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Some characterizations of fuzzy g'''- irresolute functions

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Abstract. The main purpose of this paper is to investigate characterization of fuzzy g'''-irresolute functions.

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1. Introduction

In the classical paper[15]of 1965, Zadeh generalized the usual notion of a set and introduced the important and useful notion of fuzzy sets. Several authors [1, 2, 3, 13] working in the field of fuzzy topology have shown more interest in studying the concepts of generalizations of fuzzy continuous functions. A weak form of fuzzy continuous functions called fuzzy g-continuous functions were introduced by Sundaram [12]. Recently Sudha et al [11] have introduced and studied another form of fuzzy generalized continuous functions called fuzzy ω-continuous functions respectively.

In this paper, we investigate characterization of fuzzy g'''-irresolute functions.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) (or X, Y and Z) represent fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy subset A of a space (X, τ), $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the complement of A, respectively.

We recall the following Definitions which are useful in the sequel.

Definition 2.1. A fuzzy subset A of a space (X, τ) is called:
(i) a fuzzy semi-open set [1], if $A \leq cl(int(A))$,
(ii) a fuzzy α-open set [3], if $A \leq int(cl(int(A)))$,
(iii) a fuzzy β-open set [13] (=semi-preopen set), if $A \leq cl(int(cl(A)))$.

The complements of the above mentioned open sets are called their respective closed sets. The fuzzy semi-closure [14] (resp. fuzzyα -closure [9], fuzzy semi-preclosure [13]) of a fuzzy subset A of X, denoted by $scl(A)$ (resp. $\alpha cl(A)$, $spcl(A)$) is defined to be the intersection of all fuzzy semi-closed (resp. fuzzy α-closed, fuzzy semi-preclosed) sets of (X, τ) containing A.

It is known that $scl(A)$ (resp.$\alpha cl(A)$, $spcl(A)$) is a fuzzy semi-closed (resp. fuzzy α -closed, fuzzy semi-preclosed) set.

Definition 2.2. A fuzzy subset A of a space (X, τ) is called:
(i) a fuzzy generalized closed (briefly, fuzzy g-closed) set [2], if $cl(A) \leq U$, whenever $A \leq U$ and U is fuzzy open in (X, τ). The complement of fuzzy g-closed set is called fuzzy g-open set,
(ii) a fuzzy generalized semi-closed (briefly fgs-closed) set [10], if $scl(A) \leq U$, whenever $A \leq U$ and U is fuzzy open in (X, τ). The complement of fgs-closed set is called fgs-open set,
(iii) a fuzzy g'''-closed (briefly fg'''-closed)set [5], if $cl(A) \leq U$, whenever $A \leq U$ and U is fgs-open in (X, τ). The complement of fg'''-closed set is called fg'''-open set.

Definition 2.3. A fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is called:
(i) fuzzy g'''-continuous [7], if $f^{-1}(V)$ is a fuzzy g'''-closed set in (X, τ), for every fuzzy closed set V of (Y, σ),
(ii) fuzzy g-continuous [2], if $f^{-1}(V)$ is a fuzzy g-closed set in (X, τ), for every fuzzy closed set V of (Y, σ),
(iii) fuzzy gs-continuous [10], if $f^{-1}(V)$ is a fuzzy gs-closed set in (X, τ), for every fuzzy closed set V of (Y, σ),
(iv) fuzzy g'''-closed [8], if the image of every fuzzy closed set in (X, τ) is fuzzy g'''-closed in (Y, σ),
(v) fuzzy g'''-open [8], if the image of every fuzzy open set in (X, τ) is fuzzy g'''-open in (Y, σ).

Definition 2.4. A fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is called fuzzy gs-irresolute function [6], if the inverse image of every fuzzy gs-closed (resp. fuzzy gs-open) set in (Y, σ) is fuzzy gs-closed (resp. fuzzy gs-open) in (X, τ).

Definition 2.5. A fuzzy topological space (X, τ) is called:
(i) fuzzy Tg'''-space [6], if every fuzzy g'''-closed set in it is fuzzy closed.
(ii) fuzzy $T_{1/2}$-space [2] if every fuzzy g-closed set in it is fuzzy closed.

Definition 2.6 ([6])**.** A fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is called fuzzy pre-gs-open, if $f(U)$ is fuzzy gs-open in (Y, σ), for each fuzzy gs-open set U in (X, τ).

Lemma 2.7 ([4])**.** *Let $f : (X, \tau) \to (Y, \sigma)$ be a fuzzy function. For the fuzzy sets A and B of X and Y respectively, the following statements hold:*
(1) $ff^{-1}(B) \leq B$,

(2) $f^{-1}f(A) \geq A$,
(3) $f(A^c) \geq (f(A))^c$,
(4) $f^{-1}(B^c) = (f^{-1}(B))^c$,
(5) *if f is injective, then* $f^{-1}(f(A)) = A$,
(6) *if f is surjective, then* $ff^{-1}(B) = B$,
(7) *if f is bijective, then* $f(A^c) = (f(A))^c$.

3. Fuzzy g'''-irresolute functions

We introduce the following definition:

Definition 3.1 ([6]). A fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is called a fuzzy g'''-irresolute function, if the inverse image of every fuzzy g'''-closed set in (Y, σ) is fuzzy g'''-closed in (X, τ).

Remark 3.2. The following examples show that the notions of fuzzy gs-irresolute functions and fuzzy g'''-irresolute functions are independent .

Example 3.3. Let $X = Y = \{a, b\}$ and $\tau = \{0_X,\ \alpha,\ 1_X\}$, where α is fuzzy set in X defined by $\alpha(a) = 1$, $\alpha(b) = 0$ and $\sigma = \{0_X,\ \beta,\ 1_X\}$, where β is fuzzy set in Y defined by $\beta(a) = 0.5$, $\beta(b) = 0$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Let $f : (X, \tau) \to (Y, \sigma)$ be the identity fuzzy function. Then clearly, f is fuzzy g'''-irresolute but it is not fuzzy gs-irresolute.

Example 3.4. Let $X = Y = \{a, b\}$ and $\tau = \{0_X,\ \gamma,\ 1_X\}$, where γ is fuzzy set in X defined by $\gamma(a) = 0.4$, $\gamma(b) = 0.5$ and $\sigma = \{0_X,\ \delta,\ 1_X\}$, where δ is fuzzy set in Y defined by $\delta(a) = 0.6$, $\delta(b) = 0.5$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Let $f : (X, \tau) \to (Y, \sigma)$ be the identity fuzzy function. Then clearly, f is fuzzy gs-irresolute but it is not g'''-irresolute.

Proposition 3.5. *A fuzzy function* $f : (X, \tau) \to (Y, \sigma)$ *is fuzzy* g'''*-continuous if and only if* $f^{-1}(U)$ *is fuzzy* g'''*-open in* (X, τ)*, for every fuzzy open set* U *in* (Y, σ)*.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be fuzzy g'''-continuous and U be an fuzzy open set in (Y, σ). Then U^c is fuzzy closed in (Y, σ). Since f is fuzzy g'''-continuous, $f^{-1}(U^c)$ is fuzzy g'''-closed in (X, τ). But $f^{-1}(U^c) = (f^{-1}(U))^c$. Thus $f^{-1}(U)$ is fuzzy g'''-open in (X, τ).

Conversely, assume that $f^{-1}(U)$ is fuzzy g'''-open in (X, τ), for each fuzzy open set U in (Y, σ). Let F be a fuzzy closed set in (Y, σ). Then F^c is fuzzy open in (Y, σ). Thus by assumption, $f^{-1}(F^c)$ is fuzzy g'''-open in (X, τ). Since $f^{-1}(F^c) = (f^{-1}(F))^c$, $f^{-1}(F)$ is fuzzy g'''-closed in (X, τ). So f is fuzzy g'''-continuous. □

Proposition 3.6. *A fuzzy function* $f : (X, \tau) \to (Y, \sigma)$ *is fuzzy* g'''*-irresolute if and only if the inverse of every fuzzy* g'''*-open set in* (Y, σ) *is fuzzy* g'''*-open in* (X, τ)*.*

Proof. The proof is similar to Proposition 3.5. □

Proposition 3.7. *If a fuzzy function* $f : (X, \tau) \to (Y, \sigma)$ *is fuzzy* g'''*-irresolute, then it is fuzzy* g'''*-continuous but not conversely.*

Example 3.8. Let $X = Y = \{a, b\}$ and $\tau = \{0_X,\ \alpha,\ \beta,\ 1_X\}$, where α and β are fuzzy set in X defined by $\alpha(a) = 0.5$, $\alpha(b) = 0$, $\beta(a) = 1$, $\beta(b) = 0$ and $\sigma = \{0_X,\ \beta,\ 1_X\}$, where β is fuzzy set in Y defined by $\beta(a) = 1$, $\beta(b) = 0$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Let $f : (X, \tau) \to (Y, \sigma)$ be the identity fuzzy function. Then clearly, f is fuzzy g'''-continuous but it is not fuzzy g'''-irresolute.

Proposition 3.9. *Let (X, τ) be any fuzzy topological space, (Y, σ) be a fuzzy $T_{g'''}$-space and f:$(X, \tau)\to(Y, \sigma)$ be a fuzzy function. Then the following are equivalent:*

(1) *f is fuzzy g'''-irresolute.*
(2) *f is fuzzy g'''-continuous.*

Proof. (1)⇒(2): The proof follows from Proposition 3.7.

(2)⇒(1): Let F be a fuzzy g'''-closed set in (Y, σ). Since (Y, σ) is a fuzzy $T_{g'''}$-space, F is a fuzzy closed set in (Y, σ). Then by hypothesis, $f^{-1}(F)$ is fuzzy g'''-closed in (X, τ). Thus f is fuzzy g'''-irresolute. □

Proposition 3.10. *If f:$(X, \tau)\to(Y, \sigma)$ is bijective fuzzy pre-gs-open and fuzzy g'''-continuous then f is fuzzy g'''-irresolute.*

Proof. Let A be fuzzy g'''-closed set in (Y, σ). Let U be any fuzzy gs-open set in (X, τ) such that $f^{-1}(A) \leq U$. Then $A \leq f(U)$. Since A is fuzzy g'''-closed and $f(U)$ is fuzzy gs-open in (Y, σ), $cl(A) \leq f(U)$. Thus $f^{-1}(cl(A)) \leq U$. Since f is fuzzy g'''-continuous and $cl(A)$ is fuzzy closed in (Y, σ), $f^{-1}(cl(A))$ is fuzzy g'''-closed. So $cl(f^{-1}(cl(A))) \leq U$ and thus $cl(f^{-1}(A)) \leq U$. Hence, $f^{-1}(A)$ is fuzzy g'''-closed in (X, τ). Therefore f is fuzzy g'''-irresolute. □

The following examples show that no assumption of Proposition 3.10 can be removed.

Example 3.11. The fuzzy identity function in Example 3.8 is fuzzy g'''-continuous and fuzzy bijective but not fuzzy pre-gs-open and thus f is not fuzzy g'''-irresolute.

Example 3.12. Let $X = Y = \{a, b\}$ and $\tau = \{0_X,\ \gamma,\ 1_X\}$, where γ is fuzzy set in X defined by $\gamma(a) = 0.6$, $\gamma(b) = 0.5$ and $\sigma = \{0_X,\ \delta,\ 1_X\}$, where δ is fuzzy set in Y defined by $\delta(a) = 0.4$, $\delta(b) = 0.5$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Let $f : (X, \tau) \to (Y, \sigma)$ be the identity fuzzy function. Then clearly, f is fuzzy bijective and fuzzy pre-gs-open but not fuzzy g'''-continuous. Thus f is not g'''-irresolute.

Proposition 3.13. *If f:$(X, \tau)\to(Y, \sigma)$ is fuzzy bijective closed and fuzzy gs-irresolute then the inverse function f^{-1}:$(Y, \sigma)\to(X, \tau)$ is fuzzy g'''-irresolute.*

Proof. Let A be fuzzy g'''-closed in (X, τ). Let $(f^{-1})^{-1}(A) = f(A) \leq U$, where U is fuzzy gs-open in (Y, σ). Then $A \leq f^{-1}(U)$. Since $f^{-1}(U)$ is fuzzy gs-open in (X, τ) and A is fuzzy g'''-closed in (X, τ), $cl(A) \leq f^{-1}(U)$. Thus $f(cl(A)) \leq U$. Since f is fuzzy closed and $cl(A)$ is fuzzy closed in (X, τ), $f(cl(A))$ is fuzzy closed in (Y, σ). So $f(cl(A))$ is fuzzy g'''-closed in (Y, σ). Hence $cl(f(cl(A))) \leq U$ and thus $cl(f(A)) \leq U$. Therefore $f(A)$ is fuzzy g'''-closed in (Y, σ) and hence f^{-1} is fuzzy g'''-irresolute. □

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