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Abstract. The processing of uncertainty information has gradually became one of the hot issues in artificial intelligence field, and the information measures of uncertainty information processing are of importance. Single value neutrosophic sets (SVNSs) provide us a flexible mathematical framework to process uncertainty information. In this paper, we mainly consider the measures of SVNSs. The existing information measures mostly are constructed based on the two typical inclusion relations about single value neutrosopgic sets. However, there exist some practical problems that do not apply to the two typical inclusion relations. Therefore, there exists another inclusion relation which is called the type-3 inclusion relation about SVNSs. This inclusion relation can help us to process some uncertain information in a new way. It is noteworthy that the existing information measures are not suitable for the type-3 inclusion about SVNSs. In this case, we proposed the new distance measure based on the cross-entropy about SVNSs, then the corresponding similarity measure is proposed according to the matching function between distance and similarity. Finally, the new distance measure is applied to decision-making problem with a illustrative example. The verification result show that the new distance is favorable for dealing with some practical problems.

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1. Introduction

In the real world, there exists many uncertainty, imprecise, incomplete information, so, the uncertainty information processing has gradually became one of the hot issues in many scientific field. Zadeh [45] firstly proposed the theory of fuzzy set in which one element of the universe has a membership value to the set. Atanassov [1] proposed intuitionistic fuzzy set (IFS) described by two functions, including a membership function depicting the membership value, and a non-membership function depicting the non-membership value of one object to the intuitionistic fuzzy set. Intuitionistic fuzzy sets are generalized on the basis of fuzzy sets by means of adding a non-membership function. And it has provided a more flexible mathematical framework to process these uncertainty, imprecise and incomplete information. Smarandache [26, 27] originally introduced neutrosophy and the notion of neutrosophic set in 1998. The neutrosophic set is defined by three memnership functions including truth-membership function, indeterminacy-membership function and falsity-membership which are used to expressed the degree to which an element belongs to the neutrosophic set, the degree of uncertainty and the degree of non-belonging respectively. It's noteworthy that the three membership functions of neutrosophic set are independent which is different from hesitant intuitionistic fuzzy set. Therefore, Smarandache (1998) defined the single valued neutrosophic set in the book [26], and Wang et al. [31] also given the definition of single value neutrosophic set and set-theoretic operators for better applications in real scientific and engineering fields. Single value neutrosophic sets (SVNSs) is generalized on the basis of intuitionistic fuzzy sets and provided a more facilitate tool for us to process these uncertainty, imprecise, incomplete and inconsistent information in the real world, and then it would be more suitable to be applied in indeterminate information processing and inconsistent information measures. Some researchers have shown great interests in the theory of single-value neutrosophic set and applied it to many fields including the multi-attribute decision-making, pattern recognition, image segmentation, fault diagnosis and medical diagnosis [14, 21, 22, 24, 29, 32, 33, 34, 35, 36, 37, 38, 42, 43]. Harish [10, 11, 12, 13, 18, 19, 23] has made much results in the application of neutrosophic sets environment including linguistic single-valued neutrosophic prioritized aggregation operators, prioritized muirhead mean aggregation operator and New logarithmic operational laws. Broumi [7] proposed some computing procedures in Matlab for neutrosophic operational matrices. Broumi [4, 5, 8]compared the shortest path problem with various existing algorithms and concluded the best algorithm for certain environment and proposed the shortest path problem (SPP) method in the neutrosophic environment.

Information measures are essential to decision-making in uncertain information processing, including similarity, distance or divergence measure, entropy and cross-entropy. These information measures have much applications on image processing, clustering, pattern recognition and so on. Similarity measure is mainly used to measure the level of similarity between two objects. Entropy is usually to depict uncertain degree of one object and is very important for uncertain information. Cross-entropy is used to measure the degree of discrimination of two objects and we can judge their relationship through that. Therefore, cross-entropy have been widely used in many fields as data analysis and classification, decision-making, pattern recognition and so on. Zadeh [46] firstly proposed the entropy of fuzzy events which is based on Shannon entropy. Kullback [15] was concerned with a information measure which can be called "distance" or "divergence" depicting the relation between two probability distributions. Therefore, it is known as a information measure for indicating the degree of discrimination. Furthermore, a new kind of information measure called the "cross entropy distance" of two probability distributions was introduced by Kullback and Leibler. Cross-entropy can be used to measure the degree of discrimination between two objects. Therefore, many researchers have modified cross-entropy measures. For example, Lin [16] proposed divergence based on Shannon entropy and it is a kind of modified fuzzy cross-entropy. Bhandari [2] introduced fuzzy divergence between two fuzzy sets. Shang wt al. [25] introduced the concept of fuzzy cross-entropy and a symmetric discrimination measure of fuzzy sets which was based on fuzzy divergence and is used to described the discrimination degree of fuzzy sets. Vlachos [18] presented cross-entropy on intuitionistic fuzzy sets and introduced a mathematical connection between the fuzzy entropy and intuitionistic fuzzy entropy in terms of fuzziness and intuitionism. The research on the measures of SVNSs began in 2013. In 2013, Broumi and Smarandache [6] proposed the distance measure of neutrosophic sets on the basis of Hausdorff distance, and proposed some similarity functions based on distance measure and connection function. In 2014, Majumdar and Samanta [17] put forward the distance measure and defined the entropy of the neutrosophic set. Ye [34] proposed the correlation and correlation coefficient of the neutrosophic sets based on the correlation of the intuitionistic fuzzy sets, then proposed the weighted similarity constructed by the cosine function and proposed the corresponding decision-making method. Ye [35, 37] proposed a multi-criteria decision-making method through two aggregation operators and cosine similarity, and then proposed an improved cosine similarity in 2015 for medical diagnosis. Ye [36] proposed three vector similarities including Jaccard, Dice and cosine similarities for multi-criteria decision-making (MCDM). In 2017, Ye [38] constructed cotangent similarity using cotangent function and applied it to medical diagnosis. Cross-entropy measures was generalized to the single valued neutrosophic sets and applied it to multi-criteria decision-making by Ye [33]. And then Rıdvan Şhahin [24] generalized the cross-entropy measure on interval neutrosophic sets and the application in multicriteria decision making. Garg [9] proposed some new types of distance measures of SVNSs and the applications to Pattern Recognition and Medical Diagnosis. In 2018, Wu [32] gave some methods to construct information metrics by using cosine function. Wang [30] has proved that the cross-entropy is a new kind of distance measure in fuzzy sets and single value neutrosophic sets. Nancy [20] proposed an axiomatic definition of divergence measure for SVNSs and develop a novel technique for order preference by similarity to ideal solution (TOPSIS) method for solving single-valued neutrosophic multi-criteria decision-making with incomplete weight information. Qin [22] introduced a new similarity and entropy measures of SVNSs and the application in multi-attribute decision-making. Most of the existing measures of SVNSs are based on the classical two inclusion relations which are called as type-1 and type-2 inclusion relations. In the fact, these two inclusion relations are satisfied if and only if the three membership degrees of neutrosophic sets satisfy some conditions at the same time. that is three membership functions of the neutrosophic set are equally important. However, some practical problems do not necessarily mean that the three membership degrees are equally important (for example, voting problem, we may pay more attention to the affirmative vote and dissenting vote relative to the abstentions). Therefore, Zhang [39, 40] put forward a new inclusion relation of neutrosophic sets, some operations and its algebraic structure under this inclusion relation.

The new inclusion relationship can be determined by the true membership degree and the false membership degree firstly, only when the true membership degree and the false membership degree can not be distinguished, the inclusion relationship can be determined by the degree of uncertainty. However, we find that most of the existing measures of neutrosophic sets are inappropriate the new inclusion relation. So, we will consider the new measures of SVNSs on the basis of the new inclusion relation in order to better deal with some practical problem, and the study of information measures of SVNSs based on the new inclusion relation will help us handle some practical problems in real world. In this paper, we firstly introduce some relevant notions and properties of SVNSs. In the next, we verified that the existing cross-entropy are not suitable for the new inclusion relation about SVNSs by a example. Since the cross-entropy is a kind of distance measure, this paper further considers the cross-entropy of SVNSs on the basis of new inclusion relation, and proposes a new distance measure based on the cross-entropy, and gives the corresponding similarity according to the matching function between distance measure and similarity. In the next, the new distance measure is applied to decision-making problem with a illustrative example. The information measure based on the new inclusion relation will help to solve some practical problems.

2. Preliminaries

Smarandache [26, 27] firstly proposed the definition of neutrosophic set, which is an extension of an intuitionistic fuzzy set(IFS) and an interval-valued intuitionistic fuzzy set, as follows:

Definition 2.1 ([26])**.** Let X be a universe course, where a neutrosophic set A in X is comprised of the truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$, in which $T_A(x), I_A(x), F_A(x) : X \to]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x), I_A(x), F_A(x)$. Then

$$0^- \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+.$$

It's remarkable that the three membership of neutrosophic set are independent.

Wang et al. [31] introduced the definition of single value neutrosophic set (SVNS) for the better application in the engineering field. SVNS is an extension of the IFS, and also provides another way in which to express and process uncertainty, incomplete, and inconsistent information in the real world.

Definition 2.2 ([31]). Let X be a space of points, where a single-value netrosophic set A in X is comprised of the truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0,1]$. Then a SVNS A can be denoted by:

$$A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}.$$

There is no restriction on the sum of $T_A(x), I_A(x), F_A(x)$. Thus

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

Let $SVNS(X)$ be the set of all the single value neutrosophic sets. There have three kinds of definitions of the inclusion relations of SVNSs in literature.

Definition 2.3 ([26, 28]). Let X be a universe course and let $A,\ B \in SVNS(X)$. If $A \subseteq B$, then there have $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x), F_A(x) \geq F_B(x)$, for every x in X.

This inclusion relation can be called as type-1 inclusion relation and denoted by $\subseteq_1$ [39].

Definition 2.4 ([3, 31]). Let X be a universe course and let $A,\ B \in SVNS(X)$. If $A \subseteq B$, then there have $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$, for every x in X.

This inclusion relation can be called as type-2 inclusion relation and denoted by $\subseteq_2$ [39].

An analysis about the above inclusion relations has been introduced by Zhang et al. [39, 40]. Zhang pointed that the type-1 and type-2 inclusion relations are actually divided three membership functions into two groups. That is the two inclusion relations do not really take advantage of the three membership functions; they are actually two extreme ways of handling and then determines the order relation using the method similar to intuitionistic fuzzy sets. From this point, Zhang proposed the third inclusion relation of SVNSs.

Definition 2.5 ([39]). Let X be a universe course and let $A,\ B \in SVNS(X)$. Then the type-3 inclusion relation is defined as follows:

$$\begin{aligned} A \subseteq B \text{ if and only if } \forall x \in X, &((T_A(x) < T_B(x)) \wedge (F_A(x) \geq F_B(x))) \\ \text{or } &((T_A(x) = T_B(x)) \wedge (F_A(x) > F_B(x))) \\ \text{or } &((T_A(x) = T_B(x)) \wedge (F_A(x) = F_B(x)) \wedge (I_A(x) \leq I_B(x))). \end{aligned}$$

The type-3 inclusion relation of SVNSs have the following properties: If $A \subseteq B, B \subseteq C$, then $A \subseteq C$.

Definition 2.6 ([39]). Let $N(X) = \{(x_1, x_2, x_3) | x_1, x_2, x_3 \in [0,1]\}$, $(x_1, x_2, x_3), x_1, x_2, x_3 \in [0,1]$ can be called a single value neutrosophic number. The order relation between single value neutrosophic number is introduced as follows:

$$\begin{aligned} \forall x, y \in N(X), x \leq y \Longleftrightarrow &(x_1 < y_1) \wedge (x_3 \geq y_3) \\ \text{or } &(x_1 = y_1) \wedge (x_3 > y_3) \\ \text{or } &(x_1 = y_1) \wedge (x_3 = y_3) \wedge (x_2 \leq y_2). \end{aligned}$$

3. New distance and similarity measure of single value neutrosophic sets

The cross-entropy is firstly introduced by Bhandari [2] which is also called as discrimination measure. Then, it has been modified by Shang et al. [25] on the basis of directed divergence [16]. The cross-entropy is used to measure the degree of discrimination between two objects. Wang [30] has proved that the cross-entropy is a kind of distance measure actually in fuzzy sets and single value neutrosophic sets. So, we extend the cross-entropy to a new distance measure on the new inclusion relation in SVNSs and proposed the corresponding similarity according matching function between distance and similarity measure. Firstly, the definition of distance measure is proposed.

Definition 3.1. A function $D : SVNS(X) \times SVNS(X) \to [0, 1]$ is called a distance measure for SVNSs, if the following conditions are satisfied: for any A, B, $C \in SVNS(X)$,
(i) $0 \leq D(A, B) \leq 1$,
(ii) $D(A, B) = 0$ if and only if $A = B$,
(iii) $D(A, B) = D(B, A)$,
(iv) if $A \subseteq B \subseteq C$, then $D(A, C) \geq D(A, B), D(A, C) \geq D(B, C)$.

3.1. **The cross-entropy about single value neutrosophic sets.** Ye [33] first generalized the fuzzy cross-entropy measure to the SVNSs. The information measure of neutrosophic sets are composed of the information measure of the truth-membership, indeterminacy-membership, and falsity-membership in SVNSs.

Let X be a universe course, $A, B \in SVNS(X)$, where Ye introduced the discrimination information of $T_A(x_i)$ from $T_B(x_i)$ for $(i = 1, 2, ..., n)$ on the basis of the definition of fuzzy cross-entropy [25] as the following:

$$I^T(A,B) = \sum_{i=1}^n [T_A(x_i) \ln \frac{T_A(x_i)}{1/2(T_A(x_i)+T_B(x_i))} + (1 - T_A(x_i)) \ln \frac{1-T_A(x_i)}{1-1/2(T_A(x_i)+T_B(x_i))}].$$

Then, define the following information in terms of the indeterminacy-membership function and the falsity-membership function in the same way:

$$I^I(A,B) = \sum_{i=1}^n [I_A(x_i) \ln \frac{I_A(x_i)}{1/2(I_A(x_i)+I_B(x_i))} + (1 - I_A(x_i)) \ln \frac{1-I_A(x_i)}{1-1/2(I_A(x_i)+I_B(x_i))}],$$

$$I^F(A,B) = \sum_{i=1}^n [F_A(x_i) \ln \frac{F_A(x_i)}{1/2(F_A(x_i)+F_B(x_i))} + (1 - F_A(x_i)) \ln \frac{1-F_A(x_i)}{1-1/2(F_A(x_i)+F_B(x_i))}].$$

For the symmetry, Ye also defined $E^T(A, B) = I^T(A, B) + I^T(B, A), E^I(A, B) = I^I(A, B) + I^I(B, A)$ and $E^F(A, B) = I^F(A, B) + I^F(B, A)$.

Definition 3.2 ([33])**.** The single-value neutrosophic cross-entropy about A and B, where $A, B \in SVNS(X)$ can be defined as follows:

$$I(A, B) = I^T(A, B) + I^I(A, B) + I^F(A, B).$$

Similarly, the symmetry cross-entropy can be written as:

$$E(A, B) = I(A, B) + I(B, A).$$

Lemma 1 [30]**.** $E(A, B))$ is a kind of distance measure on the basis of type-1 inclusion relation.

It's easily find $E(A, B)$ is also a distance measure according to the tape-2 inclusion relation. Then, we mainly consider if the cross-entropy satisfied the type-3 inclusion relation.

Example 1. Let X be a space of the universe course, $x, y, z \in N(X)$, where $x = (0.3, 0.4, 0.5), y = (0.5, 0.2, 0.3), z = (0.5, 0.3, 0.1)$.

It is clear that $x \leq y \leq z$ according to the type-3 inclusion relation and we can obtain the cross-entropy between single value neutrosophic number:

$E^T(x, y) = I^T(x, y) + I^T(y, x) = 0.0420,\ E^I(x, y) = I^I(x, y) + I^I(y, x) = 0.483,$
$E^F(M, T) = I^F(x, y) + I^F(y, x) = 0.0420;$

that is, $E(x, y) = E^T(x, y) + E^I(x, y) + E^F(x, y) = 0.5670.$

$E^T(y, z) = 0, E^I(y, z) = 0.0134, E^F(y, z) = 0.0648;$

that is, $E(Y, Z) = 0.0782.$

$E^T(x, z) = 0.0420, E^I(x, z) = 0.0111, E^F(x, z) = 0.2035;$

that is, $E(x, z) = 0.2566.$

It's obviously that $E(x, z) \geq E(x, y), E(x, z) \geq E(y, z)$ is no longer satisfied. That is to say, the cross-entropy is not a distance measure under the type-3 inclusion relation, the cross-entropy is not suitable for the type-3 inclusion relation or order relation $\leq$ defined in Definition 2.6. Then, we will construct a new distance measure based on cross-entropy so that it satisfied the type-3 inclusion relation or order relation $\leq$ defined in Definition 2.6.

3.2. **The new distance and similarity measures of SVNNs.** The type-1 and type-2 inclusion relation and the corresponding order relations $\leq_1, \leq_2$ about single value neutrosophic numbers (SVNNs) is based on that the three membership functions about SVNS are equally important. The three membership need to satisfy certain conditions at the same time, then, the inclusion relation of SVNSs can be obtained. However, The order relation $\leq$ on single valued neotrosophic numbers proposed by Zhang et al. [39] is essentially different from $\leq_1, \leq_2$. This order relation can be obtained through the truth-membership and the falsity-membership at first, only when the two membership are indistinguishable, the order relation or inclusion relation can determined by the indeterminacy-membership. Based the above analysis, we can construct a distance measure as the following way:

When we can get the new inclusion relation or order relation by the truth-membership degree and the falsity-membership degree, we can establish the corresponding cross-entropy just based on the two membership, only when the two membership are indistinguishable, then, we can establish the corresponding cross-entropy by the indeterminacy-membership.

Theorem 3.3. *Let $x, y, z \in N(X)$. Then $E^T(x, y) \leq 2, E^I(x, y) \leq 2, E^F(x, y) \leq 2$.*

Proof. The literate [16] have proved that $L(p_1, p_2) \leq V(p_1, p_2)$, which X is a discrete random variable and $p_1,\ p_2$ are two probability distribution of X, and $\forall x, p_1, p_2 \in [0, 1]$,

$$L(p_1, p_2) = \sum_{x \in X} p_1(x) \log \frac{p_1(x)}{1/2(p_1(x)) + p_2(x))} + p_2(x) \log \frac{p_2(x)}{1/2(p_1(x) + p_2(x))}$$

$$V(p_1,p_2)=\sum_{x\in X}|p_1(x)-p_2(x)|.$$

It's obviously $|p_1(x)-p_2(x)|\leq 1$, that is:

$p_1(x)\ln\frac{p_1(x)}{1/2(p_1(x))+p_2(x))}+p_2(x)\ln\frac{p_2(x)}{1/2(p_1(x)+p_2(x))}\leq|p_1(x)-p_2(x)|\leq 1.$

We can easily get:

$$x_i\ln\frac{x_i}{1/2(x_i+y_i)}+y_i\ln\frac{y_i}{1/2(x_i+y_i)}\leq|x_i-y_i|\leq 1 \tag{3.1}$$

similarly,

$$(1-x_i)\ln\frac{1-x_i}{1-1/2(x_i+y_i)}+(1-y_i)\ln\frac{1-y_i}{1-1/2(x_i+y_i)}\leq|(1-x_i)-(1-y_i)|\leq 1, \tag{3.2}$$

where $i=1,2,3$.
From (3.1) and (3.2), we can obtained $E^T(x,y)\leq 2$ when $i=1$, $E^I(x,y)\leq 2$ when $i=2$ and $E^F(x,y)\leq 2$ when $i=3$. □

Definition 3.4. Let X be a universe course, $x,y\in N(X)$, we can defined:

$$D_E(x,y)=\begin{cases}\dfrac{E(x_2,y_2)}{4}, & (x_1=y_1)\wedge(x_3=y_3),\\ \dfrac{4+E(x_1,y_1)+E(x_3,y_3)}{8}, & otherwise.\end{cases}$$

Theorem 3.5. *$D_E(x,y)$ is a distance measure for SVNSs under the type-3 inclusion relation.*

Proof. Let $x,y,z\in N(X)$. Then it's easy find that $E_D(x,y)\in[0,\frac{1}{2}]$ when $x_1=y_1$ and $x_3=y_3$, and $E_D(x,y)\in(\frac{1}{2},1]$ in the case of the "otherwise" from the Theorem 3.3.
It's obviously that:

(i) $E_D(x,y)\geq 0$,

(ii) $E_D(x,y)=E_D(y,x)$,

(iii) If $E_D(x,y)=0$, then this can be satisfied only in the case of $x_1=y_1$ and $x_3=y_3$, that is: $\frac{E^I(x_2,y_2)}{4}=0$, we have $x_2=y_2$. Thus $x=y$.

The next is mainly to prove the condition (iv) of distance measure defined in Definition 3.1 is satisfied.

(iv) Assume that $x=(x_1,x_2,x_3),y=(y_1,y_2,y_3),z=(z_1,z_2,z_3)$, and $x\leq y\leq z$.

Case 1. If $(x_1<y_1)\wedge(x_3\geq y_3)$ and $(y_1<z_1)\wedge(y_3\geq z_3)$, then $(x_1<y_1<z_1)\wedge(x_3\geq y_3\geq z_3)$, in this case,

$$D_E(x,y)=\frac{4+E(x_1,y_1)+E(x_3,y_3)}{8},$$

$$D_E(y,z)=\frac{4+E(y_1,z_1)+E(y_3,z_3)}{8},$$

$$D_E(x,z)=\frac{4+E(x_1,z_1)+E(x_3,z_3)}{8}.$$

. Thus from the Theorem 4 in [30], because $(x_1 < y_1 < z_1) \wedge (x_3 \geq y_3 \geq z_3)$, $E(x_1, z_1) > E(x_1, y_1), E(x_1, z_1) > E(y_1, z_1)$, $E(x_3, z_3) \geq E(x_3, y_3), E(x_3, z_3) \geq E(y_3, z_3)$. So $D_E(x, z) > D_E(x, y)$ and $D_E(x, z) > D_E(y, z)$.

Case 2. If $(x_1 < y_1) \wedge (x_3 \geq y_3)$ and $(y_1 = z_1) \wedge (y_3 > z_3)$, then $(x_1 < y_1 = z_1) \wedge (x_3 \geq y_3 > z_3)$, in this case,

$$D_E(x, y) = \frac{4 + E(x_1, y_1) + E(x_3, y_3)}{8},$$
$$D_E(y, z) = \frac{4 + E(y_1, z_1) + E(y_3, z_3)}{8},$$
$$D_E(x, z) = \frac{4 + E(x_1, z_1) + E(x_3, z_3)}{8}.$$

Similarly, since $(x_1 < y_1 = z_1) \wedge (x_3 \geq y_3 > z_3)$, $E(x_1, z_1) \geq E(x_1, y_1), E(x_1, z_1) > E(y_1, z_1) = 0$, $E(x_3, z_3) > E(x_3, y_3), E(x_3, z_3) \geq E(y_3, z_3)$. Thus $D_E(x, z) > D_E(x, y)$ and $D_E(x, z) > D_E(y, z)$.

Case 3. If $(x_1 < y_1) \wedge (x_3 \geq y_3)$ and $(y_1 = z_1) \wedge (y_3 = z_3) \wedge (y_2 \leq z_2)$, then $(x_1 < y_1 = z_1) \wedge (x_3 \geq y_3 = z_3) \wedge (y_2 \leq z_2)$, in this case,

$$D_E(x, y) = \frac{4 + E(x_1, y_1) + E(x_3, y_3)}{8},$$
$$D_E(y, z) = \frac{E(y_2, z_2)}{4},$$
$$D_E(x, z) = \frac{4 + E(x_1, z_1) + E(x_3, z_3)}{8}.$$

Similarly, since $(x_1 < y_1 = z_1) \wedge (x_3 \geq y_3 > z_3)$, $E(x_1, z_1) \geq E(x_1, y_1)$, $E(x_3, z_3) > E(x_3, y_3)$. Thus$D_E(x, z) > D_E(x, y)$. Since $D_E(y, z) = \frac{E(y_2, z_2)}{4} \in [0, \frac{1}{2}], E_D(x, z) = \frac{4+E(x_1,z_1)+E(x_1,z_1)}{8} \in (\frac{1}{2}, 1]$, $D_E(x, z) > D_E(y, z)$.

Case 4. If $(x_1 = y_1) \wedge (x_3 > y_3)$ and $(y_1 < z_1) \wedge (y_3 \geq z_3)$, then $(x_1 = y_1 < z_1) \wedge (x_3 > y_3 \geq z_3)$, in this case,

$$D_E(x, y) = \frac{4 + E(x_1, y_1) + E(x_3, y_3)}{8},$$
$$D_E(y, z) = \frac{4 + E(y_1, z_1) + E(y_3, z_3)}{8},$$
$$D_E(x, z) = \frac{4 + E(x_1, z_1) + E(x_3, z_3)}{8}.$$

Since $(x_1 = y_1 < z_1) \wedge (x_3 > y_3 \geq z_3)$, $E(x_1, z_1) > E(x_1, y_1) = 0, E(x_1, z_1) > E(y_1, z_1)$, $E(x_3, z_3) > E(x_3, y_3), E(x_3, z_3) \geq E(y_3, z_3)$. Thus $D_E(x, z) > D_E(x, y)$ and $D_E(x, z) > D_E(y, z)$.

Case 5. If $(x_1 = y_1) \wedge (x_3 > y_3)$ and $(y_1 = z_1) \wedge (y_3 > z_3)$, then $(x_1 = y_1 = z_1) \wedge (x_3 > y_3 > z_3)$, in this case,

$$D_E(x, y) = \frac{4 + E(x_1, y_1) + E(x_3, y_3)}{8},$$
$$D_E(y, z) = \frac{4 + E(y_1, z_1) + E(y_3, z_3)}{8},$$
$$D_E(x, z) = \frac{4 + E(x_1, z_1) + E(x_3, z_3)}{8}.$$

Since $(x_1 = y_1 = z_1) \wedge (x_3 > y_3 > z_3)$, $E(x_1,z_1) = E(x_1,y_1) = E(y_1,z_1) = 0$, $E(x_3,z_3) > E(x_3,y_3), E(x_3,z_3) > E(y_3,z_3)$. Thus $D_E(x,z) > D_E(x,y)$ and $D_E(x,z) > D_E(y,z)$.

Case 6. If $(x_1 = y_1) \wedge (x_3 > y_3)$ and $(y_1 = z_1) \wedge (y_3 = z_3) \wedge (y_2 \leq z_2)$, then $(x_1 = y_1 = z_1) \wedge (x_3 > y_3 = z_3)$, in this case,

$$D_E(x,y) = \frac{4 + E(x_1,y_1) + E(x_3,y_3)}{8},$$

$$D_E(y,z) = \frac{E(y_2,z_2)}{4},$$

$$D_E(x,z) = \frac{4 + E(x_1,z_1) + E(x_3,z_3)}{8}.$$

Since $(x_1 = y_1 = z_1) \wedge (x_3 > y_3 = z_3)$, $E(x_1,z_1) = E(x_1,y_1) = E(y_1,z_1) = 0$, $E(x_3,z_3) = E(x_3,y_3)$. Thus $D_E(x,z) > D_E(x,y)$. Since $D_E(y,z) = \frac{E(y_2,z_2)}{4} \in [0,\frac{1}{2}], D_E(x,z) = \frac{4+E(x_1,z_1)+E(x_3,z_3)}{8} \in (\frac{1}{2},1]$, $D_E(x,z) > D_E(y,z)$.

Case 7. If $(x_1 = y_1) \wedge (x_3 = y_3) \wedge (x_2 \leq y_2)$ and $(y_1 < z_1) \wedge (y_3 \geq z_3)$, then $(x_1 = y_1 < z_1) \wedge (x_3 = y_3 \geq z_3)$, in this case,

$$D_E(x,y) = \frac{E(x_2,y_2)}{4},$$

$$D_E(y,z) = \frac{4 + E(y_1,z_1) + E(y_3,z_3)}{8},$$

$$D_E(x,z) = \frac{4 + E(x_1,z_1) + E(x_3,z_3)}{8}.$$

Since $D_E(x,y) = \frac{E(x_2,y_2)}{4} \in [0,\frac{1}{2}], D_E(x,z) = \frac{E^T(x,z)+E^F(x,z)}{8} \in (\frac{1}{2},1]$, $D_E(x,z) > D_E(x,y)$. Since $(x_1 = y_1 < z_1) \wedge (x_3 = y_3 \geq z_3)$, $D_E(x,z) = D_E(y,z)$.

Case 8. If $(x_1 = y_1) \wedge (x_3 = y_3) \wedge (x_2 \leq y_2)$ and $(y_1 = z_1) \wedge (y_3 > z_3)$, then $(x_1 = y_1 = z_1) \wedge (x_3 = y_3 > z_3)$, in this case,

$$D_E(x,y) = \frac{E(x_2,y_2)}{4},$$

$$D_E(y,z) = \frac{4 + E(y_1,z_1) + E(y_3,z_3)}{8},$$

$$D_E(x,z) = \frac{4 + E(x_1,z_1) + E(x_3,z_3)}{8}.$$

Since $D_E(x,y) = \frac{E(x_2,y_2)}{4} \in [0,\frac{1}{2}], D_E(x,z) = \frac{E^T(x,z)+E^F(x,z)}{8} \in (\frac{1}{2},1]$, $D_E(x,z) > D_E(x,y)$. Since $(x_1 = y_1 < z_1) \wedge (x_3 = y_3 \geq z_3)$, $D_E(x,z) = D_E(y,z)$.

Case 9. If $(x_1 = y_1) \wedge (x_3 = y_3) \wedge (x_2 \leq y_2)$ and $(y_1 = z_1) \wedge (y_3 = z_3) \wedge (y_2 \leq z_2)$, then $(x_1 = y_1 = z_1) \wedge (x_3 = y_3 = z_3)$, in this case,

$$D_E(x,y) = \frac{E(x_2,y_2)}{4},$$

$$D_E(y,z) = \frac{E(y_2,z_2)}{4},$$

$$D_E(x,z) = \frac{E(x_2,z_2)}{4}.$$

Since $x_2 \leq y_2 \leq z_2$, $E(x_2, z_2) > E(x_2, y_2), E(x_2, z_2) > E(y_2, z_2)$. Thus $D_E(x, z) > D_E(x, y)$ and $D_E(x, z) > D_E(y, z)$.

In short, if $x \leq y \leq z$, then $D_E(x, z) \geq D_E(x, y)$ and $D_E(x, z) \geq D_E(y, z)$. Thus $D_E(x, y)$ is a distance measure. □

Consider Example 1 according the $D_E(x, y)$. From the Example 1,

$$x = (0.3, 0.4, 0.5), y = (0.5, 0.2, 0.3), z = (0.5, 0.3, 0.1).$$

Then $E(x_1, y_1) = 0.0420,\ E(x_2, y_2) = 0.483,\ E(x_3, y_3) = 0.0420$. Thus

$$D_E(x, y) = \frac{4 + E(x_1, y_1) + E(x_3, y_3)}{8} = 0.5105.$$

Also $E(y_1, z_1) = 0,\ E(y_2, z_2) = 0.0134,\ E(y_3, z_3) = 0.0648$. Then

$$D_E(y, z) = \frac{4 + E(y_1, z_1) + E(y_3, z_3)}{8} = 0.5081.$$

$E(x_1, z_1) = 0.0420, E(x_2, z_2) = 0.0111, E(y_3, z_3) = 0.2035$. Then

$$D_E(x, z) = \frac{4 + E(x_1, z_1) + E(y_3, z_3)}{8} = 0.5307.$$

It's clear that $D_E(x, z) > D_E(x, y)$ and $D_E(x, z) > D_E(y, z)$.

Definition 3.6. A function $S : SVNS(X) \times SVNS(X) \to [0, 1]$ is called a similarity measure for single value neutrosophic sets, if the following conditions are satisfied: for any $A,\ B,\ C \in SVNS(X)$,

(i) $0 \leq S(A, B) \leq 1$,
(ii) $S(A, B) = 1$ if and only if $A = B$,
(iii) $S(A, B) = S(B, A)$,
(iv) if $A \subseteq B \subseteq C$, then $S(A, C) \leq S(A, B), S(A, C) \leq S(B, C)$.

According to the matching function $S(A, B) = 1 - D(A, B)$ between similarity and distance measure, we can define the similarity measure as follows.

Definition 3.7. Let X be a universe course and let $x,\ y \in N(X)$. Then the similarity $S_E(x, y)$ can be defined as:

$$S_E(x, y) = \begin{cases} 1 - \dfrac{E(x_2, y_2)}{4}, & (x_1 = y_1) \wedge (x_3 = y_3), \\ \dfrac{4 - (E(x_1, y_1) + E(x_3, y_3))}{8}, & otherwise. \end{cases}$$

Theorem 3.8. *$S_E(x, y)$ is a similarity measure for single value neutrosophic number.*

The proof is evident from the proof of Theorem 3.5.

3.3. **New distance and similarity measures of SVNSs.** In this section, we mainly introduced the new distance and similarity measures about SVNSs. The distance and similarity measures about SVNSs can be defined as the following according to the distance and similarity measure about single value neutrosophic number.

Definition 3.9. Let X be a universe course and let $A,\ B \in SVNS(X)$. Then the distance measure $D_E : SVNS(X)\times SVNS(X) \to [0,1]$ between A, B can be defined as:

$$D_E(A,B) = \begin{cases} \frac{1}{n}\sum\limits_{i=1}^{n} \frac{E^I(I_A(x_i), I_B(x_i))}{4}, & (T_A(x_i) = T_B(x_i)) \wedge (F_A(x_i) = F_B(x_i)), \\ \frac{1}{n}\sum\limits_{i=1}^{n} \frac{4 + E^T(T_A(x_i), T_B(x_i)) + E^F(F_A(x_i), F_B(x_i))}{8}, & otherwise. \end{cases}$$

Definition 3.10. Let X be a universe course and let $A,\ B \in SVNS(X)$. Then the similarity measure $S_E : SVNS(X)\times$ between A, B can be defined as:

$$S_E(A,B) = \begin{cases} \frac{1}{n}\sum\limits_{i=1}^{n} (1 - \frac{E^I(I_A(x_i), I_B(x_i))}{4}), (T_A(x_i) = T_B(x_i)) \wedge (F_A(x_i) = F_B(x_i)), \\ \frac{1}{n}\sum\limits_{i=1}^{n} \frac{4 - (E^T(T_A(x_i), T_B(x_i)) + E^F(F_A(x_i), F_B(x_i)))}{8}, \quad otherwise. \end{cases}$$

Assume the weight vector $W = (\omega_1, \omega_2, \cdots, \omega_n)$ with $\omega_i \in [0,1]$ and $\sum\limits_{i=1}^{n} \omega_i = 1$. The weighed distance measure for SVNSs can be expressed as:

$$D_E(A,B) = \begin{cases} \sum\limits_{i=1}^{n} \omega_i \frac{E^I(I_A(x_i), I_B(x_i))}{4}, & (T_A(x_i) = T_B(x_i)) \wedge (F_A(x_i) = F_B(x_i)), \\ \sum\limits_{i=1}^{n} \omega_i \frac{4 + E^T(T_A(x_i), T_B(x_i)) + E^F(F_A(x_i), F_B(x_i))}{8}, & otherwise. \end{cases}$$

Similarly, the weighed similarity measure for SVNSs can be expressed as:

$$S_E(A,B) = \begin{cases} \sum\limits_{i=1}^{n} \omega_i (1 - \frac{E^I(I_A(x_i), I_B(x_i))}{4}), (T_A(x_i) = T_B(x_i)) \wedge (F_A(x_i) = F_B(x_i)), \\ \sum\limits_{i=1}^{n} \omega_i \frac{4 - (E^T(T_A(x_i), T_B(x_i)) + E^F(F_A(x_i), F_B(x_i)))}{8}, \quad otherwise. \end{cases}$$

4. Practical example

The distance and similarity measures can be used in the application of multi-criteria decision-making, which can provide theoretical support for decision-making for us. There have many multi-criteria decision-making methods have been proposed about SVNSs. In the next, the distance measures will be applied to the multi-criteria decision-making problem with the method proposed by [35].

Example 2. Let us consider the problem adapted from [35]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to choose:

(1) A_1 is a car company,
(2) A_2 is a food company,
(3) A_3 is a computer company,
(4) A_4 is an arms company.

The investment company must take a decision according to the following three criteria:

(1) C_1 is the risk,
(2) C_2 is the growth,
(3) C_3 is the environmental impact.

Then, the weight vector of the criteria is given by $W = (0.35, 0.25, 0.4)$.

For the evaluation of an alternative $A_i(i = 1, 2, 3, 4)$ with respect to a criterion $C_j(j = 1, 2, 3)$, it is obtained from the questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to a criterion C_1 , he or she may say that the possibility in which the statement is good is 0.4 and the statement is poor is 0.3 and the degree in which he or she is not sure is 0.2. In this case, it can be expressed as a neutrosophic set $\alpha_{11} = (0.4, 0.2, 0.3)$. Thus, the opinion for every alternative with respect to every criteria all can be expressed with the neutrosophic sets, and we can obtain the following simplified neutrosophic decision matrix A:

$$A = \begin{bmatrix} (0.4, 0.2, 0.3) & (0.4, 0.2, 0.3) & (0.2, 0.2, 0.5) \\ (0.6, 0.1, 0.2) & (0.6, 0.1, 0.2) & (0.5, 0.2, 0.2) \\ (0.3, 0.2, 0.3) & (0.5, 0.2, 0.3) & (0.5, 0.3, 0.2) \\ (0.7, 0.0, 0.1) & (0.6, 0.1, 0.2) & (0.4, 0.3, 0.2) \end{bmatrix}$$

In multi-criteria decision-making environments, the ideal solution can be denoted as $\alpha^* = (1, 0, 0)$, it's note that ideal solution generally does not exist in practice. Our decision can be obtained by calculating the distance between each alternative and the ideal solution. Then, by applying $D_E(A, B)$ the weighted distance measure between an alternative A_i and the ideal solution α^* can be expressed by the follows because all the alternatives and the ideal one are in the case of "otherwise" in $D_E(A, B)$:

$$D_E(A_i, \alpha^*) = \sum_{j=1}^{3} \omega_j \frac{4 + E^T(T_{A_i}(x_{ij}), 1) + E^F(F_{A_i}(x_{ij}), 0)}{8}.$$

Then, we can obtained $D_E(A_1, \alpha^*) = 063556, D_E(A_2, \alpha^*) = 0.56489, D_E(A_3, \alpha^*) = 0.59008, D_E(A_1, \alpha^*) = 0.56325$.

The ranking order of four alternatives is $A_4 \prec A_2 \prec A_3 \prec A_1$. Thus, we can see that the alternative A_4 is still the best choice among all the alternatives. So we get exactly the same results as in [35].

From the above result, we can obtain the same ranking order of alternatives as in [35]) by using the new distance measure proposed in the Definition 3.4. It shows that the new distance measure proposed in this paper are effective. Furthermore, the cross-entropy was proposed as a information measure for uncertain information which is different from distance measure. On the basis of the proof that cross-entropy proposed by [30] is a distance measure, we proposed the new distance measure according to the type-3 inclusion relation of SVNSs.

5. Conclusions

The information measure has become a hot issue in the progress of uncertain information processing. The research on information measure of SVBSs has an indispensable practical significance for us to apply SVNSs theory to decision-making, medical diagnosis and pattern recognition and so on. In the process of consulting the literature, it is found that most of the existing measures of SVNSs are based on the classical two inclusion relations(type-1 and type-2). These inclusion relations can be regarded as the consistency of the importance of the three membership functions of SVNSs. However, considering some practical problems, the importance of the three membership of SVNSs is not entirely the same. Based on such considerations, a new inclusion relation of SVNSs named as type-3 inclusion relation was proposed. In the paper, the existing cross-entropy was verified that it is not suitable for the type-3 inclusion relation about SVNSs by a example. Furthermore, we proposes a new distance measure based on the cross-entropy of SVNSs, and gives the corresponding similarity according to the matching function between distance measure and similarity. Finally, we applied the new distance measure to multi-criteria decision-making by a illustrative example and found the new distance measure is effective with the analysis of result. In short, The information measure based on the new inclusion relation will help to solve some practical problems. Therefore, we will consider other information measures of neutrosophic sets based on the new inclusion relation on one hand. On the other hand, we tend to study the application of information measures in information processing from the view of practical problems.

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