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Abstract. In this paper, we introduce the notions of L- fuzzy prime ideals, prime and maximal L-fuzzy ideals of a poset. We also study and establish some characterizations of them and give sufficient conditions for the existence of prime L-fuzzy ideals in the lattice of all L-fuzzy ideals of a poset.

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Corresponding Author: Derso Abeje Engidaw(deab02@yahoo.com)

1. Introduction

A prime ideal in a poset was introduced by Halaś and Rachůnek [9] in 1995. Next in 2006, Erné [4] did a systematic investigation and comparison of various prime and maximal ideal theorems in partially ordered sets. Also, the theory of prime ideals in a poset has been further developed by Kharat and Mokbel [11] in 2009, Joshi and Mundlik [10] in 2013 and Erné and Joshi [5] in 2015.

On the other hand, Zadeh, in his pioneering paper [21], introduced the notion of a fuzzy subset of a non-empty set X as a function from X into the unit interval $[0, 1]$ to describe, study and formulate mathematically those objects which are not well defined. In 1971, Rosenfeld [14] applied this concept to study the concept of fuzzy subgroup of a group. Since then many scholars have studied fuzzy sub algebras of several algebraic structures. Goguen [6] observed that the interval $[0, 1]$ is not enough to take the truth values of general fuzzy statements. U. M. Swamy and K. L. N. Swamy [16] introduced the concept of prime L-fuzzy ideals in rings and U. M. Swamy and D. V. Raju [17] in lattices with truth values in a complete lattice satisfying the infinite meet distributive law and latter Koguep et al. [12] discussed certain properties of prime fuzzy ideals of lattices when the truth values are taken from the interval $[0,1]$ of real numbers. The authors of this paper [1] introduced several generalizations of L-fuzzy ideal of a lattice to an arbitrary poset. In this work, by L- fuzzy ideal we mean the L- fuzzy ideal in the sense of Halaś introduced in[1].

In this paper, we introduce the notions of L- fuzzy prime ideals, prime and maximal L-fuzzy ideals of a poset whose truth values are in a complete lattice satisfying the infinite meet distributive law by applying the general theory of algebraic fuzzy systems introduced in [18] and [19]. We also study the existence of prime L-fuzzy ideals in the lattice $(\mathcal{FI}(Q), \subseteq)$ of L-fuzzy ideals of a poset.

2. Preliminaries

We briefly recall certain necessary concepts, terminologies and notations from [2], [3] and [7]. A binary relation " $\leq$ " on a set Q is called a partial order if it is reflexive, anti-symmetric and transitive. A pair $(Q, \leq)$ is called a partially ordered set or simply a poset if Q is a non-empty set and " $\leq$ " is a partial order. Let $A \subseteq Q$. Then the set $A^u = \{x \in Q : x \geq a \ \forall \ a \in A\}$ is called the upper cone of A and the set $A^l = \{x \in Q : x \leq a \ \forall \ a \in A\}$ is called the lower cone of A. A^{ul} shall mean $\{A^u\}^l$ and A^{lu} shall mean $\{A^l\}^u$. Let $a, b \in Q$. Then the upper cone $\{a\}^u$ is simply denoted by a^u and $\{a,b\}^u$ is denoted by $(a,b)^u$. Similar notations are used for lower cones. We note that $A \subseteq A^{ul}$ and $A \subseteq A^{lu}$ and if $A \subseteq B$ in Q then $A^l \supseteq B^l$ and $A^u \supseteq B^u$. Moreover, $A^{lul} = A^l$, $A^{ulu} = A^u$, $\{a^u\}^l = a^l$ and $\{a^l\}^u = a^u$.

An element x_0 in Q is called the least upper bound of A or supremum of A, denoted by $supA$ (receptively, the greatest lower bound of A or infimum of A, denoted by $infA$) if $x_0 \in A^u$ and $x_0 \leq x \ \forall x \in A^u$ (respectively, if $x_0 \in A^l$ and $x \leq x_0 \ \forall x \in A^l$). For $a, b \in Q$ we write $a \vee b$ (read as a join b) in place of $\sup\{a,b\}$ if it exists and $a \wedge b$ (read as a meet b) in place of $\inf\{a,b\}$ if it exists. An element x_0 in Q is called the largest (respectively, the smallest) element if $x \leq x_0$ (respectively, $x_0 \leq x$) for all $x \in Q$. The largest (respectively, the smallest) element if it exists in Q is denoted by 1 (respectively, by 0). A poset $(Q \leq)$ is called bounded if it has 0 and 1. Note that if $A = \varnothing$ we have $A^{lu} = (\varnothing^l)^u = Q^u$ which is either empty or consists of the largest element 1 of Q alone if it exists and $A^{ul} = (\varnothing^u)^l = Q^l$ which is either empty or consists of the smallest element 0 of Q alone if it exists. An element m in Q is said to be a maximal(respectively, minimal) element in Q if it is not contained in any other element (respectively, if it does not contain any other element) of Q. A non empty subset A of a poset Q is said to be up-directed if $A \cap (a,b)^u \neq \varnothing$ for all $a, b \in A$. Dually we have the concept of down-directed set.

Throughout this paper, L stands for a non-trivial complete lattice satisfying the infinite meet distributive law: $a \wedge \sup S = \sup\{a \wedge s : s \in S\}$ for any $a \in L$ and $S \subseteq L$ and Q stands for a poset $(Q, \leq)$ with 0 unless otherwise stated.

Now we recall some definitions and terms from a literature that we use in this paper.

Definition 2.1 ([8])**.** A subset I of a poset $(Q, \leq)$ is called an ideal in Q in the sense of Halaš, if $(a,b)^{ul} \subseteq I$, whenever $a, b \in I$. Dually we have the concept of a filter.

Note that the set of all ideals $\mathcal{I}(Q)$ of a poset Q forms a complete lattice with respect to the inclusion order "$\subseteq$" with least element $\varnothing$ and greatest element Q in which meets coincide with set intersection [9].

Definition 2.2 ([7]). Let A be any subset of a poset Q. Then the smallest ideal containing A is called an ideal generated by A and is denoted by $(A]$. The ideal generated by a singleton set $\{a\}$, denoted by $(a]$, is called a principal ideal.

Definition 2.3 ([7]). An ideal I of a poset Q is called proper, if $I \neq Q$.

Definition 2.4 ([9]). A proper ideal P of a poset Q is called prime, if for all $a, b \in Q$, $(a, b)^l \subseteq P$ implies $a \in P$ or $b \in P$.

By an L-fuzzy subset μ of a poset Q, we mean a mapping from Q into L. Note that if L is a unit interval of real numbers, then μ is the usual fuzzy subset of X originally introduced by Zadeh [21]. For each $\alpha \in L$, the α-level subset of μ denoted by μ_α is a subset of Q given by:

$$\mu_\alpha = \{x : \mu(x) \geq \alpha\}.$$

For L-fuzzy subsets μ and σ of Q, we write

$\mu \subseteq \sigma$ to mean $\mu(x) \leq \sigma(x)$, for all $x \in Q$ in the ordering of L.

It can be easily verified that the relation "$\subseteq$" is a partial order on the set L^X of L-fuzzy subsets of X and it is called the point wise ordering.

Definition 2.5 ([13]). Let μ and σ be L-fuzzy subsets of a non-empty set X.

(i) The union of fuzzy subsets μ and σ of X, denoted by $\mu \cup \sigma$, is a fuzzy subset of X defined by $(\mu \cup \sigma)(x) = \mu(x) \vee \sigma(x)$, for all $x \in X$.

(ii) The intersection of fuzzy subsets μ and σ of X, denoted by $\mu \cap \sigma$, is a fuzzy subset of X defined by $(\mu \cap \sigma)(x) = \mu(x) \wedge \sigma(x)$, for all $x \in X$.

Definition 2.6 ([20]). For each x in a poset Q and $0 \neq \alpha$ in L, the L- fuzzy subset x_α of Q defined by: for each $y \in Q$,

$$x_\alpha(y) = \begin{cases} \alpha & if\ y = x \\ 0 & if\ otherwise \end{cases}$$

is called the fuzzy point of Q.

A fuzzy point x_α of Q is said to be belongs to an L-fuzzy subset μ Q, written as $x_\alpha \in \mu$, if $\alpha \leq \mu(x)$.

Definition 2.7 ([1]). An L- fuzzy subset μ of Q is called an L- fuzzy ideal in the sense of Halaś, if it satisfies the following conditions:

(i) $\mu(0) = 1$,

(ii) for any $a, b \in Q$, $\mu(x) \geq \mu(a) \wedge \mu(b)$ for all $x \in (a, b)^{ul}$.

Lemma 2.8 ([1]). *An L- fuzzy subset μ of Q is an L- fuzzy ideal of Q if and only if μ_α is an ideal of Q in the sense of Halaś for all $\alpha \in L$.*

Lemma 2.9 ([1]). *If μ is an L- fuzzy ideal of Q, then for any $x, y \in Q$, $\mu(x) \geq \mu(y)$, whenever $x \leq y$. That is μ is anti tone.*

Theorem 2.10 ([1]). *Let $(Q, \leq)$ be a lattice. Then an L-fuzzy subset μ of Q is an L- fuzzy ideal in the poset Q if and only it an L-fuzzy ideal in the lattice Q.*

Definition 2.11 ([12])**.** The smallest L-fuzzy ideal of Q containing the L- fuzzy subset μ is called an L-fuzzy ideal of Q generated by μ and is denoted by $(\mu]$.

Definition 2.12 ([1])**.** Let μ be an L-fuzzy subset of Q and $\mathbb{N}$ be a set of positive integers. Define an L-fuzzy subset C_1^μ of Q by $C_1^\mu(x) = \sup\{\mu(a)\wedge\mu(b) : x \in (a,b)^{ul}\}$ $\forall\, x \in Q$. Inductively, let $C_{n+1}^\mu(x) = \sup\{C_n^\mu(a)\wedge C_n^\mu(b) : x \in (a,b)^{ul}\}$ for each $n \in \mathbb{N}$.

The following three results are from the authors work in[1].

Theorem 2.13. *Let μ be an L-fuzzy subset of Q. Then the set $\{C_n^\mu : n \in \mathbb{N}\}$ defined above forms a chain and the L-fuzzy ideal $(\mu]$ generated by μ is:*

$$(\mu](x) = \sup\{C_n^\mu(x) : n \in \mathbb{N}\},\ \text{for all } x \in Q$$

.

Theorem 2.14. *The set $\mathcal{FI}(Q)$ of all L-fuzzy ideal of Q forms a complete lattice, in which the supremum $sup_{i\in\Delta}\mu_i$ and the inifimum $\inf_{i\in\Delta}\mu_i$ of any family $\{\mu_i : i \in \Delta\}$ in $\mathcal{FI}(Q)$ respectively are:*

$(sup_{i\in\Delta}\mu_i)(x) = \sup\{C_n^{\bigcup_{i\in\Delta}\mu_i}(x) : n \in \mathbb{N}\}$ and $(\inf_{i\in\Delta}\mu_i)(x) = (\bigcap_{i\in\Delta}\mu_i)(x)$, for all $x \in Q$.

Corollary 2.15. *For any μ and $\theta \in \mathcal{FI}(Q)$ the supremum $\mu\vee\theta$ and the infimum $\mu\wedge\theta$ of μ and θ respectively are:*
$(\mu\vee\theta)(x) = \sup\{C_n^{\mu\cup\theta}(x) : n \in \mathbb{N}\}$ and $(\mu\wedge\theta)(x) = (\mu\cap\theta)(x)$, for all $x \in Q$.

3. Major section

Note that for any α in L, the constant L-fuzzy subset of Q which maps all elements of Q onto α is denoted by $\overline{\alpha}$.

Definition 3.1. An L-fuzzy ideal μ of a poset Q is called proper, if μ is not the constant map $\overline{1}$, that is, $\mu(x) \neq 1$, for some x in Q.

Recall that a proper L-fuzzy ideal μ of a lattice X is called L-fuzzy prime, if $\mu(a\wedge b) = \mu(a)$ or $\mu(b)$ for any $a, b \in X$ (See [12]). Now we introduce the notion of an L-fuzzy prime ideal of a poset Q.

Definition 3.2. A proper L-fuzzy ideal μ of a poset Q is called L-fuzzy prime, if $inf\{\mu(x) : x \in (a,b)^l\} = \mu(a)$ or $\mu(b)$ for any $a, b \in Q$.

The following result characterizes any L-fuzzy prime ideal of a poset in terms of its level-subset.

Theorem 3.3. *An L-fuzzy ideal μ of a poset Q is an L-fuzzy prime if and only if for any $\alpha \in L$, either $\mu_\alpha = Q$ or μ_α a prime ideal of Q.*

Proof. Suppose that μ is an L-fuzzy prime ideal of Q and $\alpha \in L$. Since μ is an L-fuzzy ideal, clearly μ_α is an ideal of Q. Suppose that $\mu_\alpha \neq Q$. Now for any $a, b \in Q$,

$$\begin{aligned}
(a,b)^l \subseteq \mu_\alpha \quad &\Rightarrow \quad \mu(x) \geq \alpha\ \forall x \in (a,b)^l \\
&\Rightarrow \quad \inf\{\mu(x) : x \in (a,b)^l\} \geq \alpha \\
&\Rightarrow \quad \mu(a) \geq \alpha \text{ or } \mu(b) \geq \alpha \\
&\Rightarrow \quad a \in \mu_\alpha \text{ or } b \in \mu_\alpha.
\end{aligned}$$

Conversely, suppose that $\mu_\alpha = Q$ or μ_α is a prime ideal of Q, for each $\alpha \in L$. Let $x \in (a,b)^l$ and put $\alpha = inf\{\mu(x) : x \in (a,b)^l\}$. Then clearly, $x \in \mu_\alpha$ $\forall x \in (a,b)^l$, that is, $(a,b)^l \subseteq \mu_\alpha$. Thus, by hypothesis, we have either $a \in \mu_\alpha$ or $b \in \mu_\alpha$. This implies that $\mu(a) \geq \alpha = inf\{\mu(x) : x \in (a,b)^l\})$ or $\mu(b) \geq \alpha = inf\{\mu(x) : x \in (a,b)^l\}$. Also since μ is anti-tone, we have

$$\mu(a) = inf\{\mu(x) : x \in (a,b)^l\} \text{ or } \mu(b) = inf\{\mu(x) : x \in (a,b)^l\}.$$

So μ is an L-fuzzy prime ideal of Q. □

Corollary 3.4. *If μ is an L-fuzzy prime ideal of Q, then the image $\mu(Q)$ of μ is a chain in L.*

Proof. Let μ be an L-fuzzy prime ideal of Q and $a, b \in Q$. Then $\mu(a), \mu(b) \in \mu(Q)$. Put $\alpha = \mu(a) \vee \mu(b)$. Now we show that $(a,b)^l \subseteq \mu_\alpha$.

$$\begin{aligned} Now\;\; x \in (a,b)^l &\Rightarrow\;\; x \leq a \; and \; x \leq b \\ &\Rightarrow\;\; \mu(x) \geq \mu(a) \; and \; \mu(x) \geq \mu(b) \\ &\Rightarrow\;\; \mu(x) \geq \mu(a) \vee \mu(b) = \alpha \\ &\Rightarrow\;\; x \in \mu_\alpha. \end{aligned}$$

Thus $(a,b)^l \subseteq \mu_\alpha$. Again since $\mu_\alpha = Q$ or a prime ideal of Q, we have either $a \in \mu_\alpha$ or $b \in \mu_\alpha$. This implies $\mu(a) \geq \alpha = \mu(a) \vee \mu(b) \geq \mu(b)$ or $\mu(b) \geq \alpha = \mu(a) \vee \mu(b) \geq \mu(b)$. So $\mu(Q)$ is a chain in L. □

Remark 3.5. The converse of the above corollary is not true. For example consider the poset $(Q, \leq)$ depicted in the figure 2 on page 12 . Define a fuzzy subset $\mu : Q \longrightarrow [0,1]$ by: $\mu(0) = 1$, $\mu(a) = \mu(b) = \dfrac{1}{2}$ and $\mu(1) = 0$. Then $\mu(Q)$ is a chain but not an L-fuzzy prime ideal of Q.

The following result also characterizes an L-fuzzy prime ideals of a poset Q.

Corollary 3.6. *Let μ be a proper L-fuzzy ideal of a poset Q such that $\mu(Q)$ is a chain in L. Then μ is an L-fuzzy prime ideal if and only if for any $a, b \in Q$ $\mu(a) \vee \mu(b) = \inf\{\mu(x) : x \in (a,b)^l\}$.*

Lemma 3.7. *Let μ be an L- fuzzy ideal of Q. Then for any $a, b \in Q$,*

$$inf\{\mu(x) : x \in (a,b)^l\} = \mu(a \wedge b),$$

whenever $a \wedge b$ exists in Q.

Proof. Put $X = \{\mu(x) : x \in (a,b)^l\}$.

$$\begin{aligned} Now \; x \in (a,b)^l &\Rightarrow\;\; x \leq a \; and \; x \leq b \\ &\Rightarrow\;\; x \leq a \wedge b \\ &\Rightarrow\;\; \mu(x) \geq \mu(a \wedge b) \end{aligned}$$

Then $\mu(x) \geq \mu(a \wedge b)$ for all $x \in (a,b)^l$. Thus $\mu(a \wedge b)$ is a lower bound of X. Let α be any lower bound of X. Then $\alpha \leq \mu(x)$, for all $x \in (a,b)^l$. Since $a \wedge b \in (a,b)^l$, we have $\alpha \leq \mu(a \wedge b)$. Thus $inf\{\mu(x) : x \in (a,b)^l\} = \mu(a \wedge b)$. □

Corollary 3.8. *Let $(Q, \leq)$ be a lattice. Then an L-fuzzy ideal μ of Q is an L- fuzzy prime ideal in the poset Q if and only it an L-fuzzy prime ideal in the lattice Q.*

Now we introduce a prime L- fuzzy ideal of a poset Q which is a prime element in the lattice $\mathcal{FI}(Q)$ of L-fuzzy ideals of Q. Recall that an element $\alpha \neq 1$ in L is said to be prime if for any $t, s \in L$, $t \wedge s \leq \alpha$ implies either $s \leq \alpha$ or $t \leq \alpha$.

Definition 3.9. A proper L-fuzzy ideal μ of a poset Q is called a prime L-fuzzy ideal, if for any L-fuzzy ideals σ and θ of Q,

$$\sigma \cap \theta \subseteq \mu \text{ implies } \sigma \subseteq \mu \text{ or } \theta \subseteq \mu.$$

Lemma 3.10. *Let $x \in Q$ and $\alpha \in L$. Define an L- fuzzy subset $(\alpha, 0)_{(x]}$ of Q by*

$$(\alpha, 0)_{(x]}(y) = \begin{cases} 1 & if\ y = 0 \\ \alpha & if\ y \in (x] - \{0\} \\ 0 & if\ y \notin (x], \end{cases}$$

for all $y \in Q$. Then $(\alpha, 0)_{(x]} = (x_\alpha]$, which is an L-fuzzy ideal of Q generated by the fuzzy point x_α.

Proof. We claim that $(\alpha, 0)_{(x]}$ is the smallest L-fuzzy ideal containing the fuzzy point x_α. By the definition of $(\alpha, 0)_{(x]}$, it is clear that $(\alpha, 0)_{(x]}(0) = 1$. Let $a, b \in Q$ and $y \in (a, b)^{ul}$. Let $a, b \in (x] - \{0\}$. Then we have $(a, b)^{ul} \subseteq (x]$. Thus $(\alpha, 0)_{(x]}(y) \geq \alpha$. Since $a \neq 0$ and $b \neq 0$, we have $(\alpha, 0)_{(x]}(a) = \alpha = (\alpha, 0)_{(x]}(b)$. So $(\alpha, 0)_{(x]})(y) \geq \alpha = \alpha \wedge \alpha = (\alpha, 0)_{(x]})(a) \wedge (\alpha, 0)_{(x]})(b)$.

If $a = 0$ or $b = 0$, then we have $y \in (a, b)^{ul} = \{0, a\}\ or \{0, b\}$. Thus $(\alpha, 0)_{(x]})(y) \geq (\alpha, 0)_{(x]})(a)$ or $(\alpha, 0)_{(x]})(y) \geq (\alpha, 0)_{(x]})(b)$. So $(\alpha, 0)_{(x]})(y) \geq (\alpha, 0)_{(x]})(a) \wedge (\alpha, 0)_{(x]})(b)$.

If $a \notin (x]$ or $b \notin (x]$, then we have $(\alpha, 0)_{(x]})(a) \wedge (\alpha, 0)_{(x]})(b) = 0$. Thus $(\alpha, 0)_{(x]})(y) \geq 0 = (\alpha, 0)_{(x]})(a) \wedge (\alpha, 0)_{(x]})(b)$. So in all cases, we have $(\alpha, 0)_{(x]})(y) \geq (\alpha, 0)_{(x]})(a) \wedge (\alpha, 0)_{(x]})(b)$, for all $y \in (a, b)^{ul}$. Hence $(\alpha, 0)_{(x]}$ is an L-fuzzy ideal.

Again since $x \in (x]$, we have $\alpha \leq (\alpha, 0)_{(x]})(x)$. Then $x_\alpha \in (\alpha, 0)_{(x]}$. Let μ be any L-fuzzy ideal of Q such that $x_\alpha \in \mu$. Then $\alpha \leq \mu(x)$. Now we show $(\alpha, 0)_{(x]}) \subseteq \mu$. Now for any $y \in Q$, if $y \notin (x]$, then $(\alpha, 0)_{(x]})(y) = 0 \leq \mu(y)$. Let $y \in (x]$. Then if $y = 0$, then $(\alpha, 0)_{(x]})(y) = 1 = \mu(y)$ and if $y \neq 0$, then $(\alpha, 0)_{(x]})(y) = \alpha \leq \mu(x) \leq \mu(y)$. Thus in all cases, we have $(\alpha, 0)_{(x]})(y) \leq \mu(y)$, for all $y \in Q$. So $(\alpha, 0)_{(x]}) \subseteq \mu$. Hence the claim is true. Therefore $(\alpha, 0)_{(x]} = (x_\alpha]$. $\square$

In the following theorem we characterize prime L-fuzzy ideals using fuzzy points of a poset Q.

Theorem 3.11. *A proper L-fuzzy ideal μ of a poset Q is prime L- fuzzy ideal if and only if for any fuzzy points x_α and y_β of Q:*

$$x_\alpha \wedge y_\beta \in \mu \Rightarrow \textit{either } x_\alpha \in \mu \textit{ or } y_\beta \in \mu$$

Proof. Suppose that μ is a prime L- fuzzy ideal of Q. Let x_α and y_β be L-fuzzy points in Q such that $x_\alpha \wedge y_\beta \in \mu$. Then

$$\begin{aligned} x_\alpha \wedge y_\beta \in \mu \quad &\Rightarrow \quad (x_\alpha \wedge y_\beta] \subseteq \mu \\ &\Rightarrow \quad (\alpha \wedge \beta, 0)_{(x,y)^l} \subseteq \mu \\ &\Rightarrow \quad (\alpha, 0)_{(x]} \cap (\beta, 0)_{(y]} \subseteq \mu \\ &\Rightarrow \quad (\alpha, 0)_{(x]} \subseteq \mu \textit{ or } (\beta, 0)_{(y]} \subseteq \mu \\ &\Rightarrow \quad x_\alpha \in (\alpha, 0)_{(x]} \subseteq \mu \textit{ or } \; y_\beta \in (\beta, 0)_{(y]} \subseteq \mu \\ &\Rightarrow \quad x_\alpha \in \mu \textit{ or } y_\beta \in \mu. \end{aligned}$$

Conversely, suppose that the given condition holds. Let σ and θ be fuzzy ideals of Q such that $\sigma \nsubseteq \mu$ and $\theta \nsubseteq \mu$. Then there exist $x, y \in Q$ such that $\sigma(x) \nleq \mu(x)$ and $\theta(y) \nleq \mu(y)$. If we put $\alpha = \sigma(x)$ and $\beta = \theta(y)$, then x_α and y_β are fuzzy points of Q such that $x_\alpha \in \sigma$ but $x_\alpha \notin \mu$ and $y_\beta \in \theta$ but $y_\beta \notin \mu$. Thus $x_\alpha \wedge y_\beta \in \sigma \cap \theta$. By hypothesis, we have $x_\alpha \wedge y_\beta \notin \mu$. So $\sigma \cap \theta \nsubseteq \mu$. Hence μ is a prime L- fuzzy ideal
. □

Definition 3.12. An L-fuzzy subset η of Q is said to be L fuzzy down directed, if for any $a, b \in Q$, there exists $x \in (a, b)^l$ such that

$$\eta(x) \geq \eta(a) \wedge \eta(b).$$

Now we prove the following theorem which is analogous to Stone's Prime ideal Theorem in distributive lattices[15].

Theorem 3.13. *Let the lattice $(\mathcal{FI}(Q), \subseteq)$ of all L-fuzzy ideals of Q is distributive, $\mu \in \mathcal{FI}(\mathcal{Q})$ and λ is a prime element in L. If η be an L-fuzzy down directed subset of Q such that such that $\mu \cap \eta \subseteq \overline{\lambda}$, then there exists a prime L-fuzzy ideal θ of Q such that $\mu \subseteq \theta$ and $\theta \cap \eta \subseteq \overline{\lambda}$.*

Proof. Let $\mathcal{S} = \{\sigma \in \mathcal{FI}(\mathcal{Q}) : \mu \subseteq \sigma \textit{ and } \sigma \cap \eta \subseteq \overline{\lambda}\}$. Since $\mu \in \mathcal{S}$, $\mathcal{S}$ is non empty, it forms a poset under the point wise ordering " $\subseteq$ " of fuzzy sets. By applying Zorn's lemma, we can choose a maximal element say θ in $\mathcal{S}$. Now we show θ is a prime L-fuzzy ideal of Q. Let x_α and y_β be L-fuzzy points in Q such that $x_\alpha \wedge y_\beta \in \theta$. This implies $(\alpha \wedge \beta, 0)_{(x,y)^l} \subseteq \theta$. Suppose that $x_\alpha \notin \theta$ and $y_\beta \notin \theta$. Put $\theta_1 = \theta \vee (\alpha, 0)_{(x]}$ and $\theta_2 = \theta \vee (\beta, 0)_{(y]}$. Then clearly, θ_1 and θ_2 are L-fuzzy containing θ properly. Thus by maximality of θ, both θ_1 and θ_2 do not belong to $\mathcal{S}$. So there exist $a, b \in Q$ such that $(\theta_1 \cap \eta)(a) \nleq \lambda$ and $(\theta_2 \cap \lambda)(b) \nleq \lambda$. Let $z \in (a, b)^l$. Then $((\theta_1 \cap \eta)(z) \nleq \lambda$ and $(\theta_2 \cap \lambda)(z) \nleq \lambda$. Since λ a prime element in L, we have $(\theta_1 \cap \theta_2) \cap \eta)(z) \nleq \lambda$. Thus $x_\alpha \wedge y_\beta \in \theta$ implies

$$\begin{aligned} ((\theta_1 \cap \theta_2) \cap \eta)(z) \nleq \lambda \quad &\Rightarrow \quad ((\theta \vee (\alpha, 0)_{(x]}) \cap (\theta \vee (\beta, 0)_{(y]})) \cap \eta)(z) \nleq \lambda \\ &\Rightarrow \quad ((\theta \vee (\alpha \wedge \beta, 0)_{(x,y)^l})) \cap \eta)(z) \nleq \lambda \\ &\Rightarrow \quad (\theta \cap \eta)(z) \nleq \lambda \; (\textit{since } (\alpha \wedge \beta, 0)_{(x,y)^l} \subseteq \theta) \end{aligned}$$

which is a contradiction to the fact that $\theta \cap \eta \subseteq \overline{\lambda}$. So $x_\alpha \wedge y_\beta \in \theta$ implies $x_\alpha \in \theta$ or $y_\beta \in \theta$. Hence by theorem 3.11, θ is a prime L-fuzzy ideal. □

Corollary 3.14. *Let μ be in the distributive lattice $(\mathcal{FI}(\mathcal{Q}), \leq)$ of all L-fuzzy ideals of Q and $a \in Q$. If $\mu(a) \leq \lambda$, where λ is a prime element in L, then there exists a prime L-fuzzy ideal θ of Q such that $\mu \subseteq \theta$ and $\theta(a) \leq \lambda$.*

In the following we characterize a prime L-fuzzy ideal of a poset Q in terms of prime ideals of Q and prime elements of L.

Lemma 3.15. *Let I be an ideal of a poset Q and $1 \neq \alpha \in L$. Then the L-fuzzy subset α_I of a poset Q defined by*

$$\alpha_I(x) = \begin{cases} 1 & if\ x \in I \\ \alpha & if\ x \notin I \end{cases},$$

for all $x \in Q$ is an L-fuzzy ideal of Q.

We call the L-fuzzy ideal α_I defined above as the α-level L-fuzzy ideal of Q corresponding to the ideal I.

Corollary 3.16. *If I and J are ideals in Q and $1 \neq \alpha, \beta \in L$, then $\alpha_I \subseteq \beta_J$ if and only if $I \subseteq J$ and $\alpha \leq \beta$.*

Theorem 3.17. *Let P be an ideal of a poset Q and $1 \neq \alpha \in L$. Then α_P is a prime L-fuzzy ideal of Q if and only if P is a prime ideal of Q and α is a prime element in L.*

Proof. Suppose that α_P is a prime L-fuzzy ideal of Q. We show that P is a prime ideal of Q and α is a prime element in L. Since α_P is proper, we have $P \neq Q$ and $\alpha \neq 1$. Let $a, b \in Q$ such that $(a, b)^l \subseteq P$.

$$\begin{aligned} Now\ (a,b)^l \subseteq P &\Rightarrow \alpha_{(a,b)^l} \subseteq \alpha_P \\ &\Rightarrow \alpha_{(a] \cap (b]} \subseteq \alpha_P \\ &\Rightarrow \alpha_{(a]} \cap \alpha_{(b]} \subseteq \alpha_P \\ &\Rightarrow \alpha_{(a]} \subseteq \alpha_P\ or\ \alpha_{(b]} \subseteq \alpha_P \\ &\Rightarrow (a] \subseteq P\ or\ (b] \subseteq P \\ &\Rightarrow a \in P\ or\ b \in P. \end{aligned}$$

Then P is a prime ideal of Q.

Again let $\beta, \gamma \in L$ such that $\beta \wedge \gamma \leq \alpha$.

$$\begin{aligned} Now\ \beta \wedge \gamma \leq \alpha &\Rightarrow (\beta \wedge \gamma)_P \subseteq \alpha_P \\ &\Rightarrow \beta_P \cap \gamma_P \subseteq \alpha_P \\ &\Rightarrow \beta_P \subseteq \alpha_P\ or\ \gamma_P \subseteq \alpha_P \\ &\Rightarrow \beta \leq \alpha\ or\ \gamma \leq \alpha. \end{aligned}$$

Thus α is a prime element in L.

Conversely, suppose that P is a prime ideal of Q and α is a prime element in L. Then clearly, α_P is an L-fuzzy ideal of Q. Let μ and σ be any L-fuzzy ideals of Q such that $\mu \nsubseteq \alpha_P$ and $\sigma \nsubseteq \alpha_P$. Then there exist $a, b \in Q$ such that $\mu(a) \nleq \alpha_P(a)$ and $\sigma(b) \nleq \alpha_P(b)$, This implies $\mu(a) \nleq \alpha$ and $\sigma(b) \nleq \alpha$ and $a \notin P$ and $b \notin P$. Since α is prime element in L and P is a prime ideal of Q, we have $\mu(a) \wedge \sigma(b) \nleq \alpha$ and $(a, b)^l \nsubseteq P$. Thus there exists $y \in (a, b)^l$ such that $y \notin P$. So we have $(\mu \wedge \sigma)(y) =$

$\mu(y) \wedge \sigma(y) \geq \mu(a) \wedge \sigma(b)$. Hence $(\mu \wedge \sigma)(y) \nleq \alpha = \alpha_P(y)$. Therefore $\mu \cap \sigma \nsubseteq \alpha_P$ and hence α_P is a prime L-fuzzy ideal of Q. □

Theorem 3.18. *Let μ be an L-fuzzy ideal of Q. Then μ is a prime L-fuzzy ideal of Q if and only if there exist prime ideal of P of Q and prime element α in L such that $\mu = \alpha_P$.*

Proof. Suppose that μ is a prime L-fuzzy ideal of Q. Since μ is proper it assumes at least two values. Since $\mu(0) = 1$, 1 is necessarily in $Im(\mu)$. Suppose that $\alpha, \beta \in Im(\mu)$ other than 1. Now we claim $\alpha = \beta$. Now $\alpha, \beta \in Im(\mu)$ implies there exist $a, b \in Q$ such that $\mu(a) = \alpha$ and $\mu(b) = \beta$. Now put $P = \mu_1 = \{x \in Q : \mu(x) = 1\}$. Now for all $x \in Q$, define L-fuzzy subsets of Q by;

$$\chi_{(a]}(x) = \begin{cases} 1 & if\ x \in (a] \\ 0 & if\ x \notin (a] \end{cases}$$

and

$$\theta(x) = \begin{cases} 1 & if\ x \in P \\ \alpha & if\ x \notin P. \end{cases}$$

Then clearly, $\chi_{(a]}$ and θ are L-fuzzy ideals of Q. Now we show $\chi_{(a]} \cap \theta \subseteq \mu$. Let $x \in Q$. If $x \in (a]$, then $\alpha = \mu(a) \leq \mu(x)$. Now in this case, if $x \in P$, then $\theta(x) = 1 = \mu(x)$. Thus $(\chi_{(a]} \cap \theta)(x) = \chi_{(a]}(x) \wedge \theta(x) = 1 \wedge 1 = 1 = \mu(x)$. If $x \notin P$, then $(\chi_{(a]} \cap \theta)(x) = \chi_{(a]}(x) \wedge \theta(x) = 1 \wedge \alpha = \alpha \leq \mu(x)$ and hence $(\chi_{(a]} \cap \theta)(x) \leq \mu(x)$ if $x \in (a]$. Again if $x \notin (a]$, then we have $(\chi_{(a]} \cap \theta)(x) = \chi_{(a]}(x) \wedge \theta(x) = 0 \wedge \theta(x) = 0 \leq \mu(x)$. Thus in either cases, we have $(\chi_{(a]} \cap \theta)(x) \leq \mu(x)$, for all $x \in Q$. So $\chi_{(a]} \cap \theta \subseteq \mu$. Since μ is a prime L-fuzzy ideal of Q, we have either $\chi_{(a]} \subseteq \mu$ or $\theta \subseteq \mu$. But since $\chi_{(a]}(a) = 1 \neq \alpha = \mu(a)$, $\chi_{(a]} \nsubseteq \mu$. Hence $\theta \subseteq \mu$. In particular, since $b \notin P$, we get that $\alpha = \theta(b) \leq \mu(b) = \beta$. Then $\alpha \leq \beta$. Similarly, we can show $\beta \leq \theta$. Thus $\alpha = \beta$. So μ assumes exactly one value other than 1 and hence $\mu = \alpha_P$.

Now we remain to show that α is a prime element in L and P a prime ideal of Q. Let $\beta, \gamma \in L$ such that $\beta \wedge \gamma \leq \alpha$. This implies $\beta_P \cap \gamma_P = (\beta \wedge \gamma)_P \subseteq \alpha_P = \mu$. Since μ is a prime L-fuzzy ideal, we have either $\beta_P \subseteq \mu$ or $\gamma_P \subseteq \mu$ and since $\mu(a) = \alpha \neq 1$, $a \notin P$. Then we have $\beta = \beta_P(a) \leq \mu(a) = \alpha$ or $\gamma = \gamma_P(a) \leq \mu(a) = \alpha$. Thus α a is a prime element in in L. Again to show P is a prime ideal, let $a, b \in Q$ such that $(a, b)^l \subseteq P$. Then $\chi_{(a,b)^l} \subseteq \chi_P$. This implies $\chi_{(a]} \cap \chi_{(b]} = \chi_{(a,b)^l} \subseteq \chi_P \subseteq \mu$, where $\chi_{(a]}$ and $\chi_{(b]}$ are the characteristic maps of $(a]$ and $(b]$ respectively. Since μ is prime, we have either $\chi_{(a]} \subseteq \mu$ or $\chi_{(b]} \subseteq \mu$ which imply that $(a] \subseteq \mu_1 = P$ or $(b] \subseteq \mu_1 = P$ that is, either $a \in P$ or $b \in P$. Thus P is a prime ideal of Q. The converse part of this theorem follows from theorem 3.17. This completes the proof. □

Corollary 3.19. *Let $L = [0, 1]$. Then a proper ideal P of Q is prime if and only if its characteristic map χ_P is a prime L-fuzzy ideal of Q.*

Note that we write α_P for the prime L-fuzzy ideal of Q corresponding to the pair (P, α) and $\mathcal{PFI}(Q)$ for the set of all prime L-fuzzy ideal of Q. Now the following result from the above theorem.

Corollary 3.20. *There is a one-to-one correspondence between the class $\mathcal{PFI}(\mathcal{Q})$ of all prime L-fuzzy ideals of Q and the collection of all pairs (P, α), where P is a prime ideal of Q and α is a prime element in L.*

Example 3.21. Consider the poset $(Q, \leq)$ depicted in the figure below. Define a fuzzy subset $\mu : Q \longrightarrow [0,1]$ by: $\mu(0) = \mu(a) = \mu(b) = 1$, $\mu(c) = \mu(d) = \mu(e) = \mu(1) = 0.5$. Then μ is a prime L-fuzzy ideal of Q.

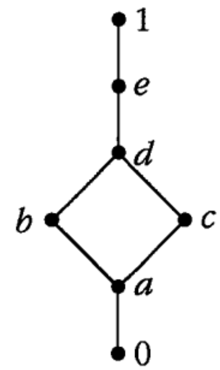

Figure 1

Theorem 3.22. *Every prime L-fuzzy ideal of a poset is an L-fuzzy prime ideal.*

Proof. Let μ be a prime L-fuzzy ideal of a poset Q. Then there exist a prime ideal P of Q and a prime element α of L such that $\mu = \alpha_P$. Since $\mu(Q) = \{\alpha, 1\}$ and $\alpha \lneq 1$, $\mu(Q)$ is a chain and μ is proper. Let $a, b \in Q$. If $(a,b)^l \subseteq P$, then $\mu(x) = 1$, for all $x \in Q$. Again since P is prime $(a,b)^l \subseteq P$ implies either $a \in P$ or $b \in P$, either $\mu(a) = 1$ or $\mu(b) = 1$. Thus $\mu(a) \vee \mu(b) = 1 = \inf\{\mu(x) : x \in (a,b)^l\}$. Again if $(a,b)^l \nsubseteq P$, then there exists $y \in (a,b)^l$ such that $y \notin P$. Thus $\mu(y) = \alpha = \inf\{\mu(x) : x \in (a,b)^l\}$. Again $(a,b)^l \nsubseteq P$ implies $a \notin P$ and $b \notin P$. Otherwise if either $a \in P$ or $b \in P$, then $y \in P$ which is a contradiction. Thus $\mu(a) = \mu(b) = \alpha$. So $\mu(a) \vee \mu(b) = \alpha$. hence in either cases, $\mu(a) \vee \mu(b) = \inf\{\mu(x) : x \in (a,b)^l\}$. Therefore, by corollary 3.6, μ is an L-fuzzy prime ideal. □

Remark 3.23. The converse of the above theorem is not true. For example consider the poset $(Q \leq\ )$ depicted in the figure 1 above and define a fuzzy subset $\mu : Q \longrightarrow [0,1]$ by $\mu(0) = 1$, $\mu(a) = \mu(b) = 0.8$, $\mu(c) = \mu(d) = \mu(e) = \mu(1) = 0$. Then μ is an L-fuzzy prime ideal but not a prime L-fuzzy ideal.

Now we introduce the notion of maximal L-fuzzy ideal of a poset which is a maximal element in the set of all proper L-fuzzy ideals of Q.

Definition 3.24. A proper L-fuzzy ideal μ of a poset Q is said to be a maximal L-fuzzy ideal, if μ is a maximal element in the set of all proper L-fuzzy ideals of Q under point wise ordering "$\subseteq$". That is, if there is no proper L-fuzzy ideal θ of Q such that $\mu \subsetneqq \theta$.

Lemma 3.25. *Let μ be an L-fuzzy ideal of Q and $\alpha \in L$. Then $\mu \cup \overline{\alpha}$ is an L-fuzzy ideal of Q containing μ.*

Proof. Clearly $\mu \subseteq \mu \cup \overline{\alpha}$. Since $(\mu \cup \overline{\alpha})(0) = \mu(0) \vee \alpha = 1 \vee \alpha = 1$, we have $(\mu \cup \overline{\alpha})(0) = 1$. Again let $a, b \in Q$ and $x \in (a, b)^{ul}$. Then

$$\begin{aligned}(\mu \cup \overline{\alpha})(x) &= \mu(x) \vee \alpha \\ &\geq (\mu(a) \wedge \mu(b)) \vee \alpha \\ &= (\mu(a) \vee \alpha) \wedge (\mu(b)) \vee \alpha) \\ &= (\mu \cup \overline{\alpha})(a) \wedge (\mu \cup \overline{\alpha})(b).\end{aligned}$$

Thus $\mu \cup \overline{\alpha}$ is an L-fuzzy ideal of Q containing μ. □

Lemma 3.26. *Let μ be a maximal L-fuzzy ideal of Q. Then $Im(\mu)$ is a chain.*

Proof. Let $\alpha, \beta \in Im(\mu)$. Then there exist $a, b \in Q$ such that $\mu(a) = \alpha$ and $\mu(b) = \beta$. By the lemma 3.25 , $\mu \cup \overline{\alpha}$ is an L-fuzzy ideal of Q. Since μ is maximal L-fuzzy ideal and $\mu \subseteq \mu \cup \overline{\alpha}$, we have either $\mu = \mu \cup \overline{\alpha}$ or $\mu \cup \overline{\alpha} = \overline{1}$. If $\mu = \mu \cup \overline{\alpha}$, then we have $\beta = \mu(b) = (\mu \cup \overline{\alpha})(b) = \mu(b) \vee \alpha = \beta \vee \alpha$ and hence $\alpha \leq \beta$. If $\mu \vee \overline{\alpha} = \overline{1}$, then we have $(\mu \cup \overline{\alpha})(a) = 1 = (\mu \cup \overline{\alpha})(b)$. This implies $\mu(a) \vee \alpha = \mu(b) \vee \alpha$, that is, $\alpha = \beta \vee \alpha$. Thus $\beta \leq \alpha$. So $Im(\mu)$ is a chain. □

Lemma 3.27. *Let μ be a maximal L-fuzzy ideal of Q. Then μ attains exactly one value other than 1.*

Proof. Since μ is an L-fuzzy ideal of Q, we have $\mu(0) = 1$. Let $a, b \in Q$ such that $\mu(a) \neq 1$ and $\mu(b) \neq 1$. Put $\mu(a) = \alpha$ and $\mu(b) = \beta$. Then $\mu \cup \overline{\alpha}$ and $\mu \cup \overline{\beta}$ are L-fuzzy ideals of Q containing μ. Since $(\mu \cup \overline{\alpha})(a) = \mu(a) \vee \alpha = \alpha \vee \alpha = \alpha \neq 1 = \overline{1}(a)$ and $(\mu \cup \overline{\beta})(b) = \mu(b) \vee \beta = \beta \vee \beta = \beta \neq 1 = \overline{1}(b)$, by maximality of μ, we have $\mu = \mu \cup \overline{\alpha} = \mu \cup \overline{\beta}$. In particular, $\beta = \mu(b) = (\mu \cup \overline{\alpha})(b) = \mu(b) \vee \alpha = \beta \vee \alpha$ and $\alpha = \mu(a) = (\mu \cup \overline{\beta})(a) = \mu(a) \vee \beta = \alpha \vee \beta$. Thus $\alpha = \beta$. So μ assumes exactly one value other than 1. □

Recall that an element $\alpha \in L$ is said to be a dual atom if there is no $\beta \in L$ such that $\alpha < \beta < 1$. Now we give a characterization of a maximal L-fuzzy ideal of a poset Q.

Theorem 3.28. *An L-fuzzy subset μ of Q is a maximal L-fuzzy ideal of Q if and only if there exist a maximal ideal M of Q and a dual atom α in L such that $\mu = \alpha_M$.*

Proof. Suppose that μ is a maximal L-fuzzy ideal of Q. Put $M = \{x \in Q : \mu(x) = 1\}$. Then by lemma 3.27, μ assumes exactly one value, say α other than 1. Thus $\mu = \alpha_M$. Now we remain to show that M is a maximal ideal of Q and α is a dual element in L. Since μ is proper, it is clear that $\varnothing \neq M \subsetneq Q$. Let I be a proper ideal of Q such that $M \subseteq I$. Then $\mu = \alpha_M \subseteq \alpha_I \subset \overline{1}$. By maximality of μ, we have that $\alpha_M = \alpha_I$. Thus $M = I$. So M is a maximal ideal of Q.

Again let $\beta \in L$ such that $\alpha \leq \beta < 1$. Then $\mu = \alpha_M \subseteq \beta_M \subset \overline{1}$. Thus by the maximality of μ, we have $\alpha_M = \beta_M$. So $\alpha = \beta$. Hence α is a dual atom in L.

Conversely, suppose $\mu = \alpha_M$, where M is a maximal ideal in Q and α is a dual atom in L. Since M is proper, there exists $a \in Q$ such that $a \notin M$. Then $\mu(a) = \alpha_M(a) = \alpha < 1$. Thus μ is proper. Let θ be any proper L-fuzzy ideal of Q such that $\mu \subseteq \theta \subset \overline{1}$. Then $M = \mu_1 \subseteq \theta_1 \subset Q$. Thus by the maximality of M, we have $M = \theta_1 = \{x \in Q : \theta(x) = 1\}$. Let $x \in Q$. If $x \in M$, then $\mu(x) = 1 = \theta(x)$. If

$x \notin M$, then we have $\mu(x) = \alpha \leq \theta(x) < 1$. Since α is a dual atom in L, we have $\mu(x) = \alpha = \theta(x)$. Thus $\mu = \alpha_M = \theta$. So μ is a maximal L-fuzzy ideal of Q. □

Corollary 3.29. *There is a one-to-one correspondence between the class of all maximal L-fuzzy ideals of Q and the collection of all pairs (M, α), where M is a maximal ideal of Q and α is a dual atom in L.*

Example 3.30. Consider the poset $(Q, \leq)$ depicted in the figure 1 above and the distributive lattice L in the figure 2 below. Define a fuzzy subset $\mu : Q \longrightarrow L$ by: $\mu(0) = \mu(a) = \mu(b) = \mu(c) = \mu(d) = \mu(e) = 1$ and $\mu(1) = a$. Then μ is a maximal L-fuzzy as $\mu = \alpha_M$, where $\alpha = a$ is a dual atom in L and $M = \{0, a, b, c, d, e\}$ is a maximal ideal of Q.

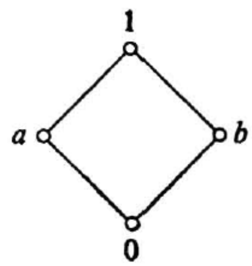

Figure 2

Since L is a distributive lattice, every dual atom in L is prime and hence we have the following.

Corollary 3.31. *If Q is a poset in which every maximal ideal is a prime ideal then every maximal L-fuzzy ideal is a prime L-fuzzy ideal.*

Remark 3.32. The converse of the above corollary is not true. Example 3.18, which is given above, is a prime L-fuzzy ideal but not a maximal L-fuzzy ideal of the given poset as there is no dual atom in $L = [0, 1]$.

4. Conclusions

In this paper, we have studied the notions of L-fuzzy prime ideals, prime L-fuzzy ideals and maximal L-fuzzy ideals of a poset, which generalize the notions of these terms in lattices. This study can be extended to other concepts of fuzzy sub algebra of a lattice to an arbitrary poset.

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References

[1] B. A. Alaba, M. A. Taye and D. A. Engidaw, L-fuzzy ideals of a poset, Ann. Fuzzy Math. Inform. 16 (3) (2018) 285–299.
[2] G. Birkhoof, Lattice theory, Amer. Math. Soc. Colloquium publications, Vol. 24 1967.
[3] B. A. Davey and H. A. Priestley, Introduction to lattices and order, Cambridge Univ. Press, Cambridge 1990.
[4] M. Erné, Prime and maximal ideals of partially ordered sets, Math. Slovaca 56 (1) (2006) 1–22.
[5] M. Erné and V. Joshi, Ideals in atomic posets, Discrete Mathematics 338 (2015) 954–971.

[6] J. A. Goguen, L-fuzzy sets, J. Math. Anal. Appl. 18 (1967) 145–174.
[7] G.Grätzer, General lattice theory, Academic Press, New York, 1978.
[8] R. Halaś, Annihilators and ideals in ordered sets, Czechoslovak Math. J. 45 (120) (1995) 127–134.
[9] R. Halaś and J. Rachůnek, Polars and prime ideals in ordered sets. Discuss. Math., Algebra and Stochastic Methods 15 (1995) 43–59.
[10] V. Joshi and N.Mundlik , Prime ideals in 0-distributive posets. Cent Eur J. Math 11 (5) (2013) 940–955.
[11] V. S. Kharat and K. A. Mokbel, Primeness and semiprimeness in posets, Math. Bohem. 134 (1) (2009) 19–30.
[12] B. B. N. Koguep, C. Lele, On fuzzy prime ideals of lattices, SJPAM 3 (2008) 1–11.
[13] J. N. Mordeson and D. S. Malik, Fuzzy commutative algebra, World Sci. Publ., Singapore 1998.
[14] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512–517.
[15] M. H. Stone, The theory of representations for boolean algebras. Trans. Amer. Math. Soc. 40 (1936) 37–111.
[16] K. L. N. Swamy and U. M. Swamy, Fuzzy prime ideals of rings, J. Math. Anal. and Appl. 134 (1988) 94–103.
[17] U. M. Swamy and D. V. Raju, Fuzzy ideals and congruences of lattices, Fuzzy Sets and Systems 95 (1998) 249–253.
[18] U. M. Swamy and D. V. Raju, Algebraic fuzzy systems, Fuzzy Sets and Systems 41 (1991) 187–194.
[19] U. M. Swamy and D. V. Raju, Irreducibility in algebraic fuzzy systems, Fuzzy Sets and Systems 41 (1991) 233–241.
[20] C. K. Wong. Fuzzy point and local properties of fuzzy topology, J. Math. Anal. Appl. 46 (1974) 316–328.
[21] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.

Berhanu Assaye Alaba (berhanu_assaye@yahoo.com)
Department of Mathematics, Bahir Dar University, Bahir Dar, Ethiopia

Mihret Alamneh Taye (mihretmahlet@yahoo.com)
Department of Mathematics, Bahir Dar University, Bahir Dar, Ethiopia

Derso Abeje Engidaw (deab02@yahoo.com)
Department of Mathematics, University of Gondar , Gondar, Ethiopia