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Abstract. The more general form of soft separation axioms are defined in soft topological spaces and its interrelationship with existing soft separation axioms are studied. It was interesting to go through separation axiom as in [23] shown that there are limited relation between T_i axioms (i = 0,1,2,3). In this paper, it is shown that these axioms are stronger than the existing separation axioms in soft topological spaces.

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1. Introduction

The notion of soft set theory was introduced by Moldstov as a new approach for modelling uncertainties [20] to resolve many complicated problems existing in engineering, economics and enviormental areas. Mathematical tools like Fuzzy set, Rough set, Vague set, etc. have some difficulties and drawback while dealing with uncertainties which are explained in [19, 21]. Moldstov [20, 21] successfully applied soft set theory in game theory, operation research, soft analysis, probability, Perron integration, Riemann integration and theory of measurement. For interested reader who are looking for further work of soft sets one may refer the following paper for better understanding of soft set in decision making and fuzzy soft sets in [17].

Maji et.al. [19] have defined and studied different operations on soft sets like soft intersection, soft union, soft subset, etc. Then Pei-Miao [22] and Chen [9] extended the work of Maji et.al.[19]. The properties and applications of soft set theory were studied increasingly in [4]. Căgman and Enginoglu [7] redefined the operations of soft sets and constructed a uni-int decision-making method by using these new operations and developed new variant of soft set theory. Then to make compaction with the operations of soft sets easy, they presented soft matrix theory and set up the soft max and min decision-making method [8]. These decision-making methods can be successfully applied to many problems that contain uncertainties. Recently, in [23] Shabir-Naz introduced and studied the notion of soft topological spaces furthermore, Min [16] gave a short note on soft topological space which provides useful information for new readers.

Ningxin Xie [25] introduced the concept of soft points in soft set and defined neighbourhood of soft points and reveal the structure of soft topological spaces. Tantawy et.al. [24] defined seperation axioms on soft topological spaces by using the notion of soft points. A soft compact subset of a soft T_2-space defined by Tantawy et.al. [24] or Shabir et.al. [23] may not be soft closed. This is a major drawback of these soft seperation axioms. Although the work of G*ü*zide was more useful to link soft sets with real number and soft subspace for better understanding. Reader may refer following work for better understanding soft real numbers in [10, 11, 12, 13].

In this paper our purpose chiefly focussed on following aim. First, is to reintroduce the concept of soft separation axioms by using the concept of soft points given by Xie et.al. [25]. We compared the new soft separation axioms with the soft separation axioms defined by Shabir et.al. [23] and Tantawy et.al. [24] and study their various properties. Then these separation axioms may be used to explain Urysohn Metrization theorem on soft sets and can generalise the theory of soft spheres [10] with respect to soft points. In section 3, we redefined the definition of soft T_0 space and studied its relation with the existing soft T_0-axioms. In section 4, we introduced soft T_1 and their topological properties have been investigated. In section 5, we reintroduced soft T_2 and soft T_3 space and their properties and behaviour have been given briefly. Moreover, soft compact subset of a soft T_2-space defined by Shabir et.al. may not be soft closed. In fact, this results investigated in [3]. Second, some mistakes founded in some works which related to soft separation axioms were corrected in [1, 2]. The relationships between soft topological spaces and their parametric topological spaces in terms of soft separation axioms are studied in detail in [14, 15].

2. Preliminaries

Throughout this paper we assume X to be any non empty set which is referred as an initial universe set and E be the non empty set of parameters.

Definition 2.1 ([20])**.** A pair (F, E) is said to be a soft set over the initial universe X, if F is a mapping from E to $P(X)$, where $P(X)$ is the collection of subsets of X.

Definition 2.2 ([19])**.** Let (F, A) and (G, B) be two soft sets over a common universe X and over a common parameter E.

(i) Then (F, A) is a soft subset of (G, B), denoted by $(F, A)\tilde{\subset}(G, B)$, if

(a) $A \subseteq B$, and

(b) $F(e) \subseteq G(e)$, for each $e \in A$,

where A and B are subsets of E.

(ii) Two soft sets (F, E) and (G, E)over a common universe X are said to be soft equal, if $(F, E)\tilde{\subset}(G, E)$ and $(G, E)\tilde{\subset}(F, E)$.

(iii) A soft set (F, E) of X is called a null soft set, denoted by $\tilde{\varnothing}$, if $F(e) = \varnothing$, for each $e \in E$.

(iv) A soft set (F, E) of X is called an absolute soft set, denoted by $\tilde{X}$, if $F(e) = X$, for each $e \in E$.

(v) The union of two soft sets (F, A) and (G, B) over a common universe X is the soft set (H, C), where $C = A \cup B$, is defined by:

$$H(e) = \begin{cases} F(e), & \text{if, } e \in A - B \\ G(e), & \text{if, } e \in B - A \\ F(e) \cup G(e), & \text{if, } e \in B \cap A, \end{cases}$$

for each $e \in C$.

It is denoted by $(H, C) = (F, A)\tilde{\cup}(G, B)$.

(vi) The intersection of two soft sets (F, A) and (G, B) over a common universe X is the soft set (H, C), where $C = A \cap B$, is defined by $H(e) = F(e) \cap G(e)$, for each $e \in C$. It is denoted by $(H, C) = (F, A)\tilde{\cap}(G, B)$.

Definition 2.3 ([23])**.** Relative complement of a soft set (F, A), where $A \subseteq E$, is defined by $(F, A)^c$ =(F^c, A), where $F^c : A \to P(X)$ is a mapping given by $F^c(e) =$ X - $F(e)$, for each $e \in A$.

Definition 2.4 ([23, 26])**.** Let X be a non empty set, a collection of soft sets over X is said to be a soft topology on X if it satisfies the following axioms:

(i) $\tilde{\varnothing}, \tilde{X} \in \tau$,

(ii) Arbitrary union of soft sets in τ belongs to τ,

(iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.5. [23] Let (X, τ, E) be a soft topological space and (G, E) be a soft set over X.

(i) The soft closure of (G, E) is the soft set, $Cl(G, E)$, defined by:

$Cl(G, E) = \overline{(G, E)} = \tilde{\bigcap}\{(P, E) : (P, E)\text{is soft closed in } \tau \text{ and } \}(G, E)\tilde{\subseteq}(P, E)\}$.

(ii) $x \in (F, E)$, whenever $x \in F(e)$ for each $e \in E$ and $x \notin (F, E)$, whenever $x \notin F(e)$ for some $e \in E$.

(iii) (x, E) denotes the soft set over X for which $x(e) = \{x\}$, for all $e \in E$. It is referred to as absolute soft point.

Definition 2.6 ([25])**.** A soft set (F, E) is said to be a soft point over X, if there exist $t \in E$ and $x \in X$ such that

$$F(t) = \begin{cases} \{x\}, & \text{if } t = e, \\ \varnothing, & \text{if } t \in E - \{e\}. \end{cases}$$

In this case, x is called the support point of the soft point, $\{x\}$ is called the support set of the soft point and e is called the expressive parameter. Throughout this paper, we denote a soft point with support x and expressive parameter e by x_e and complement of x_e by x_e^c.

We can represent an absolute soft point (x, E) in terms of soft points which is represented by $(x, E) = \tilde{\bigcup}_{e \in E} x_e$.

Definition 2.7. Let (F, E) be a soft set over X. A soft point x_e belongs to (F, E), if $x \in F(e)$.

Remark 2.8. Two soft points x_e and y_t in X are distinct if and only if the support points are different or expressive parameters are different or both.

That is $x_e \neq y_t$ if and only if $x \neq y$ or $e \neq t$ or $x \neq y$ and $e \neq t$.

Definition 2.9 ([25])**.** Let (X, τ, E) be a soft topological space over X and (F, E) be any soft set over X. Let x_e be a soft point in X with support point x.

(i) (F, E) is called a neighborhood of x_e, if there exists $(G, E) \in \tau$ such that

$$x_e \tilde{\in} (G, E) \tilde{\subseteq} (F, E).$$

(ii) (F, E) is called an open neighborhood of x_e, if $(F, E) \in \tau$ and (F, E) is a neighborhood of x_e.

Definition 2.10 ([25])**.** Let (X, τ, E) be a soft topological space over X and let $\mathfrak{B} \subseteq \tau$. $\mathfrak{B}$ is called a base for (X, τ, E), if for any $(F, E) \in \tau$, there exists $\mathfrak{B}' \subseteq \mathfrak{B}$ such that

$$(F, E) = \tilde{\bigcup} \{(B, E) : (B, E) \tilde{\in} \mathfrak{B}'\}.$$

Definition 2.11 ([23])**.** Let (X, τ, E) be a soft topological space over X. Y be a non-empty subset of X. Then $\tau_Y = \{(F, E) \tilde{\cap} \tilde{Y} : (F, E) \in \tau\ \}$ is called the relative soft topology on Y and (Y, τ_Y, E) is called soft subspace topology of (X, τ, E).

Proposition 2.12 ([23])**.** (1) *Let (Y, τ_Y, E) be a soft subspace of a soft topological space (X, τ, E) and (F, E) be a soft open set in Y. If $\tilde{Y} \in \tau$, then $(F, E) \in \tau$.*

(2) *Let(Y, τ_Y, E) be a soft subspace of soft topological space (X, τ, E) and (F, E) be a soft set over X. Then*

(a) *(F, E) is soft open in Y if and only if $(F, E) = \tilde{Y} \tilde{\cap} (G, E)$, for some $(G, E) \in \tau$,*

(b) *(F, E) is soft closed in Y if and only if $(F, E) = \tilde{Y} \tilde{\cap} (G, E)$, for some soft closed set (G, E) in X.*

Proposition 2.13 ([23])**.** *Let (X, τ, E) ba a soft space over X. Then for each $\alpha \in E$, collection $\tau_\alpha = \{F(\alpha) : (F, E) \in \tau\}$ defines a topology on X.*

Proposition 2.13 shows that a soft topology gives a parameterized family of topologies on X.

Definition 2.14 ([18])**.** Let $SS(X)_A$ and $SS(Y)_B$ be collection of all soft sets in X and Y respectively, $f : X \to Y$ and $g : A \to B$ be mappings. Let $f_g : SS(X)_A \to SS(Y)_B$ be a mapping.

(i) If $(F, A) \tilde{\in} SS(X)_A$. Then the image of (F, A) under f_g, written as $f_g(F, A) = (f(F), g(A))$, is a soft set in $SS(Y)_B$ such that

$$f_g(F, A)(b) = \begin{cases} \cup_{a \in g^{-1}(b) \cap A} f(F(a)), & \text{if } g^{-1}(b) \cap A \neq \varnothing\ ; \\ \varnothing, & \text{otherwise.} \end{cases}$$

(ii) If $(G,B)\tilde{\in}SS(Y)_B$. Then the inverse image of (G,B) under f_g, written as $f_g^{-1}(G,B) = (f^{-1}(G), g^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$$f_g^{-1}(G,B)(a) = \begin{cases} f^{-1}(G(g(a))), & \text{if } g(a) \in B \text{ ;} \\ \varnothing, & \text{otherwise.} \end{cases}$$

The soft mapping f_g is called injective, if f and g are injective.The soft mapping f_g is called surjective, if f and g are surjective.

Definition 2.15. [5] Let $f_p : SS(X)_A \to SS(Y)_B$ and $g_q : SS(Y)_B \to SS(Z)_C$, then the composition of f_p and g_q is denoted by $f_p \circ g_q$ and defined by $f_p \circ g_q = f \circ g_{p \circ q}$.

Theorem 2.16 ([18])**.** *Let X and Y crisp sets and $f_g : SS(X)_A \to SS(Y)_B$, $(F_i, A)\tilde{\in}SS(X)_A$ and $(G_i, B)\tilde{\in}SS(Y)_B$ where i in any index set.*

(1) *If $(F_1, A)\tilde{\subseteq}(F_2, A)$, then $f_g(F_1, A)\tilde{\subseteq}f_g(F_2, A)$.*
(2) *If $(G_1, B)\tilde{\subseteq}(G_2, B)$, then $f_g^{-1}(G_1, B)\tilde{\subseteq}f_g^{-1}(G_2, B)$.*
(3) *$(F, A)\tilde{\subseteq}f_g^{-1}(f_g(F, A))$, the equality holds if f_g injective.*
(4) *$f_g(f_g^{-1}(F, A))\tilde{\subseteq}(F, A)$, the equality holds if f_g surjective.*
(5) *$f_g^{-1}((G, B)^c) = (f_g^{-1}(G, B))^c$.*
(6) *$f_g((F, A)\tilde{\cup}(F_1, A)) = f_g(F, A)\tilde{\cup}f_g(F_1, A)$.*
(7) *$f_g((F, A)\tilde{\cap}(F_1, A))\tilde{\subseteq}f_g(F, A)\tilde{\cap}f_g(F_1, A)$.*
(8) *$f_g^{-1}((G, B)\tilde{\cup}f_g^{-1}(G_1, B)) = f_g^{-1}(G, B)\tilde{\cup}f_g^{-1}(G_1, B)$.*
(9) *$f_g^{-1}((G, B)\tilde{\cap}(G_1, B)) = f_g^{-1}(G, B)\tilde{\cap}f_g^{-1}(G_1, B)$.*
(10) *$f_g(\tilde{\varnothing}_X) = \tilde{\varnothing}_Y$.*
(11) *$f_g^{-1}(\tilde{\varnothing}_Y) = \tilde{\varnothing}_X$.*

Definition 2.17 ([6])**.** Let X and Y be two nonempty sets and $(F, A)\tilde{\in}SS(X)_A$ and $(G, B)\tilde{\in}SS(Y)_B$. The Cartesian Product $(F, A)\times(G, B)$ is defined by $(F\times G, A\times B)$, where

$$(F \times G, A \times B) = F(e) \times G(k), \forall (e, k) \in A \times B$$

Definition 2.18 ([5])**.** The soft mappings $(p_q)_i, i \in \{1, 2\}$, is called soft projection mapping from $SS(X_1)_{A_1} \times SS(X_2)_{A_2}$ to $SS(X_i)_{A_i}$ and is defined by $(p_q)_i((F_1, A_1)\times(F_2, A_2)) = p_i(F_1 \times F_2)_{q_i(A_1\times A_2)} = (F_i, A_i)$, where $(F_1, A_1)\tilde{\in}SS(X_1)_{A_1}$ and $(F_2, A_2)\tilde{\in}SS(X_2)_{A_2}$ and $p_i : X_1 \times X_2 \to X_i$ and $q_i : A_1 \times A_2 \to A_i$ are projection mapping in classical meaning.

Definition 2.19. [26] A soft topological space (X, τ, E) is called soft compact space if each soft open cover of $\tilde{X}$ has a finite soft subcover.

Shabir et.al. [23] defined the notion of soft T_i-axiom $(i = 0, 1, 2, 3)$ in a soft topological space by using absolute soft points. Tantawy [24] introduced new set for soft T_i-axioms. In these axioms they consider distinct soft points with the same expressive parameters. Hence, for the sake of convenience and distinguish various notions with same name, throughout this paper the notion of soft T_i-axiom defined by Shabir et.al. would be referred as absolute soft T_i axiom $(i = 0,\ 1,\ 2,\ 3)$ and that by Tantawy et. al. as parametric soft T_i axiom $(i = 0,\ 1,\ 2,\ 3)$.

Definition 2.20 ([23])**.** Let (X, τ, E) be a soft topological space over X. Then (X, τ, E) is called:

(i) an absolute soft T_0-space, if for any $x, y \in X$ such that $x \neq y$, there exist soft open sets (F, E) and (G, E) such that

$$x \in (F, E),\ y \notin (F, E) \text{ or } y \in (G, E),\ x \notin (G, E),$$

(ii) an absolute soft T_1-space, if for any $x, y \in X$ such that $x \neq y$, there exist soft open sets (F, E) and (G, E) such that

$$x \in (F, E),\ y \notin (F, E) \text{ and } y \in (G, E),\ x \notin (G, E),$$

(iii) an absolute soft T_2-space, if for any $x, y \in X$ such that $x \neq y$, there exist soft open sets (F, E) and (G, E) such that

$$x \in (F, E),\ x \in (F, E) \text{ and } (G, E)\tilde{\cap}(F, E) = \tilde{\varnothing},$$

(iv) an absolute soft regular space, if for each soft closed set (G, E) in X and each $x \in X$ such that $x \notin (G, E)$, there exist soft open sets (F_1, E) and (F_2, E) such that

$$x \in (F_1, E),\ (G, E)\tilde{\subseteq}(F, E) \text{ and } (F_1, E)\tilde{\cap}(F_2, E) = \tilde{\varnothing},$$

(v) an absolute T_3 space, if it is soft regular and soft T_1 space.

Definition 2.21 ([24])**.** A soft topological space (X, τ, E) is said to be:

(i) parametric soft T_0-space, if for every two soft points x_e, y_e such that $x \neq y$, there exists $(G, E) \in \tau$ such that $x_e\tilde{\in}(G, E), y_e\tilde{\notin}(G, E)$ or there exists $(H, E) \in \tau$ such that $y_e\tilde{\in}(H, E), x_e\tilde{\notin}(H, E)$,

(ii) parametric soft T_1-space, if for every two soft points x_e, y_e such that $x \neq y$, there exists $(G, E), (H, E) \in \tau$ such that

$$x_e\tilde{\in}(G, E),\ y_e\tilde{\notin}(G, E) \text{ and } y_e\tilde{\in}(H, E),\ x_e\tilde{\notin}(H, E),$$

(iii) parametric soft T_2-space, if for every two soft points x_e, y_e such that $x \neq y$, there exists $(G, E), (H, E) \in \tau$ such that

$$x_e\tilde{\in}(G, E),\ y_e\tilde{\in}(H, E) \text{ and } (G, E)\tilde{\cap}(H, E) = \tilde{\varnothing},$$

(iv) parametric soft T_3-space, if it is a soft regular space and a parametric soft T_1-space.

In these sections, we will introduce new notions of soft separation axioms, which depends on the soft points defined by Ningxin [25] and study its various properties. We also compare these soft separation axioms with the corresponding soft separation axioms introduced by Shabhir et al. [23] and Tantawy et al. [24].

3. Soft T_0-Spaces

Definition 3.1. A soft topological space (X, τ, E) is said to be soft T_0-space, if for each pair of distinct soft points x_t and y_u, where $x, y \in X$ and $t, u \in E$, there exist soft open sets (F, E) or (G, E) containing one of the soft point and not containing the other soft point.

Theorem 3.2. *A soft topological space (X, τ, E) is soft T_0-space if and only if for each pair of distinct soft points x_t and y_u,there exists a soft open neighborhoods (U, E) and (V, E) containing one of the soft point and not containing the other soft point.*

Proof. Let (X, τ, E) be a soft T_0-space. Then there exists a soft open set (F, E) and (G, E) such that $x_t \tilde{\in} (F, E)$ but $y_u \tilde{\notin} (F, E)$ or $y_u \tilde{\in} (G, E)$ and $x_t \tilde{\notin} (G, E)$. Since every soft open set is a soft neighborhood of each of its soft point, this implies that (F, E) is a soft open neighborhood of x_t which does not contain y_u.

Conversely, suppose that if $x_t \neq y_u$, then there exists soft open neighborhoods (U, E) such that $x_t \tilde{\in} (U, E)$ but $y_u \tilde{\notin} (U, E)$ or $y_u \tilde{\in} (U, E)$ but $x_t \tilde{\notin} (U, E)$. Since (U, E) is soft open neighborhood, there exists such that $(F, E) \tilde{\subseteq} (U, E)$. Then (X, τ, E) is a soft T_0-space □

The above theorem shows that each pair of soft points is soft topologically distinguishable in a soft T_0-space.

Theorem 3.3. *A soft topological space,* (X, τ, E) *is soft* T_0 *if and only if*

$$\overline{x_t} = \overline{y_u} \Rightarrow x_t = y_u$$

Proof. Let (X, τ, E) be a soft T_0-space. Suppose that $\overline{x_t} = \overline{y_u}$. If possible, let us assume that $x_t \neq y_u$. Since (X, τ, E) is soft T_0, there exists a soft neighborhood N_E of one of the soft points, say, x_t which does not contain y_u. This implies $x_t \notin \overline{y_u}$. Then $\overline{x_t} \neq \overline{y_u}$ which is a contradiction. Thus $x_t = y_u$.

Conversely, let $x_t \neq y_u$ and suppose if possible X is not soft T_0-space then every neighborhood of x_t contains y_u and every neighborhood of y_u contains x_t. This implies $x_t \tilde{\in} \overline{y_u}$, and $y_u \tilde{\in} \overline{x_t}$ which in turn implies that $\overline{x_t} \tilde{\subseteq} \overline{y_u}$, and $\overline{y_u} \tilde{\subseteq} \overline{x_t}$ It follows $\overline{x_t} = \overline{y_u}$ which implies $x_t = y_u$.This yields a contradiction. Then (X, τ, E) is a soft T_0-space. □

Example 3.4. Suppose $X = \mathbb{R}$, E=$\{e_1, e_2\}$, define the soft sets $F_x : E \to 2^X$ by

$$F_x(e_i) = \begin{cases} R_x^+, & \text{if i = 1} \\ \{x\} \cup ((x - \epsilon, x + \epsilon) \cap \mathbb{Q}^c), & \text{if i = 2,} \end{cases}$$

where $R_x = (x, \infty)$ and $R_x^+ = R_x \cup \{x\}$ and $\mathbb{Q}^c$, collection of all irrationals.

Let (X, τ, E) be the soft topological space generated by the collection of soft set $\mathfrak{S} = \{(F_x, E) : x \in \mathbb{R}\}$. Then (X, τ, E) is a soft T_0 space.

Theorem 3.5. *soft* $T_0 \Rightarrow$ *absolute soft* T_0*.*

Proof. Let (X, τ, E) be a soft T_0-space and x and y be two distinct absolute soft points. Then $x = \bigcup_{e \in E} x_e$ and $y = \bigcup_{e \in E} y_e$. Since (X, τ, E) is soft T_0, the following cases arise:

(1) **Case 1.** For each $e \in E$, we can find soft open set (U_e, E) containing x_e and not containing y_{e_i} for each $e_i \in E$. Then $x \in \bigcup_{e \in E} (U_e, E)$ but $y \notin \bigcup_{e \in E} (U_e, E)$.

(2) **Case 2.** There exist a $e_0 \in E$ such that for each $e \in E$, we do not have a soft open set (U, E) such that $x_{e_0} \tilde{\in} (U, E)$ but $y_e \tilde{\notin} (U, E)$. In this case, since x_{e_0} and y_e are two distinct soft points for $e \in E$, there exist a soft open set,(V_e, E) such that the soft point $x_{e_0} \tilde{\notin} (V_e, E)$ but $y_e \tilde{\in} (V_e, E)$. Then $(V, E) = \bigcup_{e \in E} (V_e, E)$ is a soft open set such that $y \in (V, E)$ but $x \notin (V, E)$ as $x_{e_0} \notin (V_e, E)$ for each $e \in E$.

Hence (X, τ, E) is absolute soft T_0. □

Remark 3.6. Converse of the Theorem 3.5 may not be true as illustrated in the following example.

Example 3.7. Let $X = \{a, b, c\}$ and $E = \{e_1, e_2\}$.
Let $\tau = (\tilde{\varnothing}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E))$, where

$F_1(e_1) = \{a\}, F_1(e_2) = \{a\}$,
$F_2(e_1) = \{b\}, F_2(e_2) = \{b\}$,
$F_3(e_1) = \{c\}, F_3(e_2) = \{c\}$,
$F_4(e_1) = \{a, c\}, F_4(e_2) = \{c, a\}$,
$F_5(e_1) = \{a, b\}, F_5(e_2) = \{b, a\}$,
$F_6(e_1) = \{b, c\}, F_6(e_2) = \{b, c\}$.

Then (X, τ, E) is an absolute soft T_0-space. Since a_{e_1} and a_{e_2} are two distinct soft points in (X, τ, E) but there does not exist any soft open set (F_i, E) $(i = 1, 2, 3, 4, 5, 6)$ such that it contains any one of the soft points a_{e_i} but not the other.

Remark 3.8. Following examples show that parametric soft T_0-axiom and absolute soft-T_0 axiom are not comparable.

Example 3.9. Let (X, τ, E) be a soft topological space, where $X = \{x, y\}$ and $E = \{e_1, e_2\}$. Let $\tau = \{\tilde{\varnothing}, \tilde{X}, (F_1, E)\}$, where (F_1, E) is the soft set defined by $F_1(e_1) = \{x\}; F_1(e_2) = \{y\}$. Then (X, τ, E) is a parametric soft T_0 space, but it is not absolute soft T_0 because there does not exist any soft open set containing one of the absolute point and not the other.

Example 3.10. Let (X, τ, E) be a soft topological space, where $X = \{x, y\}$, $E = \{e_1, e_2\}$. Let $\tau = \{\tilde{\varnothing}, \tilde{X}, (F_1, E)\}$ where (F_1, E) is defined by $F_1(e_1) = X; F_1(e_2) = \{y\}$. Then (X, τ, E) is an absolute soft T_0-space. But the soft topological space (X, τ, E) is not parametric soft T_0 because there does not exist a soft open set containing either of the soft points x_{e_1} or y_{e_1} and not the other.

Theorem 3.11. *Every soft T_0-space is parametric soft T_0.*

Proof. Let (X, τ, E) be a soft T_0-space and let $x_e \neq y_e$, where $x \neq y$. Since (X, τ, E) is soft T_0, there exist a soft open set (U, E) such that $x_e \tilde{\in} (U, E)$ but $y_e \tilde{\notin} (U, E)$ or $y_e \tilde{\in} (U, E)$ but $x_e \tilde{\notin} (U, E)$. Hence (X, τ, E) is a parametric soft T_0. □

The following example shows that the converse of Theorem 3.11 may not be true.

Example 3.12. Let (X, τ, E) be the soft topological space defined in Example 3.9. We can see that there does not exists a soft open set containing one of the soft points among x_{e_2} and y_{e_1} and not containing the other. This shows (X, τ, E) is not soft T_0 -space.

4. Soft T_1-Spaces

Definition 4.1. A soft topological space (X, τ, E) is said to be soft T_1, if for each pair of distinct soft points, say x_t and y_u, there exist soft open sets (U, E) and (V, E) such that $x_t \tilde{\in} (U, E)$ but $y_u \tilde{\notin} (U, E)$ and $y_u \tilde{\in} (V, E)$ but $x_t \tilde{\notin} (V, E)$.

Theorem 4.2. *A soft topological space (X, τ, E) is soft T_1 if and only if every soft point is closed.*

Proof. Let X be a soft T_1-space and y_u be a soft point in X and $x_t \tilde{\in} y_u^c$. Since X is soft T_1, there exist a soft open set (U_x, E) such that $x_t \tilde{\in} (U_x, E)$ and which does not contain y_u. This implies that $x_t \tilde{\in} (U_x, E) \tilde{\subseteq} y_u^c$. Which in turn implies that y_u^c is a soft neighborhood of each of its soft point and hence y_u is soft closed.

Conversely, let $x_t \neq y_u$ and every soft point is soft closed in (X, τ, E). Then $(U, E) = y_u^c$ and $(V, E) = x_t^c$ are soft open set such that $x_t \tilde{\in} (U, E)$ and $y_u \tilde{\in} (V, E)$. □

Example 4.3. Let $X = \mathbb{R}$, E=$\{e_1, e_2\}$. Let $(F_{x,y}, E), x < y$ be the soft set on $\mathbb{R}$ defined by

$$F_{x,y}(e_1) = \mathbb{R} \setminus \{x\} \text{ and } F_{x,y}(e_2) = \mathbb{R} \setminus \{y\}.$$

Let τ be the soft topological space generated by the subbasis

$$\mathcal{S} = \{(F_{x,y}, E), x < y : x, y \in \mathbb{R}\}.$$

Then (X, τ, E) is a soft T_1-space.

Theorem 4.4. *Every soft T_1-space is a soft T_0-space.*

Proof. Proof is obvious. □

Remark 4.5. Converse of Theorem 4.4 may not be true (See Example 3.4).

Theorem 4.6. *Every soft T_1-space is an absolute soft T_1-space.*

Proof. Let (X, τ, E) be a soft T_1-space and let $x \neq y$ be two distinct absolute soft points. Then $x_{e_i} \neq y_{e_i}$ for each $e_i \in E$. Since (X, τ, E) is soft T_1-space, there exist soft open sets (U_i, E), (V_i, E) such that $x_{e_i} \tilde{\in} (U_i, E)$ but $y_{e_i} \tilde{\notin} (U_i, E)$, for each $e_i \in E$ and $y_{e_i} \tilde{\in} (V_i, E)$ but $x_{e_i} \tilde{\notin} (V_i, E)$, for each $e_i \in E$. Let $(U, E) = \bigcup\limits_{e_i \in E} (U_i, E)$ and $(V, E) = \bigcup\limits_{\forall e_i \in E} (V_i, E)$. Then $x \in (U, E)$ but $y \notin (U, E)$ and $y \in (V, E)$ but $x \notin (V, E)$. This shows that (X, τ, E) is absolute soft T_1. □

Converse of the Theorem 4.6 may not be true as shown in the following example.

Example 4.7. Let (X, τ, E) be the soft topology defined in Example 3.7. Since each absolute soft points is a soft open set in this soft topology, (X, τ, E) is an absolute soft T_1-space . It is already established that (X, τ, E) is not a soft T_0-space, it follows that (X, τ, E) is not a soft T_1-space.

Remark 4.8. The following example shows that every absolute soft T_1-space may not be a parametric soft T_1-space.

Example 4.9. Let (X, τ, E) be a soft topological space, where $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{\varnothing}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ such that (F_i, E) $(i = 1, 2, 3)$ is defined by:

$F_1(e_1) = X; F_1(e_2) = \{y\}$,
$F_2(e_1) = \{x\}; F_2(e_2) = X$,

$F_3(e_1) = \{x\}; F_3(e_2) = \{y\}$.
Then (X, τ, E) is absolute soft T_1-space, but it is not parametric soft T_1 because x_{e_1} and y_{e_1} are two distinct soft points which can not be separated by soft open sets.

Theorem 4.10. *Every soft T_1-space is a parametric soft T_1-space.*

Proof. Let (X, τ, E) be a soft T_1-space and let $x_e \neq y_e$ be two distinct soft points with same expressive parameters. Since X satisfies the soft T_1-axiom, there exist soft open sets $(U, E), (V, E)$ such that $x_e \tilde{\in} (U, E)$ but $y_e \tilde{\notin} (U, E)$ and $y_e \tilde{\in} (V, E)$ but $x_e \tilde{\notin} (V, E)$. Then (X, τ, E) is a parametric soft T_1-space. □

Remark 4.11. The following example shows that every parametric soft T_1-space may not be soft T_1 also every parametric soft T_1-space may not be absolute soft T_1.

Example 4.12. Let (X, τ, E) be a soft topological space as defined in (Example 3.7 [24]), where $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{\varnothing}, \tilde{X}, (F_1, E), (F_2, E)\}$ such that
$F_1(e_1) = \{x\}; F_1(e_2) = \{y\}$,
$F_2(e_1) = \{y\}; F_2(e_2) = \{x\}$.
Then (X, τ, E) is a parametric soft T_1-space, which is neither soft T_1 nor absolute soft T_1.

5. Soft T_2 and soft T_3-Spaces

Definition 5.1. A soft topological space, (X, τ, E) is said to be a soft T_2-space, if for each pair of distinct soft points x_t, y_u there exist soft open sets (V, E) and (U, E) such that $x_t \tilde{\in} (U, E)$, $y_u \tilde{\in} (V, E)$ and $(U, E) \tilde{\cap} (V, E) = \tilde{\varnothing}$

Theorem 5.2. *Every soft T_2-space is a soft T_1-space.*

Proof. Obvious by definition. □

Converse of Theorem 5.2 may not be true as exhibited in the following example.

Example 5.3. The soft topological space (X, τ, E) defined in Example 4.3 is a soft T_1-space but it is obvious that (X, τ, E) is not a soft T_2-space.

Theorem 5.4. *If (X, τ, E) is a soft T_2-space with a finite set of parameters E then (X, τ, E) is an absolute soft T_2-space.*

Proof. Let (X, τ, E) be a soft T_2-space with the set of parameter E is finite. Since X is soft T_2, for each pair of distinct soft points x_{e_i} and y_{e_j}, there exist soft open sets (U_{e_i}, E) and (V_{e_j}, E) such that $x_{e_i} \tilde{\in} (U_{e_j}, E), y_{e_j} \tilde{\in} (V_{e_j}, E)$ and $(U_{e_j}, E) \tilde{\cap} (V_{e_j}, E) = \tilde{\varnothing}$, for each $e_j \in E$. Let $(V_i, E) = \tilde{\bigcap}_{e_j \in E} (U_{e_j}, E) \in \tau$ and $(V_j, E) = \tilde{\bigcup}_{e_j \in E} (V_{e_j}, E)$. Then clearly, $(U_i, E) \tilde{\cap} (V_j, E) = \tilde{\varnothing}$ such that $y \tilde{\in} (V_j, E)$ and $x_{e_i} \tilde{\in} (U_i, E)$. Thus, the absolute point $x \in \tilde{\bigcup}_{e_i \in E} (U_i, E)$ and the absolute point $y \in \tilde{\bigcap}_{e_j \in E} (V_j, E)$ such that these two open sets are disjoint. So X is an absolute soft T_2-space. □

Following example shows that converse of Theorem 5.4 may not be true.

Example 5.5. Let $X = \mathbb{R}$ and $E = \mathbb{N}$. Let $(F^i_{x,y}, E), i \in E$ be the soft set defined by:

$$F^i_{x,y}(n) = \begin{cases} (x-\delta, x+\delta), & \text{if n = i} \\ \varnothing, & \text{otherwise,} \end{cases}$$

where $\delta = \begin{cases} \dfrac{\mid x-y \mid}{3}, & \text{if } x \neq y \\ \text{a fixed finite positive number,} & \text{if x = y.} \end{cases}$

Let τ be the soft topology generated by the collection $\mathcal{B} = \{(F^i_{x,y}, E) : i \in E$ and $x, y \in \mathbb{R}\}$. Then (X, τ, E) is a soft T_2-space and also an an absolute soft T_2-space. This is an example of a soft T_2-space which is a non discrete soft topological space with the set of parameters is infinite. This is also an absolute soft T_2-space.

Example 5.6. Let $X = \mathbb{R}$ and $E = \mathbb{N}$. Let $(F^i_{x,y}, E), i \in E$ be the soft set defined by:

$$F^i_{x,y}(n) = \begin{cases} (x-\dfrac{\delta}{2^i}, x+\dfrac{\delta}{2^i}), & \text{if n = i} \\ \varnothing, & \text{otherwise,} \end{cases}$$

where $\delta = \begin{cases} \dfrac{\mid x-y \mid}{3}, & \text{if } x \neq y \\ \text{a fixed finite positive number,} & \text{if x = y.} \end{cases}$

Let τ be the soft topology generated by the collection $\mathcal{B} = \{(F^i_{x,y}, E) : i \in E$ and $x, y \in \mathbb{R}\}$. Hence, (X, τ, E) is a soft T_2-space but not an absolute soft T_2-space.

Example 5.7. [24] Let (X, τ, E) be the soft topology defined in Example 4.12. Then, (X, τ, E) is a parametric soft T_2-space. It is clear from the definition of (F_1, E) and (F_2, E) that (X, τ, E) is not an absolute soft T_2-space.

Theorem 5.8. *Every soft T_2-space is parametric soft T_2-space.*

Proof. Let (X, τ, E) be a soft T_2-space and let $x_e \neq y_e$. Since $(X.\tau.E)$ is a soft T_2-space, there exist soft open sets $(U, E), (V, E)$ such that $x_e \tilde{\in} (U, E)$, $y_e \tilde{\in} (V, E)$ and $(U, E) \tilde{\cap} (V, E) = \tilde{\varnothing}$. Hence (X, τ, E) is parametric soft T_2. □

Converse of Theorem 5.8 may not be true as it is exhibited by the following example.

Example 5.9. Let (X, τ, E) be the soft topological space defined in the Example 4.12. Then (X, τ, E) is not soft T_2, because the soft points x_{e_1} and y_{e_2} can not be separated by disjoint soft open sets.

Theorem 5.10. *In a soft T_2-space, every soft compact subset is soft closed.*

Proof. Let (X, τ, E) be a soft T_2-space and (P, E) be a soft compact set in X. It is sufficient to prove that $(P, E)^c$ is soft open. Let $x_t \tilde{\in} (P, E)^c$. Then for each $y_u \tilde{\in} (P, E)$, there exist disjoint soft open sets (U_{y_u}, E) and (V_{y_u}, E) such that

$$x_t \tilde{\in} (U_{y_u}, E),\ y_u \tilde{\in} (V_{y_u}, E) \ \text{ and } \ (U_{y_u}, E) \tilde{\cap} (V_{y_u}, E) = \tilde{\varnothing}.$$

Then the collection $\mathcal{C} = \{(V_{y_u}, E) : y_u \tilde{\in} (P, E)\}$ forms a soft open covering for (P, E). Since (P, E) is soft compact, $\mathcal{C}_k = \{(V_{(y_i)_{u_i}}, E) : (y_i)_{u_i} \tilde{\in} (P, E)$ and $i = 1, 2, ..., k\}$ is a finite subcover for (P, E). For each $(V_{(y_i)_{u_i}}, E) \in \mathcal{C}_k$, $(U_{(y_i)_{u_i}}, E)$

be the corresponding soft open set containing x_t. Let $(U,E) = \tilde{\cap}_{i=1}^{k}(U_{(y_i)_{u_i}},E)$ and (U,E) is a soft open set containing x_t and $(U,E)\tilde{\cap}(P,E) = \tilde{\varnothing}$. This implies that $(U,E)\tilde{\subseteq}(P,E)^c$. This shows that for each $x_t\tilde{\in}(P,E)^c$, there exist a soft open set (U,E) such that $x_t\tilde{\in}(U,E)\tilde{\subseteq}(P,E)^c$, which in turn implies that $(P,E)^c$ is soft open. □

Remark 5.11. Soft compact sets in an absolute soft T_2-space may not be soft closed as shown in the following example.

Example 5.12. Let $X = \{a,b,c\}$, $E = \{e_1,e_2\}$ and
$\tau = \{\tilde{\varnothing},\tilde{X},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E),(F_8,E),$
$(F_9,E),(F_{10},E)\}$, where the soft set (F_i,E), $(i = 1,2,3,4,5,6,7,8,9,10)$ is defined by:

$F_1(e_1) = \{a\}; F_1(e_2) = \{a\}$,
$F_2(e_1) = \{b\}; F_2(e_2) = \{b\}$,
$F_3(e_1) = \{c\}; F_3(e_2) = \{c\}$,
$F_4(e_1) = \{a,c\}; F_4(e_2) = \{c,a\}$,
$F_5(e_1) = \{a,b\}; F_5(e_2) = \{b,a\}$,
$F_6(e_1) = \{b,c\}; F_6(e_2) = \{b,c\}$,
$F_7(e_1) = \{a\}; F_7(e_2) = \varnothing$,
$F_8(e_1) = \{a,b\}; F_8(e_2) = \{b\}$,
$F_9(e_1) = \{a,c\}; F_9(e_2) = \{c\}$,
$F_{10}(e_1) = X; F_{10}(e_2) = \{b,c\}$.

Then (X,τ,E) is an absolute soft T_2-space. The soft set (F_7,E) is soft compact in (X,τ,E) but it is not soft closed.

Remark 5.13. Soft compact sets in a parametric soft T_2-space may not be soft closed as shown in the following example.

Example 5.14. Let $X = \{a,b\}$, $E = \{e_1,e_2\}$ and
$\tau = \{\tilde{\varnothing},\tilde{X},(F_1,E),(F_2,E),(F_3,E),(F_4,E)\}$, where the soft set $(F_i,E), i = 1,2,3,4$ are defined by:

$F_2(e_1) = \{b\}; F_2(e_2) = \{a\}$,
$F_3(e_1) = \{a\}; F_3(e_2) = \varnothing$,
$F_4(e_1) = X; F_4(e_2) = \{a\}$.

Then (X,τ,E) is a parametric soft T_2-space. The soft set (F_3,E) is soft compact in (X,τ,E) but it is not soft closed.

The above two examples show that a soft compact space in parametric soft T_2-space as well as in absolute soft T_2-space may not be soft closed. But in soft T_2-space every soft compact set is soft closed.

Theorem 5.15. *Let (X,τ,A) be a soft topological space and (Y,δ,B) be a soft T_i-space(i=0,1,2). If the soft function $f_p : (X,\tau,A) \to (Y,\delta,B)$ is soft continuous injective function, where $p : A \to B$ and $f : X \to Y$ then (X,τ,A) is a soft T_i-space $(i = 0,1,2)$.*

Proof. We prove the theorem for $i = 2$, for $i = 0$ and 1, the proof is same as for $i = 2$. Let $f_p : (X,\tau,A) \to (Y,\delta,B)$ be a soft continuous injective function from a soft topological space (X, τ, A) into a soft T_2-space (Y, δ, B). Let x_t, y_u be two distinct soft points in (X, τ, A). Since f_p is soft injective, $f_p(x_t), f_p(y_u)$ are two distinct soft points in the soft T_2-space (Y, δ, B). Then there exists soft disjoint open sets (U_1, E) and (U_2, E) such that $f_p(x_t)\tilde{\in}(U_1, E)$ and $f_p(y_u)\tilde{\in}(U_2, E)$. Thus $f_p^{-1}(U_1, E)$ is soft open in X containing x_t and $f_p^{-1}(U_2, E)$ is soft open in X containing y_u. Also,$f_p^{-1}(U_1, E)\tilde{\cap}\ f_p^{-1}(U_2, E) = f_p^{-1}((U_1, E)\tilde{\cap}(U_2, E)) = \tilde{\varnothing}$. So (X, τ, E) is soft T_2-space. □

Definition 5.16 ([24]). A soft topological space (X, τ, E) is said to be soft regular space, if for all closed soft sets (F, E) and soft points x_e such that $x_e\tilde{\notin}(F, E)$, there exist $(G, E), (H, E) \in \tau$ such that

$$x_t\tilde{\in}(G, E),\ (F, E)\tilde{\subseteq}(H, E) \text{ and } (G, E)\tilde{\cap}(H, E) = \tilde{\varnothing}.$$

Definition 5.17. A soft T_1-space (X, τ, E) is said to be soft T_3-space, if it is soft regular space.

Example 5.18. Every soft discrete topological space is soft T_3.

The following examples shows that absolute soft T_3 and soft T_3 axioms are independent to each other.

Example 5.19. The soft topological space (X, τ, E) defined in Example 3.7 is an absolute soft T_3-space but it is not a soft T_3 because it is not a soft T_1-space

Example 5.20. Let (X, τ, E) be the soft discrete topological space defined on the universal set $X = \{a, b, c\}$ and $E = \{0, 1\}$ be the set of parameters. Then each (F, E) is soft open as well as soft closed. Consider the soft set (G, E) defined by:

$$G(0) = X,\ G(1) = \varnothing.$$

. Then (G, E) is soft closed and the absolute point $a \notin (G, E)$. Now we cannot find a pair of soft open sets which separates the absolute soft point and soft closed set, (G, E). Thus (X, τ, E) is not an absolute soft T_3-space.

The following examples show that parametric soft T_3-axiom and absolute soft T_3-axiom are independent to each other.

Example 5.21. Let (X, τ, E) be the soft topological space defined in Example 4.12. Since (X, τ, E) is parametric soft T_1 and soft regular space, it satisfies the parametric soft T_3-axiom. However it is obvious that (X, τ, E) is not satisfying the absolute soft T_3-axiom.

Example 5.22. The soft topological space (X, τ, E) defined in Example 9 in [23] is an absolute soft T_3-space. This soft space is not parametric soft T_3, because $h_{2_{e_2}}$ and $h_{3_{e_2}}$ are two distinct soft points which cannot be separated by any soft open sets. This implies (X, τ, E) is not parametric soft T_1-space. Then absolute soft T_3-axiom does not imply parametric soft T_3-space.

Theorem 5.23. *A soft topological space (X, τ, E) is soft T_3 then X is soft T_2-space.*

Proof. It is sufficient to prove

$$\text{soft } T_3 \Leftrightarrow \text{soft regular} + \text{soft } T_1 \Rightarrow \text{soft } T_2$$

Let X be a soft T_3-space and $x_t \neq y_u$. Since (X, τ, E) is soft T_1-space, every soft singleton are soft closed. Then $x_t \tilde{\notin} Cl\{y_u\}$. Since (X, τ, E) is soft regular, there exist disjoint soft open sets, say,(U, E) and (V, E) such that $x_t \tilde{\in} (U, E)$ and $y_u \tilde{\in} Cl\{y_u\} \tilde{\subseteq} (V, E)$. Thus (X, τ, E) is a soft T_2-space. □

Theorem 5.24. *Let (X, τ, E) be a soft T_i-space $(i = 0, 1, 2, 3)$, then for each $\alpha \in E$, (X, τ_α) is a T_i-space $(i = 0, 1, 2, 3)$.*

Proof. We prove the theorem for $i = 0$ and for $i = 1, 2, 3$, the proof follows in similar way. Let (X, τ, E) be a soft T_0-space and $x_\alpha \neq y_\alpha$, for $\alpha \in E$, be two distinct soft points in X. Then there exists a soft open set (U, E) such that one of the soft point belongs to the soft set and not containing the other. Let $x_\alpha \tilde{\in} (U, E)$ and $y_\alpha \tilde{\notin} (U, E)$, which in turn implies that $x \in U(\alpha)$ but $y \notin U(\alpha)$. Thus (X, τ_α) is T_0-space. □

Theorem 5.25. *Let $\{(X, T_\alpha) : \alpha \in E\}$ be a collection of topological spaces. Then there exists a soft topology τ on X such that $\tau_\alpha = T_\alpha$ for each $\alpha \in E$.*

Proof. For each $U \in T_\alpha$, define a soft set $(F_{U,\alpha}, E)$ by:

$$F_{U,\alpha} : E \to 2^X \text{ such that } F_{U,\alpha}(\alpha) = U \text{ and } F_{U,\alpha}(\beta) = \varnothing, \text{ for each } \beta \neq \alpha.$$

Let $\mathcal{S}_\alpha = \{(F_{U,\alpha}, E) : U \in T_\alpha\}$ and $\mathcal{S} = \tilde{\bigcup}_{\alpha \in E}\{\mathcal{S}_\alpha\}$. Let τ be the soft topology generated by the collection $\mathcal{S}$. Then it is obvious that $\tau_\alpha = T_\alpha$, for each $\alpha \in E$. □

Note: The above defined soft topology is called the topologically generated soft topological space.

(X, τ_{e_i}) is a T_0-space then there exist open sets U and V such that $x \in U$ but $y \notin U$ or $y \in V$ but $x \notin V$. Then $x_{e_i} \tilde{\in} (F_{U,e_i}, E)$ but $y_{e_i} \tilde{\notin} (F_{U,e_i}, E)$ or $y_{e_i} \tilde{\in} (F_{V,e_i}, E)$ but $x_{e_i} \tilde{\notin} (F_{V,e_i}, E)$.

Theorem 5.26. *Let $\{(X, T_\alpha) : \alpha \in E\}$ be a collection of T_i-spaces $(i = 0, 1, 2, 3)$. Then there exists a soft topology τ on X such that (X, τ, E) is a soft T_i-space $(i = 0, 1, 2, 3)$ with $\tau_\alpha = T_\alpha$ for each $\alpha \in E$.*

Proof. We prove the theorem for $i = 0$ and for $i = 1, 2, 3$, the proof follows in similar way. Let (X, τ, E) be the soft topological space defined in Theorem 5.25 and $x_{e_1} \neq y_{e_2}$ be any two distinct soft points

Case 1. $e_1 \neq e_2$. Then by Proposition 5.25, there exists a soft open set $(F_{X,e_1}, E) \in \mathcal{S}_{e_1}$ such that $x_{e_1} \tilde{\in} (F_{X,e_1}, E)$ but $y_{e_2} \tilde{\notin} (F_{X,e_1}, E)$.

Case 2. $e_1 = e_2 (= e$, say$)$. Then $x \neq y$, since (X, T_e) is a T_0-space, there exists an open set U which contains one of the point and not the other. Let $x \in U$ but $y \notin U$. Then $x_e \tilde{\in} (F_U, E)$ but $y_e \tilde{\notin} (F_U, E)$. Thus (X, τ, E) is a soft T_0-space. □

Following Theorem shows that soft T_i-spaces $(i = 0, 1, 2, 3)$ satisfies hereditary property.

Theorem 5.27. *If X is soft-T_i space(i=0,1,2,3) then every subspace of X is soft-T_i space(i=0,1,2,3).*

Proof. We prove the theorem for $i = 2$ and for $i = 0, 1, 3$, the proof follows in similar way. Let (X, τ, E) be a soft T_2-space and let A be a subset of X. Let a_t, b_u be two distinct soft points in A. Then these soft points are distinct soft points in X. Since (X, τ, E) is soft T_2, there exist nonempty disjoint soft open sets (U, E) and (V, E) containing a_t and b_u, respectively in (X, τ, E). Thus $(U, E)\tilde{\cap}\tilde{A}$ and $(V, E)\tilde{\cap}\tilde{A}$ are disjoint soft open sets containing a_t and b_u, respectively in (A, τ_A, E). So (A, τ_A, E) is soft T_2-space. □

Following Theorem shows that soft T_i-spaces $(i = 0, 1, 2, 3)$ satisfies expansive property.

Theorem 5.28. *A soft topological space (X, τ, E) is soft T_i-space (i=0,1,2,3) then every coarser soft topology on X also satisfy soft T_i-axiom (i=0,1,2,3).*

Proof. Let (X, τ_1, E) and (X, τ_2, E) be two soft topological space over same parameter set E and $\tau_1 \tilde{\subseteq} \tau_2$. If τ_1 satisfy soft T_0-axiom, then for every pair of distinct soft points x_t, y_u in X, there exist soft open sets (U, E), (V, E) such that $x_t \tilde{\in} (U, E)$ but $y_u \tilde{\notin} (U, E)$ or $y_u \tilde{\in} (V, E)$ but $x_t \tilde{\notin} (V, E)$. Since $\tau_1 \tilde{\subseteq} \tau_2$, (U, E), $(V, E) \in \tau_2$. Thus (X, τ_2, E) is soft T_0. In similar fashion, we can show that it is true for other soft T_i-axioms $(i = 1, 2, 3)$. □

6. Conclusion

In this paper we have defined a more general form of soft separation axioms, We studied the relationship between these soft T_i-axioms $(i = 0, 1, 2, 3)$ and corresponding soft T_i-axioms $(i = 0, 1, 2, 3)$ defined by various authors([23],[16],[24]) and summarized the relations. It is shown that different soft T_i-axioms $(i = 0, 1, 2, 3)$ in the topological spaces are carry forward to the topologically generated soft topological space. The relations between various soft T_i-axioms are given below:

(1) soft $T_3 \Rightarrow$ soft $T_2 \Rightarrow$ soft $T_1 \Rightarrow$ soft T_0 but
soft $T_3 \not\Leftarrow$ soft $T_2 \not\Leftarrow$ soft $T_1 \not\Leftarrow$ soft T_0.
(2) soft $T_i \Rightarrow$ absolute soft T_i for each $i = 0, 1$ but
soft $T_i \not\Leftarrow$ absolute soft T_i for each $i = 0, 1, 2, 3$.
(3) Every soft compact space in a soft T_2-space is soft closed.
(4) If (X, τ, E) is any soft topological space with finite set of parameters then soft $T_2 \Rightarrow$ absolute soft T_2.
(5) soft $T_i \Rightarrow$ parametric soft T_i for each $i = 0, 1, 2$ but
soft $T_i \not\Leftarrow$ parametric soft T_i for each $i = 0, 1, 2, 3$.
(6) If (X, τ, E) is soft T_i-space for $i = 0, 1, 2, 3$ then each of the parametric topology generated from τ is T_i-space $(i = 0, 1, 2, 3)$.

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References

[1] T.M. Al-shami, Corrigendum to "Separation axioms on soft topological spaces, Ann. Fuzzy Math. Inform. 11 (4) (2016) 511-525", Annals of Fuzzy Mathematics and Informatics, 15 (3) (2018) 309-312.

[2] T.M. Al-shami, Comments on "Soft mappings spaces", The Scientific World Journal, Vol 2019, Article ID 6903809 (2019)

[3] T.M. Al-shami and L. D. R. Koaćinac, The equivalence between the enriched and extended soft topologies, Applied and Computational Mathematics, 18 (2) (2019) 149-162.

[4] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57 (2009) 1547–1553.

[5] A. Aygunoglu and H. Aygun, Some notes on soft topological spaces, Neural Comput and applications 21 (2012) 113–119.

[6] K. V. Babitha and Sunil J. John, Soft set relations and functions, Comput. Math. Appl. 60 (2010) 1840–1849.

[7] N. Căgman and S. Enginoglu, Soft set theory and uni-int decision making, Europ. J. Operat. Res. 207 (2010) 848–855.

[8] N. Căgman and S. Enginoglu, Soft matrix theory and its decision making, Comput. Math. Appl. 59 (2010) 3308–3314.

[9] D. Chen, The parametrization reduction of soft sets and its applications, Computers and Mathematics with Applications 49 (2005) 757–763.

[10] Senel Güzide, A New Construction of Spheres Via Soft Real Numbers and Soft Points, Mathematics Letters 4 (3) (2018) 39–43.

[11] Senel Güzide, A Comparative Research on the Definition of Soft Point, International Journal of Computer Applications 163 (2) (2017) 1–4.

[12] Senel Güzide and N. Căgman, Soft topological subspaces. Ann. Fuzzy Math. Informa. 10 (4) (2015) 525–535.

[13] Senel Güzide and N. Căgman, Soft Closed Sets on Soft Bitopological Space, Journal of New Results in Science 3 (5) (2014) 57–66.

[14] M. E. El-Shafei, M. Abo-Elhamayel and T. M. Al-shami, Two notes on "On soft Haus- dorff spaces", Annals of Fuzzy Mathematics and Informatics, 16 (3) (2018) 333-336.

[15] M. E. El-Shafei, M. Abo-Elhamayel and T. M. Al-shami, Partial soft separation axioms and soft compact spaces, Filomat, 32 (13) (2018) 4755-4771.

[16] Won Keun Min, A note on soft topological spaces, Comput. Math. Appl. 62 (2011) 3524–3528.

[17] Azadeh Zahedi Khameneh, Adem Kilicman and Abdul Razak Salleh, Fuzzy soft boundary, Ann. Fuzzy Math. Inform. 8 (5) (2014) 687–703.

[18] A. Kharal and B. Ahmad, Soft mappings on soft classes, New mathematics and Natural Computation 7 (3) (2011) 471–481.

[19] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.

[20] D. Molodtsov, Soft set theory first results, Comput. Math. Appl. 37 (4-5) (1999) 19–31.

[21] D. A. Molodtsov, V. Y. Leonov and D. V. Kovkov, Soft sets technique and its application, Nechetkie Sistemy i Myagkie Vychisleniya 1 (2006) 8–39.

[22] D. Pie and D. Miao, From soft sets to information systems, Granular computing, IEEE Inter. Conf. (2005) 617–621.

[23] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786–1799.

[24] O. Tantawy, S. A. El-Sheikh and S. Hamde, Separation axioms on soft topological spaces, Ann. Fuzzy Math. Inform. 11 (4) (2016) 511–525.

[25] Ningxin Xie, Soft points and the structure of soft topological spaces, Ann. Fuzzy Math. Inform. 10 (2) (2015) 309–322.

[26] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. (2012) 171–185.

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