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Novel concepts of soft sets with applications

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Abstract. We introduce different concepts of soft sets, including soft σ-ring, soft algebra, and soft σ-algebra. We present different types of set functions, including soft finitely sub-additive, soft countably sub-additive, soft finitely additive, soft countably additive and soft monotone. We study the concept of soft outer measure and soft Lebesgue outer measure. We also describe interesting applications of soft mappings to decision-making.

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1. Introduction

In 1999, Molodtsov [5] originated the concept of soft set theory as a mathematical instrument to tackle imprecise data and decision making problems. There are many practical utilizations of soft set theory in various fields of sciences including social sciences, physics, engineering, economics, computer science and medical sciences. Kharal and Ahmad [1] established mappings on soft classes, the images of soft sets, and the inverse images of soft sets. In [2], Mukherjee *et al.* studied measurable soft sets. Khameneh and Kilicman [3] discussed Soft σ-Algebras in connection with soft probability space. Chen *et al.* [4] have discussed some pioneering work on some applications of soft sets. Feng *et al.* studied soft sets and soft rough sets and presented attribute analysis of information systems based on elementary soft implications. They also established an approach to fuzzy soft set based decision-making (See [6, 7, 8, 9]). Zorlutuna and Çakir [10] worked on soft continuity, soft open-ness and soft closed-ness of soft mappings and also investigated the behavior of soft separation axioms and generalized the pasting lemma in view of soft set theory. In [11, 12] Hur *et al.* studied Fuzzy equivalence relations, fuzzy partitions, fuzzy functions and fuzzy partially ordered sets. Ali *et al.* [15] suggested some operations on soft sets which became very useful in the field of soft set theory. Shabir and Naz

[18] studied soft topological spaces. Cagman *et al.* [19, 20] proposed the theory of soft topological spaces and accomplished various properties regarding soft topological spaces. Maji *et al.* [22, 23] employed soft sets theory in decision making problems and defined many soft sets operations.
Akram and Adeel [13, 14, 21] intoduced representation of labeling tree based on m-polar fuzzy sets and they introduced soft intersection Lie algebras. Samanta and Majumdar [24] proposed the notion of soft groups, and discussed soft mappings on soft sets. Samanta and Das proposed fundamental properties of soft real sets and soft real numbers. They also studied soft elements, soft points and soft metric spaces (See [27, 28, 29]). In [32], Rong discussed the soft countable spaces, soft separable spaces and soft Lindölof spaces and investigated some interesting results using these notions. Abdullah *et al.* [25, 26] studied fuzzy soft set over a fuzzy topological space and semigroups characterized by the properties of $(\alpha, \beta)^*$-fuzzy ideals. Roy and Samanta[30] discussed some interesting results in the literature utilizing the ideas of soft base and soft sub-base. Çetkin *et al.* [31] presented a new approach in handling soft decision making problems. Riaz and Fatima [16] used soft sets, soft elements and soft points to explore soft dense, nowhere soft dense sets, soft first category, soft second category and soft Baire space for soft metric spaces and established the Baire's category theorem for soft metric spaces. Riaz and Naeem [17] studied Measurable Soft Mappings and some applications of sot set theory.
In continuation to the remarkable work done by the aforementioned mathematicians, we explore, in the following pages, various properties of soft ring of sets and soft sigma algebras. We introduce various set functions like soft finitely sub-additive, soft countably sub-additive, soft finitely additive, soft countably additive, and soft monotone.

2. Preliminaries

Definition 2.1 ([5])**.** Let X be a universe and E a non-empty collection of decision variables. Suppose that 2^X is the aggregate of all subsets of X and $A(\neq \phi) \subseteq E$. The doublet (T, A), where $T : A \to 2^X$ is a mapping, is known as a soft set over X. Mathematically speaking, it may be expressed as

$$(T, A) = \{(\eta, T_A(\eta)) \ : \ \eta \in A, T_A(\eta) \in 2^X\}.$$

Definition 2.2 ([27])**.** Let $\mathfrak{B}(\mathbb{R})$ be the collection of all non-void bounded subsets of the set $\mathbb{R}$ of real numbers. Assume that A is the collection of decision variates. The map $T : A \to \mathfrak{B}(\mathbb{R})$ is known as a soft real set, designated by (T, A). If (T, A) is a soft set comprising only one soft element , then after recognizing (T, A) with the corresponding soft element, it is termed as a soft real number.

We express a soft real number by $\tilde{r}$, whereas $\bar{r}$ will represent the particular type of soft real numbers such that $\bar{r}(\eta) = r$, for all $\eta \in A$.

We use the notation $\widetilde{\mathbb{R}}$ to represent the set of soft real numbers. If we choose unbounded subsets of the set $\mathbb{R}_\infty$ of extended real numbers, then the corresponding set is termed as the set of extended real numbers and is designated as $\widetilde{\mathbb{R}}_\infty$.

Definition 2.3 ([1])**.** Let $f : X \to Y$ and $u : E_1 \to E_2$ be mappings. Then a soft mapping $\psi_{fu} : (X, E_1) \to (Y, E_2)$, where (X, E_1) and (Y, E_2) are soft classes, is defined as: For a soft set (F, A) in (X, E_1), $(\psi_{fu}(F, A), B)$, $B = u(A) \subseteq E_2$ is a soft set in (Y, E_2) given by

$$\psi_{fu}(F,A)(\eta_2) = \begin{cases} f(\bigcup_{\eta_1 \in u^{-1}(\eta_2)\cap A} F(\eta_1)), & \text{if} \quad u^{-1}(\eta_2) \cap A \neq \phi \\ \phi, & \text{otherwise} \end{cases}$$

for $\eta_2 \in B \subseteq E_2$. $(\psi_{fu}(F, A), B)$ is called soft image of a soft set (F, A).

3. Soft σ-Algebra

Definition 3.1. Let X and E be respectively the universe and aggregate of attributes. Assume that $\widetilde{\mathcal{R}}$ is the non-null collection of soft subsets of (X, E). Then $\widetilde{\mathcal{R}}$ is called a soft ring of sets, if for all $T_A, T_B \widetilde{\in} \widetilde{\mathcal{R}}$, $T_A \widetilde{\cup} T_B \widetilde{\in} \widetilde{\mathcal{R}}$ and $T_A \widetilde{\setminus} T_B \widetilde{\in} \widetilde{\mathcal{R}}$.

Definition 3.2. A soft ring of sets $\widetilde{\mathcal{R}}$ is called a soft σ-ring of sets, if $\widetilde{\cup}_{n=1}^{\infty} T_{A_n} \widetilde{\in} \widetilde{\mathcal{R}}$ for any soft sequence $\{T_{A_n}\}$ of soft sets in $\widetilde{\mathcal{R}}$.

Remark 3.3. It follows from definition that the soft symmetric difference $T_A \widetilde{\triangle} T_B$ and the soft intersection $T_A \widetilde{\cap} T_B$ of two soft sets in $\widetilde{\mathcal{R}}$ are also in $\widetilde{\mathcal{R}}$.

Example 3.4. Let $\widetilde{\mathcal{R}}$ be the family of all finite soft subsets of the soft set $\widetilde{\mathbb{N}}$ of soft natural numbers. Then $\widetilde{\mathcal{R}}$ is a soft ring of sets but not a soft σ-ring.

Definition 3.5. A collection $\widetilde{\mathcal{A}}$ of soft subsets of a soft set $\breve{X}$ is called a Soft Boolean Algebra or Soft Algebra of sets, if for all $T_A, T_B \widetilde{\in} \widetilde{\mathcal{A}}$, $T_A \widetilde{\cup} T_B \widetilde{\in} \widetilde{\mathcal{A}}$ and $T_A^c \widetilde{\in} \widetilde{\mathcal{A}}$.

Definition 3.6 ([3])**.** A collection $\widetilde{\mathcal{A}}$ of soft subsets of $\breve{X}$ is called a *soft σ-algebra* on $\breve{X}$ if it satisfies the following conditions:
(i) $T_\phi \widetilde{\in} \widetilde{\mathcal{A}}$.
(ii) If $T_A \widetilde{\in} \widetilde{\mathcal{A}}$. then $T_A^c \widetilde{\in} \widetilde{\mathcal{A}}$.
(iii) If $T_{A_1}, T_{A_2}, \ldots$ is a countable collection of soft sets in $\widetilde{\mathcal{A}}$, then $\widetilde{\cup}_{i=1}^{\infty} \widetilde{\mathcal{A}}_i \widetilde{\in} \widetilde{\mathcal{A}}$.
The pair $(\breve{X}, \widetilde{\mathcal{A}})$ is called a soft measurable space and $T_{A_i} \widetilde{\in} \widetilde{\mathcal{A}}$ is called a measurable soft set.

Since

$$T_A \widetilde{\setminus} T_B = T_A \widetilde{\cap} T_B^c = (T_A^c \widetilde{\cup} T_B)^c,$$

it follows that any soft algebra (soft σ-algebra) $\widetilde{\mathcal{A}}$ is a soft ring (soft σ-ring).

The converse to this statement, however, is not always true. It follows from Example 3.4, cited above, that a soft ring may not be a soft algebra. Let $\widetilde{\mathcal{A}}$ be the aggregate of all countable soft subsets of an uncountable soft set $\breve{X}$. Then, $\widetilde{\mathcal{A}}$ is a soft σ-algebra but not a soft σ-ring.

Lemma 3.7. *A soft ring $\widetilde{\mathcal{R}}$ of soft subsets of $\breve{X}$ is a soft algebra if and only if $\breve{X} \widetilde{\in} \widetilde{\mathcal{R}}$.*

Proof. Let $T_A \widetilde{\in} \widetilde{\mathcal{R}}$. Then $T_A^c \widetilde{\in} \widetilde{\mathcal{R}}$. Thus $\breve{X} = T_A \widetilde{\cup} T_A^c \widetilde{\in} \widetilde{\mathcal{R}}$.

Conversely, suppose that $\breve{X} \widetilde{\in} \widetilde{\mathcal{R}}$. Then, for any $T_A \widetilde{\in} \widetilde{\mathcal{R}}$, we have $T_A^c = \breve{X} \widetilde{\setminus} T_A \widetilde{\in} \widetilde{\mathcal{R}}$. Then $\widetilde{\mathcal{R}}$ is soft algebra. □

Example 3.8. If $\breve{X}$ is a non-null soft set, then $\{T_\phi, \breve{X}\}$ and $\widetilde{P}(X)$ are soft σ-algebras. In fact, these are extreme examples of soft σ-algebras in the sense that any other soft σ-algebra lies between these two extremes, i.e., if $\widetilde{\mathcal{A}}$ is any other soft σ-algebra on $\breve{X}$, then $\{T_\phi, \breve{X}\} \widetilde{\subset} \widetilde{\mathcal{A}} \widetilde{\subset} \widetilde{P}(\breve{X})$.

Example 3.9. Let $\widetilde{\mathcal{A}}$ be the aggregate of all soft subsets $T_A \widetilde{\subseteq} \breve{X} \neq \phi$ such that either T_A or its complement T_A^c is a countable soft set. Then $\widetilde{\mathcal{A}}$ is a soft σ-algebra. In fact, if $\{T_{A_n}\}$ is a sequence and all T_{A_n} are soft countable, then so is $\cup_n T_{A_n}$. Otherwise $(\cup_n T_{A_n})^c$ is soft countable.

Example 3.10. Let $X = \{g, r, s\}$ and $E = \{\eta_1, \eta_2\}$. Assume that

$T_{A_1} = T_\phi$,
$T_{A_2} = \{(\eta_1, \{g\}), (\eta_2, \{\})\}$,
$T_{A_3} = \{(\eta_1, \{r\}), (\eta_2, \{g, s\})\}$,
$T_{A_4} = \{(\eta_1, \{s\}), (\eta_2, \{r\})\}$,
$T_{A_5} = \{(\eta_1, \{g, r\}), (\eta_2, \{g, s\})\}$,
$T_{A_6} = \{(\eta_1, \{r, s\}), (\eta_2, \{g, r, s\})\}$,
$T_{A_7} = \{(\eta_1, \{g, s\}), (\eta_2, \{r\})\}$, and
$T_{A_8} = \breve{X}$.

Then $\widetilde{\mathcal{A}} = \{T_{A_i} : i = 1, 2, 3, ..., 8\}$ is a soft σ-algebra over X.

Theorem 3.11. *The soft intersection of any collection of soft σ-algebras on $\breve{X}$ forms again a soft σ-algebra on $\breve{X}$.*

Proof. Let $\{\widetilde{\mathcal{A}}_i\}$ be a collection of soft σ-algebras on $\breve{X}$. Let $\widetilde{\mathcal{A}} = \widetilde{\cap}_i \widetilde{\mathcal{A}}_i$. Since $T_\phi \widetilde{\in} \widetilde{\mathcal{A}}_i$ for each i, $T_\phi \widetilde{\in} \widetilde{\mathcal{A}}$.

If $T_A \widetilde{\in} \widetilde{\mathcal{A}}$, then $T_A \widetilde{\in} \widetilde{\mathcal{A}}_i$ for each i and so is T_A^c. Thus $T_A^c \widetilde{\in} \widetilde{\mathcal{A}}$.

Finally, if $T_{A_n} \widetilde{\in} \widetilde{\mathcal{A}}$ for $n \in \mathbb{N}$, then $T_{A_n} \widetilde{\in} \widetilde{\mathcal{A}}_i$ for every n, i and $\widetilde{\cup}_{n=1}^{\infty} T_{A_n} \widetilde{\in} \widetilde{\mathcal{A}}_i$ for every i. This implies that $\widetilde{\cup}_{n=1}^{\infty} T_{A_n} \widetilde{\in} \widetilde{\mathcal{A}}$. □

Theorem 3.12. *The soft difference of two soft σ-algebras on $\breve{X}$ is again a soft σ-algebra on $\breve{X}$.*

Proof. Let $\widetilde{\mathcal{A}}$ be a soft σ-algebra and $T_{A_1}, T_{A_2} \widetilde{\in} \widetilde{\mathcal{A}}$. Since $T_{A_2} \widetilde{\in} \widetilde{\mathcal{A}}$, so by definition $T_{A_2}^c \widetilde{\in} \widetilde{\mathcal{A}}$. The result follows now quickly from the identity $T_{A_1} \widetilde{\setminus} T_{A_2} = T_{A_1} \widetilde{\cap} T_{A_1}^c$. □

Remark 3.13. The soft union of two soft σ-algebras may not enjoy the same property. For an illustration, consider the following example:

Let $X = \{g, r, s, x\}$ be the initial universe, $E = \{\eta_i, i = 1, 2, 3\}$ be the family of decision variables and $A = \{\eta_1, \eta_3\} \subset E$. Let

$$T_A = \{(\eta_1, \{g, r, s, x\}), (\eta_3, \{g, r, s, x\})\}.$$

Then

$$\widetilde{\mathcal{A}}_1 = \{T_\phi, \{(\eta_1, \{g\})\}, \{(\eta_1, \{r, s, x\})\}, T_A\},$$

and

$$\widetilde{\mathcal{A}}_2 = \{T_\phi, \{(\eta_3, \{r\})\}, \{(\eta_3, \{g, s, x\})\}, T_A\}$$

are two soft σ-algebras over X. However, their soft union

$$\widetilde{\mathcal{A}}_1 \widetilde{\cup} \widetilde{\mathcal{A}}_2 = \{T_\phi, \{(\eta_1, \{g\})\}, \{(\eta_1, \{r, s, x\})\}, \{(\eta_3, \{r\})\}, \{(\eta_3, \{g, s, x\})\}, T_A\}$$

is not a soft σ-algebra over X, for $\{(\eta_1, \{g\})\}, \{(\eta_1, \{r, s, x\})\} \widetilde{\in} \widetilde{\mathcal{A}}_1 \widetilde{\cup} \widetilde{\mathcal{A}}_2$, but their soft union

$$\{(\eta_1, \{g\})\} \widetilde{\cup} \{(\eta_1, \{r, s, x\})\} = \{(\eta_1, \{g, r, s, x\})\} \widetilde{\notin} \widetilde{\mathcal{A}}_1 \widetilde{\cup} \widetilde{\mathcal{A}}_2.$$

The following result provides a basic technique for the construction of σ-algebra.

Theorem 3.14. *Let $\widetilde{\mathcal{G}}$ be the collection of soft subsets of $\check{X}$. Then there is a smallest soft σ-algebra containing $\widetilde{\mathcal{G}}$.*

Proof. Let $\widetilde{\mathcal{F}} = \{\widetilde{\mathcal{A}} : \widetilde{\mathcal{A}}$ is a soft σ-algebra and $\widetilde{\mathcal{G}} \widetilde{\subseteq} \widetilde{\mathcal{A}}\}$. Then $\widetilde{\mathcal{F}} \neq T_\phi$, because $\widetilde{P}(\check{X})$ is a soft σ-algebra such that $\widetilde{\mathcal{G}} \widetilde{\subseteq} \widetilde{P}(\check{X})$.

Let's write $\widetilde{\mathcal{H}} = \widetilde{\cap}_{\widetilde{\mathcal{A}} \widetilde{\in} \widetilde{\mathcal{F}}} \widetilde{\mathcal{A}}$. Since arbitrary soft intersection of soft σ-algebras is also a soft σ-algebra, $\widetilde{\mathcal{H}}$ is a soft σ-algebra. Since $\widetilde{\mathcal{G}} \widetilde{\subseteq} \widetilde{\mathcal{A}}, \forall \widetilde{\mathcal{A}} \widetilde{\in} \widetilde{\mathcal{F}}, \widetilde{\mathcal{G}} \widetilde{\subseteq} \widetilde{\cap}_{\widetilde{\mathcal{A}} \widetilde{\in} \widetilde{\mathcal{F}}} \widetilde{\mathcal{A}} = \widetilde{\mathcal{H}}$, i.e., $\widetilde{\mathcal{H}}$ is a soft σ-algebra containing $\widetilde{\mathcal{G}}$. Suppose that $\widetilde{\mathcal{H}'}$ is a soft σ-algebra containing $\widetilde{\mathcal{G}}$. It follows by definition of $\widetilde{\mathcal{F}}$ that $\widetilde{\mathcal{H}'} \widetilde{\in} \widetilde{\mathcal{F}}$. Thus, $\widetilde{\cap}_{\widetilde{\mathcal{A}} \widetilde{\in} \widetilde{\mathcal{F}}} \widetilde{\mathcal{A}} \widetilde{\subseteq} \widetilde{\mathcal{H}'}$ gives that $\widetilde{\mathcal{H}} \widetilde{\subseteq} \widetilde{\mathcal{H}'}$. So $\widetilde{\mathcal{H}}$ is the smallest soft σ-algebra containing $\widetilde{\mathcal{G}}$. □

Definition 3.15. The smallest soft σ-algebra $\widetilde{\mathcal{H}}$ containing some soft collection $\widetilde{\mathcal{G}}$ of soft subsets of $\check{X}$ (whose existence is guaranteed in Theorem 3.14) is called the soft σ-algebra generated by $\widetilde{\mathcal{G}}$.

Example 3.16. Let $\widetilde{\mathcal{G}} = \{T_A : T_A \neq T_\phi \wedge T_A \widetilde{\subsetneq} \check{X}\}$. Then $\widetilde{\mathcal{H}} = \{T_\phi, T_A, T_A^c, \check{X}\}$ is the smallest soft σ-algebra containing T_A because if $T_A \widetilde{\in} \widetilde{\mathcal{A}}$, a soft σ-algebra, then $T_\phi, \check{X}, T_A^c \widetilde{\in} \widetilde{\mathcal{A}}$. Thus $\widetilde{\mathcal{H}} \widetilde{\subseteq} \widetilde{\mathcal{A}}$.

Theorem 3.17. *Let $\{T_{A_i}\}$ be a sequence of soft sets in a soft σ-algebra $\widetilde{\mathcal{A}}$. Then there exits a sequence $\{T_{B_i}\}$ of pairwise disjoint soft sets such that $\widetilde{\cup}_i T_{B_i} = \widetilde{\cup}_i T_{A_i}$.*

Proof. Let's define

$$\begin{aligned}
T_{B_1} &= T_{A_1}, \\
T_{B_n} &= T_{A_n} \widetilde{\setminus} (T_{A_1} \widetilde{\cup} T_{A_2} \widetilde{\cup} \ldots \widetilde{\cup} T_{A_{n-1}}),\ \forall n > 1, \\
&= T_{A_n} \widetilde{\cap} (T_{A_1} \widetilde{\cup} T_{A_2} \widetilde{\cup} \ldots \widetilde{\cup} T_{A_{n-1}})^c\ (\because\ T_A \widetilde{\setminus} T_B = T_A \widetilde{\cap} T_B^c), \\
&= T_{A_n} \widetilde{\cap} (T_{A_1}^c \widetilde{\cap} T_{A_2}^c \widetilde{\cap} \ldots \widetilde{\cap} T_{A_{n-1}}^c).
\end{aligned}$$

Then clearly, $T_{B_n} \widetilde{\in} \widetilde{\mathcal{A}}, \forall n$.

Now we show that T_{B_i}'s are pairwise soft disjoint by showing that $T_{B_m} \widetilde{\cap} T_{B_n} = T_\phi$ for $m \neq n$. Without any loss of generality, we may assume that $m < n$. Then by definition of T_{B_m}, $T_{B_m} \widetilde{\subseteq} T_{A_m}$. Thus $T_{B_m} \widetilde{\cap} T_{B_n} \widetilde{\subseteq} T_{A_m} \widetilde{\cap} T_{A_n}$. But

$$\begin{aligned}
& T_{B_m} \widetilde{\cap} T_{B_n} \\
&= T_{A_m} \widetilde{\cap} (T_{A_n} \widetilde{\cap} T_{A_1}^c \widetilde{\cap} T_{A_1}^c \widetilde{\cap} \ldots \widetilde{\cap} T_{A_{n-1}}^c) \\
&= T_{A_m} \widetilde{\cap} T_{A_m}^c \widetilde{\cap} (T_{A_n} \widetilde{\cap} T_{A_1}^c \widetilde{\cap} T_{A_2}^c \widetilde{\cap} \ldots \widetilde{\cap} T_{A_{m-1}}^c \widetilde{\cap} T_{A_{m+1}}^c \ldots \widetilde{\cap} T_{A_{n-1}}^c) \\
&= T_\phi \widetilde{\cap} (T_{A_n} \widetilde{\cap} T_{A_1}^c \widetilde{\cap} T_{A_2}^c \widetilde{\cap} \ldots \widetilde{\cap} T_{A_{m-1}}^c \widetilde{\cap} T_{A_{m+1}}^c \widetilde{\cap} \ldots \widetilde{\cap} T_{A_{n-1}}^c) \\
&= T_\phi.
\end{aligned}$$

So $T_{B_m} \widetilde{\cap} T_{B_n} = T_\phi$ for $m \neq n$.

Now we show that $\widetilde{\cup}_i T_{B_i} = \widetilde{\cup}_i T_{A_i}$. Since $T_{B_i} \widetilde{\subseteq} T_{A_i}, \forall i, \widetilde{\cup}_i T_{B_i} \widetilde{\subseteq} \widetilde{\cup}_i T_{A_i}$.

Let $P_e^x \widetilde{\in} \widetilde{\cup}_i T_{A_i}$. Then $P_e^x \widetilde{\in} T_{A_i}$ for some i. Let j be the smallest positive integer for which $P_e^x \widetilde{\in} T_{A_j}$ so that $P_e^x \widetilde{\notin} T_{A_1}, T_{A_2}, \ldots, T_{A_{j-1}}$, i.e.,

$$P_e^x \widetilde{\in} T_{A_j} \widetilde{\setminus} (T_{A_1} \widetilde{\cup} T_{A_2} \widetilde{\cup} \ldots \widetilde{\cup} T_{A_{j-1}}) = T_{B_j}.$$

Then, $P_e^x \widetilde{\in} \widetilde{\cup}_j T_{B_j}$. Thus $\widetilde{\cup}_i T_{B_i} = \widetilde{\cup}_i T_{A_i}$. □

Definition 3.18. Let $\widetilde{\mathcal{A}}$ be a soft σ-algebra of soft subsets over X and $\widetilde{\mu}$ be a soft real-valued mapping on $\widetilde{\mathcal{A}}$. Let $\{T_{A_i}\}$ be a sequence of soft sets in $\widetilde{\mathcal{A}}$. The soft mapping $\widetilde{\mu}$ is called:

(i) finitely soft sub-additive, if $\widetilde{\mu}(\widetilde{\cup}_{i=1}^{n} T_{A_i}) \widetilde{\leq} \Sigma_{i=1}^{n} \widetilde{\mu}(T_{A_i})$,

(ii) countably soft sub-additive, if $\widetilde{\mu}(\widetilde{\cup}_{i=1}^{\infty} T_{A_i}) \widetilde{\leq} \Sigma_{i=1}^{\infty} \widetilde{\mu}(T_{A_i})$,

(iii) finitely soft additive, if $\widetilde{\mu}(\widetilde{\cup}_{i=1}^{n} T_{A_i}) = \Sigma_{i=1}^{n} \widetilde{\mu}(T_{A_i})$, where T_{A_i}'s are pairwise soft disjoint,

(iv) countably soft additive or soft σ-additive, if $\widetilde{\mu}(\widetilde{\cup}_{i=1}^{\infty} T_{A_i}) = \Sigma_{i=1}^{\infty} \widetilde{\mu}(T_{A_i})$, where T_{A_i}'s are pairwise soft disjoint,

(v) soft monotone, if $T_A \widetilde{\subseteq} T_B \Rightarrow \widetilde{\mu}(T_A) \widetilde{\leq} \widetilde{\mu}(T_B)$, $\forall\ T_A, T_B \widetilde{\in} \widetilde{\mathcal{A}}$.

Remark 3.19. It follows from above definition that every countably soft additive (countably soft sub-additive) set mapping is finitely soft additive (finitely soft sub-additive).

Definition 3.20. Let $\widetilde{\mathcal{A}}$ be a soft σ-algebra of soft subsets of a set X and $\widetilde{\mu}$ be an extended soft real-valued mapping on $\widetilde{\mathcal{A}}$. Then $\widetilde{\mu}$ is called a *soft measure* on $\widetilde{\mathcal{A}}$, if

(i) $\widetilde{\mu}(T_\phi) = \overline{0}$,

(ii) $\widetilde{\mu}(T_A) \widetilde{\geq} \overline{0}$ for each $T_A \widetilde{\in} \widetilde{\mathcal{A}}$,

(iii) $\widetilde{\mu}$ is countably soft additive, i.e., $\widetilde{\mu}(\widetilde{\cup}_{i=1}^{\infty} T_{A_i}) = \Sigma_{i=1}^{\infty} \widetilde{\mu}(T_{A_i})$, T_{A_i}'s being pairwise soft disjoint.

If $\widetilde{\mu}$ is a soft measure on a soft σ-algebra $\widetilde{\mathcal{A}}$, then the triplet $(X, \widetilde{\mathcal{A}}, \widetilde{\mu})$ is called a soft measure space.

Example 3.21. Let $X = \widetilde{\mathbb{N}}$ and $\widetilde{\mathcal{A}} = \widetilde{2^{\mathbb{N}}}$. Let $\widetilde{\mu} : \widetilde{\mathcal{A}} \to \widetilde{\mathbb{R}}$ be defined as

$$\widetilde{\mu}(T_A) = \begin{cases} \text{number of soft points in } T_A, & \text{if } T_A \text{ is soft finite} \\ \overline{\infty}\ , & \text{if } T_A \text{ is soft infinite.} \end{cases}$$

Then the first two axioms of Definition 3.20 follow directly from definition of $\widetilde{\mu}$. For third condition, let $\{T_{A_n}\}$ be a sequence of pairwise soft disjoint sets in $\widetilde{\mathcal{A}}$. There arise two cases:

If each T_{A_n} is soft finite, then $\widetilde{\mu}(\cup_{n=1}^{\infty} T_{A_n}) = \Sigma_{n=1}^{\infty} \widetilde{\mu}(T_{A_n})$ is obvious.

Suppose that there is an integer n_0 such that $T_{A_{n_0}}$ is an infinite soft set. Then $\widetilde{\mu}(T_{A_{n_0}}) = \overline{\infty}$. Thus $\Sigma_{n=1}^{\infty} \widetilde{\mu}(T_{A_n}) = \overline{\infty}$. Since $\widetilde{\cup}_{n=1}^{\infty} T_{A_n}$ is an infinite soft set, $\widetilde{\mu}(\cup_{n=1}^{\infty} T_{A_n}) = \overline{\infty}$. So, $\widetilde{\mu}(\cup_{n=1}^{\infty} T_{A_n}) = \Sigma_{n=1}^{\infty} \widetilde{\mu}(T_{A_n})$. Since $\widetilde{\mu}$ satisfies all the three condition given in Definition 3.20, $\widetilde{\mu}$ is a soft measure and is known as the counting soft measure.

Theorem 3.22. *Let $(X, \widetilde{\mathcal{A}}, \widetilde{\mu})$ be a soft measure space.*

(1) *If there exists $T_A \widetilde{\in} \widetilde{\mathcal{A}}$ such that $\widetilde{\mu}(T_A) \widetilde{<} +\overline{\infty}$, then $\widetilde{\mu}(T_\phi) = \overline{0}$.*

(2) *$\widetilde{\mu}$ is soft monotone.*

(3) *If $\{T_{A_n}\}$ is a sequence of pairwise soft disjoint sets in $\widetilde{\mathcal{A}}$, then*

$$\widetilde{\mu}(T_G) = \Sigma_{n=1}^{\infty} \widetilde{\mu}(T_{A_n} \widetilde{\cap} T_G) + \widetilde{\mu}(T_G \widetilde{\cap} (\widetilde{\cup}_{n=1}^{\infty} T_{A_n})^c),$$

for any $T_G \widetilde{\in} \widetilde{\mathcal{A}}$.

Proof. (1) Since $\widetilde{\mu}(T_A) = \widetilde{\mu}(T_A \,\widetilde{\cup}\, T_\phi) = \widetilde{\mu}(T_A) + \widetilde{\mu}(T_\phi)$,
$\widetilde{\mu}(T_\phi) = \widetilde{\mu}(T_A) - \widetilde{\mu}(T_A) = \overline{0}$ ($\because \widetilde{\mu}(A) \,\widetilde{<}\, +\overline{\infty}$).

2) Let $T_A, T_B \,\widetilde{\in}\, \widetilde{\mathcal{A}}$ such that $T_A \,\widetilde{\subseteq}\, T_B$. Then, $T_B = (T_B \widetilde{\setminus} T_A) \,\widetilde{\cup}\, T_A$ yields $\widetilde{\mu}(T_B) = \widetilde{\mu}(T_B \widetilde{\setminus} T_A) + \widetilde{\mu}(T_A)$. Thus $\widetilde{\mu}(T_B) - \widetilde{\mu}(T_A) = \widetilde{\mu}(T_B \widetilde{\setminus} T_A) \,\widetilde{\geq}\, \overline{0}$. So $\widetilde{\mu}(T_B) \,\widetilde{\geq}\, \widetilde{\mu}(T_A)$, i.e., $\widetilde{\mu}(T_A) \,\widetilde{\leq}\, \widetilde{\mu}(T_B)$.

(3) Let $T_G \,\widetilde{\in}\, \widetilde{\mathcal{A}}$. Then

$$\begin{aligned} T_G &= T_G \,\widetilde{\cap}\, X \\ &= T_G \,\widetilde{\cap}\, [(\widetilde{\cup}_{n=1}^{\infty} T_{A_n}) \,\widetilde{\cup}\, (\widetilde{\cup}_{n=1}^{\infty} T_{A_n})^c] \\ &= [T_G \,\widetilde{\cap}\, (\widetilde{\cup}_{n=1}^{\infty} T_{A_n})] \,\widetilde{\cup}\, [T_G \,\widetilde{\cap}\, (\widetilde{\cup}_{n=1}^{\infty} T_{A_n})^c]. \end{aligned}$$

Thus

$$\begin{aligned} \widetilde{\mu}(T_G) &= \widetilde{\mu}([T_G \,\widetilde{\cap}\, (\widetilde{\cup}_{n=1}^{\infty} T_{A_n})] \,\widetilde{\cup}\, [T_G \,\widetilde{\cap}\, (\widetilde{\cup}_{n=1}^{\infty} T_{A_n})^c]) \\ &= \widetilde{\mu}(T_G \,\widetilde{\cap}\, (\widetilde{\cup}_{n=1}^{\infty} T_{A_n})) + \widetilde{\mu}(T_G \,\widetilde{\cap}\, (\widetilde{\cup}_{n=1}^{\infty} T_{A_n})^c) \\ &= \Sigma_{n=1}^{\infty}\, \widetilde{\mu}(T_{A_n} \,\widetilde{\cap}\, T_G) + \widetilde{\mu}(T_G \,\widetilde{\cap}\, (\widetilde{\cup}_{n=1}^{\infty} T_{A_n})^c). \end{aligned}$$

□

Definition 3.23. A soft measure space $(X, \widetilde{\mathcal{A}}, \widetilde{\mu})$ is said to be soft finite, if $\widetilde{\mu}(X) \,\widetilde{<}\, \overline{\infty}$. In general, a soft set T_A is said to be of finite soft measure, if $T_A \,\widetilde{\in}\, \widetilde{\mathcal{A}}$ with $\widetilde{\mu}(T_A) \,\widetilde{<}\, \overline{\infty}$.

Definition 3.24. A soft measure space $(X, \widetilde{\mathcal{A}}, \widetilde{\mu})$ is called soft σ-finite, if there is a sequence $\{T_{A_n}\}$ of soft sets in $\widetilde{\mathcal{A}}$ such that $X = \widetilde{\cup}_n\, T_{A_n}$ and $\widetilde{\mu}(T_{A_n}) \,\widetilde{<}\, \overline{\infty}, \forall n$.

Example 3.25. Let $\widetilde{\mathcal{A}}$ be a soft σ-algebra of all soft subsets of X. Then the soft measure $\widetilde{\mu}$ defined by

$$\widetilde{\mu}(T_A) = \begin{cases} \overline{0}, & \text{if } T_A = T_\phi \\ \overline{\infty}, & \text{if } T_A \neq T_\phi \end{cases}$$

is neither soft finite nor soft σ-finite.

Example 3.26. Let $\widetilde{\mathcal{A}}$ be a soft σ-algebra of all soft subsets of X. Let P_e^x be a fixed element of T_A. Then the soft measure $\widetilde{\mu}$ defined as

$$\widetilde{\mu}(T_A) = \begin{cases} \overline{0}, & \text{if } P_e^x \,\widetilde{\notin}\, T_A \\ \overline{1}, & \text{if } P_e^x \,\widetilde{\in}\, T_A \end{cases}$$

is soft finite.

Example 3.27. The counting soft measure $\widetilde{\mu}$ on $\widetilde{\mathbb{N}}$, given in Example 3.21 is soft σ-finite but not soft finite because $\widetilde{\mathbb{N}} = \widetilde{\cup}_{\overline{n} \,\widetilde{\in}\, \widetilde{\mathbb{N}}}\, \{\overline{n}\}$ with $\widetilde{\mu}(\{\overline{n}\}) = \overline{1} \,\widetilde{<}\, \infty$.

Remark 3.28. Every soft finite measure may be regarded as a soft σ-finite measure. However, its converse need not hold.

Definition 3.29. A non-negative soft extended real-valued set function $\widetilde{\mu}^*$ defined on 2^X is called a soft outer measure, if

(i) $\widetilde{\mu}^*(T_\phi) = \overline{0}$,
(ii) $\widetilde{\mu}^*$ is soft monotone,
(iii) $\widetilde{\mu}^*$ is countably soft sub-additive, i.e., $\widetilde{\mu}^*(\widetilde{\cup}_{i=1}^{\infty}\, T_{A_i}) \,\widetilde{\leq}\, \Sigma_{i=1}^{\infty}\, \widetilde{\mu}^*\, (T_{A_i})$.

Example 3.30. Let $\widetilde{\mu}^* : 2^X \to [\overline{0}, \overline{\infty}]$ be defined as

$$\widetilde{\mu}^*(T_A) = \begin{cases} \overline{0}, & \text{if } T_A = T_\phi \\ \overline{1}, & \text{if } T_A \neq T_\phi. \end{cases}$$

Then clearly, $\widetilde{\mu}^*$ is an extended soft real-valued set function.

(i) $\widetilde{\mu}^*(T_\phi) = \overline{0}$.

(ii) Let $T_A, T_B \widetilde{\in} 2^X$ such that $T_A \widetilde{\subseteq} T_B$. There arise three cases:

If $T_A = T_B = T_\phi$, then $\widetilde{\mu}^*(T_A) = \widetilde{\mu}^*(T_B) = \overline{0}$.

If $T_A = T_\phi$ and $T_B \neq T_\phi$, then $\widetilde{\mu}^*(T_A) = \overline{0}$ and $\widetilde{\mu}^*(T_B) = \overline{1}$. Thus $\widetilde{\mu}^*(T_A) \widetilde{<} \widetilde{\mu}^*(T_B)$.

If $T_A \neq T_\phi$ and $T_B \neq T_\phi$, then $\widetilde{\mu}^*(T_A) = \widetilde{\mu}^*(T_B) = \overline{1}$.

So, in all cases, $\widetilde{\mu}^*(T_A) \widetilde{\leq} \widetilde{\mu}^*(T_B)$.

(iii) Let $\{T_{A_n}\}$ be a sequence of pairwise soft disjoint sets in 2^X. If $T_{A_n} \neq T_\phi$ for each n, then $\widetilde{\cup}_{n=1}^{\infty} T_{A_n} \neq T_\phi$. Thus $\widetilde{\mu}^*(\widetilde{\cup}_{n=1}^{\infty} T_{A_n}) = \overline{1}$. Also $\widetilde{\mu}^*(T_{A_n}) = \overline{1}$, $\forall n$, yields $\Sigma_{n=1}^{\infty} \widetilde{\mu}^*(T_{A_n}) = \overline{1} + \overline{1} + \overline{1} + ... = \overline{\infty}$. So $\widetilde{\mu}^*(\widetilde{\cup}_{n=1}^{\infty} T_{A_n}) \widetilde{<} \Sigma_{n=1}^{\infty} \widetilde{\mu}^*(T_{A_n})$. In all other cases, we have $\widetilde{\mu}^*(\widetilde{\cup}_{n=1}^{\infty} T_{A_n}) \widetilde{\leq} \Sigma_{n=1}^{\infty} \widetilde{\mu}^*(T_{A_n})$.

Hence $\widetilde{\mu}^*$ is a soft outer measure.

Remark 3.31. In the above example, if T_{A_i} consists of more than one soft point, then $\widetilde{\mu}^*$ is not countably soft additive. For an illustration, let $X = \{g, r, s\}$ and $E = \{\eta_1, \eta_2\}$. Suppose that

$T_{A_1} = \{(\eta_1, \{g\}), (\eta_2, \{r\})\}$,
$T_{A_2} = \{(\eta_1, \{r\}), (\eta_2, \{g\})\}$, and
$T_{A_3} = \{(\eta_1, \{s\}), (\eta_2, \{s\})\}$.

Then, $\cup_{n=1}^{3} T_{A_n} = \{(\eta_1, \{g, r, s\}), (\eta_2, \{g, r, s\})\}$ so that $\widetilde{\mu}^*(\cup_{n=1}^{3} T_{A_n}) = \overline{1}$ whereas $\Sigma_{n=1}^{3} \widetilde{\mu}^*(T_{A_n}) = \overline{3}$ and hence $\widetilde{\mu}^*(\cup_{n=1}^{3} T_{A_n}) \neq \Sigma_{n=1}^{3} \widetilde{\mu}^*(T_{A_n})$.

4. Applications of soft mappings to decision-making

In this section, we present two applications of soft mappings to decision-making.

Example 4.1. Suppose that a country is facing decaying economic conditions. The chief executive (CE) of that country makes a committee of experts to find core issues. The committee found the following problems:

Slow capital formation, poor technology/low productivity, poor quality of human resources, inflation, unemployment, political instability, defective planning, health problems and lack of efficient administration.

The CE makes another committee of experts to rate these issues from high importance to low importance and reasons behind those issues so as to address the issues one by one. The report given by the experts may be encoded in the form of soft set as follows:

$$(T, A) = \begin{cases} \text{High significance} = \{\text{slow Capital formation, poor technology, poor quality of HRs}\}, \\ \text{Medium significance} = \{\text{inflation, unemployment, political instability}\}, \\ \text{Low significance} = \{\text{defective planning, health problems, lack of efficient administration}\} \end{cases}$$

The term "significance" means relative importance of the issue that should be addressed at first, second and third step.

The economical acquaintance may be instructed in the form of a computerized representation of the concept of mapping in mathematics called look-up table. Assume that the experts provided following knowledge:

u(Slow Capital formation) = low saving, low investment,
u(poor technology) = lack of capital,
u(poor quality of human resources) = illiteracy, unskilled workers, poverty,
u(inflation) = increase in money supply, fall in production,

u(unemployment) = rapid population growth, slow industrial growth,
u(political instability) = long spell of military interventions,
u(defective planning) = unreliable statistical data,
u(health problems) = low income level, lack of awareness,
u(lack of efficient admin) = inefficient and irresponsible government machinery,
and
v(high significance) = inadequate governance,
v(low significance) = frequent changes in economical policies of the government.
For convenience in further mathematical manipulation, we represent the information gathered by means of signs and symbols as under:

α_1 = Slow Capital formation,
α_2 = poor technology,
α_3 = poor quality of human resources,
α_4 = inflation,
α_5 = unemployment,
α_6 = political instability,
α_7 = defective planning,
α_8 = health problems,
α_9 = lack of efficient administration;
η_1 = high significance,
η_2 = medium significance,
η_3 = low significance

and

β_1 = low saving, low investment,
β_2 = lack of capital,
β_3 = illiteracy, unskilled workers, poverty,
β_4 = increase in money supply, fall in production,
β_5 = rapid population growth, slow industrial growth,
β_6 = long spell of military interventions
β_7 = unreliable statistical data,
β_8 = low income level, lack of awareness,
β_9 = lack of efficient administration;
$\eta_1^{'}$ = high significance,
$\eta_2^{'}$ = medium significance.

Then we are endowed with two soft classes (X_1, E) and $(X_2, E^{'})$, where $X_1 = \{\alpha_1, \alpha_2, ..., \alpha_9\}$, $X_2 = \{\beta_1, \beta_2, ..., \beta_9\}$ and the corresponding sets of decision variables $E = \{\eta_i, i = 1, 2, 3\}$, $E^{'} = \{\eta_i^{'}, i = 1, 2\}$. The pair (X_1, E) represents the soft class of issues and their significance for the country, whereas $(X_2, E^{'})$ denotes reasons and economical preference for resolving those issues. The soft set of report by the experts may be represented as:

$$(T, A) = \{(\eta_1, \{\alpha_i : i = 1, 2, 3\}), (\eta_2, \{\alpha_i : i = 4, 5, 6\}), (\eta_3, \{\alpha_i : i = 7, 8, 9\})\}.$$

Thus the soft set T_A may be tabulated as follows:

Attributes	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9
η_1	1	1	1	0	0	0	0	0	0
η_2	0	0	0	1	1	1	0	0	0
η_3	0	0	0	0	0	0	1	1	1

As a first task of the fiscal expert system, stored economical comprehension is to be employed on given situation. This information, in computer programming language, is given as look-up tables. The maps $u : X_1 \to X_2$ and $v : E \to E^{'}$ are defined as:

$u(\alpha_i) = \beta_i,\ i = 1, 2, ..., 9,$
$v(\eta_i) = \eta_i^{'},\ i = 1, 2, 3.$

Calculations yield:

$$f(T, A) = \{(\eta_1^{'}, \{\beta_1, \beta_2, \beta_3\}), (\eta_2^{'}, \{\beta_4, \beta_5, \beta_6\})\},$$

i.e., $f(T,A) = \{$(high significance, {low saving, low investment; lack of capital; illiteracy, unskilled workers, poverty}), (medium significance, {increase in money supply, fall in production; rapid population, growth, slow industrial growth; long spell of military interventions})$\}$

Example 4.2. Suppose that a patient suffering from a recent tragedy visits a doctor in a very bad mode and narrated following issues:

"I have severe migraine, fatigue, anxiety, depression and Sleeplessness. Besides, I also feel Semi consciousness, backache and have high blood pressure."

The symptoms/complaints narrated by the patient may be encoded in the form of soft set (See [1]) as follows:

$$(T, A) = \begin{cases} \text{High significance} = \{\text{Anxiety, Depression, Sleeplessness, Semi consciousness}\}, \\ \text{Medium significance} = \{\text{High blood pressure, Fatigue}\}, \\ \text{Low significance} = \{\text{Migraine, Backache}\}. \end{cases}$$

Then the medical acquaintance may be instructed in the form of a computerized representation of the concept of mapping in mathematics called look-up table.
Assume that our medical professionals provide us the following information:

u(anxiety) = low energy level,
u(depression) = low energy level,
u(sleeplessness) = acidity,
u(semi-conscious sleep) = fatigue,
u(high blood pressure) = unbalanced food,
u(fatigue) = low energy level,
u(migraine) = low energy level,
u(backache) = wrong posture

and

v(high significance) = infrequent high potency,
v(medium significance) = frequent high potency.

For convenience in mathematical manipulation, we represent the symptoms and gradations by signs and symbols as under:

α_1 = anxiety,
α_2 = depression,
α_3 = sleeplessness,
α_4 = semi-conscious sleep,

α_5 = high blood pressure,
α_6 = fatigue,
α_7 = migraine,
α_8 = backache;
η_1 = high significance,
η_2 = medium significance,
η_3 = low significance

and

β_1 = low energy level,
β_2 = acidity,
β_3 = fatigue,
β_4 = unbalanced food,
β_5 = wrong posture;
$\eta_1^{'}$ = infrequent high potency,
$\eta_2^{'}$ = frequent high potency.

Thus we are bestowed with two soft classes (X_1, E) and $(X_2, E^{'})$, where $X_1 = \{\alpha_1, ..., \alpha_8\}$, $X_2 = \{\beta_1, ..., \beta_5\}$ and the corresponding sets of attributes $E = \{\eta_i, i = 1, 2, 3\}$, $E^{'} = \{\eta_i^{'}, i = 1, 2\}$. The pair (X_1, E) represents the soft class of narrated symptoms and their weight for patient, whereas $(X_2, E^{'})$ denotes reasons and medical priority for treatment. The soft set of narration by the patient may be represented as:

$$T_A = \{(\eta_1, \{\alpha_i : i = 1, ..., 4\}), (\eta_2, \{\alpha_i : i = 5, 6\}), (\eta_3, \{\alpha_i : i = 7, 8\})\}.$$

The tabular representation of the soft set T_A is

Attributes	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
η_1	1	1	1	1	0	0	0	0
η_2	0	0	0	0	1	1	0	0
η_3	0	0	0	0	0	0	1	1

As a first task of the medical specialist system, recorded medical information is to be employed on the given case. This information, in computer programming language, is given as look-up tables. The mappings $u : X_1 \to X_2$ and $v : E \to E^{'}$ are defined as:

$u(\alpha_1) = \beta_1$, $u(\alpha_2) = \beta_1$, $u(\alpha_3) = \beta_2$, $u(\alpha_4) = \beta_3$,
$u(\alpha_5) = \beta_4$, $u(\alpha_6) = \beta_1$, $u(\alpha_7) = \beta_1$, $u(\alpha_8) = \beta_5$,
$v(\eta_1) = \eta_1^{'}$, $v(\eta_2) = \eta_2^{'}$.

Calculations yield:

$$f(T_A) = \{(\eta_1^{'}, \{\beta_1, \beta_2, \beta_3\}), (\eta_2^{'}, \{\beta_1, \beta_5\})\},$$

i.e., $f(T_A) = \{(\text{infrequent high potency}, \{\text{low energy level}, \text{acidity}, \text{fatigue}\}),$
$(\text{frequent high potency}, \{\text{low energy level}, \text{wrong posture}\})\}.$

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