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N-Intuitionistic fuzzy metric spaces and various types of mappings

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Abstract. In this paper, the concept of N-Intuitionistic Fuzzy Metric Space (NIFMS) is introduced and analysed some of their properties. Also defined the various types of mapping over N-intuitionistic fuzzy metric spaces illustrated with examples. Moreover the convergence sequence and cauchy sequence of the above concepts are introduced.

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1. Introduction

The theory of fuzzy sets was introduced by L.A. Zadeh [28] in 1965. Kramosil and Michalek [10] introduced the fuzzy metric spaces (FM-spaces) by generalizing the concept of probabilistic metric spaces to fuzzy situation. George and Veeramani [7] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek [10]. In [3] Abbas et al. discussed some fixed point theorems of non-compatible mappings in fuzzy metric space. On the other hand, Atanassov [2] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Park[19] has introduced and studied the notion of intuitionistic fuzzy metric spaces. Further, Saadati et al.[20] proposed the idea of a continuous t-representable under the name modified intuitionistic fuzzy metric space which is a milestone in developing fixed point theory. In a series of papers the authors [1, 8, 9, 11, 12, 13, 17, 18, 27] proved several fixed point results in intuitionistic fuzzy metric space. In [4] Alaca, Turkoglu and Yildiz, proved the well known fixed point theorems of Banach and Edelstein in intuitionistic fuzzy metric spaces with the help of Grabiec [5]. Sharma, Sharma and Iseki [21] studied for the first time contraction type mappings in 2-metric space.

In 1992, Dhage [6] in introduced a new class of generalized metric space called D-metric space. Singh and Chouhan [25] defined S-metric space by using the concept of D-metric space. However, Mustafa and Sims in [14, 15] have pointed out that most of the results claimed by Dhage and others in D-metric spaces are invalid, they introduced a new concept of generalized metric space called G-metric space [26]. Using the concept of G-metric space, Sun and Yang [26] introduced the notion of Q-Fuzzy metric space. Sedghi et. al. in [24] introduced D^*-metric space which is a generalization of G-metric space. Using the concept of D^*-metric space, Sedghi and Shobe [22] defined M-fuzzy metric space. Sedghi et. al [23] defined S-metric space which is a generalization of D^*-metric space and G-metric space. Recently, Malviya [16] initiated the concept of N fuzzy metric space.

In this paper, the notion of an N-intuitionistic fuzzy metric space is introduced and defined the various types of mapping over N-intuitionistic fuzzy metric space. Some properties of an N-intuitionistic fuzzy metric space are studied.

2. Preliminaries

Definition 2.1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t-norm, if $*$ is satisfying the following conditions:

(1) $*$ is commutative and associative,
(2) $*$ is continuous,
(3) $a * 1 = a$, for all $a \in [0,1]$,
(4) $a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Definition 2.2. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm, if $\diamond$ is satisfying the following conditions:

(1) $\diamond$ is commutative and associative,
(2) $\diamond$ is continuous,
(3) $a \diamond 0 = a$, for all $a \in [0,1]$,
(4) $a \diamond b \leq c \diamond d$, whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$.

Definition 2.3. The 5-tuple $(X, M, N, *, \diamond)$ is said to be intuitionistic fuzzy metric space, if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X,\ s, t > 0$,

(1) $M(x,y,t) + N(x,y,t) \leq 1$;
(2) $M(x,y,t) > 0$;
(3) $M(x,y,t) = 1 \Leftrightarrow x = y$;
(4) $M(x,y,t) = M(y,x,t)$;
(5) $M(x,z,t+s) \geq M(x,y,t) * M(y,z,s)$;
(6) $M(x,y,.) : (0,\infty) \to (0,1]$ is left-continuous;
(7) $\lim_{t\to\infty} M(x,y,t) = 1$;
(8) $N(x,y,t) > 0$;
(9) $N(x,y,t) = 0 \Leftrightarrow x = y$;
(10) $N(x,y,t) = N(y,x,t)$;
(11) $N(x,z,t+s) \leq N(x,y,t) \diamond N(y,z,s)$;
(12) $N(x,y,.) : (0,\infty) \to (0,1]$ is right-continuous;

(13) $\lim_{t\to\infty} N(x,y,t) = 0$;

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

3. N-intuitionistic fuzzy metric space

Definition 3.1. The 5-tuple $(X, N_1, N_2, *, \diamond)$ is said to be N - intuitionistic fuzzy metric space, if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and N_1,N_2 are fuzzy sets on $X^3 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$, $r, s, t > 0$,

(1) $N_1(x, y, z, t) + N_2(x, y, z, t) \leq 1$;
(2) $N_1(x, y, z, t) > 0$;
(3) $N_1(x, y, z, t) = 1$ if and only if $x = y = z$;
(4) $N_1(x, y, z, r + s + t) \geq N_1(x, x, a, r) * N_1(y, y, a, s) * N_1(z, z, a, t)$;
(5) $N_1(x, y, z, .) : (0, \infty) \to (0, 1]$ is left-continuous;
(6) $N_2(x, y, z, t) > 0$;
(7) $N_2(x, y, z, t) = 0$ if and only if $x = y = z$;
(8) $N_2(x, y, z, r + s + t) \leq N_2(x, x, a, r) \diamond N_2(y, y, a, s) \diamond N_2(z, z, a, t)$;
(9) $N_2(x, y, z, .) : (0, \infty) \to (0, 1]$ is right-continuous;

Then (N_1, N_2) is called an N-intuitionistic fuzzy metric on X.

Example 3.2. Consider $X = R$ is a real line and S is an S-metric on X defined by

$$S(x, y, z) = |x - z| + |y - z|.$$

Define $a * b * c = \min\{a, b, c\}$ and $a \diamond b \diamond c = \max\{a, b, c\}$ for every $a, b, c \in [0, 1]$ and let N_1, N_2 be the function on $X^3 \times [0, \infty)$ is defined by

$$\begin{cases} N_1(x,y,z,t) = \frac{t}{t+S(x,y,z)} \\ N_2(x,y,z,t) = \frac{S(x,y,z)}{t+S(x,y,z)} \end{cases} \tag{3.1}$$

for all $x, y, z \in X$ and $t > 0$.
Then $(R, N_1, N_2, *, \diamond)$ is an N-intuitionistic fuzzy metric space.

4. Topology induced by N-intuitionistic fuzzy metric

Definition 4.1. Let $(X, N_1, N_2, *, \diamond)$ be an N- intuitionistic fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$B(x, r, t) = \{y \in X : N_1(y, y, x, t) > 1 - r, N_2(y, y, x, t) < r\}.$$

Proposition 4.2. *Let $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric space. Define $\tau = \{A \subset X : x \in A$ if and only if there exist $t > 0$ and r, $0 < r < 1$ such that $B(x, r, t) \subset A\}$. Then τ is a topology on X.*

Proof. Obviously, ϕ and X belong to τ.

Let $A_1, A_2, ..., A_i \in \tau$. Take $U = \cup_{i\in I} A_i$ and let $a \in U$. Then $a \in \cup_{i\in I} A_i$. Thus $a \in A_i$ for some $a \in A_i$ and for some $i \in I$. Since $A_i \in \tau$, there exists $0 < r < 1, t > 0$ such that $B(a, r, t) \subset A_i$. So $B(a, r, t) \subset A_i \subset \cup_{i\in I} A_i = U$. Hence $U \in \tau$.

Let $A_1, A_2, A_3, ..., , A_n \in \tau$ and $U = \cap_{i \in I} A_i$. Let $a \in U$. Then $a \in A_i, \forall i \in I$. Thus for each $i \in I$, there exists $0 < r_i < 1, t_i > 0$ such that $B(a, r_i, t_i) \subset A_i$. Let $r = \min\{r_i, i \in I\}$ and $t = \max\{t_i, i \in I\}$. Then $r \leq r_i, \forall i \in I$. Thus $r \leq r_i$ for all $i \in I$,$1 - r \leq 1 - r_i$ for all $i \in I$. Also, $t > 0$. So $B(a, r, t) \subseteq A_i, \forall i \in I$. Hence $B(a, r, t) \subset \cap_{i \in I} A_i = U$. Therefore $U \in \tau$. □

Theorem 4.3. *Let $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric space. Let $B(x, r_1, t)$ and $B(x, r_2, t)$ be open balls with the same center $x \in X$ and $t > 0$ with radius $0 < r_1 < 1$ and $0 < r_2 < 1$ respectively. Then we either have $B(x, r_1, t) \subset B(x, r_2, t)$ or $B(x, r_2, t) \subset B(x, r_1, t)$.*

Proof. Let $x \in X$ and $t > 0$. Consider the open balls $B(x, r_1, t)$ and $B(x, r_2, t)$ with $0 < r_1 < 1, 0 < r_2 < 1$.

If $r_1 = r_2$, then the result is trivial.

Let us assume that $r_1 \neq r_2$. Without loss of generality, assume that $0 < r_1 < r_2 < 1$. Then $1 - r_2 < 1 - r_1$. Now, let $a \in B(x, r_1, t)$. It follows that

$$\begin{aligned} N_1(a, a, x, t) &> 1 - r_1 \\ &> 1 - r_2 \\ N_2(a, a, x, t) &< r_1 \\ &< r_2 \end{aligned} \tag{4.1}$$

Thus $a \in B(a, r_2, t)$. This implies that $B(a, r_1, t) \subset B(a, r_2, t)$.

Assuming that $0 < r_2 < r_1 < 1$. Then $1 - r_1 < 1 - r_2$. Now, let $a \in B(x, r_2, t)$. It follows that

$$\begin{aligned} N_1(a, a, x, t) &> 1 - r_2 \\ &> 1 - r_1 \\ N_2(a, a, x, t) &< r_2 \\ &< r_1 \end{aligned} \tag{4.2}$$

Thus $a \in B(a, r_1, t)$. This implies that $B(a, r_2, t) \subset B(a, r_1, t)$. □

Theorem 4.4. *Let $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric space, then for all $x, y, z \in X$ and $t > 0$, we have $N_1(x, x, y, t) = N_1(y, y, x, t)$ and $N_2(x, x, y, t) = N_2(y, y, x, t)$.*

Proof. Since N-intuitionistic fuzzy metric is induced by S-metric and in $S-$metric space $S(x, x, y) = S(y, y, x), \forall x, y \in X$. Then in N-intuitionistic fuzzy metric space, $N_1(x, x, y, t) = N_1(y, y, x, t)$ and $N_2(x, x, y, t) = N_2(y, y, x, t), \forall x, y \in X, t > 0$. □

Remark 4.5. $N_1(x, x, y, .)$ is non-decreasing and $N_2(x, x, y, .)$ is non-increasing, for all $x, y \in X$.

Proposition 4.6. *Every N-intuitionistic fuzzy metric space is Hausdorff.*

Proof. Let $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric space. Let x, y be two distinct points of X. Then $0 < N_1(x, x, y, t) < 1$ and $0 < N_2(x, x, y, t) < 1$. Let $N_1(x, x, y, t) = r_1, N_2(x, x, y, t) = r_2$ and $r = \max\{r_1, r_2\}$. Then for each $r_0 \in (r, 1)$, there exist r_3 and r_4 such that $r_3 * r_3 * r_3 \geq r_0$ and $(1-r_4) \diamond (1-r_4) \diamond (1-r_4) \leq (1-r_0)$.

Put $r_5 = \max\{r_3, r_4\}$ and consider the open balls $B(x, 1-r_5, \frac{t}{3})$ and $B(y, 1-r_5, \frac{t}{3})$. Then clearly

$$B(x, 1-r_5, \frac{t}{3}) \cap B(y, 1-r_5, \frac{t}{3}) = \phi.$$

For if there exists $z \in B(x, 1-r_5, \frac{t}{3}) \cap B(y, 1-r_5, \frac{t}{3})$, then

$$\begin{aligned} r_1 = N_1(x,x,y,t) &\geq N_1(x,x,y,\frac{t}{3}) * N_1(x,x,y,\frac{t}{3}) * N_1(x,x,y,\frac{t}{3}) \\ &\geq r_5 * r_5 * r_5 \\ &\geq r_3 * r_3 * r_3 \\ &\geq r_0 < r_1 \\ r_2 = N_2(x,x,y,t) &\leq N_2(x,x,y,\frac{t}{3}) \diamond N_2(x,x,y,\frac{t}{3}) \diamond N_2(x,x,y,\frac{t}{3}) \\ &\leq (1-r_5) \diamond (1-r_5) \diamond (1-r_5) \\ &\leq (1-r_4) \diamond (1-r_4) \diamond (1-r_4) \\ &\leq (1-r_0) < r_2. \end{aligned} \tag{4.3}$$

which is a contradiction. Thus $(X, N_1, N_2, *, \diamond)$ is Hasudorff □

Definition 4.7. A sequence x_n in $(X, N_1, N_2, *, \diamond)$ is converges to $x \in X$ if

$$N_1(x_n, x_n, x, t) = 1 \text{ and } N_2(x_n, x_n, x, t) = 0$$
$$\text{or}$$
$$N_1(x, x, x_n, t) = 1 \text{ and } N_2(x, x, x_n, t) = 0$$

as $n \to \infty$ for each $t > 0$. That is for each $\epsilon > 0$ and $t > 0$ there exists $n \in N$ such that for all $n \geq n_0$,

$$N_1(x_n, x_n, x, t) > 1-\epsilon \text{ and } N_2(x_n, x_n, x, t) < \epsilon$$
$$\text{or}$$
$$N_1(x, x, x_n, t) > 1-\epsilon \text{ and } N_2(x, x, x_n, t) < \epsilon.$$

Lemma 4.8. *Let $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric space, where $*$ is minimum t-norm and $\diamond$ is maximum t-conorm. Let $\{x_n\}$ be a sequence in X. If $\{x_n\}$ converges to x and $\{x_n\}$ also converges to y then $x = y$. That is the limit of $\{x_n\}$ if exists is unique.*

Proof. Let $\{x_n\}$ converges to x and y. Then $N_1(x, x, x_n, r) \to 1$ and $N_2(x, x, x_n, r) \to 0$ as $n \to \infty$ for each $r > 0$ and $N_1(y, y, x_n, t-2r) \to 1$ and $N_2(y, y, x_n, t-2r) \to 0$ as $n \to \infty$ for each $t - 2r > 0$.

$$\begin{aligned} N_1(x,x,y,t) &\geq N_1(x,x,x_n,r) * N_1(x,x,x_n,r) * N_1(y,y,x_n,t-2r) \\ &= 1 * 1 * 1 \quad as \ \ n \to \infty \\ &\to 1 \\ N_2(x,x,y,t) &\leq N_2(x,x,x_n,r) \diamond N_2(x,x,x_n,r) \diamond N_2(y,y,x_n,t-2r) \\ &= 0 \diamond 0 \diamond 0 \quad as \ \ n \to \infty \\ &\to 0. \end{aligned} \tag{4.4}$$

□

Definition 4.9. Let $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric space and $\{x_n\}$ be a sequence in X is called Cauchy sequence. If for each $\epsilon > 0$ and $t > 0$ there exists $n_0 \in \mathbb{N}$ such that

$$N_1(x_n, x_n, x_m, t) > 1 - \epsilon \text{ and } N_2(x_n, x_n, x_m, t) < \epsilon$$
$$\text{or}$$
$$N_1(x_m, x_m, x_n, t) > 1 - \epsilon \text{ and } N_2(x_m, x_m, x_n, t) < \epsilon$$

for all $n, m \geq n_o$.

Definition 4.10. Let $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric space. If every Cauchy sequence in X is convergent in X, then X is called a complete N-intuitionistic fuzzy metric space.

Lemma 4.11. *Let $(X, N_1, N_2, *, \diamond)$ be an N-fuzzy metric space, where $*$ and $\diamond$ are the minimum t-norm and maximum t-conorm respectively, and $\{x_n\}$ be a sequence in X. If $\{x_n\}$ converges to x, then $\{x_n\}$ is a Cauchy sequence.*

Proof. For each $r, t > 0$, there is $p \in \mathbb{N}$. Then

$$N_1(x_n, x_n, x, r) \to 1,\ N_2(x_n, x_n, x, r) \to 0 \text{ as } n \to \infty$$

and

$$N_1(x_{n+p}, x_{n+p}, x, t - 2r) \to 1, N_2(x_{n+p}, x_{n+p}, x, t - 2r) \to 0$$

as $n \to \infty$ for each $t - 2r > 0$.
On the other hand,

(4.5)
$$\begin{aligned} N_1(x_n, x_n, x_{n+p}, r) &\geq N_1(x_n, x_n, x, r) * N_1(x_n, x_n, x, r) * N_1(x_{n+p}, x_{n+p}, x, t - 2r) \\ &= 1 * 1 * 1 \quad as\ \ n \to \infty \\ &= 1 \quad [\text{where } \ 1 * 1 * 1 = \min\{1, 1, 1\}] \\ N_2(x_n, x_n, x_{n+p}, r) &\leq N_2(x_n, x_n, x, r) \diamond N_2(x_n, x_n, x, r) \diamond N_2(x_{n+p}, x_{n+p}, x, t - 2r) \\ &= 0 \diamond 0 \diamond 0 \quad as\ \ n \to \infty \\ &= 0 \quad [\text{where } \ 0 \diamond 0 \diamond 0 = \max\{0, 0, 0\}]. \end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence. □

Definition 4.12. Let $(X, N_1, N_2, *, \diamond)$ and $(X', N_1', N_2', *, \diamond)$ be N-intuitionistic fuzzy metric spaces. Then a function $f : X \to X'$ is said to be continuous at a point $x \in X$ if and only if it is sequentially continuous at x, that is whenever $\{x_n\}$ is convergent to x we have $\{f(x_n)\}$ is convergent to $f(x)$.

Lemma 4.13. *Let $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric space. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X and suppose $\{x_n\} \to x, \{y_n\} \to y$ as $n \to \infty$ and $N_1(x, x, y, t_n) \to N_1(x, x, y, t)$, $N_2(x, x, y, t_n) \to N_2(x, x, y, t)$ as $n \to \infty$. Then $N_1(x_n, x_n, y_n, t_n) \to N_1(x, x, y, t)$ and $N_2(x_n, x_n, y_n, t_n) \to N_2(x, x, y, t)$as $n \to \infty$.*

Proof. Since, $\lim_{n\to\infty} x_n = x, \lim_{n\to\infty} y_n = y$ and $\lim_{n\to\infty} N_1(x, x, y, t_n) = N_1(x, x, y, t)$, $\lim_{n\to\infty} N_2(x, x, y, t_n) = N_2(x, x, y, t)$, there is $n_0 \in \mathbb{N}$ such that $|t - t_n| < \delta$ for

$n \geq n_0$ and $\delta < \frac{t}{2}$.
Since $N_1(x,x,y,t)$ is non-decreasing with respect to t,

$$
\begin{aligned}
N_1(x_n,x_n,y_n,t) &\geq N_1(x_n,x_n,y_n,t-\delta) \\
&\geq N_1(x_n,x_n,x,\frac{\delta}{3}) * N_1(x_n,x_n,x,\frac{\delta}{3}) * N_1(y_n,y_n,x,t-\frac{5\delta}{3}) \\
&\geq N_1(x_n,x_n,x,\frac{\delta}{3}) * N_1(x_n,x_n,x,\frac{\delta}{3}) * N_1(y_n,y_n,x,\frac{\delta}{6}) \\
&* N_1(y_n,y_n,y,t-\frac{\delta}{6}) * N_1(y,y,x,t-2\delta).
\end{aligned} \tag{4.6}
$$

Since $N_2(x,x,y,t)$ is non-increasing with respect to t,

$$
\begin{aligned}
N_2(x_n,x_n,y_n,t) &\leq N_2(x_n,x_n,y_n,t-\delta) \\
&\leq N_2(x_n,x_n,x,\frac{\delta}{3}) \diamond N_2(x_n,x_n,x,\frac{\delta}{3}) \diamond N_2(y_n,y_n,x,t-\frac{5\delta}{3}) \\
&\leq N_2(x_n,x_n,x,\frac{\delta}{3}) \diamond N_2(x_n,x_n,x,\frac{\delta}{3}) \\
&\diamond N_2(y_n,y_n,x,\frac{\delta}{6}) \diamond N_2(y_n,y_n,y,t-\frac{\delta}{6}) \diamond N_2(y,y,x,t-2\delta).
\end{aligned} \tag{4.7}
$$

Combining the arbitrariness of δ and the continuity of $N_1(x,x,y,.), N_2(x,x,y,.)$ w.r.to t. For large enough n, we have

$$
\begin{aligned}
&N_1(x,x,y,t) \geq N_1(x_n,x_n,y_n,t_n) \geq N_1(y,y,x,t) \\
&N_1(x,x,y,t) \geq N_1(x_n,x_n,y_n,t_n) \geq N_1(x,x,y,t) \qquad \text{Using } 4.4 \\
&\Rightarrow \lim_{n\to\infty} N_1(x_n,x_n,y_n,t_n) \to N_1(x,x,y,t) \\
&N_2(x,x,y,t) \leq N_2(x_n,x_n,y_n,t_n) \leq N_2(y,y,x,t) \\
&N_2(x,x,y,t) \leq N_2(x_n,x_n,y_n,t_n) \leq N_2(x,x,y,t) \qquad \text{Using } 4.4 \\
&\Rightarrow \lim_{n\to\infty} N_2(x_n,x_n,y_n,t_n) \to N_2(x,x,y,t).
\end{aligned} \tag{4.8}
$$

□

Lemma 4.14. *Let $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric space. If there exists $g \in (0,1)$ such that $N_1(x,x,y,gt) \geq N_2(x,x,y,t), N_2(x,x,y,gt) \leq N_2(x,x,y,t)$, for all $x,y \in X, t > 0$ and*

$$\lim_{n\to\infty} N_1(x,y,z,t) = 1,$$
$$\lim_{n\to\infty} N_2(x,y,z,t) = 0$$

Then $x = y$.

Proof. Suppose that there exists $g \in (0,1)$ such that $N_1(x,x,y,gt) \geq N_1(x,x,y,t)$, $N_2(x,x,y,gt) \leq N_2(x,x,y,t)$ for all $x,y \in X$ and $t > 0$.
Then $N_1(x,x,y,t) \geq N_1(x,x,y,\frac{t}{g})$. Thus $N_1(x,x,y,t) \geq N_1(x,x,y,\frac{t}{g^n})$
Similarly, $N_2(x,x,y,t) \leq N_2(x,x,y,\frac{t}{g})$. So

$$N_2(x,x,y,t) \leq N_2(x,x,y,\tfrac{t}{g^n}), \text{ for positive integer } n.$$

Taking limit as $n \to \infty$, $N_1(x,x,y,t) \geq 1, N_2(x,x,y,t) \leq 0$. Hence $x = y$. □

5. The various types of mapping in N-intuitionistic fuzzy metric spaces

Definition 5.1. Let S and T maps from an N-intuitionistic fuzzy metric space $(X, N_1, N_2, *, \diamond)$ into itself. The maps S and T are said to be compatible, if for all $t > 0$,

$\lim_{n\to\infty} N_1(STx_n, STx_n, TSx_n, t) = 1$ and $\lim_{n\to\infty} N_2(STx_n, STx_n, TSx_n, t) = 0$

Example 5.2. Let $X = [2, 20)$. For each $t \in (0, \infty)$ and for all $x, y, z \in X$, define

$N_1(x, y, z, t) = \frac{t}{t+|x-z|+|y-z|}$ and $N_2(x, y, z, t) = \frac{|x-z|+|y-z|}{t+|x-z|+|y-z|}$.

Clearly $(X, N_1, N_2, *, \diamond)$ is an N-intuitionistic fuzzy metric space, where $*$ is defined by $a * b * c = \min\{a, b, c\}$ and $\diamond$ is defined by $a \diamond b \diamond c = \max\{a, b, c\}$. Let S and T be self maps of X defined as

$$S(x) = \begin{cases} 2 & \text{if } x = 2 \text{ or } x > 5, \\ 6 & \text{if } 2 < x \le 5 \end{cases} \qquad T(x) = \begin{cases} 2 & \text{if } x = 2 \text{ or } x > 5, \\ 12 & \text{if } 2 < x \le 5 \\ \frac{x+1}{3} & \text{if } x > 5. \end{cases} \tag{5.1}$$

Let sequence $\{x_n\}$ be defined as $x_n = 5 + \frac{1}{n}, n \ge 1$. Then we have $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = 2$. Thus, S and T satisfy the property $(E.A.)$. Also,

$$\lim_{n\to\infty} N_1(STx_n, STx_n, TSx_n, t) = \frac{t}{t+|2-2|+|2-2|} = \frac{t}{t+0} = 1,$$
$$\lim_{n\to\infty} N_2(STx_n, STx_n, TSx_n, t) = \frac{|2-2|+|2-2|}{t+|2-2+|2-2|} = \frac{0}{t+0} = 0.$$

So, S and T are compatible.

Definition 5.3. Let S and T be maps from an N-intuitionistic fuzzy metric space $(X, N_1, N_2, *, \diamond)$ into itself. The maps are said to be weakly compatible, if they commute at their coincidence points, that is, $Sz = Tz$ implies that $STz = TSz$.

Definition 5.4. Let S and T be maps from an N-intuitionistic fuzzy metric space $(X, N_1, N_2, *, \diamond)$ into itself. The maps S and T are said to be semicompatible, if for all $t > 0$. $\lim_{n\to\infty} N_1(STx_n, STx_n, Tz, t) = 1$ and $\lim_{n\to\infty} N_2(STx_n, STx_n, Tz, t) = 0$ whenever$\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $x \in X$.

Remark 5.5. The semicompatible of the pair (S, T), need not imply the semicompatible of (T, S).

Example 5.6. Let $X = [0, 1]$ and $(X, N_1, N_2, *, \diamond)$ be the N-intuitionistic fuzzy metric space with

$N_1(x, y, z, t) = [\exp \frac{[|x-z|+|y-z|]}{t}]^{-1}$

and

$N_2(x, y, z, t) = 1 - [\exp \frac{[|x-z|+|y-z|]}{t}]^{-1}$

for all $x, y, z, \in X, t > 0$.

Define self-map as follows:

$$S(x) = \begin{cases} x & \text{if } 0 \le x < \frac{1}{2} \\ 1 & \text{if } x \ge \frac{1}{2}. \end{cases} \tag{5.2}$$

Let I be the identity map on X and $x_n = \frac{1}{2} - \frac{1}{n}$. Then, $\{ISx_n\} = \{Sx_n\} = \frac{1}{2} \ne S\{\frac{1}{2}\}$. Thus (I, S) is not semicompatible. Again as (I, S) is commuting, it is compatible. Further, for any sequence $\{x_n\}$ in X such that $\{x_n\} \to x$ and $\{Sx_n\} \to x$, we have $\{SIx_n\} = \{Sx_n\} \to x = Ix$. Hence (S, I) is always semicompatible.

Remark 5.7. The above example gives an important aspect of semicompatibility as the pair of self-maps (I, S) is commuting, hence it is weakly commuting, compatible and weak compatible yet it is not semicompatible. Further, it is to be noted that the pair (S, I) is semicompatible but (I, S) is not semicompatible here.

The following is an example of a pair of self- maps (A, S) which is semicompatible but not compatible.

Example 5.8. Let $X = [0, 2]$ and $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric, where $*$ is defined by $a * b * c = \min\{a, b, c\}$ and $\diamond$ is defined by $a \diamond b \diamond c = \max\{a, b, c\}$. $N_1(x, y, z, t) = \frac{t}{t+|x-z|+|y-z|}$ and $N_2(x, y, z, t) = \frac{|x-z|+|y-z|}{t+|x-z|+|y-z|}$. Define self-maps A and S as follows:

$$A(x) = \begin{cases} 2 & \text{if } 0 \le x \le 1 \\ \frac{x}{2} & \text{if } 1 < x \le 2 \end{cases} \qquad S(x) = \begin{cases} 2 & \text{if } x = 1 \\ \frac{x+3}{5} & \text{otherwise} \end{cases} \tag{5.3}$$

and $x_n = 2 - \frac{1}{2n}, n \ge 1$. Then we have $S(1) = A(1) = 2$ and $S(2) = A(2) = 1$. Also $SA(1) = AS(1) = 1$ and $SA(2) = AS(2) = 2$. Thus, (A, S) is weak compatible.

Again, $Ax_n = 1 - \frac{1}{4n}, Sx_n = 1 - \frac{1}{10n}$. Then, $Ax_n \to 1$, $Sx_n \to 1$. Thus $u = 1$. Furthermore, $SAx_n = \frac{4}{5} - \frac{1}{20n}, ASx_n = 2$.
Now, $\lim_{n\to\infty} N_1(ASx_n, ASx_n, Su, t) = \lim_{n\to\infty} N_1(2, 2, 2, t) = 1$
and

$$\lim_{n\to\infty} N_2(ASx_n, ASx_n, Su, t) = \lim_{n\to\infty} N_2(2, 2, 2, t) = 0.$$

So

$$\begin{aligned} \lim_{n\to\infty} N_1(ASx_n, ASx_n, SAx_n, t) &= \lim_{n\to\infty} N_1(2, 2, \frac{4}{5} - \frac{1}{20n}, t) \\ &= \frac{t}{t + |2 - \frac{4}{5}| + |2 - \frac{4}{5}|} \\ &= \frac{t}{t + \frac{12}{5}} < 1 \quad \forall t > 0 \end{aligned} \tag{5.4}$$

and

$$\begin{aligned} \lim_{n\to\infty} N_2(ASx_n, ASx_n, SAx_n, t) &= \lim_{n\to\infty} N_2(2, 2, \frac{4}{5} - \frac{1}{20n}, t) \\ &= \frac{|2 - \frac{4}{5}| + |2 - \frac{4}{5}|}{t + |2 - \frac{4}{5}| + |2 - \frac{4}{5}|} \\ &= \frac{1}{1 + \frac{5t}{12}} > 0 \quad \forall t > 0. \end{aligned} \tag{5.5}$$

Hence (A, S) is semicompatible but it is not compatible.

Definition 5.9. Let S and T be two self-maps of an N-intuitionistic fuzzy metric space $(X, N_1, N_2, *, \diamond)$. We say that S and T satisfy the property (E.A.), if there exists a sequence $\{x_n\}$ such that $\lim_{n\to\infty}\{Sx_n\} = \lim_{n\to\infty}\{Tx_n\} = z$ for some $z \in X$.

Example 5.10. Let $(X, N_1, N_2, *, \diamond)$ be an N-intuitionistic fuzzy metric space where $X = [0, 1]$, $*$ is defined by $a * b * c = \min\{a, b, c\}$ and $\diamond$ is defined by $a \diamond b \diamond c = \max\{a, b, c\}$ and $N_1(x, y, z, t) = \frac{t}{t+|x-z|+|y-z|}$ and $N_2(x, y, z, t) = \frac{|x-z|+|y-z|}{t+|x-z|+|y-z|}$. Define the self-maps $S, T : X \to X$ by

$$S(x) = \begin{cases} 1 - x & \text{if } \; x \in [0, \frac{1}{2}] \\ 0 & \text{if } \; x \in (\frac{1}{2}, 1] \end{cases} \quad T(x) = \begin{cases} \frac{1}{2} & \text{if } \; x \in [0, \frac{1}{2}] \\ \frac{3}{4} & \text{if } \; x \in (\frac{1}{2}, 1] \end{cases} \tag{5.6}$$

Then for the sequence $\{x_n\} = \{\frac{1}{2} - \frac{1}{n}\}, n \geq 2$, we have

$\lim_{n\to\infty} S(\frac{1}{2} - \frac{1}{n}) = \lim_{n\to\infty} \frac{1}{2} + \frac{1}{n} = \frac{1}{2} = \lim_{n\to\infty} T(\frac{1}{2} - \frac{1}{n})$.

Thus the pair (S, T) satisfies property (E.A.).

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