle C_3, 0.6, 0.45, 0.3\rangle,\\ \langle C_4, 0.7, 0.4, 0.4\rangle, \langle C_5, 0.5, 0.7, 0.4\rangle\end{array}\right\},\\
(a_1,\ b_2) = \left\{\begin{array}{c}\langle C_1, 0.7, 0.5, 0.7\rangle, \langle C_2, 0.5, 0.7, 0.5\rangle, \langle C_3, 0.5, 0.55, 0.5\rangle,\\ \langle C_4, 0.8, 0.45, 0.6\rangle, \langle C_5, 0.6, 0.6, 0.5\rangle\end{array}\right\},\\
(a_2,\ b_1) = \left\{\begin{array}{c}\langle C_1, 0.7, 0.3, 0.5\rangle, \langle C_2, 0.6, 0.6, 0.3\rangle, \langle C_3, 0.6, 0.45, 0.5\rangle,\\ \langle C_4, 0.7, 0.35, 0.6\rangle, \langle C_5, 0.5, 0.5, 0.5\rangle\end{array}\right\},\\
(a_2,\ b_2) = \left\{\begin{array}{c}\langle C_1, 0.7, 0.5, 0.7\rangle, \langle C_2, 0.6, 0.8, 0.4\rangle, \langle C_3, 0.5, 0.55, 0.5\rangle,\\ \langle C_4, 0.7, 0.4, 0.6\rangle, \langle C_5, 0.6, 0.4, 0.5\rangle\end{array}\right\},\\
(a_3,\ b_1) = \left\{\begin{array}{c}\langle C_1, 0.6, 0.3, 0.6\rangle, \langle C_2, 0.7, 0.5, 0.4\rangle, \langle C_3, 0.6, 0.5, 0.6\rangle,\\ \langle C_4, 0.7, 0.45, 0.7\rangle, \langle C_5, 0.5, 0.6, 0.6\rangle\end{array}\right\},\\
(a_3,\ b_2) = \left\{\begin{array}{c}\langle C_1, 0.6, 0.5, 0.7\rangle, \langle C_2, 0.6, 0.7, 0.4\rangle, \langle C_3, 0.5, 0.6, 0.6\rangle,\\ \langle C_4, 0.7, 0.5, 0.7\rangle, \langle C_5, 0.6, 0.5, 0.6\rangle\end{array}\right\}
\end{array}\right\}.$$

Definition 3.11. (OR-product operation of two neutrosophic soft sets) Let $(F,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over a common universe set U. The OR-product of $(F,\ A)$ and $(G,\ B)$ denoted by $(F,\ A)\widetilde{\bigvee}(G,\ B)$ is defined as $(F,\ A)\widetilde{\bigvee}(G,\ B) = (J,\ C)$, where $C = A \times B$ and the truth-membership, indeterminacy-membership and false-membership of $(H,\ C)$ are as follows:

$$T_{J(e_1,e_2)}(x) = max\left(T_{F(e_1)}(x), T_{G(e_2)}(x)\right),$$

$I_{J(e_1,e_2)}(x) = \frac{I_{F(e_1)}(x)+I_{G(e_2)}(x)}{2}$ and
$F_{J(e_1,e_2)}(x) = min\left(F_{F(e_1)}(x), F_{G(e_2)}(x)\right), \quad \forall\ e_1 \in A,\ \forall\ e_2 \in B.$

Example 3.9. Consider Example 3.6. Then $(F,\ A)\widetilde{\bigvee}(G,\ B) = (J,\ C)$. Let $a_1 =$ beautiful$a_2 =\ costly, a_3 = model$ 2010 and $b_1 = made\ in\ Japan, b_2 = model$ 2010. Then

$$(J,\ C) = \left\{\begin{array}{l} (a_1,\ b_1) = \left\{\begin{array}{c}\langle C_1, 0.8, 0.3, 0.5\rangle, \langle C_2, 0.7, 0.5, 0.3\rangle, \langle C_3, 0.8, 0.45, 0.3\rangle, \\ \langle C_4, 0.9, 0.4, 0.1\rangle, \langle C_5, 0.7, 0.7, 0.1\rangle\end{array}\right\}, \\ (a_1,\ b_2) = \left\{\begin{array}{c}\langle C_1, 0.8, 0.5, 0.7\rangle, \langle C_2, 0.6, 0.7, 0.4\rangle, \langle C_3, 0.8, 0.55, 0.3\rangle, \\ \langle C_4, 0.9, 0.45, 0.4\rangle, \langle C_5, 0.7, 0.6, 0.1\rangle\end{array}\right\}, \\ (a_2,\ b_1) = \left\{\begin{array}{c}\langle C_1, 0.8, 0.3, 0.5\rangle, \langle C_2, 0.7, 0.6, 0.3\rangle, \langle C_3, 0.6, 0.45, 0.3\rangle, \\ \langle C_4, 0.7, 0.35, 0.1\rangle, \langle C_5, 0.6, 0.5, 0.4\rangle\end{array}\right\}, \\ (a_2,\ b_2) = \left\{\begin{array}{c}\langle C_1, 0.8, 0.5, 0.5\rangle, \langle C_2, 0.6, 0.8, 0.3\rangle, \langle C_3, 0.6, 0.55, 0.5\rangle, \\ \langle C_4, 0.8, 0.4, 0.6\rangle, \langle C_5, 0.6, 0.4, 0.5\rangle\end{array}\right\}, \\ (a_3,\ b_1) = \left\{\begin{array}{c}\langle C_1, 0.8, 0.3, 0.5\rangle, \langle C_2, 0.7, 0.5, 0.3\rangle, \langle C_3, 0.8, 0.5, 0.3\rangle, \\ \langle C_4, 0.7, 0.45, 0.1\rangle, \langle C_5, 0.8, 0.6, 0.4\rangle\end{array}\right\}, \\ (a_3,\ b_2) = \left\{\begin{array}{c}\langle C_1, 0.8, 0.5, 0.6\rangle, \langle C_2, 0.7, 0.7, 0.4\rangle, \langle C_3, 0.8, 0.6, 0.5\rangle, \\ \langle C_4, 0.8, 0.5, 0.6\rangle, \langle C_5, 0.8, 0.5, 0.5\rangle\end{array}\right\} \end{array}\right\}.$$

Proposition 3.3. Let $(F,\ A), (G,\ B)$ *and* $(H,\ C)$ be three neutrosophic soft over a common universe set U. Then

(1) $(F,\ A)\widetilde{\bigwedge}\left((G,\ B)\widetilde{\bigwedge}(H,\ C)\right) = \left((F,\ A)\widetilde{\bigwedge}(G,\ B)\right)\widetilde{\bigwedge}(H,\ C)$,
(2) $(F,\ A)\widetilde{\bigvee}\left((G,\ B)\widetilde{\bigvee}(H,\ C)\right) = \left((F,\ A)\widetilde{\bigvee}(G,\ B)\right)\widetilde{\bigvee}(H,\ C)$,
(3) $(F,\ A)\widetilde{\bigwedge}(F,\ A) = (F,\ A)$.
(4) $(F,\ A)\widetilde{\bigvee}(F,\ A) = (F,\ A)$

Proof. (1) By Definition 3.10,

$$(F,\ A)\widetilde{\bigwedge}\left((G,\ B)\widetilde{\bigwedge}(H,\ C)\right) = (F,\ A)\widetilde{\bigwedge}(G,\ B \times C) = (N, A \times\ B \times C),$$

where for all $(b,\ c) \in B \times C$, $M(b,c) =\ G(b) \cap H(c)$ and for all $(a,\ b,\ c) \in A \times B \times C$, $N(a,\ b,\ c) = F(a) \cap\ M(b,c) =\ F(a) \cap (G(b) \cap H(c)) =\ (F(a) \cap G(b)) \cap H(c) = Q(a,b) \cap H(c)$ with $Q(a,b) =\ F(a) \cap G(b)$. Then

$$(Q,\ A \times B)\widetilde{\bigwedge}(H,\ C) = \left((G,\ B)\widetilde{\bigwedge}(H,\ C)\right)\widetilde{\bigwedge}(H,\ C).$$

Thus (1) holds.

(2) The proof is similar to proof of (1).

The proofs of (3) and (4) are straightforward. □

Definition 3.12. (Extended intersection of two neutrosophic soft set) Let $(F,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over a common universe set U. The extended intersection of $(F,\ A)$ and $(G,\ B)$ denoted by $(F,\ A)\widetilde{\cap}_E(G,\ B)$ is defined as $(F,\ A)\widetilde{\cap}_E(G,\ B) = (W,\ C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and false-membership of $(W,\ C)$ are as follows:

$$\begin{aligned} T_{W(e)}(x) &= T_{F(e)}(x), & if\ e \in A/B \\ &= T_{G(e)}(x) & if\ e \in B/A \\ &= min\left(T_{F(e)}(x), T_{G(e)}(x)\right) & if\ e \in A \cap B, \\ I_{W(e)}(x) &= I_{F(e)}(x), & if\ e \in A/B \\ &= I_{G(e)}(x) & if\ e \in B/A \\ &= \frac{I_{F(e)}(x)+I_{G(e)}(x)}{2} & if\ e \in A \cap B, \\ F_{W(e)}(x) &= F_{F(e)}(x), & if\ e \in A/B \\ &= F_{G(e)}(x) & if\ e \in B/A \\ &= max\left(F_{F(e)}(x), F_{G(e)}(x)\right) & if\ e \in A \cap B. \end{aligned}$$

Example 3.10. Consider Example 3.6. Then the extended intersection of $(F,\ A)$ and $(G,\ B)$ is $(W,\ C)$,

$$(W,\ C) = \left\{ \begin{array}{c} beautiful = \left\{ \begin{array}{c} \langle C_1, 0.7, 0.4, 0.7\rangle, \langle C_2, 0.5, 0.6, 0.5\rangle, \langle C_3, 0.8, 0.4, 0.3\rangle, \\ \langle C_4, 0.9, 0.5, 0.4\rangle, \langle C_5, 0.7, 0.8, 0.1\rangle \end{array} \right\}, \\ costly = \left\{ \begin{array}{c} \langle C_1, 0.7, 0.4, 0.5\rangle, \langle C_2, 0.6, 0.8, 0.3\rangle, \langle C_3, 0.6, 0.4, 0.5\rangle, \\ \langle C_4, 0.7, 0.4, 0.6\rangle, \langle C_5, 0.6, 0.4, 0.5\rangle \end{array} \right\}, \\ Model\ 2010 = \left\{ \begin{array}{c} \langle C_1, 0.6, 0.5, 0.7\rangle, \langle C_2, 0.6, 0.7, 0.4\rangle, \langle C_3, 0.5, 0.6, 0.6\rangle, \\ \langle C_4, 0.7, 0.5, 0.7\rangle, \langle C_5, 0.6, 0.5, 0.6\rangle \end{array} \right\}, \\ made\ in\ Japan = \left\{ \begin{array}{c} \langle C_1, 0.8, 0.2, 0.5\rangle, \langle C_2, 0.7, 0.4, 0.3\rangle, \langle C_3, 0.6, 0.5, 0.3\rangle, \\ \langle C_4, 0.7, 0.3, 0.1\rangle, \langle C_5, 0.5, 0.6, 0.4\rangle \end{array} \right\} \end{array} \right\}.$$

Definition 3.13. (Restricted union of two neutrosophic soft sets) Let $(F,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over a common universe set U. The restricted union of $(F,\ A)$ and $(G,\ B)$ denoted by $(F,\ A)\cup_{\mathrm{R}}(G,\ B)$ is defined as $(F,\ A)\cup_{\mathrm{R}}(G,\ B) = (V,\ C)$, where $C = A \cap B \neq \varnothing$ and the truth-membership, indeterminacy-membership and false-membership of $(V,\ C)$ are as follows:

$$T_{V(e)}(x) = max\left(T_{F(e)}(x), T_{G(e)}(x)\right),$$
$$I_{V(e)}(x) = \frac{I_{F(e)}(x)+I_{G(e)}(x)}{2} \text{ and}$$
$$F_{V(e)}(x) = min\left(F_{F(e)}(x), F_{G(e)}(x)\right), \quad \forall\ e \in C.$$

Example 3.11. Consider Example 3.6. Then $(F,\ A)\cup_{\mathrm{R}}(G,\ B) = (V,\ C)$, where

$$(V,\ C) = \left\{ Model\ 2010 = \left\{ \begin{array}{c} \langle C_1, 0.8, 0.5, 0.6\rangle, \langle C_2, 0.7, 0.7, 0.4\rangle, \langle C_3, 0.8, 0.6, 0.5\rangle, \\ \langle C_4, 0.8, 0.5, 0.6\rangle, \langle C_5, 0.8, 0.5, 0.5\rangle \end{array} \right\} \right\}.$$

Proposition 3.4. Suppose that $(F,\ A)$, $(G,\ B)$ and $(H,\ C)$ are three neutrosophic soft sets over the same universe set U. Then

(1) $(F,\ A)\cup_{\mathrm{R}}(F,\ A) = (F,\ A)$,
(2) $(F,\ A)\cup_{\mathrm{R}}((G,\ B)\cup_{\mathrm{R}}(H,\ C)) = ((F,\ A)\cup_{\mathrm{R}}(G,\ B))\cup_{\mathrm{R}}(H,\ C)$,
(3) $(F,\ A)\cup_{\mathrm{R}}(G,\ B)_{\varnothing} = (F,\ A)$,
(4) $(F,\ A)\cup_{\mathrm{R}}(G,\ B)_{\varnothing} = \left\{ \begin{array}{ll} (F,\ A), & if\ A = B \\ (R,\ D), & otherwise \end{array} \right.$ where $D = A \cap B$,
(5) $(F,\ A)\widetilde{\cap}_E(F,\ A) = (F,\ A)$,
(6) $(F,\ A)\widetilde{\cap}_E\left((G,\ B)\widetilde{\cap}_E(H,\ C)\right) = \left((F,\ A)\widetilde{\cap}_E(G,\ B)\right)\widetilde{\cap}_E(H,\ C)$,
(7) $(F,\ A)\widetilde{\cap}_E(G,\ B)_{\varnothing} = (F,\ A)$.

Proof. The proofs are straightforward. □

Theorem 3.1. Let $(F,\ A)$, $(G,\ B)$ and $(H,\ C)$ be neutrosophic soft sets over a common universe set U. such that $\left(\hat{H},\ C\right) \widetilde{\subseteq} \left(\hat{F},\ A\right)$. Then

1. $(F,\ A)\, \widetilde{\cap}_E \left((G,\ B)\, \widetilde{\cup}\ \left(\hat{H},\ C\right)\right) \widetilde{\subseteq} \left((F,\ A)\, \widetilde{\cap}_E\ (G,\ B)\right) \widetilde{\cup}\, (H,\ C)$,
2. $(F,\ A)\, \widetilde{\cap}_E \left((G,\ B)\, \widetilde{\cup}\ \left(\hat{H},\ C\right)\right) = \left((F,\ A)\, \widetilde{\cap}_E\ (G,\ B)\right) \widetilde{\cup}\, (H,\ C)$, if $A \subseteq B$.

Proof. The proofs are straightforward. □

Theorem 3.2. If $(F,\ A)$, $(G,\ B)$ and $(H,\ C)$ are three neutrosophic soft sets over a common universe set U. Then

1. $(F,\ A)\, \widetilde{\cup}_R\, ((G,\ B) \Cap (H,\ C)) = \left((F,\ A)\, \widetilde{\cup}_R\ (G,\ B)\right) \Cap \left((F,\ A)\, \widetilde{\cup}_R\ (H,\ C)\right)$,
2. $(F,\ A) \Cap \left((G,\ B)\, \widetilde{\cup}_R\ (H,\ C)\right) = ((F,\ A) \Cap (G,\ B))\, \widetilde{\cup}_R\ ((F,\ A) \Cap (H,\ C))$,

Proof. The proofs are straightforward. □

Theorem 3.3. Let $(F,\ A)$ and $(G,\ B)$ be two neutrosophic soft sets over U. Then the following holds.

1. $(F,\ A) \Cap (G,\ B)$ is a neutrosophic soft set over U, if it is non-null,
2. $(F,\ A)\, \widetilde{\cap}_E\, (G,\ B)$ is a neutrosophic soft set over U, if it is non-null,
3. $(F,\ A) \widetilde{\cup}_R\ (G,\ B)$ is a neutrosophic soft set over U, whenever, it is non-null and if $F\,(x)$ and $G\,(x)$ are ordered by inclusion relation for all
$$x \in supp((F,\ A)\, \widetilde{\cup}_R\ (G,\ B)),$$
4. $(F,\ A)\, \widetilde{\bigwedge}\, (G,\ B)$ is a neutrosophic soft set over U, if it is non-null,
5. $(F,\ A)\, \widetilde{\cup}(G,\ B)$ is a neutrosophic soft set over U, if it is non-null and if A and B are disjoint,
6. $(F,\ A) \bigvee (G,\ B) = (N,\ A \times B)$ is a neutrosophic soft set over U, if it is non-null and if $F\,(x)$ and $G\,(y)$ are ordered by inclusion relation for all
$$(x,\ y) \in supp(N,\ A \times B),$$
7. $(F,\ A)\, \tilde{+}\, (G,\ B)$ is a neutrosophic soft set over U, if it is non-null.

Proof. (1) Let $(F,\ A) \Cap (G,\ B) = (K,\ C)$, where $K\,(x) = F(x) \cap G(x)$, for all $x \in C = A \cap B\ \neq \varnothing$. By hypothesis, $(K,\ C)$ is a non-null neutrosophic soft set over U. If $x \in supp(K,\ C)$, then $K\,(x) = F(x) \cap G(x) \neq \varnothing$. It follows that $F(x) \neq \varnothing$ and $G(x) \neq \varnothing$ are both neutrosophic set over U. Thus $K\,(x)$ is a neutrosophic set over U, for all $x \in supp(K,\ C)$. So $(K,\ C)$ is a neutrosophic soft set over U.

(2) Let $(F,\ A)\, \widetilde{\cap}_E\, (G,\ B) = (M,\ A \cup B)$, where

$$M\,(x) = \begin{cases} F\,(x)\,, & if\ x \in A - B \\ G\,(x)\,, & if x \in B - A \\ F\,(x) \cap G\,(x)\,, & if\ x \in A \cap B, \end{cases}$$

for all $x \in A \cup B$. Then by the hypothesis, $(M,\ A \cup B)$ is a non-null neutrosophic soft set over U. Let $x \in supp\,(M,\ A \cup B)$. If $x \in A - B$, then $\varnothing \neq M\,(x) = F\,(x)$. If $x \in B - A$, then $\varnothing \neq M\,(x) = G\,(x)$, and if $x \in A \cap B$, then $M\,(x) =$

$F(x) \cap G(x) \neq \varnothing$. Since $F(x) \neq \varnothing$ and $G(x) \neq \varnothing$ are both neutrosophic set over U, $M(x)$ is a neutrosophic set over U, for all $x \in supp(M,\ A \cup B)$. Thus $(F,\ A)\widetilde{\cap}_E(G,\ B) = (M,\ A \cup B)$ is a neutrosophic soft set over U.

(3) Let $(F,\ A)\widetilde{\cup}_R\ (G,\ B) = (R,\ A \cap B)$, where $R(x) = F(x) \cup G(x)$, for all $x \in A \cap B \neq \varnothing$. Then by hypothesis, $(R,\ A \cap B)$ is a non-null neutrosophic soft set over U. if $x \in supp(R,\ A \cap B)$, then $R(x) = F(x) \cup G(x) \neq \varnothing$. Since, $F(x)$ and $G(x)$ are ordered by inclusion relation for all $x \in supp(R,\ A \cap B)$, $F(x) \cup G(x) = F(x)$ or $F(x) \cup G(x) = G(x)$. Since $F(x) \neq \varnothing$ and $G(x) \neq \varnothing$ are both neutrosophic set over U, $R(x)$ is a neutrosophic set over U, for all $x \in supp(R,\ A \cap B)$. Thus $(R,\ A \cap B)$ is a neutrosophic soft set over U.

(4) Let $(F,\ A)\widetilde{\bigwedge}(G,\ B) = (Q,\ A \times B)$, where $Q(x,y) = F(x) \cap G(y)$, for all $(x,y) \in A \times B$. Then by hypothesis, $(Q,\ A \times B)$ is a non-null neutrosophic soft set over U. If $(x,\ y) \in supp(Q,\ A \times B)$, then $Q(x,\ y) = F(x) \cap G(y) \neq \varnothing$. It follows that $F(x) \neq \varnothing$ and $G(y) \neq \varnothing$ are both neutrosophic set over U. Thus $Q(x,\ y)$ is a neutrosophic set over U for all $(x,\ y) \in supp(Q,\ A \times B)$. So $(F,\ A)\widetilde{\bigwedge}(G,\ B)$ is a neutrosophic soft set over U.

(5) Let $(F,\ A)\widetilde{\cup}(G,\ B) = (V,\ A \cup B)$, where

$$V(x) = \begin{cases} F(x), & if\ x \in A - B \\ G(x), & if x \in B - A \\ F(x) \cup G(x), & if\ x \in A \cap B. \end{cases}$$

For all $x \in A \cup B$ and $A \cap B = \varnothing$, it follows that either $x \in A - B$ or $x \in B - A$, for all $x \in A \cup B$. If $x \in A - B$, then $V(x) = F(x)$ is a neutrosophic set over U and if $x \in B - A$, then $V(x) = G(x)$ is a neutrosophic set over U. Thus $(F,\ A)\widetilde{\cup}(G,\ B)$ is a neutrosophic soft set over U.

(6) Let $(F,\ A)\widetilde{\bigvee}(G,\ B) = (N,\ A \times B)$, where $N(x,\ y) = F(x) \cup G(y)$, for all $(x,\ y) \in A \times B$. Then by hypothesis, $(N,\ A \times B)$ is a non-null neutrosophic soft set over U. If $(x,\ y) \in supp(N,\ A \times B)$, then $N(x,\ y) = F(x) \cup G(y) \neq \varnothing$. Since $F(x)$ and $G(y)$ are ordered by inclusion relation for all $(x,\ y) \in supp(N,\ A \times B)$, $F(x) \cup G(y) = F(x)$ or $F(x) \cup G(y) = G(y)$. It is clear that $F(x) \neq \varnothing$ and $G(y) \neq \varnothing$ are both neutrosophic set over U for all $(x,\ y) \in supp(N,\ A \times B)$. Thus $(F,\ A)\widetilde{\bigvee}(G,\ B)$ is a neutrosophic soft set over U.

(7) $(F,\ A)\tilde{+}(G,\ B) = (H,\ A \times B)$, where $H(x,\ y) = F(x) + G(y)$, for all $(x,y) \in A \times B$. Then by the hypothesis, $(H,\ A \times B)$ is a non-null neutrosophic soft set over U. Suppose $(x,\ y) \in supp(H,\ A \times B)$. Then $H(x,\ y) = F(x) + G(y) \neq \varnothing$. It means that $F(x) \neq \varnothing$ and $G(y) \neq \varnothing$ are both neutrosophic set over U. Thus $H(x,\ y)$ is an neutrosophic set over U for all $(x,\ y) \in supp(H,\ A \times B)$. So $(F,\ A)\tilde{+}(G,\ B)$ is a neutrosophic soft set over U. □

4. De Morgan's laws and inclusions

Theorem 4.1. *Suppose* $(F,\ A)$ *and* $(G,\ B)$ *are two neutrosophic soft sets over a common universe set* U*. Then*

(1) $\left((F,\ A)\widetilde{\cup}_R(G,\ B)\right)^C = (F,\ A)^C \Cap (G,\ B)^C$,

(2) $\left((F,\ A) \Cap (G,\ B)\right)^C = (F,\ A)^C \widetilde{\cup}_R (G,\ B)^C$.

Proof. (1) Let $(F,\ A) = \left\{\left\langle c,\ T_{F(x)}(c), I_{F(x)}(c),\ F_{F(x)}(c)\right\rangle : c \in U\right\}$ and

$(G,\ B) = \left\{\langle c,\ T_{G(x)}(c), I_{G(x)}(c),\ F_{G(x)}(c)\rangle : c \in U\right\}$ be two neutrosophic soft sets over the common universe set U. Also, let $(F,\ A)\widetilde{\cup}_R(G,\ B) = (V,\ C)$, where $C = A \cap B \neq \varnothing$ and

$$V(e) = \left\{\left\langle \begin{array}{c} c, max\left(T_{F(e)}(c),\ T_{G(e)}(c)\right),\ \frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}, \\ min(F_{F(e)}(c),\ F_{G(e)}(c)) \end{array} \right\rangle : c \in U\right\}.$$

Then

$$\left((F,\ A)\widetilde{\cup}_R(G,\ B)\right)^C = (V,\ C)^C$$
$$= \left\{\left\langle \begin{array}{c} c, min\left(F(c),\ F_{G(e)}(c)\right),\ \frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}, \\ max(T_{F(e)}(c),\ F_{G(e)}(c)) \end{array} \right\rangle : c \in U\right\} \quad \textbf{(4.1)}$$

Again,

$$(F,\ A)^C \Cap (G,\ B)^C$$
$$= \left\{\left\langle \begin{array}{c} c, min(F_{F^c(e)}(c), F_{F^c(e)}(c)),\ \frac{I_{F^c(e)}(c)+I_{F^c(e)}(c)}{2}, \\ max(T_{F^c(e)}(c), T_{F^c(e)}(c)) \end{array} \right\rangle : c \in U\right\}$$
$$= \left\{\left\langle \begin{array}{c} c, max\left(T_{F(e)}(c),\ T_{G(e)}(c)\right),\ \frac{I_{F(e)}(c)+I_{G(e)}(c)}{2}, \\ min(F_{F(e)}(c),\ F_{G(e)}(c)) \end{array} \right\rangle : c \in U\right\}^C$$
$$= \left\{\left\langle \begin{array}{c} c, min\left(F_{F(e)}(c),\ F_{G(e)}(c)\right),\ \frac{I_{F(e)}(c),+I_{G(e)}(c)}{2}, \\ max(T_{F(e)}(c),\ T_{G(e)}(c)) \end{array} \right\rangle : c \in U\right\}. \quad \textbf{(4.2)}$$

It is obvious that (4.1) = (4.2). Then (1) holds.

(2) Let $(F,\ A) \Cap (G,\ B) = (H,\ C)$, where $C = A \cap B \neq \varnothing$ and for all $e \in C$. Then we have

$$H(e) = \left\{\left\langle \begin{array}{c} c, min\left(T_{F(e)}(c),\ T_{G(e)}(c)\right),\ \frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}, \\ max(F_{F(e)}(c),\ F_{G(e)}(c)) \end{array} \right\rangle : c \in U\right\}.$$

Thus

$$\left((F,\ A) \Cap (G,\ B)\right)^C = (H,\ C)^C$$
$$= \left\{\left\langle \begin{array}{c} c, max\left(F_{F(e)}(c),\ F_{G(e)}(c)\right),\ \frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}, \\ min(T_{F(e)}(c),\ T_{G(e)}(c)) \end{array} \right\rangle : c \in U\right\} \quad \textbf{(4.3)}$$

Again,

$$(F,\ A)^C \widetilde{\cup}_R (G,\ B)^C$$
$$= \left\{\left\langle \begin{array}{c} c, max(F_{F^c(e)}(c), F_{F^c(e)}(c)),\ \frac{I_{F^c(e)}(c)+I_{F^c(e)}(c)}{2}, \\ min(T_{F^c(e)}(c), T_{F^c(e)}(c)) \end{array} \right\rangle : c \in U\right\}$$
$$= \left\{\left\langle \begin{array}{c} c, min\left(T_{F(e)}(c),\ T_{G(e)}(c)\right),\ \frac{I_{F(e)}(c)+I_{G(e)}(c)}{2}, \\ max(F_{F(e)}(c),\ F_{G(e)}(c)) \end{array} \right\rangle : c \in U\right\}^C$$
$$= \left\{\left\langle \begin{array}{c} c, max\left(F_{F(e)}(c),\ F_{G(e)}(c)\right),\ \frac{I_{F(e)}(c)+I_{G(e)}(c)}{2}, \\ min(T_{F(e)}(c),\ T_{G(e)}(c)) \end{array} \right\rangle : c \in U\right\}. \quad \textbf{(4.4)}$$

It is clear that (4.3) = (4.4). Then (2) holds. □

Theorem 4.2. *Suppose* $(F,\ A)$ *and* $(G,\ B)$ *are two neutrosophic soft sets over a common universe set* U*. Then*

(1) $\left((F,\ A)\widetilde{\cup}\ (G,\ B)\right)^C = (F,\ A)^C \widetilde{\cap}_E (G,\ B)^C$,

(2) $\left((F,\ A)\widetilde{\cap}_E(G,\ B)\right)^C = (F,\ A)^C \widetilde{\cup} (G,\ B)^C$.

Proof. (1) Let $(F,\ A)\,\widetilde{\cup}\ (G,\ B) = (K,\ C)$

$$= \left\{ \begin{array}{c} \langle \text{c},\ \left(max\left(T_{F(e)}\left(c\right),\ T_{G(e)}\left(c\right)\ if\ e \in A\cap B\right)\ \right), T_{F(e)}\left(c\right)\ if\ e \in A/B, \\ T_{G(e)}\left(c\right)\ if\ e \in B/A), (\frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}\ \ if\ e \in A\cap B, I_{F(e)}\left(c\right) \\ if\ e \in A/B, I_{G(e)}\left(c\right)\ if\ e \in B/A), (min\left(F_{F(e)}\left(c\right),\ F_{G(e)}\left(c\right)\ if\ e \in A\cap B\right) \\ F_{F(e)}\left(c\right)\ if\ e \in A/B, F_{G(e)}\left(c\right)\ if\ e \in B/A)\rangle\text{: c}\in U \end{array} \right\}.$$

Then

$$\left((F,\ A)\,\widetilde{\cup}\ (G,\ B)\right)^C = \ (K,\ C)^C$$

$$= \left\{ \begin{array}{c} \langle \text{c},\ \left(min\left(F_{F(e)}\left(c\right),\ F_{G(e)}\left(c\right)\ if\ e \in A\cap B\right)\ \right), F_{F(e)}\left(c\right)\ if\ e \in A/B, \\ F_{G(e)}\left(c\right)\ if\ e \in B/A), (\frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}\ \ if\ e \in A\cap B, I_{F(e)}\left(c\right) \\ if\ e \in A/B, I_{G(e)}\left(c\right)\ if\ e \in B/A), (max\left(T_{F(e)}\left(c\right),\ T_{G(e)}\left(c\right)\ if\ e \in A\cap B\right) \\ T_{F(e)}\left(c\right)\ if\ e \in A/B, T_{G(e)}\left(c\right)\ if\ e \in B/A)\rangle\text{: c}\in U \end{array} \right\} \quad \textbf{(4.5)}$$

and

$$(F,\ A)^C\widetilde{\cap}_E(G,\ B)^{\text{C}}$$

$$= \left\{ \begin{array}{c} \langle \text{c},\ \left(min\left(F_{F^c(e)}\left(c\right), F_{G^c(e)}\left(c\right)\ if\ e \in A\cap B\right)\ \right), F_{F^c(e)}\left(c\right)\ if\ e \in A/B, \\ F_{G^c(e)}\left(c\right)\ if\ e \in B/A), (\ \frac{I_{F^c(e)}(\text{c})+I_{G^c(e)}(\text{c})}{2}\ \ if\ e \in A\cap B, I_{F^c(e)}\left(\text{c}\right) \\ if\ e \in A/B, I_{G^c(e)}\left(\text{c}\right)\ if\ e \in B/A), (max\left(T\left(c\right), T_{G^c(e)}\left(c\right)\ if\ e \in A\cap B\right) \\ T_{F^c(e)}\left(c\right)\ if\ e \in A/B, T_{G^c(e)}\left(c\right)\ if\ e \in B/A)\rangle\text{: c}\in U \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \langle \text{c},\ \left(max\left(T_{F(e)}\left(c\right),\ T_{G(e)}\left(c\right)\ if\ e \in A\cap B\right)\ \right), T\left(c\right)\ if\ e \in A/B, \\ T_{G(e)}\left(c\right)\ if\ e \in B/A), (\frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}\ \ if\ e \in A\cap B, I_{F(e)}\left(c\right) \\ if\ e \in A/B, I_{G(e)}\left(c\right)\ if\ e \in B/A), (min\left(F_{F(e)}\left(c\right),\ F_{G(e)}\left(c\right)\ if\ e \in A\cap B\right) \\ F_{F(e)}\left(c\right)\ if\ e \in A/B, F_{G(e)}\left(c\right)\ if\ e \in B/A)\rangle\text{: c}\in U \end{array} \right\}^C$$

$$= \left\{ \begin{array}{c} \langle \text{c},\ \left(min\left(F_{F(e)}\left(c\right),\ F_{G(e)}\left(c\right)\ if\ e \in A\cap B\right)\ \right), F_{F(e)}\left(c\right)\ if\ e \in A/B, \\ F_{G(e)}\left(c\right)\ if\ e \in B/A), (\frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}\ \ if\ e \in A\cap B, I_{F(e)}\left(c\right) \\ if\ e \in A/B, I_{G(e)}\left(c\right)\ if\ e \in B/A), (max\left(T_{F(e)}\left(c\right),\ T_{G(e)}\left(c\right)\ if\ e \in A\cap B\right) \\ T_{F(e)}\left(c\right)\ if\ e \in A/B, T_{G(e)}\left(c\right)\ if\ e \in B/A)\rangle\text{: c}\in U \end{array} \right\} \quad \textbf{(4.6)}$$

It is obvious that (4.5) = (4.6). Then (1) holds.

(2) Let $(F,\ A)\,\widetilde{\cap}_E\,(G,\ B) = (K,\ C)$

$$= \left\{ \begin{array}{c} \langle \text{c},\ \left(min\left(T_{F(e)}\left(c\right),\ T_{G(e)}\left(c\right)\ if\ e \in A\cap B\right)\ \right), T_{F(e)}\left(c\right)\ if\ e \in A/B, \\ T_{G(e)}\left(c\right)\ if\ e \in B/A), (\frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}\ \ if\ e \in A\cap B, I_{F(e)}\left(c\right) \\ if\ e \in A/B, I_{G(e)}\left(c\right)\ if\ e \in B/A), (max\left(F_{F(e)}\left(c\right),\ F_{G(e)}\left(c\right)\ if\ e \in A\cap B\right) \\ F_{F(e)}\left(c\right)\ if\ e \in A/B, F_{G(e)}\left(c\right)\ if\ e \in B/A)\rangle\text{: c}\in U \end{array} \right\}.$$

Then

$$\left((F,\ A)\,\widetilde{\cap}_E\ (G,\ B)\right)^C = \ (K,\ C)^C$$

$$= \left\{ \begin{array}{c} \langle \text{c},\ \left(max\left(F_{F(e)}\left(c\right),\ F_{G(e)}\left(c\right)\ if\ e \in A\cap B\right)\ \right), F_{F(e)}\left(c\right)\ if\ e \in A/B, \\ F_{G(e)}\left(c\right)\ if\ e \in B/A), (\frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}\ \ if\ e \in A\cap B, I_{F(e)}\left(c\right) \\ if\ e \in A/B, I_{G(e)}\left(c\right)\ if\ e \in B/A), (min\left(T_{F(e)}\left(c\right),\ T_{G(e)}\left(c\right)\ if\ e \in A\cap B\right) \\ T_{F(e)}\left(c\right)\ if\ e \in A/B, T_{G(e)}\left(c\right)\ if\ e \in B/A)\rangle\text{: c}\in U \end{array} \right\} \quad \textbf{(4.7)}$$

and

$$(F,\ A)^C\widetilde{\cup}(G,\ B)^{\text{C}}$$

$$= \left\{ \begin{array}{c} \langle \text{c},\ \left(max\left(F_{F^c(e)}\left(c\right), F_{G^c(e)}\left(c\right)\ if\ e \in A\cap B\right)\ \right), F_{F^c(e)}\left(c\right)\ if\ e \in A/B, \\ F_{G^c(e)}\left(c\right)\ if\ e \in B/A), (\ \frac{I_{F^c(e)}(\text{c})+I_{G^c(e)}(\text{c})}{2}\ \ if\ e \in A\cap B, I_{F^c(e)}\left(\text{c}\right) \\ if\ e \in A/B, I_{G^c(e)}\left(\text{c}\right)\ if\ e \in B/A), (min\left(T\left(c\right), T_{G^c(e)}\left(c\right)\ if\ e \in A\cap B\right) \\ T_{F^c(e)}\left(c\right)\ if\ e \in A/B, T_{G^c(e)}\left(c\right)\ if\ e \in B/A)\rangle\text{: c}\in U \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \langle \text{c}, \ \left(min\left(T_{F(e)}\left(c\right), \ T_{G(e)}\left(c\right) \ if \ e \in A \cap B\right)\ \right), T_{F(e)}\left(c\right) \ if \ e \in A/B, \\ T_{G(e)}\left(c\right) \ if \ e \in B/A), (\frac{I_{F(e)}(c)+ \ I_{G(e)}(c)}{2} \ \ if \ e \in A \cap B, I_{F(e)}\left(c\right) \\ if \ e \in A/B, I_{G(e)}\left(c\right) \ if \ e \in B/A), (max\left(F_{F(e)}\left(c\right), \ F_{G(e)}\left(c\right) \ if \ e \in A \cap B\right) \\ F_{F(e)}\left(c\right) \ if \ e \in A/B, F_{G(e)}\left(c\right) \ if \ e \in B/A)\rangle \text{: c} \in U \end{array} \right\}^{C}$$

$$= \left\{ \begin{array}{c} \langle \text{c}, \ \left(max\left(F_{F(e)}\left(c\right), \ F_{G(e)}\left(c\right) \ if \ e \in A \cap B\right)\ \right), F_{F(e)}\left(c\right) \ if \ e \in A/B, \\ F_{G(e)}\left(c\right) \ if \ e \in B/A), (\frac{I_{F(e)}(c)+ \ I_{G(e)}(c)}{2} \ \ if \ e \in A \cap B, I_{F(e)}\left(c\right) \\ if \ e \in A/B, I_{G(e)}\left(c\right) \ if \ e \in B/A), (min\left(T_{F(e)}\left(c\right), \ T_{G(e)}\left(c\right) \ if \ e \in A \cap B\right) \\ T_{F(e)}\left(c\right) \ if \ e \in A/B, T_{G(e)}\left(c\right) \ if \ e \in B/A)\rangle \text{: c} \in U \end{array} \right\} \quad \textbf{(4.8)}$$

It is clear that (4.7) = (4.8). Then (2) holds. □

Theorem 4.3. *Suppose $(F,\ A)$ and $(G,\ B)$ are two neutrosophic soft sets over a common universe set U. Then*

(1) $((F,\ A) \Cap \ (G,\ B))^{C} \widetilde{\subseteq} (F,\ A)^{C} \Cap (G,\ B)^{\text{C}}$,

(2) $(F,\ A)^{C} \widetilde{\cup} (G,\ B)^{\text{C}} \widetilde{\subseteq} \ ((F,\ A) \widetilde{\cup} (G,\ B))^{C}$.

Proof. (1) Let $(F,\ A) \Cap \ (G,\ B) = (H,\ C)$, where $C = A \cap B \neq \varnothing$ and for all $e \in C$. Then we have

$$H\left(e\right) = \left\{ \left\langle \begin{array}{c} c, min\left(T_{F(e)}\left(c\right), \ T_{G(e)}\left(c\right)\right), \ \frac{I_{F(e)}(c)+ \ I_{G(e)}(c)}{2}, \\ max(F_{F(e)}\left(c\right), \ F_{G(e)}\left(c\right)) \end{array} \right\rangle : c \in U \right\}.$$

Thus

$$((F,\ A) \Cap \ (G,\ B))^{C} = \ (H,\ C)^{C}$$
$$= \left\{ \left\langle \begin{array}{c} c, max\left(F_{F(e)}\left(c\right), \ F_{G(e)}\left(c\right)\right), \ \frac{I_{F(e)}(c)+ \ I_{G(e)}(c)}{2}, \\ min(T_{F(e)}\left(c\right), \ T_{G(e)}\left(c\right)) \end{array} \right\rangle : c \in U \right\}. \quad \textbf{(4.9)}$$

and

$$(F,\ A)^{C} \Cap (G,\ B)^{\text{C}}$$
$$= \left\{ \left\langle \begin{array}{c} \text{c}, min(F_{F^{c}(e)}\left(\text{c}\right), F_{F^{c}(e)}\left(\text{c}\right)), \ \frac{I_{F^{c}(e)}(\text{c})+I_{F^{c}(e)}(\text{c})}{2}, \\ max(T_{F^{c}(e)}\left(\text{c}\right), T_{F^{c}(e)}\left(\text{c}\right)) \end{array} \right\rangle : c \in U \right\}$$
$$= \left\{ \left\langle \begin{array}{c} c, max\left(T_{F(e)}\left(c\right), \ T_{G(e)}\left(c\right)\right), \ \frac{I_{F(e)}(c)+I_{G(e)}(c)}{2}, \\ min(F_{F(e)}\left(c\right), \ F_{G(e)}\left(c\right)) \end{array} \right\rangle : c \in U \right\}^{C}$$
$$= \left\{ \left\langle \begin{array}{c} c, min\left(F_{F(e)}\left(c\right), \ F_{G(e)}\left(c\right)\right), \ \frac{I_{F(e)}(c)+I_{G(e)}(c)}{2}, \\ max(T_{F(e)}\left(c\right), \ T_{G(e)}\left(c\right)) \end{array} \right\rangle : c \in U \right\}. \quad \textbf{(4.10)}$$

It is clear that (4.9) $\widetilde{\subseteq}$ (4.10). Then (1) holds.

(2) $(F,\ A)^{C} \widetilde{\cup} (G,\ B)^{\text{C}}$

$$= \left\{ \begin{array}{c} \langle \text{c}, \ \left(max\left(F_{F^{c}(e)}\left(c\right), F_{G^{c}(e)}\left(c\right) \ if \ e \in A \cap B\right)\ \right), F_{F^{c}(e)}\left(c\right) \ if \ e \in A/B, \\ F_{G^{c}(e)}\left(c\right) \ if \ e \in B/A), (\ \frac{I_{F^{c}(e)}(c)+I_{G^{c}(e)}(c)}{2} \ \ if \ e \in A \cap B, I_{F^{c}(e)}\left(c\right) \\ if \ e \in A/B, I_{G^{c}(e)}\left(c\right) \ if \ e \in B/A), (min\left(T\left(c\right), T_{G^{c}(e)}\left(c\right) \ if \ e \in A \cap B\right) \\ T_{F^{c}(e)}\left(c\right) \ if \ e \in A/B, T_{G^{c}(e)}\left(c\right) \ if \ e \in B/A)\rangle \text{: c} \in U \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \langle \text{c}, \ \left(min\left(T_{F(e)}\left(c\right), \ T_{G(e)}\left(c\right) \ if \ e \in A \cap B\right)\ \right), T_{F(e)}\left(c\right) \ if \ e \in A/B, \\ T_{G(e)}\left(c\right) \ if \ e \in B/A), (\frac{I_{F(e)}(c)+ \ I_{G(e)}(c)}{2} \ \ if \ e \in A \cap B, I_{F(e)}\left(c\right) \\ if \ e \in A/B, I_{G(e)}\left(c\right) \ if \ e \in B/A), (max\left(F_{F(e)}\left(c\right), \ F_{G(e)}\left(c\right) \ if \ e \in A \cap B\right) \\ F_{F(e)}\left(c\right) \ if \ e \in A/B, F_{G(e)}\left(c\right) \ if \ e \in B/A)\rangle \text{: c} \in U \end{array} \right\}^{C}$$

$$= \left\{ \begin{array}{c} \langle \text{c}, \ \left(max \left(F_{F(e)}(c), \ F_{G(e)}(c) \ if \ e \in A \cap B \right) \right), F_{F(e)}(c) \ if \ e \in A/B, \\ F_{G(e)}(c) \ if \ e \in B/A), (\frac{I_{F(e)}(c) + \ I_{G(e)}(c)}{2} \ \ if \ e \in A \cap B, I_{F(e)}(c) \\ if \ e \in A/B, I_{G(e)}(c) \ if \ e \in B/A), (min \left(T_{F(e)}(c), \ T_{G(e)}(c) \ if \ e \in A \cap B \right) \\ T_{F(e)}(c) \ if \ e \in A/B, T_{G(e)}(c) \ if \ e \in B/A) \rangle \text{: c} \in U \end{array} \right\} \quad \textbf{(4.11)}$$

Let $(F, \ A) \widetilde{\cup} \ (G, \ B) = (K, \ C)$

$$= \left\{ \begin{array}{c} \langle \text{c}, \ \left(max \left(T_{F(e)}(c), \ T_{G(e)}(c) \ if \ e \in A \cap B \right) \right), T_{F(e)}(c) \ if \ e \in A/B, \\ T_{G(e)}(c) \ if \ e \in B/A), (\frac{I_{F(e)}(c) + \ I_{G(e)}(c)}{2} \ \ if \ e \in A \cap B, I_{F(e)}(c) \\ if \ e \in A/B, I_{G(e)}(c) \ if \ e \in B/A), (min \left(F_{F(e)}(c), \ F_{G(e)}(c) \ if \ e \in A \cap B \right) \\ F_{F(e)}(c) \ if \ e \in A/B, F_{G(e)}(c) \ if \ e \in B/A) \rangle \text{: c} \in U \end{array} \right\}^{C}$$

Then

$$\left((F, \ A) \widetilde{\cup} \ (G, \ B) \right)^{C} = \ (K, \ C)^{C}$$

$$= \left\{ \begin{array}{c} \langle \text{c}, \ \left(min \left(F_{F(e)}(c), \ F_{G(e)}(c) \ if \ e \in A \cap B \right) \right), F_{F(e)}(c) \ if \ e \in A/B, \\ F_{G(e)}(c) \ if \ e \in B/A), (\frac{I_{F(e)}(c) + \ I_{G(e)}(c)}{2} \ \ if \ e \in A \cap B, I_{F(e)}(c) \\ if \ e \in A/B, I_{G(e)}(c) \ if \ e \in B/A), (max \left(T_{F(e)}(c), \ T_{G(e)}(c) \ if \ e \in A \cap B \right) \\ T_{F(e)}(c) \ if \ e \in A/B, T_{G(e)}(c) \ if \ e \in B/A) \rangle \text{: c} \in U \end{array} \right\} \quad \textbf{(4.12)}$$

It is obvious that (4.11) $\widetilde{\subseteq}$ (4.12). Then (2) holds. □

Theorem 4.4. *Suppose* $(F, \ A)$ *and* $(G, \ B)$ *are two neutrosophic soft sets over a common universe set* U*. Then*

(1) $(F, \ A)^{C} \Cap (G, \ B)^{\text{C}} \widetilde{\subseteq} \left((F, \ A) \widetilde{\cup} \ (G, \ B) \right)^{C}$,

(2) $\left((F, \ A) \Cap (G, \ B) \right)^{C} \widetilde{\subseteq} \ (F, \ A)^{C} \widetilde{\cup} (G, \ B)^{\text{C}}$.

Proof. (1) $(F, \ A)^{C} \Cap (G, \ B)^{\text{C}}$

$$= \left\{ \left\langle \begin{array}{c} \text{c}, min \left(F_{F^{\text{c}}(e)}(\text{c}), F_{F^{\text{c}}(e)}(\text{c}) \right), \ \frac{I_{F^{\text{c}}(e)}(\text{c}) + I_{F^{\text{c}}(e)}(\text{c})}{2}, \\ max(T_{F^{\text{c}}(e)}(\text{c}), T_{F^{\text{c}}(e)}(\text{c})) \end{array} \right\rangle : c \in U \right\}$$

$$= \left\{ \left\langle \begin{array}{c} c, max \left(T_{F(e)}(c), \ T_{G(e)}(c) \right), \ \frac{I_{F(e)}(c) + I_{G(e)}(c)}{2}, \\ min(F_{F(e)}(c), \ F_{G(e)}(c)) \end{array} \right\rangle : c \in U \right\}^{C}$$

$$= \left\{ \left\langle \begin{array}{c} c, min \left(F_{F(e)}(c), \ F_{G(e)}(c) \right), \ \frac{I_{F(e)}(c) + I_{G(e)}(c)}{2}, \\ max(T_{F(e)}(c), \ T_{G(e)}(c)) \end{array} \right\rangle : c \in U \right\} \quad \textbf{(4.13)}$$

Let $(F, \ A) \widetilde{\cup} \ (G, \ B) = (K, \ C)$

$$= \left\{ \begin{array}{c} \langle \text{c}, \ \left(max \left(T_{F(e)}(c), \ T_{G(e)}(c) \ if \ e \in A \cap B \right) \right), T_{F(e)}(c) \ if \ e \in A/B, \\ T_{G(e)}(c) \ if \ e \in B/A), (\frac{I_{F(e)}(c) + \ I_{G(e)}(c)}{2} \ \ if \ e \in A \cap B, I_{F(e)}(c) \\ if \ e \in A/B, I_{G(e)}(c) \ if \ e \in B/A), (min \left(F_{F(e)}(c), \ F_{G(e)}(c) \ if \ e \in A \cap B \right) \\ F_{F(e)}(c) \ if \ e \in A/B, F_{G(e)}(c) \ if \ e \in B/A) \rangle \text{: c} \in U \end{array} \right\}$$

Then

$$\left((F, \ A) \widetilde{\cup} \ (G, \ B) \right)^{C} = \ (K, \ C)^{C}$$

$$= \left\{ \begin{array}{c} \langle \text{c}, \ \left(min \left(F_{F(e)}(c), \ F_{G(e)}(c) \ if \ e \in A \cap B \right) \right), F_{F(e)}(c) \ if \ e \in A/B, \\ F_{G(e)}(c) \ if \ e \in B/A), (\frac{I_{F(e)}(c) + \ I_{G(e)}(c)}{2} \ \ if \ e \in A \cap B, I_{F(e)}(c) \\ if \ e \in A/B, I_{G(e)}(c) \ if \ e \in B/A), (max \left(T_{F(e)}(c), \ T_{G(e)}(c) \ if \ e \in A \cap B \right) \\ T_{F(e)}(c) \ if \ e \in A/B, T_{G(e)}(c) \ if \ e \in B/A) \rangle \text{: c} \in U \end{array} \right\} \quad \textbf{(4.14)}$$

It is obvious that (4.13) $\widetilde{\subseteq}$ (4.14). Then (1) holds.

(2) Let $(F,\ A) \Cap (G,\ B) = (H,\ C)$, where $C = A \cap B \neq \varnothing$ and for all $e \in C$. Then we have

$$H(e) = \left\{ \left\langle \begin{array}{c} c, min\left(T_{F(e)}(c),\ T_{G(e)}(c)\right),\ \frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}, \\ max(F_{F(e)}(c),\ F_{G(e)}(c)) \end{array} \right\rangle : c \in U \right\}.$$

Thus

$$((F,\ A) \Cap (G,\ B))^C = (H,\ C)^C$$

$$= \left\{ \left\langle \begin{array}{c} c, max\left(F_{F(e)}(c),\ F_{G(e)}(c)\right),\ \frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}, \\ min(T_{F(e)}(c),\ T_{G(e)}(c)) \end{array} \right\rangle : c \in U \right\} \quad \textbf{(4.15)}$$

and

$$(F,\ A)^C \widetilde{\cup} (G,\ B)^C$$

$$= \left\{ \begin{array}{c} \langle \mathrm{c},\ \left(max\left(F_{F^c(e)}(c), F_{G^c(e)}(c)\ if\ e \in A \cap B\right)\right), F_{F^c(e)}(c)\ if\ e \in A/B, \\ F_{G^c(e)}(c)\ if\ e \in B/A), \left(\frac{I_{F^c(e)}(\mathrm{c})+I_{G^c(e)}(\mathrm{c})}{2}\ if\ e \in A \cap B, I_{F^c(e)}(\mathrm{c})\right. \\ if\ e \in A/B, I_{G^c(e)}(c)\ if\ e \in B/A), (min\left(T(c), T_{G^c(e)}(c)\ if\ e \in A \cap B\right) \\ T_{F^c(e)}(c)\ if\ e \in A/B, T_{G^c(e)}(c)\ if\ e \in B/A)\rangle\mathrm{: c}{\in}U \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \langle \mathrm{c},\ \left(min\left(T_{F(e)}(c),\ T_{G(e)}(c)\ if\ e \in A \cap B\right)\right), F_{F(e)}(c)\ if\ e \in A/B, \\ F_{G(e)}(c)\ if\ e \in B/A), \left(\frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}\ if\ e \in A \cap B, I_{F(e)}(c)\right. \\ if\ e \in A/B, I_{G(e)}(c)\ if\ e \in B/A), (max\left(F_{F(e)}(c),\ F_{G(e)}(c)\ if\ e \in A \cap B\right) \\ F_{F(e)}(c)\ if\ e \in A/B, F_{G(e)}(c)\ if\ e \in B/A)\rangle\mathrm{: c}{\in}U \end{array} \right\}^C$$

$$= \left\{ \begin{array}{c} \langle \mathrm{c},\ \left(max\left(F_{F(e)}(c),\ F_{G(e)}(c)\ if\ e \in A \cap B\right)\right), F_{F(e)}(c)\ if\ e \in A/B, \\ F_{G(e)}(c)\ if\ e \in B/A), \left(\frac{I_{F(e)}(c)+\ I_{G(e)}(c)}{2}\ if\ e \in A \cap B, I_{F(e)}(c)\right. \\ if\ e \in A/B, I_{G(e)}(c)\ if\ e \in B/A), (min\left(T_{F(e)}(c),\ T_{G(e)}(c)\ if\ e \in A \cap B\right) \\ T_{F(e)}(c)\ if\ e \in A/B, T_{G(e)}(c)\ if\ e \in B/A)\rangle\mathrm{: c}{\in}U \end{array} \right\} \textbf{(4.16)}$$

It is clear that (4.15) $\widetilde{\subseteq}$ (4.16). Then (2) holds. □

5. Neutrosophic soft set based decision making

Like most of the decision making problems, neutrosophic soft set based decision making involves the evaluation of all the objects which are decision options. Most of these problems are essentially humanistic and therefore subjective in nature (that is based on human understanding and ability to see). In general, there actually does not exist a uniform criterion for the evaluation of decision options.

5.1. **Level soft sets of neutrosophic soft set.**

In this section, we present an approach to neutrosophic soft set based decision making problems. This is based on the following concept called level soft set.

Definition 5.1.1. Let $\Psi = (F, A)$ be a neutrosophic soft set over U, where $A \subseteq E$ and E is a set of decision parameters. For $s, t,\ v \in [0,\ 1]$, the $(\boldsymbol{s}, \boldsymbol{t}, \boldsymbol{v}) - \boldsymbol{level}$ soft set of Ψ is a crisp soft set $L(\Psi; s, t, v) = \langle F_{(s,\ t,v)},\ A\rangle$ defined by: for all $a \in A$,

$$F_{(s,\ t,v)}(a) = L(F(a); s,\ t, v)$$
$$= \left\{ x \in U : \mu_{F(a)}(x) \geq s,\ \Omega_{F(a)}(x) \geq t\ and\ \lambda_{\hat{F}(a)}(x) \leq v \right\}.$$

This definition is clearly an extension of level soft sets of fuzzy soft sets. That is, $s \in [0,\ 1]$ and $t \in [0,\ 1]$ can be viewed as a given **least threshold** on membership values, indeterminate values respectively and $v \in [0,\ 1]$ can be seen as a given **greatest threshold** on non-membership values. In a real-life application of neutrosophic soft sets based decision making, normally the thresholds are predetermined by the decision maker(s) and represent their requirements on truth-membership levels, indeterminate-membership levels and falsity-membership levels.

For $(\boldsymbol{s}, \boldsymbol{t}, \boldsymbol{v})$-level soft sets, let us consider Example 3.1 and $\Psi = (\ F,\ A)$ with tabular representation in Table 2. It is obvious that the $(\boldsymbol{s}, \boldsymbol{t}, \boldsymbol{v})$-threshold of $\Psi = (F,\ (A)$ is neutrosophic set:
Now, let us take $s = 0.6$, $t = 0.3$ and $v = 0.3$, then we obtain the following:

$$L\left(F\left(a_1\right); 0.7,\ 0.3,\ 0.3\right) = \{c_1, c_3, c_4\},$$
$$L\left(F\left(a_2\right); 0.7,\ 0.3,\ 0.3\right) = \{\ \},$$
$$L\left(F\left(a_3\right); 0.7,\ 0.3, 0.3\right) = \{c_2\},$$
$$L\left(F\left(a_4\right); 0.7,\ 0.3, 0.3\right) = \{c_3\}.$$

Then the $(0.7,\ 0.3,\ 0.3)$ $-$level soft set $\Psi = (\ F,\ A)$ is a soft set $L\left(\Psi\mathbf{;}0.7,\ 0.3,\ 0.3\right) = \left\langle F_{(0.7, 0.3,\ 0.3)},\ A\right\rangle$, where the set- valued mapping $F_{(0.7,0.3,\ 0.3)} : A\ \rightarrow P(U)$ is defined by:

$$F_{(0.7,0.3,\ 0.3)}\left(a_i\right) = L(F\left(a_i\right); 0.7,\ 0.3,\ 0.3) \text{ for } i = 1,\ 2,\ 3,\ 4.$$

Then Table 2 gives the tabular representation of the $(0.7, 0.3, 0.3)$-level soft set $L\left(\Psi\mathbf{;}0.7, 0.3, 0.3\right)$ with choice value.

Table 2: Tabular representation of the (s, t, v)-level soft set $L\left(\Psi;0.7,\ 0.3,\ 0.3\right)$ with choice value

U/A	a_1	a_2	a_3	a_4	*Choice value* (c_i)
c_1	1	0	0	0	1
c_2	0	0	1	0	1
c_3	1	0	0	1	2
c_4	1	0	0	0	1
c_5	0	0	0	0	0

From Table 2, the maximum choice value is $c_3 = 2$ and this corresponds to car c_3.

Decision: The optimum decision for Mr X is to select car c_3 for transportation for his wedding celebration.

Definition 5.1.2. Let $\Psi = (\ F, A)$ be a neutrosophic soft set over U, where $A \subseteq E$ and E is a set of decision parameters. Let $h : A\ \rightarrow [0,\ 1] \times [0,\ 1] \times [0,\ 1]$ be a neutrosophic set in A which is called a Threshold neutrosophic set. The level soft set of Ψ with respect to h is a crisp soft set $L\left(\Psi, h\right) = (F_h,\ A)$ defined by
$F_h\left(a\right) = \ L\left(F\left(a\right);\ h\left(a\right)\right) = \ \left\{x \in U :\ \mu_{F(a)}\left(x\right) \geq\ \mu_h\left(a\right),\ \Omega_{F(a)}\left(x\right) \geq \Omega_h(a) \text{ and } \lambda_{F(a)}\left(x\right) \leq \lambda_h(a)\right\}$ for all $a \in A$. Clearly, the level soft sets of neutrosophic soft sets

with respect to a neutrosophic set are extensions of the level soft set.

Definition 5.1.3. (The mid-level soft set of a neutrosophic soft set) Let $\Psi = (F,\ A)$ be a neutrosophic soft set over a universe set U, where $A \subseteq E$ and E is a set of decision parameters. Based on the neutrosophic soft set $\Psi = (F,\ A)$, we can define a neutrosophic set $mid_\Psi : A \rightarrow [0,\ 1] \times [0,\ 1] \times [0,\ 1]$ define by: for all $a \in A$'

$$\mu_{mid\Psi}(a) = \frac{1}{|U|} \sum_{x \in U} \mu_{F(a)}(x),$$

$$\Omega_{mid\Psi}(a) = \frac{1}{|U|} \sum_{x \in U} \Omega_{F(a)}(x)$$

and

$$\lambda_{mid\Psi}(a) = \frac{1}{|U|} \sum_{x \in U} \lambda_{F(a)}(x).$$

Then the neutrosophic set mid_Ψ is called the mid-threshold of the neutrosophic soft set Ψ. In addition, the level soft set of Ψ with respect to the mid-threshold neutrosophic set mid_Ψ, namely $L(\Psi,\ mid_\Psi)$ is called the mid-level soft set of Ψ and is represented simply by $L(\Psi; mid)$. In what follows the mid-level decision rule will mean using the mid-threshold and considering the mid-level soft set in neutrosophic soft sets based decision making.

Algorithm 1

(1) Input the neutrosophic soft set $\Psi = (F,\ A)$.
(2) Input a threshold neutrosophic set $h : A \rightarrow [0,\ 1] \times [0,\ 1] \times [0,\ 1]$ (or give a threshold value triple $(s,\ t, v) \in [0,1] \times [0,\ 1] \times [0,\ 1]$; or choose a mid-level decision rule; or choose the triple-top-level decision rule; or choose the triple-bottom-level decision rule) for decision making.
(3) Compute the level soft set L(h;h) with respect to the threshold neutrosophic set h (or the (s, t, v) $-$level soft set $L(\Psi; s, t, v)$; or the mid-level soft set $L(\Psi; mid)$; or the triple-top-level soft set $L(\Psi; tripletop)$ or the triple-bottom-level soft set $L(\Psi; triplebottom)$).
(4) Present the level soft set $L(\Psi; h)$ (or $L(\Psi; s,\ t, v)$; $L(\Psi; mid)$; $or L(\Psi; tripletop)$; $or\ L(\Psi; triplebottom)$) in tabular form and compute the choice value c_i of o_i, for all i.
(5) The optimal decision is to select o_k if $c_k = max_i c_i$.
(6) If k has more than one value then any one of o_k may be chosen.

For mid-level soft sets, let us consider Example 3.1 and $\Psi = (F,\ A)$ with tabular representation in Table 3. It is obvious that, the mid-threshold of $(F,\ A)$ is neutrosophic set:

$$mid_{(F,\ A)} = \left\{ \begin{array}{c} \langle a_1,\ 0.64,\ 0.5,\ 0.36\rangle, \langle a_2,\ 0.76,\ 0.34,\ 0.42\rangle, \langle a_3,\ 0.66,\ 0.44,\ 0.44\rangle, \\ \langle a_4,\ 0.74,\ 0.6,\ 0.48\rangle \end{array} \right\}.$$

Table 3: Tabular representation of the mid-level soft set $L((F,\ A);mid)$ with choice value

U/A	a_1	a_2	a_3	a_4	*Choice value* (c_i)
c_1	0	0	1	0	1
c_2	0	0	0	0	0
c_3	0	0	0	0	0
c_4	0	0	0	0	0
c_5	0	0	0	0	0

From Table 3, it is clear that the maximum choice value is $c_1 = 1$ and this corresponds to car c_1.

Decision: The optimum decision for Mr X is to select car c_1 for transportation for his wedding celebration.

Definition 5.1.4. (The Triple-Top-level soft set and Triple-Bottom-level soft set of a neutrosophic soft set) Let $\Psi = (\ F,\ A)$ be a neutrosophic soft set over the universe set U, where $A \subseteq E$ and E is a set of decision parameters. Based on the neutrosophic soft set $\Psi = (\ F,\ A)$, we can define a neutrosophic set $\boldsymbol{tripletop}_{\Psi}$: $A \ \to [0,\ 1] \times [0,\ 1] \times [0,\ 1]$ by: for all $a \in A$,

$$\mu_{tripletop\ \Psi}(a) = \max_{x \in U} \mu_{F(a)}(x)\ ,$$

$$\Omega_{tripletop\Psi}(a) = \max_{x \in U} \Omega_{F(a)}(x)$$

and

$$\lambda_{tripletop\ \Psi}(a) = \max_{x \in U} \lambda_{F(a)}\ (x).$$

For triple-top-level soft sets, let us consider Example 3.1 and $\Psi = (\ F,\ A)$ with tabular representation in Table 4. It is obvious that, the triple-top-threshold of $(F,\ A)$ is neutrosophic set:

$$tripletop_{(F,\ A)} = \left\{ \begin{array}{c} \langle a_1, 0.8, 0.8, 0.5\rangle, \langle a_2, 0.9, 0.5, 0.6\rangle, \langle a_3, 0.8, 0.7, 0.6\rangle, \\ \langle a_4, 0.8, 0.8, 0.6\rangle \end{array} \right\}.$$

Table 4: Tabular representation of the triple-top-level soft set $L((F,\ A);$ $tripletop)$ with choice value

U/A	a_1	a_2	a_3	a_4	*Choice value* (c_i)
c_1	0	0	0	0	0
c_2	0	0	0	0	0
c_3	0	0	0	0	0
c_4	0	0	0	0	0
c_5	0	0	0	0	0

From Table 4, there is no maximum choice for Mr X. Here it means that, if Mr X is a high income earner, no car meets his requirement.
Also, neutrosophic set $\boldsymbol{triplebottom}_\Psi : A \rightarrow [0,\ 1] \times [0,\ 1] \times [0,\ 1]$ is define by: for all $a \in A$,

$$\mu_{triplebottom\ \Psi}(a) = \min_{x \in U} \mu_{F(a)}(x),$$

$$\Omega_{triplebottom\Psi}(a) = \min_{x \in U} \Omega_{F(a)}(x)$$

and

$$\lambda_{triplebottom\ \Psi}(a) = \min_{x \in U} \lambda_{F(a)}\ .$$

To illustrate the above definitions, we shall consider the following Example 3.1.
For triple-bottom-level soft sets, let us consider Example 3.1 and $\Psi = (\ F,\ A)$ with tabular representation in Table 5. It is obvious that, the triple-bottom-threshold of $(F,\ A)$ is neutrosophic set:

$$triplebottom_{(F,\ A)} = \left\{ \begin{array}{c} \langle a_1, 0.4, 0.3, 0.3\rangle, \langle a_2, 0.7, 0.2, 0.3\rangle, \langle a_3, 0.5, 0.3, 0.3\rangle, \\ \langle a_4, 0.6, 0.6, 0.3\rangle \end{array} \right\}.$$

Table 5: Tabular representation of the triple-bottom-level soft set $L((F,\ A);triplebottom)$ with choice value

U/A	a_1	a_2	a_3	a_4	*Choice value* (c_i)
c_1	1	0	0	0	1
c_2	0	0	1	0	1
c_3	1	1	0	1	3
c_4	1	1	0	0	2
c_5	0	0	0	0	0

From Table 5, we can easily see that the maximum choice value is $c_3 = 3$ and this corresponds to car c_3.

Decision: The optimum decision for Mr X is to select car c_3 for transportation for his wedding celebration.

6. Weighted neutrosophic soft set based decision making

In 1996, Lin [12] defined a new theory of mathematical analysis, namely the weighted soft sets (W-soft sets). In accordance with Lin's style, Maji *et al* [13] defined the weighted table of a soft set. A weighted table of a soft set is presented by having $d_{ij} = w_j \times h_{ij}$ instead of 0 and 1 only, where h_{ij} are entries in the table of the soft set and w_j are the weights of the attributes e_j. The weighted choice value of an object o_i is $\bar{c}_i$, given by $\bar{c}_i = \sum_j d_{ij}$. By imposing weights on choice parameters, a revised algorithm for arriving at the final optimal decisions was established in [13]. In tandem with this idea, we introduce the notion of weighted neutrosophic soft set and present its application to multicriteria decision making problems.

Definition 6.1. [10]. A weighted neutrosophic soft set is a triple $\eta = (F,\ A,\ w)$,where $(F,\ A)$ is a neutrosophic soft set over U, and $w : A \longrightarrow [0,\ 1]$ is a weight function specifying the weight $w_j = w(a_j)$, for each attribute $a_j \in A$.

By definition, every neutrosophic soft set can be considered as a weighted neutrosophic soft set. clearly the concept of weighted neutrosophic soft set provides a mathematical framework for modeling and analyzing the decision making problems in which all the choice parameters may not be of equal significance. The difference between the importance of parameters are characterized by the weight function in a weighted neutrosophic soft set.

Algorithm 1 can be revised to deal with decision making problems based on weighted neutrosophic soft sets (See Algorithm 2). In the revised algorithm, we take the weights of parameters in to consideration and compute the weighted choice values $\overline{c}_i$ instead of choice values c_i. Note that for a weighted neutrosophic soft set $\eta = (F,\ A,\ w)$ the weight function $w : A \longrightarrow [0,\ 1]$ can be used as a threshold neutrosophic set, which implies that one can consider the level soft set $L((F,\ A)\,; w\ )$. This will be called decision making approach based on the weight function decision rule in what follows. Sometimes it is much reasonable to use this decision rule since the decision maker may need higher membership levels on the parameters he puts on more preference.

Algorithm 2.

(1) Input the weighted neutrosophic soft set $\eta = (F,\ A,\ w)$.
(2) Input a threshold neutrosophic set $\lambda : A \rightarrow [0,\ 1]$, (or give a threshold value $(s, t, v) \in [0,\ 1] \times [0,\ 1] \times [0,\ 1]$; or choose the mid-level decision rule; or choose the triple-top-level decision rule or choose the weight function decision rule) for decision making.
(3) Compute the level soft set $L(\eta; \lambda\ )$ with respect to the threshold neutrosophic set λ (or the $(s, t, v)-$level soft set $L(\eta; (s, t, v)\ )$; or the mid-level soft set $L(\eta; mid\ )$ or the triple-top-level soft set $L\,(\eta; max\ )$ or the triple-bottom-level soft set $L\,(\eta; min\ )\, or\ L((F,\ A)\,,\ w))$.
(4) Present the level soft set $L(\eta; \lambda\ )$(or $L\,(\eta; (s, t, v)\ )$; or $L(\eta; mid)$ or $L\,(\eta; max)$or $L\,(\eta; min\ )$ or $L((F,\ A)\,,\ w)\ )$ in tabular form and compute the weighted choice value $\overline{c}_i$ of o_i for all i.
(5) The optimal decision is to select o_k if $\overline{c}_k = \ max_i \overline{c}_i$.
(6) If k has more than one value, then any one of o_k may be chosen.

Note that in the last step of algorithm 2, if too many optimal choices are obtained, one can go back to the third step and change the threshold (or decision rule) previously used so as to adjust the final optimal decision.
To illustrate the above concept, we reconsider Example 3.1 for the $(\boldsymbol{s}, \boldsymbol{t}, \boldsymbol{v})$-level soft set $\boldsymbol{L}\,(\eta;\mathbf{0.7,\ 0.3,\ 0.3})$ now assume that Mr X have imposed the following weights for the parameters in A: for the parameter "beautiful", $w_1 = 0.7$; for the parameter "costly", $w_2 = 0.4$; for the parameter "model 2010", $w_3 = 0.5$; for the parameter "made in Japan", $w_4 = 0.3$.

Table 6: Tabular representation of the level soft set $L((F,\ A),\ w)$

U/A	$a_1(0.7)$	$a_2(0.4)$	$a_3(0.5)$	$a_4(0.3)$	*Choice value* (c_i)
c_1	1	0	0	0	0.7
c_2	0	0	1	0	0.5
c_3	1	0	0	1	1.0
c_4	1	0	0	0	0.7
c_5	0	0	0	0	0.0

From Table 6, it is obvious that the maximum choice value is $\widetilde{\mathbf{c}}_{\mathbf{3}} = \mathbf{1}$.

Decision: The optimum decision is for Mr X to purchase car c_3 for transportation during his wedding celebration.

Also, let us consider the case of a triple-bottom-level soft sets, Example 3.1 and $\Psi = (\ F,\ A)$ with tabular representation in Table 7. It is obvious that, the triple-bottom-threshold of $(F,\ A)$ is neutrosophic set:

$$triplebottom_{(F,\ A)} = \left\{ \begin{array}{c} \langle a_1, 0.4, 0.3, 0.3\rangle, \langle a_2, 0.7, 0.2, 0.3\rangle, \langle a_3, 0.5, 0.3, 0.3\rangle, \\ \langle a_4, 0.6, 0.6, 0.3\rangle \end{array} \right\}.$$

Let us assume that Mr X imposed the following weights for the parameters in A: for the parameter "beautiful", $w_1 = 0.3$; for the parameter "costly", $w_2 = 0.25$; for the parameter "model 2010", $w_3 = 0.8$; for the parameter "made in Japan", $w_4 = 0.2$.

Table 7: Tabular representation of the level soft set $L((F,\ A),\ w)$

U/A	$a_1(0.3)$	$a_2(0.25)$	$a_3(0.8)$	$a_4(0.2)$	*Choice value* (c_i)
c_1	1	0	0	0	0.30
c_2	0	0	1	0	0.80
c_3	1	1	0	1	0.75
c_4	1	1	0	0	0.55
c_5	0	0	0	0	0.00

From Table 7, It is clear that the maximum choice value is $\widetilde{\mathbf{c}}_{\mathbf{2}} = \mathbf{0.8}$.

Decision: The optimum decision is for Mr X to purchase car c_2 for transportation during his wedding celebration.

7. Conclusion

In this paper, the basic soft set operations were defined, this includes restricted intersection, restricted union, and extended intersection in neutrosophic soft set context supported with basic and illustrative examples. Some basic results on neutrosophic soft set were presented. We stated De Morgan's laws and inclusions and proved them in details. We also presented some detailed results on restricted union, union, restricted intersection, extended intersection, AND and OR products with respect to the various operations.

Finally, an adjustable approach to multicriteria decision making problem using level soft set of a neutrosophic soft set was presented with relevant and illustrative example. Weighted neutrosophic soft set and its application in decision making problem was presented. The advantage the method of multicriteria approach adopted over the Maji et al. [13] approach is that in the adjustable approach in multicriteria decision making, it is easy to arrive at the optimum decision within a shortest time possible compared to that of Maji et al. that requires a lot of computations.

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