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Abstract. Intuitionistic fuzzy set theory and intuitionistic fuzzy operators have many applications on Engineering, Space sciences, Social science and Genetic (See[6, 8, 9, 12, 15]). There is no study by using intuitionisitc fuzzy set theory and intuitionistic fuzzy operators on how optimum pesticide and fertilization must be applied on an agricultural area on which a smart system is built. For this reason, in this paper, the optimum amount of pesticide and fertilization that trees need is examined according to 7 periods (Periods which are determined by regarding the periods of a tree). Another essential point is to protect plants from the effect of pesticide and fertilization which are given to plants more than they need. Also, this method paves the way for farmers to make more economical agricultural and use less chemical substances than traditional agricultural.

2010 AMS Classification: Primary 05C38, 15A15; Secondary 05A15, 15A18

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1. Introduction

The concept of fuzzy sets was introduced by Zadeh [17] as an extension of crisp sets by expanding the truth value set to the real unit interval [0,1]. Let X be a set. The function $\mu \to [0,1]$ is called a fuzzy set over X. A fuzzy set is denoted by (FS(X)). For, $x \in X$ is the membership degree of x and the non-membership degree is $1-\mu(x)$. Intuitionistic fuzzy sets have been introduced by Atanassov [1], as an extension of fuzzy sets. If X is a universal set then an intuitionistic fuzzy set A, the membership and non-membership degree for each $x \in X$ respectively, $\mu_A(x)$ ($\mu_A : X \to [0,1]$ and $\nu_A(x)$ ($\nu_A : X \to [0.1]$ such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The class of intuitionistic fuzzy sets on X is denoted by IFS(X). While the sum of membership degree and non-membership degree is 1 on FS, this sum is equal to 1 or less than 1 on IFS.

2. Preliminaries

Definition 2.1 ([5])**.** Let $L = [0,1]$. Then

$$L^* = \left\{(x_1, x_2) \in [0,1]^2 : x_1 + x_2 \le 1\right\}$$

is a lattice with

$$(x_1, x_2) \le (y_1, y_2) :\Longleftrightarrow \text{"}x_1 \le y_1 \text{ and } x_2 \ge y_2\text{"}.$$

For (x_1, x_2), $(y_1, y_2) \in L^*$ the operators $\wedge$ and $\vee$ on $(L^*, \le)$ are defined as following:

$$(x_1, x_2) \wedge (y_1, y_2) = \min(x_1, x_2),\ \max(y_1, y_2),$$

$$(x_1, x_2) \vee (y_1, y_2) = \max_1(x_1, x_2),\ \max(\ y_1, y_2),$$

$$\sup J = (\sup\{x \in [0,1] : (y \in [0,1]((x,y) \in J)\},\ \inf\{y \in [0,1] : (x \in [0,1])((x,y) \in J)\},$$

$$\inf J = (\inf\{x \in [0,1] : (y \in [0,1])((x,y) \in J)\},\ \sup\{y \in [0,1] : (x \in [0,1]((x,y) \in J)\}.$$

Definition 2.2 ([1])**.** An intuitionistic fuzzy set (shortly IFS) on a set X is an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

where $\mu_A(x), (\mu_A : X \to [0,1])$ is called the "degree of membership of x in A ", $\nu_A(x), (\nu_A : X \to [0,1])$is called the " degree of non-membership of x in A ", and μ_A and ν_A satisfy the following condition:

$$\mu_A(x) + \nu_A(x) \le 1, \text{ for all } x \in X.$$

Definition 2.3 ([7])**.** Let X be a universal set and $A \in IFS(X)$, $\alpha, \beta \in [0,1]$. Then

$$E_{\alpha,\beta}(A) = \{\langle x, \beta(\alpha\mu_A(x) + 1 - \alpha),\ \alpha(\beta\nu_A(x) + 1 - \beta)\rangle : x \in X\}.$$

Definition 2.4 ([9])**.** Let X be a universal set and $A \in IFS(X)$, $\alpha, \beta, \omega \in [0,1]$. Then

$$Z^{\omega}_{\alpha,\beta}(A) = \{\langle x, \beta(\alpha\mu_A(x) + \omega - \omega.\alpha), \alpha(\beta\nu_A(x) + \omega - \omega.\beta)\rangle : x \in X\}.$$

Definition 2.5 ([4])**.** Let X be a universal set and $A \in IFS(X)$, $\alpha, \beta \in [0,1]$. Then

$$F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha\pi_A(x), \nu_A(x) + \beta\pi_A(x)\rangle : x \in X\}.$$

Definition 2.6 ([11])**.** Let X be a universal set and $A \in IFS(X)$, $\alpha, \beta \in [0,1]$. Then

$$B_{\alpha,\beta}(A) = \{\langle x, \beta(\alpha\mu_A(x) + 1 - \alpha)\nu_A(x), \alpha((1-\beta)\mu_A(x) + \nu_A(x))\rangle : x \in X\}.$$

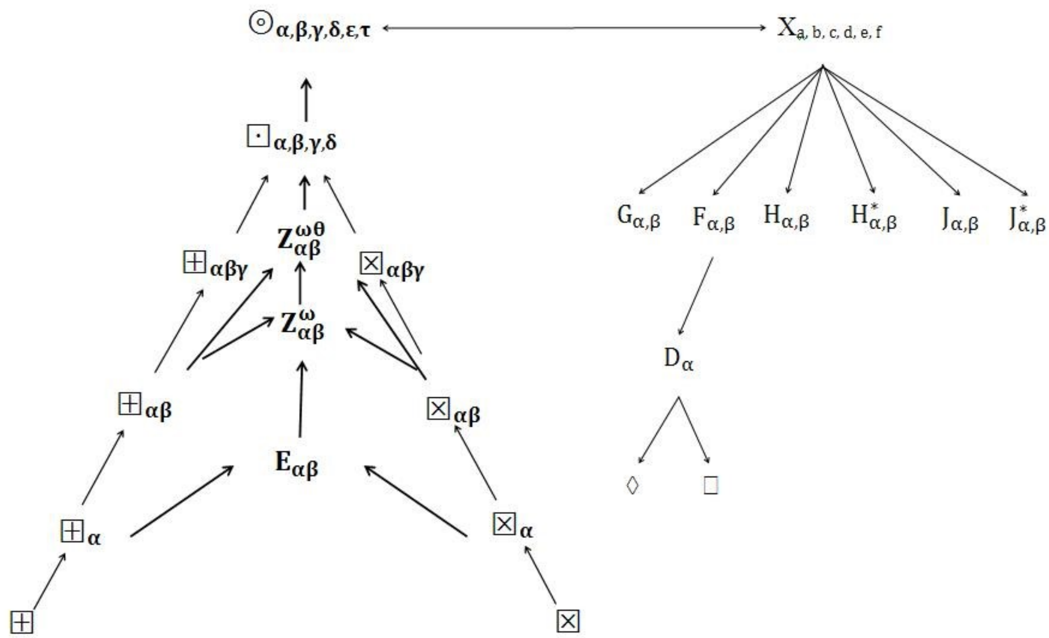

Figure 1.

3. The application of intuitionistic fuzzy modal operator $E_{\alpha,\beta}$ on agriculture

Intuitionistic Fuzzy Set Theory [2, 3] makes appropriate algorithms on the analysis of complex systems and decision processes because intuitionistic fuzzy set theory takes natural variables instead of random variables on uncertainty models.

We use the regulation operator (INT) [16] to separate an agricultural area in different parts because every part of it does not need same amount of pesticide and fertilization. The operator which is written below for this purpose is used.

The regulation operator (INT) on fuzzy set A creates fuzzy set.

$$A' = INT(A) \tag{3.1}$$

is a membership function such that

if $0 \leq \mu_A(x) \leq 0.5$, then $\mu_{A'}(x) = \mu_{INT(A)}(x) = 2\mu_A(x)^2$,

if $0.5 \leq \mu_A(x) \leq 1$, then$\mu_{A'}(x) = \mu_{INT(A)}(x) = 1 - 2(1 - \mu_A(x))^2$.

The regulation operator decreases the fuziness value of an element of set A if the membership function $\mu_A(x)$ is smaller than 0.5 and increases fuziness value of an element of set A if the membership function $\mu_A(x)$ is bigger than 0.5. The image of agricultural area is enhanced by the help of this operator after process that is mentioned above. The results of operators $\mu_{A'}(x)$ and $E_{\alpha,\beta}$ are compared in [10]

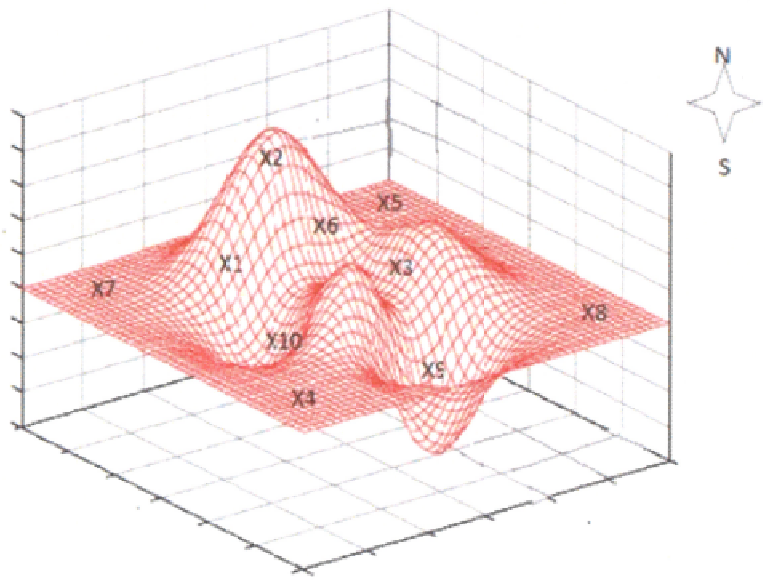

FIGURE 2. Different elevations on the land

Shortly, respectively, the level of Basic Pesticide-Fertilization and the level of Risk Pesticide-Fertilization is marked BPFL and RPFL.

Let agricultural area have same properties like Figure2. Assume that, different kinds of trees are grown on every different parts of this agricultural area. On this circumstance, agricultural area is separated different parts according to tree variety, direction and elevation.

According to the definiton of intuitionistic fuzzy modal operator $E_{\alpha,\beta}$, membership degree is smaller than β and non-membership degree is smaller than α. In addition, the values α and β are between 0 and 1, the values of α and β are independent from each other.

The problem of optimum pesticide-fertilization regarding to different parts of agricultural area is easily solved by the help of some properties $E_{\alpha,\beta}$ of modal operator [10].

For this reason, assume that, same kind of trees are grown on agricultural area that has different directions and elevations. In this situation, variables are directions and elevations on agricultural area. Naturally, this two characteristic properties (directions, elevations) create different rate of pesticide-fertilization that trees need, but the amount of pesticide and fertilization that trees need is same. So, critical value can be determined. Because, trees which are not on the same part of area do not have critical value. This critical value is marked by β which belongs to $E_{\alpha,\beta}$. Critical value creates risk and this variable is marked α.

	$\mu_A(x)$	$\mu_{A'}(x)$	$E_{0.3,0.4}(A)$	$\mu_{E_{0.3,0.4}(A)}(x)$	$\nu_{E_{0.3,0.4}(A)}(x)$	$E_{0.3,0.4}(A')$
x_1	0.0	0.00	0.28	0.300	0.1568	0.2800 0.3000
x_2	0.1	0.02	0.292	0.288	0.1705	0.2824 0.2976
x_3	0.2	0.08	0.304	0.276	0.1849	0.2896 0.2904
x_4	0.3	0.18	0.316	0.264	0.1997	0.3016 0.2784
x_5	0.4	0.32	0.328	0.252	0.2151	0.3184 0.2616
x_6	0.5	0.5	0.34	0.24	0.2312	0.3400 0.2400
x_7	0.6	0.68	0.352	0.228	0.2478	0.3616 0.2184
x_8	0.7	0.82	0.364	0.216	0.255	0.3784 0.2016
x_9	0.8	0.92	0.376	0.204	0.2828	0.3904 0.1896
x_{10}	0.9	0.98	0.388	0.192	0.3010	0.3976 0.1824
x_{11}	1.0	1.00	0.400	0.180	0.3200	0.4000 0.1800

Figure 3.

	$\mu_A(x)$	$\mu_{A'}(x)$	$E_{0.1,0.8}(A)$	$\mu_{E_{0.1,0.8}(A)}(x)$	$\nu_{E_{0.1,0.8}(A)}(x)$	$E_{0.1,0.8}(A')$
x_1	0.0	0.00	0.720	0.100	0.8432	0.7200 0.1000
x_2	0.1	0.02	0.728	0.092	0.8520	0.7216 0.0984
x_3	0.2	0.08	0.736	0.084	0.8606	0.7264 0.0936
x_4	0.3	0.18	0.744	0.076	0.8689	0.7344 0.0856
x_5	0.4	0.32	0.752	0.068	0.8770	0.7456 0.0744
x_6	0.5	0.50	0.760	0.060	0.8848	0.7600 0.0600
x_7	0.6	0.68	0.768	0.052	0.8923	0.7744 0.0456
x_8	0.7	0.82	0.776	0.044	0.8996	0.7856 0.0344
x_9	0.8	0.92	0.784	0.036	0.9066	0.7936 0.0264
x_{10}	0.9	0.98	0.792	0.028	0.9134	0.7984 0.0216
x_{11}	1.0	1.00	0.800	0.020	0.9200	0.8000 0.0200

Figure 4.

From Figure 3, the level of Basic Pesticide-Fertilization BPFL=0.4, the level of Risk Pesticide-Fertilization RPFL=0.3, system interferes step by step to measure the amount of appropriate pesticide and fertilization on different parts according to fourth column in Figure 3. Specially, we take the values of RPFL and BPFL respectively 0.3 and 0.4 for the agricultural area. If the conditions of agricultural area change, then chosen of RPFL and BPFL easily is made.

According to Figure 4, if, BPFL and RPFL is not close, values change fastly when the amount of pesticide-fertilization that different parts of agricultural area need is

	$\mu_A(x)$	$\mu_{A'}(x)$	$E_{0.8,0.1}(A)$	$\mu_{E_{0.8,0.1}(A)}(x)$	$\nu_{E_{0.8,0.1}(A)}(x)$	$E_{0.8,0.1}(A')$
x_1	0.0	0.00	0.020	0.800	0.0008	0.0200 0.8000
x_2	0.1	0.02	0.028	0.792	0.0016	0.0216 0.7984
x_3	0.2	0.08	0.036	0.784	0.0026	0.0264 0.7936
x_4	0.3	0.18	0.044	0.776	0.0039	0.0344 0.7856
x_5	0.4	0.32	0.052	0.768	0.0054	0.0456 0.7744
x_6	0.5	0.5	0.06	0.760	0.0072	0.0600 0.7600
x_7	0.6	0.68	0.068	0.752	0.0093	0.0744 0.7456
x_8	0.7	0.82	0.076	0.744	0.0116	0.0856 0.7344
x_9	0.8	0.92	0.084	0.736	0.0141	0.0936 0.7264
x_{10}	0.9	0.98	0.092	0.728	0.0169	0.0984 0.7216
x_{11}	1.0	1.00	0.100	0.720	0.0200	0.1000 0.7200

Figure 5.

close to RPFL. The speed of values decreases and respond in a controlled manner if values are close to BPFL.

In Figure 4, system fastly rules out the value of RPFL in order to get appropriate amount of pesticide-fertilization after this process, BPFL makes an important role to decrease the speed of system.

According to the relationship between $E_{\alpha,\beta}$ and $E_{\beta,\alpha}$ operators, the dual of $E_{\alpha,\beta}$ is risky and necessity value of amount of pesticide that is given from soil and leaves.

It is understood in Figure 5, these values decrease the speed of pesticide. It is easily seen that system regulates the amount of pesticide-fertilization to decrease the effect of chemical substances.

The interpretation for Figure 4 and Figure 5, the demand of pesticide-fertilization decreases when effect of pesticide decreases on plants.

4. Application of some intuitionistic fuzzy modal operators on agricultural areas

In this paper, our assumption is to separate agricultural areas into different parts like separation of images into parts to enhance [16]. In addition to other authors' results, we use the result of two-dimensional [13, 14] image enhancement to separate agricultural areas into different parts by using intuitionistic fuzzy modal operator $E_{\alpha,\beta}$.

Some variables are used to separate agricultural areas. Such as; temperature, the density of afforestation, the kind of trees, the properties of soil, the moisture of soil, the slope of area, the moisture of air etc. are different values. We choose the kind of soil, moisture of air, and the moisture of soil from other values.

Figure 6.

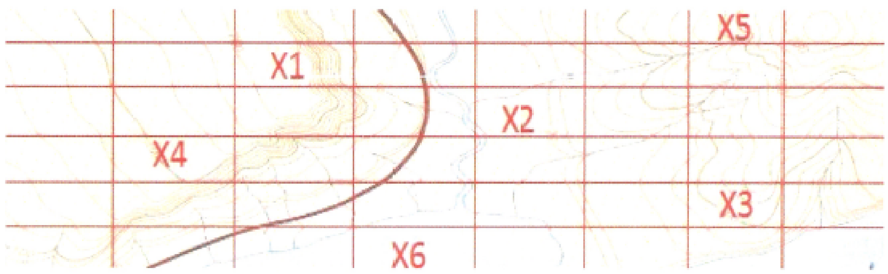

Figure 7.

In the light of this purpose, we use enhancement algorithms which is defined on membership function and intuitionistic fuzzy modal operators $Z^{\omega}_{\alpha,\beta}$, $F_{\alpha,\beta}$, $B_{\alpha,\beta}$. These operators are used to make appropriate arrangements.

Intuitionistic fuzzy modal operators give appropriate algorthims on different areas. These operators also are used to estimate the agricultural parameters. Pesticide and fertilization must be made step by step to protect the plants from chemical effect.

Intuitionistic fuzzy modal operators are used to determine the level of pesticide-fertilization. Two or more steps are used on agricultural areas. In Figure 6, the sloping area is demonstrated.

It is seen that in Figure 7, after these processes, every parts are separated regarding to variables.

The definition of $Z^{\omega}_{\alpha,\beta}(A)$, $F_{\alpha,\beta}(A)$, $B_{\alpha,\beta}(A)$ and some values of α, β, ω are used to calculate the number of steps. In addition, $\alpha, \beta, \omega \in [0, 1]$ and the values of α, β, ω do not affect each others.

5. Using of intuitionistic fuzzy modal operators on agricultural Area

It is given that the interpretation of $Z^{\omega}_{\alpha,\beta}(A)$, $F_{\alpha,\beta}(A)$, $B_{\alpha,\beta}(A)$ modal operators for pesticide and fertilization of trees in *Graphic* 1, *Graphic* 2 and *Graphic* 3. It is easily seen from graphics that it is not appropriate to use $F_{\alpha,\beta}(A)$, $B_{\alpha,\beta}(A)$ modal operators for calculating appropriate pesticide-fertilization because these operators

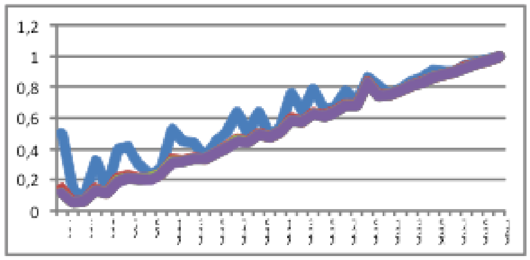

FIGURE 8.
Graphic 1 Model of pesticide-fertilization with $F_{\alpha,\beta}(A)$

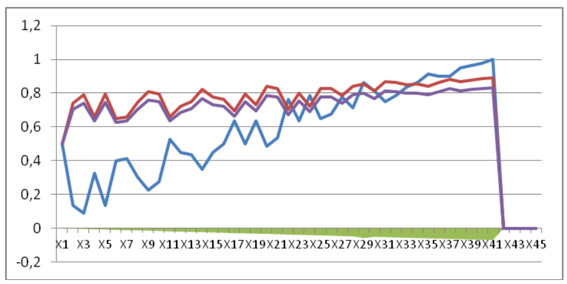

FIGURE 9.
Graphic 2 Model of pesticide-fertilization with $B_{\alpha,\beta}(A)$

are not useful to stabilize the level of pesticide-fertilization. It is seen that $Z^{\omega}_{\alpha,\beta}(A)$ modal operators can be used. For example; trees have different RPFL because of their location in the area. So; $Z^{\omega}_{\alpha,\beta}(A)$ interferes the system to give optimum amount of pesticide-fertilization according trees which are in different parts of area. That is why, in this paper, $Z^{\omega}_{\alpha,\beta}(A)$ is used to calculate the appropriate ratio of pesticide-fertilization.

The interpreation of Graphic 1; numbers which are on the vertical of the graphics represent the RPFL of trees on the same area. In steps, $F_{\alpha,\beta}(A)$ interferes the system but it is not successful to stabilize BPFL.

In Graphic 2; $B_{\alpha,\beta}(A)$ also does not stabilize RPFL of trees because every trees need BPFL, but $B_{\alpha,\beta}(A)$ is not successful in the Graphic 2.

According to Graphic 3; it is easily seen that $Z^{\omega}_{\alpha,\beta}(A)$ stabilizes BPFL of trees. Because, trees which have different RPFL are given pesticide and fertilization until all of them reaches BPFL.

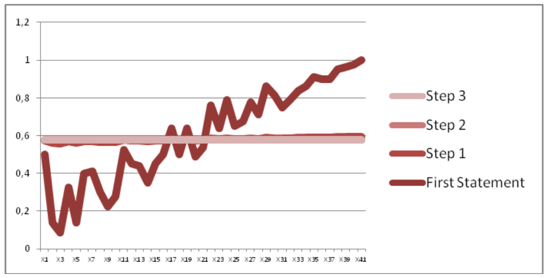

FIGURE 10.
Graphic 3 Model of pesticide-fertilization with $Z^{\omega}_{\alpha,\beta}(A)$

6. Using of $Z^{\omega}_{\alpha,\beta}$ to calculate models for agricultural areas

It is separated into 7 groups: First Period, Second Period, Third Period, Fourth Period, Fifth Period, Sixth Period, Seventh Period regarding to tree periods, flower and fruit period.

Every periods has different time that tress need pesticide-fertilization. It is necessary to determine the level of pesticide-fertilization for every plant periods.

α in $Z^{\omega}_{\alpha,\beta}(A)$ is used as a parameter for plant period. Such as; for 6th Period, $\alpha \leq 0.2$ and for 2nd Period, $\alpha \geq 0.5$.

The need of water of 2nd Period is more than other periods. In addition, 2nd Period does not need pesticide-fertilization so, plants must be watered little by little and often.

The level of 0.65 is based on the study. The way of problem solution is given below.

First of all, choosing of α, β, ω in $Z^{\omega}_{\alpha,\beta}(A)$ is given below to answer essential need of pesticide-fertilization.

Theorem 6.1 ([10]). *The partial sum sequence of coefficients of n-th degree of $Z^{\omega}_{\alpha\beta}(A)$ is convergent.*

Proof.
(6.1)

$$\left(Z^{\omega}_{\alpha\beta}(A)\right)^n = \left\{\left\langle x, \alpha^n\beta^n\mu(A) + \beta\omega(1-\alpha)\sum_{i=0}^{n-1}(\alpha\beta)^i, \alpha^n\beta^n\nu(A) + \alpha\omega(1-\beta)\sum_{i=0}^{n-1}(\alpha\beta)^i\right\rangle : x \in X\right\}$$

Let $a = \beta\omega(1-\alpha)$, $r = \alpha\beta$. Then sequentially, we obtain

$$S_k = a\sum_{i=0}^{k} r^i = a + ar + ... + ar^k, \tag{6.2}$$

$$rS_k = ar + ar^2 + ... + ar^{k+1},$$

$$S_k = a\frac{1-r^{k+1}}{1-r},$$

$$S_{n-1} = a\frac{1-r^n}{1-r}.$$

So it is obtained that:

$$for\ S_{n-1},\ \frac{a}{1-r} = \frac{\beta\omega\left(1-\alpha\right)}{1-\alpha\beta}.$$

It is calculated in the same way for non-membership degree:

$$\frac{\alpha\omega\left(1-\beta\right)}{1-\alpha\beta}.$$

□

Remark 6.2. The required coefficient of moisture (RM) is calculated for kind of soil and plant:

$$the\ membership\ degree\ of\ \left(Z^{\omega}_{\alpha\beta}\left(A\right)\right)\ is\ \frac{\beta\omega\left(1-\alpha\right)}{1-\alpha\beta},$$

$$the\ non-membership\ degree\ of\ \left(Z^{\omega}_{\alpha\beta}\left(A\right)\right)\ is\ \frac{\alpha\omega\left(1-\beta\right)}{1-\alpha\beta},$$

$$rm = \frac{1+\beta\omega-2\alpha\beta}{2\left(1-\alpha\beta\right)}.$$

Example 6.3. Let X be an agricultural area. α in $Z^{\omega}_{\alpha\beta}\left(A\right)$ represents Plant Period and $|\beta-\omega|$ represents the require of pesticide and fertilization of soil. Then, the require of pesticide and fertilization decreases, if $|\beta-\omega|$ is small, require of pesticide and fertilization increases, if $|\beta-\omega|$ is big.

Example 6.4. For $Z^{\omega}_{\alpha\beta}\left(A\right)$ modal operator let $\alpha = 0.1$, $\beta = 0.9$, $\omega = 0.9$. The require level of moisture is calculated below for this area. It is easily seen that in Figure 11. The result given below is obtained by Theorem 1:

$$\frac{\beta\omega\left(1-\alpha\right)}{1-\alpha\beta} = \frac{0.9\times 0.9\left(1-0.1\right)}{1-\left(0.1\times 0.9\right)} = \frac{0.81\times 0.9}{1-0.09} = \frac{0.729}{0.91} = 0.801098. \tag{6.3}$$

(6.4) It converges membership degree.

$$\frac{\alpha\omega\left(1-\beta\right)}{1-\alpha\beta} = \frac{0.1\times 0.9\left(1-0.9\right)}{1-\left(0.1\times 0.9\right)} = \frac{0.09\times 0.1}{1-0.09} = \frac{0.009}{0.91} = 0.009801. \tag{6.5}$$

(6.6) It converges non-membership degree.

$$rm = \frac{1+\beta\omega-2\alpha\beta}{2\left(1-\alpha\beta\right)} = \frac{0.801098+0.990199}{2} = 0.8956485. \tag{6.7}$$

It is shown that in Graphic 4; required moisture for the solid is calculated.

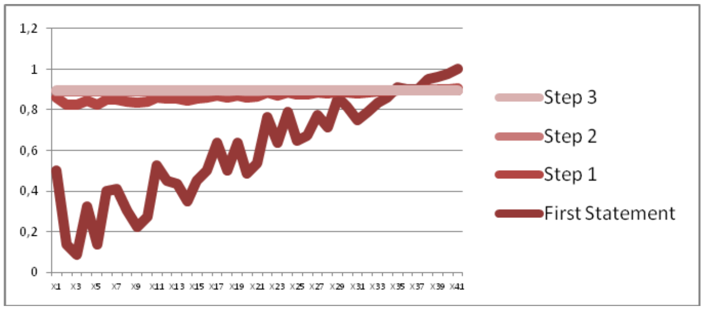

FIGURE 11. *Graphic* 4 The example of calculation level of required moisture

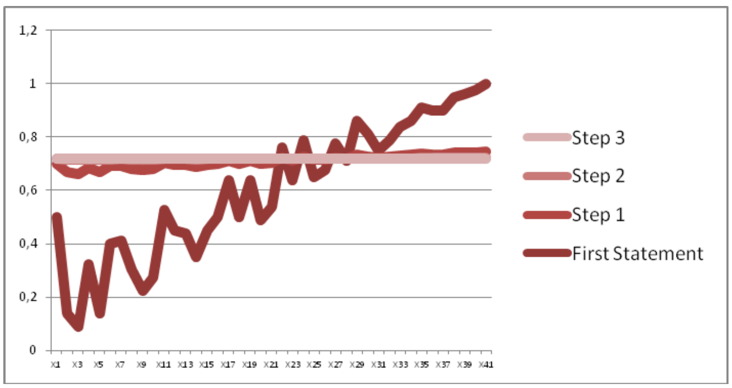

FIGURE 12. *Graphic* 5 The graphic of pesticide-fertilization of 6 Period.

In Graphic 5, 6th period is studied. It is shown that it is calculated appropriate moisture for agricultural area by using $Z^{\omega}_{\alpha\beta}(A)$ modal operator. Respectively, the value of α, β, ω is 0.1 , 0.5 and 0.5, for $Z^{\omega}_{\alpha\beta}(A)$.

It is easily seen that pesticide-fertilization is made in three steps from Graphic 5. Pesticide and fertilization are made fastly in first step. This process goes slowly in other steps. It is found in Figure 5 appropriate pesticide and fertilization for sixth period.

Example 6.5. Third period is examined in Graphic 6. The absorption of pesticide and fertilization in second period is not good. So, more steps must be made to absorp more pesticide and fertilizer. So, 5 steps are used for third period. The value $\alpha = 0.5$, $\beta = 0.9$, $\omega = 0.5$ in $Z^{\omega}_{\alpha\beta}(A)$ modal operators for second period. The absorption of pesticide and fertilizer goes down so; the required of pesticide and fertilizer that plants need decrease. Pesticide and fertilization are made in healthy and economical way by the help of this process.

Example 6.6. The result is found in Graphic 7, if $\alpha = 0.3$, $\beta = 0.9$, $\omega = 0.5$ are chosen for seventh period.

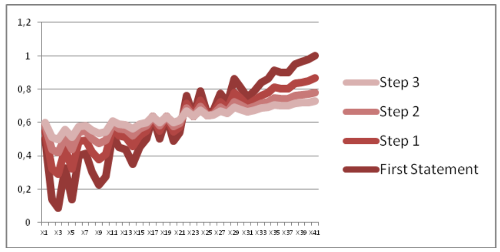

FIGURE 13. *Graphic* 6 : Pesticide-fertilization for 3rd Period

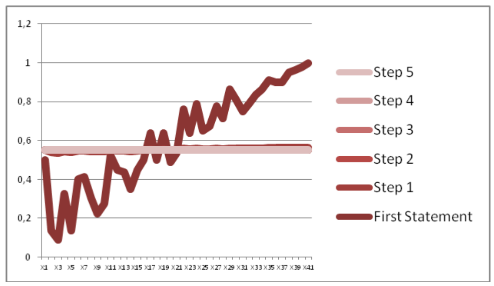

FIGURE 14. *Graphic* 7 : Pesticide-fertilization for 7th Period

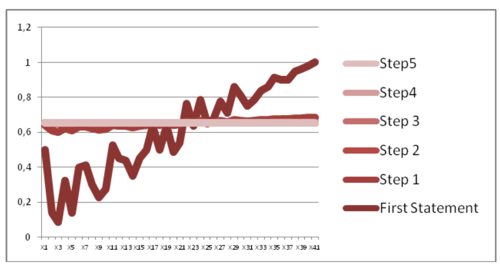

FIGURE 15. *Graphic* 8 : Pesticide-fertilization for 3rd Period

Example 6.7. The result is found in Graphic 8, if $\alpha = 0.6$, $\beta = 0.9$, $\omega = 0.45$ are chosen for third period.

The critical level of pesticide and fertilization is different for every parts of agricultural area, in this paper, we separate agricultural are into several parts regarding to temperature, soil moisture, air moisture and slope of area.

In general situation, we consider that agricultural area has same kind of plant. $E_{\alpha,\beta}$ modal operator was used in paper [10], $Z^{\omega}_{\alpha,\beta}$ modal operator can be used for watering agricultural area regarding to slope, temperature, air moisture and soil moisture in spite of using $E_{\alpha,\beta}$. The system that is recommended in this paper paves the way for using less chemical substance and more economical than traditional agricultural methods.

μ	v	MDZ 0,1 0,5 0,5	NDZ 0,1 0,5 0,5	average	MDZ 0,1 0,5 0,5	NDZ 0,1 0,5 0,5	average	MDZ 0,1 0,5 0,5	NDZ 0,1 0,5 0,5	average
0	0	0,225	0,025	0,1	0,23625	0,02625	0,005	0,236813	0,026313	0,00025
0,03	0,75	0,2263	0,0625	0,444	0,236313	0,028125	0,022219	0,236816	0,026406	0,0011109
0,05	0,875	0,2275	0,0688	0,492	0,236375	0,028438	0,024594	0,236819	0,026422	0,0012297
0,08	0,425	0,2288	0,0463	0,266	0,236438	0,027313	0,013313	0,236822	0,026366	0,0006656
0,1	0,825	0,23	0,0663	0,444	0,2365	0,028313	0,022219	0,236825	0,026416	0,0011109
0,13	0,325	0,2313	0,0413	0,195	0,236563	0,027063	0,00975	0,236828	0,026353	0,0004875
0,15	0,325	0,2325	0,0413	0,183	0,236625	0,027063	0,009156	0,236831	0,026353	0,0004578
0,18	0,57	0,2338	0,0535	0,288	0,236688	0,027675	0,014381	0,236834	0,026384	0,0007191
0,2	0,75	0,235	0,0625	0,361	0,23675	0,028125	0,018063	0,236838	0,026406	0,0009031
0,23	0,675	0,2363	0,0588	0,314	0,236813	0,027938	0,015688	0,236841	0,026397	0,0007844
0,25	0,2	0,2375	0,035	0,076	0,236875	0,02675	0,003813	0,236844	0,026338	0,0001906
0,28	0,375	0,2388	0,0438	0,148	0,236938	0,027188	0,007375	0,236847	0,026359	0,0003687
0,3	0,425	0,24	0,0463	0,159	0,237	0,027313	0,007969	0,23685	0,026366	0,0003984
0,33	0,625	0,2413	0,0563	0,243	0,237063	0,027813	0,012125	0,236853	0,026391	0,0006063
0,35	0,45	0,2425	0,0475	0,148	0,237125	0,027375	0,007375	0,236856	0,026369	0,0003688
0,38	0,375	0,2438	0,0438	0,1	0,237188	0,027188	0,005	0,236859	0,026359	0,00025
0,4	0,125	0,245	0,0313	-0,03	0,23725	0,026563	-0,00153	0,236863	0,026328	0,000077
0,43	0,425	0,2463	0,0463	0,1	0,237313	0,027313	0,005	0,236866	0,026366	0,00025
0,45	0,175	0,2475	0,0338	-0,03	0,237375	0,026688	-0,00153	0,236869	0,026334	0,000077
0,48	0,5	0,2488	0,05	0,112	0,237438	0,0275	0,005594	0,236872	0,026375	0,0002797
0,5	0,425	0,25	0,0463	0,064	0,2375	0,027313	0,003219	0,236875	0,026366	0,0001609
0,53	0	0,2513	0,025	-0,15	0,237563	0,02625	-0,00747	0,236878	0,026313	-0,000373
0,55	0,275	0,2525	0,0388	-0,03	0,237625	0,026938	-0,00153	0,236881	0,026347	-0,000077
0,58	0	0,2538	0,025	-0,17	0,237688	0,02625	-0,00866	0,236884	0,026313	-0,000433
0,6	0,3	0,255	0,04	-0,04	0,23775	0,027	-0,00213	0,236888	0,02635	-0,000106
0,63	0,275	0,2563	0,0388	-0,07	0,237813	0,026938	-0,00331	0,236891	0,026347	-0,000166
0,65	0,1	0,2575	0,03	-0,16	0,237875	0,0265	-0,00806	0,236894	0,026325	-0,000403
0,68	0,25	0,2588	0,0375	-0,1	0,237938	0,026875	-0,00509	0,236897	0,026344	-0,000255
0,83	0,1	0,2663	0,03	-0,24	0,238313	0,0265	-0,01222	0,236916	0,026325	-0,000611
0,73	0,1	0,2613	0,03	-0,2	0,238063	0,0265	-0,00984	0,236903	0,026325	-0,000492
0,75	0,25	0,2625	0,0375	-0,14	0,238125	0,026875	-0,00687	0,236906	0,026344	-0,000344
0,78	0,2	0,2638	0,035	-0,17	0,238188	0,02675	-0,00866	0,236909	0,026338	-0,000433
0,8	0,125	0,265	0,0313	-0,22	0,23825	0,026563	-0,01103	0,236913	0,026328	-0,000552
0,83	0,1	0,2663	0,03	-0,24	0,238313	0,0265	-0,01222	0,236916	0,026325	-0,000611
0,85	0,025	0,2675	0,0263	-0,29	0,238375	0,026313	-0,01459	0,236919	0,026316	-0,00073
0,88	0,075	0,2688	0,0288	-0,28	0,238438	0,026438	-0,014	0,236922	0,026322	-0,0007
0,9	0,1	0,27	0,03	-0,28	0,2385	0,0265	-0,014	0,236925	0,026325	-0,0007
0,93	0,025	0,2713	0,0263	-0,33	0,238563	0,026313	-0,01638	0,236928	0,026316	-0,000819
0,95	0,025	0,2725	0,0263	-0,34	0,238625	0,026313	-0,01697	0,236931	0,026316	-0,000848
0,98	0,02	0,2738	0,026	-0,35	0,238688	0,0263	-0,01768	0,236934	0,026315	-0,000884
1	0	0,275	0,025	-0,38	0,23875	0,02625	-0,01875	0,236938	0,026313	-0,000938

Figure 16.

µ	v	MDZ 0,3 0,9 0,5	NDZ 0,3 0,9 0,5	average	average	average	average	average
0	0	0,315	0,015	0,15	0,0405	0,010935	0,002952	0,003556
0,03	0,75	0,3218	0,2175	0,414625	0,111949	0,030226	0,008161	0,005501
0,05	0,875	0,3285	0,2513	0,451125	0,121804	0,032887	0,008879	0,005784
0,08	0,425	0,3353	0,1298	0,27775	0,074993	0,020248	0,005467	0,00454
0,1	0,825	0,342	0,2378	0,414625	0,111949	0,030226	0,008161	0,005555
0,13	0,325	0,3488	0,1028	0,223	0,06021	0,016257	0,004389	0,004178
0,15	0,325	0,3555	0,1028	0,213875	0,057746	0,015591	0,00421	0,004129
0,18	0,57	0,3623	0,1689	0,294175	0,079427	0,021445	0,00579	0,004732
0,2	0,75	0,369	0,2175	0,35075	0,094703	0,02557	0,006904	0,005161
0,23	0,675	0,3758	0,1973	0,31425	0,084848	0,022909	0,006185	0,004914
0,25	0,2	0,3825	0,069	0,13175	0,035573	0,009605	0,002593	0,003603
0,28	0,375	0,3893	0,1163	0,1865	0,050355	0,013596	0,003671	0,004019
0,3	0,425	0,396	0,1298	0,195625	0,052819	0,014261	0,00385	0,004104
0,33	0,625	0,4028	0,1838	0,2595	0,070065	0,018918	0,005108	0,004587
0,35	0,45	0,4095	0,1365	0,1865	0,050355	0,013596	0,003671	0,004073
0,38	0,375	0,4163	0,1163	0,15	0,0405	0,010935	0,002952	0,003825
0,4	0,125	0,423	0,0488	0,049625	0,013399	0,003618	0,000977	0,003113
0,43	0,425	0,4298	0,1298	0,15	0,0405	0,010935	0,002952	0,003861
0,45	0,175	0,4365	0,0623	0,049625	0,013399	0,003618	0,000977	0,003149
0,48	0,5	0,4433	0,15	0,159125	0,042964	0,0116	0,003132	0,003964
0,5	0,425	0,45	0,1298	0,122625	0,033109	0,008939	0,002414	0,003716
0,53	0	0,4568	0,015	-0,041625	-0,01124	-0,00303	-0,00082	0,002538
0,55	0,275	0,4635	0,0893	0,049625	0,013399	0,003618	0,000977	0,00322
0,58	0	0,4703	0,015	-0,059875	-0,01617	-0,00436	-0,00118	0,002441
0,6	0,3	0,477	0,096	0,0405	0,010935	0,002952	0,000797	0,00319
0,63	0,275	0,4838	0,0893	0,02225	0,006007	0,001622	0,000438	0,003075
0,65	0,1	0,4905	0,042	-0,05075	-0,0137	-0,0037	-0,001	0,002561
0,68	0,25	0,4973	0,0825	-0,005125	-0,00138	-0,00037	-0,0001	0,002911
0,83	0,1	0,5378	0,042	-0,114625	-0,03095	-0,00836	-0,00226	0,002222
0,73	0,1	0,5108	0,042	-0,078125	-0,02109	-0,0057	-0,00154	0,002416
0,75	0,25	0,5175	0,0825	-0,0325	-0,00877	-0,00237	-0,00064	0,002766
0,78	0,2	0,5243	0,069	-0,059875	-0,01617	-0,00436	-0,00118	0,002585
0,8	0,125	0,531	0,0488	-0,096375	-0,02602	-0,00703	-0,0019	0,002337
0,83	0,1	0,5378	0,042	-0,114625	-0,03095	-0,00836	-0,00226	0,002222
0,85	0,025	0,5445	0,0218	-0,151125	-0,0408	-0,01102	-0,00297	0,001974
0,88	0,075	0,5513	0,0353	-0,142	-0,03834	-0,01035	-0,00279	0,002058
0,9	0,1	0,558	0,042	-0,142	-0,03834	-0,01035	-0,00279	0,002076
0,93	0,025	0,5648	0,0218	-0,1785	-0,0482	-0,01301	-0,00351	0,001829
0,95	0,025	0,5715	0,0218	-0,187625	-0,05066	-0,01368	-0,00369	0,00178
0,98	0,02	0,5783	0,0204	-0,198575	-0,05362	-0,01448	-0,00391	0,001718
1	0	0,585	0,015	-0,215	-0,05805	-0,01567	-0,00423	0,001617

Figure 17.

In Figure 16, it is the table of the amount of pesticide-fertilization, if $\alpha = 0.1$, $\beta = 0.5$, $\omega = 0.5$ in $Z^{\omega}_{\alpha,\beta}$ are chosen. In Figure 17, it is the table of the amount of pesticide-fertilization, if $\alpha = 0.3$, $\beta = 0.9$, $\omega = 0.5$ in $Z^{\omega}_{\alpha,\beta}$ are chosen.

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