Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 17, No. 3, (June 2019) pp. 303–312
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr
https://doi.org/10.30948/afmi.2019.17.3.303

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Reprinted from the
Annals of Fuzzy Mathematics and Informatics
Vol. 17, No. 3, June 2019

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Received 20 December 2018; Revised 24 January 2019; Accepted 19 March 2019

Abstract. In this paper, we introduce and study some new soft properties namely, soft R_0 and soft $R_1(SR_i$, for short $i = 0, 1)$ by using the concept of distinct soft points and we obtain some of their properties. We show how they relate to some soft separation axioms in [21]. Also we, show that the properties SR_0, SR_1 are special cases of soft regularity. We further, show that in the case of soft compact spaces, SR_1 is equivalent to soft regularity. Finally, the relations between these properties in soft topologies and that in crisp topologies are studied. Moreover, some counterexamples are given.

2010 AMS Classification: 06D72, 54A40, 54D10

Keywords: Soft set, Soft points, Soft topology, SR_0 and SR_1 spaces.

1. Introduction

Molodtsov [13] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. Research works on soft set theory and its applications are progressing rapidly in various fields, for example topology [3, 5, 7, 12, 17, 18, 19, 20, 22], algebra [1, 2, 6, 9], decision making [4, 10, 11] and so on. In [20], Shabir and Naz introduced the concept of soft topological spaces and studied some related concepts. Soft separation axioms studied in some papers (see, for example, [12, 14, 16, 20]). Recently, O. Tantawy et. al [21] defined soft separation axioms by using soft pints and studied some of their properties. In this work, we have continued to study soft separation axioms, we generalize some soft separations axioms [21] by defining the properties $SR_i, i = 0, 1$, investigating some nice results and relations for them. We mention that this work is a soft version of our paper [15].

This paper is organized as follows. In section 2, we recall some basic definitions which will be needed throughout this work. In section 3, we present new definitions and characterizations concerning the properties SR_i , $i = 0, 1$, and study some of their properties. In section 4, some nice results and relations of them are obtained with some counterexamples.

2. Some basic definitions

Throughout this work, X refers to an initial universe set, E be the set of all parameters for X and $P(X)$ is the power set of X.

Definition 2.1 ([13]). A soft set $F_E = (F, E)$ over X is a mapping $F : E \longrightarrow P(X)$. Then F_E can be represented by the set $F_E = \{(e, F(e)) : e \in E \text{ and } F(e) \in P(X)\}$. We denote the family of all soft sets over X by $SS(X, E)$.

Definition 2.2 ([11, 12, 13]). Let F_E, G_E are soft sets over X and $x \in X$.

(i) F_E is called a null denoted, by $\varnothing_E$, if $F(e) = \varnothing$ for all $e \in E$ and is called an absolute soft set, denoted by X_E, if $F(e) = X$ for all $e \in E$.

(ii) If $F(e) = \{x\}$ and $F(e^{'}) = \varnothing$ for all $e^{'} \in E - \{e\}$, then F_E is called a soft point in X_E, denoted by x_e. The complement of a soft point x_e is a soft set over X denoted by $(x_e)^c$. The soft point $x_e \widetilde{\in} F_E$, if for the element $e \in E$, $x \in F(e)$. The set of all soft points in X_E is denoted by $SP(X, E)$.

(iii) The soft pints x_e, y_e in X_E are called distinct, if $x \neq y$.

(iv) The complement of F_E denoted by F_E^c, where $F^c : E \longrightarrow P(X)$ is a mapping given by, $(F(e))^c = X - F(e)$ for all $e \in E$. Clearly, $(F_E^c)^c = F_E$.

(v) F_E is a soft subset of G_E, denoted by $F_E \widetilde{\subseteq} G_E$, if $F(e) \subseteq G(e)$ for all $e \in E$.

(vi) F_E and G_E are equal, if $F_E \widetilde{\subseteq} G_E$ and $G_E \widetilde{\subseteq} F_E$. It is denoted by $F_E = G_E$.

(vii) The soft union of F_E and G_E is a soft set H_E given by $H(e) = F(e) \cup G(e)$ for all $e \in E$ and denoted by, $F_E \widetilde{\cup} G_E$.

(viii) The soft intersection of F_E and G_E is a soft set H_E defined by $H(e) = F(e) \cap G(e)$ for all $e \in E$ and denoted by $F_E \widetilde{\cap} G_E$.

Definition 2.3 ([17, 20]). Let $F_E \in SS(X, E)$, $\varnothing \neq Y \subset X$ and $x \in X$.

(i) $x \widetilde{\in} F_E$, if $x \in F(e)$ for all $e \in E$, and $x \notin F_E$, if $x \widetilde{\notin} F(e)$ for some $e \in E$.

(ii) If $F(e) = \{x\}$ for all $e \in E$, then F_E is called a soft singleton point, denoted by x_E. And we have, $x_E \widetilde{\subseteq} F_E \Longleftrightarrow x \widetilde{\in} F_E \Longleftrightarrow x_e \widetilde{\in} F_E$ for all $e \in E$.

(iii) $Y_E = (Y, E)$ denotes the soft set over X for which, $Y(e) = Y$ for all $e \in E$.

Definition 2.4 ([20, 22]). A soft topological space is the triple (X, τ^*, E), where X is universe set, E is the fixed set of parameters and τ^* is the collection of soft sets over X, which are satisfies:

(i) $\varnothing_E, X_E \in \tau^*$,

(ii) the soft intersection of any two soft sets in τ^* is in τ^*,

(iii) the soft union of any number of soft sets in τ^* is in τ^*.

In this case, any member in τ^* is called an open soft set in X. A soft set F_E is called closed soft in X if $F_E^c \in \tau^*$. The family of all closed soft sets in X, denoted by τ^{*c}.

Definition 2.5 ([22]). A soft set F_E is said to be a soft neighborhood of a soft point x_e in (X, τ^*, E), if there exists $U_E \in \tau^*$ such that $x_e \widetilde{\in} U_E \widetilde{\subseteq} F_E$.

Notation. For $x_e \in SP(X, E)$ the soft set O_{x_e} refers to a soft open set contains x_e and O_{x_e} is called a soft open neighborhood of x_e.

Definition 2.6 ([20]). Let (X, τ^*, E) be a soft topological space and $F_E \in SS(X, E)$. Then the soft closure of F_E, denoted by $\overline{F_E}$, is the intersection of all soft closed super sets of F_E, that is, $\overline{F_E} = \widetilde{\cap}\{G_E : G_E \in \tau^{*c}$ *and* $F_E \widetilde{\subseteq} G_E\}$.

Theorem 2.7 ([20, 21]). *Let (X, τ^*, E) be a soft topological space over X and F_E , G_E are two soft sets over X. Then*

(1) *F_E is a closed soft set* $\Longleftrightarrow \overline{F_E} = F_E$,

(2) $F_E \widetilde{\subseteq} G_E \Longrightarrow \overline{F_E} \widetilde{\subseteq} \overline{G_E}$,

(3) $x_e \widetilde{\in} \overline{F_E} \Longleftrightarrow G_E \widetilde{\cap} F_E \neq \varnothing_E$ *for all* $G_E \in \tau^*$, $x_e \widetilde{\in} G_E$.

Definition 2.8 ([20]). Let (X, τ^*, E) be a soft topological space over X and Y be a nonempty subset of X. Then $\tau^*_Y = \{Y_E \widetilde{\cap} F_E : F_E \in \tau^*\}$ is called the soft relative topology on Y and (Y, τ^*_Y, E) is called a soft subspace of (X, τ^*, E).

Proposition 2.9 ([20]). *Let (Y, τ^*_Y, E) be a soft subspace of (X, τ^*, E) and F_E be a soft set over Y. Then F_E is an open soft set in Y if and only if $F_E = Y_E \widetilde{\cap} G_E$, for some $G_E \in \tau^*$.*

Definition 2.10. (i) Let (X, τ^*, E) be a soft topological space. Then the collection $\tau_e = \{F(e) : F_E \in \tau^*\}$ for all $e \in E$, defines a topology on X (See [20]).

(ii) Let (X, τ) be a topological space. Then the family $\tau^*_\tau = \{F_E : F(e) = A \ \forall e \in E$ *and* $\forall \ A \in \tau\}$, defines a soft topology on X (See Example 3.4 in [7]).

Definition 2.11 ([14]). (i) A soft topological space (X, τ^*, E) is called a soft singleton point space if $\tau^* = \{U_E : U(e) = U \ \forall e \in E$ *and* $\forall \ U \subset X\}$. In this case, every soft singleton points x_E are open soft set.

(ii) If (X, τ^*, E) is a soft singleton point space, then every soft element of (X, τ^*, E) is both soft open, closed set, and (X, τ_e) is a discrete space for all $e \in E$.

Note. If (X, τ) is a discrete topological space, then the soft topology τ^*_τ which is defined in (ii) of Definition 2.10, is a soft singleton point topology on X.

Definition 2.12. A soft topological space (X, τ^*, E) is said to be:

(i) soft T_0 (ST_0, for short), if for every two soft points x_e, y_e with $x \neq y$, there exists $G_E \in \tau^*$ such that $x_e \widetilde{\in} G_E$, $y_e \widetilde{\notin} G_E$ or there exists $H_E \in \tau^*$ such that $y_e \widetilde{\in} H_E$, $x_e \widetilde{\notin} H_E$ [21],

(ii) soft T_1 (ST_1, for short), if for every two soft points x_e , y_e with $x \neq y$, there exist G_E , $H_E \in \tau^*$ such that $x_e \widetilde{\in} G_E$, $y_e \widetilde{\notin} G_E$ and $y_e \widetilde{\in} H_E$, $x_e \widetilde{\notin} H_E$ [21],

(iii) soft T_2 (ST_2, for short), if for every two soft points x_e , y_e with $x \neq y$, there exist G_E , $H_E \in \tau^*$ such that $x_e \widetilde{\in} G_E$, $y_e \widetilde{\in} H_E$ and $G_E \widetilde{\cap} H_E = \varnothing_E$ [21],

(iv) soft regular, if for every $F_E \in \tau^{*c}$ and $x_e \in SP(X, E)$ with $x_e \widetilde{\notin} F_E$, there exist G_E , $H_E \in \tau^*$ such that $x_e \widetilde{\in} G_E$, $F_E \widetilde{\subseteq} H_E$ and $G_E \widetilde{\cap} H_E = \varnothing_E$ [21],

(v) Soft compact, if every soft open cover of X_E has a finite subcover of X_E [22].

Theorem 2.13. (1) $ST_2 \Longrightarrow ST_1 \Longrightarrow ST_0$ [21].

(2) *Every soft closed subset of a soft compact space is compact* [22].

Definition 2.14 ([8]). A topological space (X, τ) is said to be:

(i) R_0, if for every pair of distinct points $x, y \in X$ with $\overline{x} \neq \overline{y}$, $\overline{x} \cap \overline{y} = \varnothing$,

(ii) R_1, if for every pair of distinct points $x, y \in X$ with $\overline{x} \neq \overline{y}$, there exist disjoint open sets U, V such that $x \in U$, $y \in V$.

3. Soft R_0 and soft R_1 Spaces.

Definition 3.1. A soft topological space (X, τ^*, E) is said to be:

(i) soft R_0 (SR_0, for short), if for every pair of distinct soft points x_e, y_e with $x_e \widetilde{\in} \overline{y_e}$ implies $y_e \widetilde{\in} \overline{x_e}$,

(ii) soft R_1 (SR_1, for short), if for every pair of distinct soft points x_e, y_e with $\overline{x_e} \neq \overline{y_e}$, there exist $F_E, G_E \widetilde{\in} \tau^*$ such that $x_e \widetilde{\in} G_E$, $y_e \widetilde{\in} H_E$ and $F_E \widetilde{\cap} G_E = \varnothing_E$.

In the following, we introduce some characterizations of SR_0 and SR_1 spaces.

Theorem 3.2. *Let (X, τ^*, E) be a soft topological space. Then the following properties are equivalent:*

(1) *(X, τ^*, E) is SR_0 ,*

(2) *$\overline{x_e} \widetilde{\subseteq} U_E$ for all $U_E \in \tau^*$, $x_e \widetilde{\in} U_E$.*

Proof. (1) $\Longrightarrow$ (2): Let (X, τ^*, E) be SR_0. Suppose that $\overline{x_e} \widetilde{\nsubseteq} U_E$, for some $U_E \in \tau^*$, $x_e \widetilde{\in} U_E$. Then there exists soft point y_e with $y_e \widetilde{\in} \overline{x_e}$ and $y_e \widetilde{\notin} U_E$. Thus $y_e \widetilde{\cap} U_E = \varnothing_E$, for some $U_E \in \tau^*$, $x_e \widetilde{\in} U_E$ and x_e, y_e distinct soft points. So $x_e \widetilde{\notin} \overline{y_e}$. This contradiction. Hence $\overline{x_e} \widetilde{\subseteq} U_E$, for all $U_E \in \tau^*$, $x_e \widetilde{\in} U_E$.

(2) $\Longrightarrow$ (1): Let $x_e \widetilde{\notin} \overline{y_e}$. Then there exists an open soft set V_E containing x_e with $y_e \widetilde{\cap} V_E = \varnothing_E$. Thus $y_e \widetilde{\notin} V_E$. Since $\overline{x_e} \widetilde{\subseteq} V_E$, $y_e \widetilde{\notin} \overline{x_e}$. So (X, τ^*, E) is SR_0. □

Theorem 3.3. *For a soft topological space (X, τ^*, E) and $x_e \in SP(X, E)$, the following properties are equivalent:*

(1) *(X, τ^*, E) is a SR_0 space,*

(2) *for any $F_E \in \tau^{*c}$ with $x_e \widetilde{\notin} F_E$, $\overline{x_e} \widetilde{\cap} F_E = \varnothing_E$,*

(3) *for any distinct soft points x_e , y_e, either $\overline{x_e} \neq \overline{y_e}$ or $\overline{x_e} \widetilde{\cap} \overline{y_e} = \varnothing_E$.*

Proof. (1) $\Longrightarrow$ (2): It follows directly from the above theorem.

(2) $\Longrightarrow$ (3): Let x_e, y_e are distinct soft points with $\overline{x_e} \neq \overline{y_e}$. Then there exists $z_e \widetilde{\in} \overline{x_e}$ and $z_e \widetilde{\notin} \overline{y_e}$. Thus there exists $U_E \in \tau^*$ such that $y_e \widetilde{\notin} U_E$, $z_e \widetilde{\in} U_E$. So $x_e \widetilde{\in} U_E$. Hence $x_e \widetilde{\notin} \overline{y_e}$. Therefore by (2), $\overline{x_e} \widetilde{\cap} \overline{y_e} = \varnothing_E$. The proof of the other case is similar.

(3) $\Longrightarrow$ (1): Let x_e, y_e are distinct soft points with $x_e \widetilde{\notin} \overline{y_e}$. Then $\overline{x_e} \neq \overline{y_e}$. Thus by (3), we get $\overline{x_e} \widetilde{\cap} \overline{y_e} = \varnothing_E \Longrightarrow y_e \widetilde{\in}\ \overline{y_e} \widetilde{\subset} \overline{x_e}^c \Longrightarrow y_e \widetilde{\notin} \overline{x_e}$. So the result holds. □

By using the above theorems, one can shows the following corollary.

Corollary 3.4. *A soft topological space (X, τ^*, E) is SR_0 if and only if for any distinct soft points x_e, y_e with $\overline{x_e} \neq \overline{y_e}$, implies $\overline{x_e} \widetilde{\cap} \overline{y_e} = \varnothing_E$.*

Definition 3.5. Let (X, τ^*, E) be a soft topological space and $F_E \in SS(X, E)$. The soft kernel of F_E, denoted by $SK(F_E)$, is the soft set defined by:

$$SK(F_E) = \widetilde{\cap}\{G_E \in \tau^* : F_E \widetilde{\subseteq} G_E\}.$$

In particular, the soft kernel of $x_e \in SP(X, E)$, is the soft set given by:

$$SK(x_e) = \widetilde{\cap}\{G_E \in \tau^* : x_e \widetilde{\in} G_E\}.$$

Lemma 3.6. *Let (X, τ^*, E) be a soft topological space and $F_E \in SS(X, E)$. Then*

$$SK(F_E) = \widetilde{\cup}\{x_e \in SP(X, E) : \overline{x_e} \widetilde{\cap} F_E \neq \varnothing_E\}.$$

Proof. Let $x_e \widetilde{\in} SK(F_E)$ and $\overline{x_e} \widetilde{\cap} F_E = \varnothing_E$. Then $F_E \widetilde{\subseteq} \overline{x_e}^c = U_E \in \tau^*$ and $x_e \widetilde{\notin} U_E$. This is contradiction with $x_e \widetilde{\in} SK(F_E)$. Thus $\overline{x_e} \widetilde{\cap} F_E \neq \varnothing_E$. So

$$SK(F_E) \widetilde{\subseteq} \widetilde{\cup} \{x_e \in SP(X,E) : \overline{x_e} \widetilde{\cap} F_E \neq \varnothing_E\}.$$

On other hand, let $\overline{x_e} \widetilde{\cap} F_E \neq \varnothing_E$ and $x_e \widetilde{\notin} SK(F_E)$. Then there exists $V_E \in \tau^*$ such that $F_E \widetilde{\subseteq} V_E$, $x_e \widetilde{\notin} V_E$. Let $y_e \widetilde{\in} \overline{x_e} \widetilde{\cap} F_E \neq \varnothing_E$. Then V_E is an open soft contain y_e, which $x_e \widetilde{\notin} V_E$. By this contradiction, $x_e \widetilde{\in} SK(F_E)$. Thus the result holds. □

Lemma 3.7. *Let (X, τ^*, E) be a soft topological space and $x_e \in SP(X,E)$. Then $y_e \widetilde{\in} SK(x_e)$ if and only if $x_e \widetilde{\in} \overline{y_e}$.*

Proof. It is obvious. □

Proposition 3.8. *Let (X, τ^*, E) be a soft topological space. Then the following properties are equivalent:*
(1) *(X, τ^*, E) is SR_0 ,*
(2) *$\overline{x_e} \widetilde{\subseteq} SK(x_e)$, for all $x_e \in SP(X,E)$.*

Proof. It follows from the above Lemma and Theorem 3.2. □

By using Lemma 3.7 and the above proposition, one can verify the following corollary.

Corollary 3.9. *A soft topological space (X, τ^*, E) is SR_0 if and only if for any soft point x_e in X_E, $\overline{x_e} = SK(x_e)$.*

Theorem 3.10. *Let (X, τ^*, E) be a soft topological space. Then the following properties are equivalent:*
(1) *(X, τ^*, E) is SR_0 ,*
(2) *if F_E is soft closed, then $F_E = SK(F_E)$,*
(3) *if F_E is soft closed with $x_e \widetilde{\in} F_E$, then $SK(x_e) \widetilde{\subseteq} F_E$,*
(4) *if $x_e \in SP(X,E)$, then $SK(x_e) \widetilde{\subseteq} \overline{x_e}$.*

Proof. (1) $\Longrightarrow$ (2): Let F_E be soft closed and $x_e \widetilde{\notin} F_E$. Then $x_e \widetilde{\in} F_E^c$ which is soft open set containing x_e. Since (X, τ^*, E) is SR_0, $\overline{x_e} \widetilde{\subseteq} F_E^c \Longrightarrow \overline{x_e} \widetilde{\cap} F_E = \varnothing_E$. thus by Lemma 3.6, $x_e \widetilde{\notin} SK(F_E)$. So $F_E = SK(F_E)$.

(2) $\Longrightarrow$ (3): It follows from the fact, $A_E \widetilde{\subseteq} B_E \Longrightarrow SK(A_E) \widetilde{\subseteq} SK(B_E)$.

(3) $\Longrightarrow$ (4): It is obvious.

(4) $\Longrightarrow$ (1): Let x_e , y_e are distinct soft points with $x_e \widetilde{\in} \overline{y_e}$. Then by Lemma 3.7, $y_e \widetilde{\in} SK(x_e)$. Since $x_e \widetilde{\in} \overline{x_e}$ which is soft closed, by (4), $y_e \widetilde{\in} SK(x_e) \widetilde{\subseteq} \overline{x_e}$, that is, $y_e \widetilde{\in} \overline{x_e}$. So (X, τ^*, E) is a SR_0 space. □

Lemma 3.11. *Let (X, τ^*, E) be a soft topological space and x_e , $y_e \in SP(X,E)$. Then $SK(x_e) \neq SK(y_e)$ if and only if $\overline{x_e} \neq \overline{y_e}$.*

Proof. Necessity: Let $SK(x_e) \neq SK(y_e)$. Then there exists $z_e \in SP(X,E)$ such that $z_e \widetilde{\in} SK(x_e)$ and $z_e \widetilde{\notin} SK(y_e)$. If $z_e \widetilde{\in} SK(x_e)$, then by Lemma 3.6, $x_e \widetilde{\cap} \overline{z_e} \neq \varnothing_E \Longrightarrow x_e \widetilde{\in} \overline{z_e}$, that is, $\overline{x_e} \widetilde{\subseteq} \overline{z_e}$. Similarly, if $z_e \widetilde{\notin} SK(y_e)$, then $y_e \widetilde{\notin} \overline{z_e}$. Since $\overline{x_e} \widetilde{\subset} \overline{z_e}$ and $y_e \widetilde{\notin} \overline{z_e}$, $y_e \widetilde{\notin} \overline{x_e}$. Thus $\overline{x_e} \neq \overline{y_e}$.

Conversely, let $\overline{x_e} \neq \overline{y_e}$. Then there exists $z_e \in SP(X,E)$ such that $z_e \widetilde{\in} \overline{x_e}$ and $z_e \widetilde{\notin} \overline{y_e}$. Thus there exists a soft open set containing z_e. So x_e but not y_e. Hence $y_e \notin SK(x_e)$. Therefore $SK(x_e) \neq SK(y_e)$. □

Theorem 3.12. *A soft topological space (X, τ^*, E) is SR_0 if and only if for every distinct soft points x_e, y_e with $SK(x_e) \neq SK(y_e)$, $SK(x_e) \widetilde{\cap} SK(y_e) = \varnothing_E$.*

Proof. Necessity: Let (X, τ^*, E) is SR_0 and x_e, y_e are two distinct soft points with $SK(x_e) \neq SK(y_e)$. Then by Lemma 3.11, $\overline{x_e} \neq \overline{y_e}$. Suppose $SK(x_e) \widetilde{\cap} SK(y_e) \neq \varnothing_E$. Then there exists $z_e \widetilde{\in} SK(x_e) \widetilde{\cap} SK(y_e)$. If $z_e \widetilde{\in} SK(x_e)$, then by Lemma 3.7, $x_e \widetilde{\in} \overline{z_e} \Longrightarrow \overline{x_e} \widetilde{\subset} \overline{z_e}$. Since $x_e \widetilde{\in} \overline{x_e}$, by Corollary 3.4, $\overline{x_e} = \overline{z_e}$. Similarity, if $z_e \widetilde{\in} SK(y_e)$, then $\overline{y_e} = \overline{z_e} = \overline{x_e}$. This contradiction. Thus $SK(x_e) \widetilde{\cap} SK(y_e) = \varnothing_E$.

Conversely, letx_e, y_e are distinct soft points with $\overline{x_e} \neq \overline{y_e}$. Then by Lemma 3.11, $SK(x_e) \neq SK(y_e)$. Thus by hypothesis, $SK(x_e) \widetilde{\cap} SK(y_e) = \varnothing_E$. Suppose that $\overline{x_e} \widetilde{\cap} \overline{y_e} \neq \varnothing_E$. Then there exists $z_e$in X_E such that $z_e \widetilde{\in} \overline{x_e}, z_e \widetilde{\in} \overline{y_e}$. Thus by Lemma 3.7, $x_e \widetilde{\in} SK(z_e)$ and $y_e \widetilde{\in} SK(z_e)$. By Lemma 3.6, $SK(x_e) \widetilde{\cap} SK(z_e) \neq \varnothing_E$ and $SK(y_e) \widetilde{\cap} SK(z_e) \neq \varnothing_E$. By hypothesis, $SK(x_e) = SK(z_e)$ and $SK(y_e) = SK(z_e) = SK(x_e)$. So $SK(x_e) \widetilde{\cap} SK(y_e) \neq \varnothing_E$. This contradiction. Hence $\overline{x_e} \widetilde{\cap} \overline{y_e} = \varnothing_E$. Therefore by Corollary 3.4, (X, τ^*, E) is SR_0. □

Theorem 3.13. *Let (X, τ^*, E) be a soft topological space. Then (X, τ^*, E) is* SR_1 *if and only if for every distinct soft points x_e ,y_e with $SK(x_e) \neq SK(y_e)$, there exist F_E , $G_E \in \tau^*$ such that $\overline{x_e} \widetilde{\subseteq} F_E$, $\overline{y_e} \widetilde{\subseteq} G_E$ and $F_E \widetilde{\cap} G_E = \varnothing_E$.*

Proof. It follows from Lemma 3.7. □

From Definition 3.1, Lemma 3.11 and the above theorem, one can verify the following corollary.

Corollary 3.14. *For a soft topological space (X, τ^*, E), the following properties are equivalent:*

(1) *(X, τ^*, E) is soft R_1,*

(2) *for every distinct soft points x_e, y_e with $x_e \widetilde{\notin} \overline{y_e}$, there exist $F_E, G_E \in \tau^*$ such that $x_e \widetilde{\in} F_E, y_e \widetilde{\in} G_E$ and $F_E \widetilde{\cap} G_E = \varnothing_E$.*

(3) *for every distinct soft points x_e, y_e with $\overline{x_e} \neq \overline{y_e}$, there exist $F_E, G_E \in \tau^*$ such that $\overline{x_e} \widetilde{\subseteq} F_E, \overline{y_e} \widetilde{\subseteq} G_E$ and $F_E \widetilde{\cap} G_E = \varnothing_E$.*

Theorem 3.15. *A soft singleton point space (X, τ^*, E) is SR_i, $i = 0,\ 1$.*

Proof. As a sample, we prove the case $i = 1$. Let (X, τ^*, E) be a soft singleton point space and x_e ,y_e are two distinct soft points with $\overline{x_e} \neq \overline{y_e}$. Then there exist soft open sets x_E , y_E $\in \tau^*$ such that $x_e \widetilde{\in} x_E$, $y_e \widetilde{\in} y_E$ and $x_E \widetilde{\cap} y_E = \varnothing_E$. □

Theorem 3.16. *Every SR_1 space is a SR_0 space.*

Proof. Obvious. □

Note. The following example shows that the converse of the above theorem is not necessary true.

Example 3.17. Let X be a non-empty infinite set and let $\tau_{\infty}^{*} = \{\varnothing_E\} \cup \{F_E \in SS(X,E) : (F(e))^c$ is a finite subset of X for all $e \in E\}$. The space $(X, \tau_{\infty}^{*}, E)$ is a soft topological space, called a soft cofinite space (See [21]). Now, to show that $(X, \tau_{\infty}^{*}, E)$ is SR_0 but not SR_1. Let x_e , y_e be distinct soft points with $x_e \widetilde{\in} y_e^c$. Since x_e^c , $y_e^c \in \tau_{\infty}^{*}$, x_e is closed soft set, that is, $\overline{x_e} = x_e$. thus $\overline{x_e} = x_e \widetilde{\subseteq} y_e^c$. So by Theorem 3.2, $(X, \tau_{\infty}^{*}, E)$ is SR_0 .

On other hand, suppose $(X, \tau_{\infty}^{*}, E)$ is SR_1. Then for every distinct soft points x_e , y_e with $\overline{x_e} \neq \overline{y_e}$, there exist F_E , $G_E \in \tau_{\infty}^{*}$ such that $x_e \widetilde{\in} F_E$, $y_e \widetilde{\in} G_E$ and $F_E \widetilde{\cap} G_E = \varnothing_E$. Thus $(F(e))^c \cup (G(e))^c = X$. Since $(F(e))^c$, $(G(e))^c$ are two finite subset of X, X is finite. This is contradiction. So $(X, \tau_{\infty}^{*}, E)$ is not SR_1.

Theorem 3.18. *Every soft subspace (Y, τ_Y^{*}, E) of SR_i (X, τ^{*}, E) is SR_i,$i = 0,\ 1$.*

Proof. As a sample, we prove the case, $i = 1$. Let $x_e,\ y_e$ be distinct soft points in (Y, E) with $\overline{x_e} \neq \overline{y_e}$. Then $x_e,\ y_e$ are distinct soft points in (X, E) with $\overline{x_e} \neq \overline{y_e}$. Since (X, τ^{*}, E) is SR_1, there exist F_E , $G_E \in \tau^{*}$ such that $x_e \widetilde{\in} F_E$, $y_e \widetilde{\in} G_E$ with $F_E \widetilde{\cap} G_E = \varnothing_E$. Thus there exist soft open sets, $U_E^Y = Y_E \widetilde{\cap} F_E \in \tau_Y^{*}$ and $V_E^Y = Y_E \widetilde{\cap} G_E \in \tau_Y^{*}$ which are contains $x_e,\ y_e$, respectively with $U_E^Y \widetilde{\cap} V_E^Y = \varnothing_E$. So (Y, τ_Y^{*}, E) is a SR_1 space. □

4. Some relations and results.

Theorem 4.1. *Every ST_i space (X, τ^{*}, E) is SR_{i-1}, $i = 1,\ 2$.*

Proof. For the case $i = 1$, let $U_E \widetilde{\in} \tau^{*}$ with $x_e \widetilde{\in} U_E$. We will show that $\overline{x_e} \widetilde{\subseteq} U_E$. Let $y_e \widetilde{\notin} U_E$. Then $x_e \widetilde{\notin} \overline{y_e}$ and x_e, y_e are distinct soft points. Since (X, τ^{*}, E) is ST_1, there exists $G_E \widetilde{\in} \tau^{*}$ such that $y_e \widetilde{\in} G_E$, $x_e \widetilde{\notin} G_E$. thus $y_e \widetilde{\notin} \overline{x_e}$. So $\overline{x_e} \widetilde{\subseteq} U_E$. Hence the result holds. The proof of the other case is clear. □

Note. The following example shows that the converse of the above theorem is not true in general.

Example 4.2. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\tau^{*} = \{\varnothing_E, X_E, F_E, G_E\}$, where $F_E = \{(e_1, X)\}, G_E = \{(e_2, X)\}$. Then (X, τ^{*}, E) is a soft topological space. Now one can check that (X, τ^{*}, E) is SR_0 and SR_1.

On other hand, for two distinct soft points x_{e_1}, y_{e_1}, the only open soft sets which are containing x_{e_1} are X_E and $F_E = \{(e_1, X)\}$ but also, they are containing y_{e_1}. Then (X, τ^{*}, E) not ST_1 and thus, not ST_2. Indeed, for soft points x_{e_1}, y_{e_1}, there no exist disjoint open soft sets U_E, V_E such that $x_{e_1} \widetilde{\in} U_E$ and $y_{e_1} \widetilde{\in} V_E$.

Theorem 4.3. *Let (X, τ^{*}, E) be a soft topological space. Then*
(1) (X, τ^{*}, E) *is* ST_1 if and only if *is both* SR_0 *and* ST_0,
(2) (X, τ^{*}, E) *is* ST_2 if and only if *is both* SR_1 *and* ST_0.

Proof. (1) Necessity: It follows from Theorems 4.1 and 2.13.

Conversely, let x_e, y_e are distinct soft points in X_E. Since (X, τ^{*}, E) is ST_0 and SR_0, $\overline{x_e} \neq \overline{y_e}$. Then by Corollary 3.4, $\overline{x_e} \widetilde{\cap} \overline{y_e} = \varnothing_E$. Thus $\overline{y_e}^c \widetilde{\in} \tau^{*}$ which is contain x_e, not y_e and $\overline{x_e}^c \widetilde{\in} \tau^{*}$ which contains y_e, not x_e. So (X, τ^{*}, E) is ST_1.

(2) Necessity: It follows from Theorems 4.1 and 2.13.

Conversely, let $x_e,\ y_e$ be distinct soft points of X_E with $x_e \widetilde{\notin} \overline{y_e}$. Since (X, τ^*, E) is SR_0, $y_e \widetilde{\notin} \overline{x_e}$. Thus $\overline{x_e} \neq \overline{y_e}$. Also (X, τ^*, E) is SR_1. So there exist disjoint open soft sets U_E , V_E containing x_e , y_e respectively. Hence (X, τ^*, E) is ST_2. □

Theorem 4.4. *A soft space* (X, τ^*, E) *is* ST_2 if and only if *is both* SR_1 *and* ST_1.

Proof. The proof is consequence of that of the above theorem. □

Theorem 4.5. *Every soft regular space* (X, τ^*, E) *is a* SR_i *space,* $i = 0,\ 1$.

Proof. For the case, $i = 1$. Let x_e, y_e be distinct soft points with $\overline{x_e} \neq \overline{y_e}$. Then either $x_e \widetilde{\notin} \overline{y_e}$ or $y_e \widetilde{\notin} \overline{x_e}$. Without loss of generality, assume that $x_e \widetilde{\notin} \overline{y_e}$, where $\overline{y_e}$ is closed soft with $x_e \widetilde{\notin} \overline{y_e}$. Since (X, τ^*, E) is soft regular, there exist disjoint soft open sets U_E , V_E with $x_e \widetilde{\in} U_E$ and $y_e \widetilde{\in}\ \overline{y_e} \widetilde{\subseteq} V_E$. Thus (X, τ^*, E) is SR_1.
The proof of the other case is similar. □

Note. The next example shows that the converse of the above theorem is not true in general.

Example 4.6. Let $X = \{x, y\}$ and $E = \{e_1, e_2\}$. Then the family $\tau^* = \{\varnothing_E,\ X_E,\ F_E,\ G_E,\ H_E,\ M_E\}$, where $F_E = \{(e_1, \{x\})\}, G_E = \{(e_1, \{x\}), (e_2, \{y\})\}$, $H_E = \{(e_1, \{y\}), (e_2, \{x\})\}$ and $M_E = \{(e_1, X), (e_2, \{x\})\}$ is a soft topology on X. Now one can check that (X, τ^*, E) is SR_0 and SR_1 but not soft regular. Indeed, for the soft closed set $F_E^c = \{(e_1, \{y\}), (e_2, X)\}$, $x_{e_1} \widetilde{\notin} F_E^c$, there exists $F_E \in \tau^*$ with $x_{e_1} \widetilde{\in} F_E$ and the only open soft set which is containing F_E^c is X_E, but X_E not disjoint from F_E.

Theorem 4.7. *Let* (X, τ^*, E) *be a soft compact space. Then* (X, τ^*, E) *is* SR_1 *if and only if is soft regular.*

Proof. Necessity: Let (X, τ^*, E) be a soft compact and SR_1 space. To prove that (X, τ^*, E) is soft regular, let us $H_E \in \tau^{*c}$ and $x_e \in SP(X, E)$ with $x_e \widetilde{\notin} H_E$. Now for all $y_e \widetilde{\in} H_E$, $\overline{y_e} \widetilde{\subseteq} H_E$. Since $x_e \widetilde{\notin} H_E$, $x_e \widetilde{\notin} \overline{y_e}$ and thus, $\overline{x_e} \neq \overline{y_e}$. Since (X, τ^*, E) is SR_1, for all $y_e \widetilde{\in} H_E$, there exist disjoint open soft sets $F_E^{y_e}$, $G_E^{y_e}$ such that $x_e \widetilde{\in} F_E^{y_e}$, $y_e \widetilde{\in} G_E^{y_e}$. So $\{G_E^{y_e} : y_e \widetilde{\in} H_E\}$ is an open cover of H_E. By (2) of Theorem 2.13, H_E is soft compact. Hence there exists a finite subfamily $\{G_{1E}, G_{2E}, \ldots, G_{nE}\}$ of $\{G_E^{y_e} : y_e \widetilde{\in} H_E\}$ that covers H_E. Now let $\{F_{1E}, F_{2E}, \ldots, F_{nE}\}$ a corresponding subfamily of $\{F_E^{y_e} : y_e \widetilde{\in} H_E\}$. Then it is clear that, $U_E = \widetilde{\cap}_{i=1}^n F_{iE}$ and $V_E = \widetilde{\cup}_{i=1}^n G_{iE}$ are open soft sets and U_E disjoint from V_E, because $U_E \widetilde{\subseteq} F_{iE}\ \forall i$ which is disjoint from the corresponding G_{iE} with $x_e \widetilde{\in} U_E$ and $H_E \widetilde{\subseteq} V_E$. Therefore (X, τ^*, E) is soft regular. The proof of the converse follows from Theorem 4.5. □

Theorem 4.8. *If* (X, τ^*, E) *is a* SR_i *space, then* (X, τ_e) *is* R_i , $\forall\ e \in E$, $i = 0,\ 1$.

Proof. For the case $i = 1$, let $x, y \in X$ and $x \neq y$ with $\overline{x} \neq \overline{y}$. Then either $x \notin \overline{y}$ *or* $y \notin \overline{x}$ and thus, $x_e \notin \overline{y_e}$ *or* $y_e \notin \overline{x_e}$, then $\overline{x_e} \neq \overline{y_e}$. Since (X, τ^*, E) is SR_1, there exist U_E , $V_E \widetilde{\in} \tau^*$ such that $x_e \widetilde{\in} U_E$, $y_e \widetilde{\in} V_E$ and $U_E \widetilde{\cap} V_E = \varnothing_E$. So there exist $U(e), V(e) \in \tau_e$ such that $x \in U(e)$, $y \in V(e)$ and $U(e) \cap V(e) = \varnothing$, for all $e \in E$. Hence (X, τ_e) is $R_1\ \forall e \in E$. The proof of the other case is similar. □

Note. The next example shows that the converse of the above theorem is not true in general.

Example 4.9. Let $X = \{x, y\}$ and $E = \{e_1, e_2\}$. Then the family $\tau^* = \{\varnothing_E,\ X_E,\ F_{1E}\ , F_{2E}\ ,\ F_{3E}\ ,\ F_{4E}\ \}$, where $F_{1E} = \{(e_1, \{x\})\}\ ,\ F_{2E} = \{(e_1, \{x\}), (e_2, \{x\})\}$, $F_{3E}\ = \{(e_1, \{x\}), (e_2, \{y\})\}$ and $F_{4E} = \{(e_1, \{x\}), (e_2, X)\}$ is a soft topology on X and the family $\tau_{e_2} = \{\varnothing, X, \{x\}, \{y\}\}$ is a topology on X. It is clear that (X, τ_{e_2}) is R_1 and R_0, but (X, τ^*, E) is not SR_0. Indeed, for distinct soft points $x_{e_1},\ y_{e_1}$we have, $X_E = \overline{x_{e_1}} \neq \overline{y_{e_1}} = y_{e_1}$but $\overline{x_{e_1}} \widetilde{\cap} \overline{y_{e_1}} \neq \varnothing_E$, and so, (X, τ^*, E) is not SR_1.

Corollary 4.10. *Let (X, τ^*, E) be a soft singleton point space. Then (X, τ^*, E) is SR_i if and only if (X, τ_e) is R_i, $\forall\ e \in E$, $i = 0, 1$.*

Proof. For the case $i = 1$. Necessity: It follows from Theorem 4.8.

Conversely, let $x_e,\ y_e$ be distinct soft points with $\overline{x_e} \neq \overline{y_e}$. Then $x \neq y$ with $\overline{x} \neq \overline{y}$. Since (X, τ_e) is R_1, there exist open subsets $A,\ B$ of X such that $x \in A$, $y \in B$ and $A \cap B = \varnothing$. Thus there exist U_E , $V_E \widetilde{\in} \tau^*_\tau$ such that $A = U(e)$ and $B = V(e)\ \ \forall\ e \in E$ with $x_e \widetilde{\in} U_E$, $y_e \widetilde{\in} V_E$ and $U_E \widetilde{\cap} V_E = \varnothing_E$. So the result holds. The proof of the other case is similar. □

Theorem 4.11. *A topological space (X, τ) is R_i if and only if (X, τ^*_τ, E) is SR_i, $i = 0,\ 1$.*

Proof. For the case $i = 1$. Necessity: The proof is analogues to that of the converse part in the above corollary.

Conversely, let $x,\ y \in X$ and $x \neq y$ with $\overline{x} \neq \overline{y}$. Then either $x \notin \overline{y}$ *or* $y \notin \overline{x}$ and thus, $x_e \notin \overline{y_e}$ *or* $y_e \notin \overline{x_e}$. So $\overline{x_e} \neq \overline{y_e}$. Since (X, τ^*_τ, E) is SR_1, there exist U_E , $V_E \widetilde{\in} \tau^*_\tau$ such that $x_e \widetilde{\in} U_E$, $y_e \widetilde{\in} V_E$ and $U_E \widetilde{\cap} V_E = \varnothing_E$. Hence there exist disjoint open sets $F,\ G \in \tau$ such that $x \in U(e) = F$ *and* $y \in V(e) = G$, for all $e \in E$. Therefore (X, τ) is R_1. The proof of the other case is similar. □

5. Conclusion.

In this paper, we defined and studied some new properties are called soft R_0 and R_1 properties in soft topological spaces. Some characterizations of them are obtained. Also we, studied some nice results and relations. We hope these basic results will help the researchers to enhance and promote the research on soft set theory and its applications.

Acknowledgements. The author is grateful thanks to referees and Prof. Kul Hur, Editor-in-Chief, for their valuable comments and suggestion helpful in improving this paper.

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S. Saleh (s_wosabi@yahoo.com , drsalemwosabi@gmial.com)
Department of Mathematics, Faculty of Education-Zabid, Hodeidah University, Hodeidah, Yemen