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Soft k-int-ideals of semirings and its algebraic structures

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Abstract. In this study, we first give a definition of soft int-ideal and soft k-int-ideal of a semiring by using intersection operation of sets with their properties. We then define a soft k-product of two soft left k-int-ideals and investigate their algebraic structures.

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1. Introduction

Many authors have become interested in modeling uncertainty recently. There are some testatum mathematical theories to deal with uncertainty, such as fuzzy set theory [22], rough set theory [5], etc. These theories have its connatural difficulties as indicated by Molodtsov in [18]. To deal with uncertainty, there is an another theory called soft set theory that was suggested by Molodtsov [18].

In recent years, different kinds of operations of soft sets have been defined [3, 4, 6, 8, 17, 19] to use in the theory and applications. By using these operations, works on the algebraic structure of soft set theory have been progressing rapidly. In 2007, Aktas and Cagman [2] started to work on soft algebra by defining the soft groups. After that, Acar et al. [1] studied the soft rings. Feng et al. [10] defined the notions of soft semiring and soft ideal. Jun and Park [13] introduced the concept of soft ideals and idealistic soft BCK/BCI-algebras. Jun et al. [14] proposed the soft p-ideals of soft BCI-algebras. Sun et al. [21] introduced the soft modules. Cagman et al. [7] introduced the concept of soft int-groups. Then Citak and Cagman [9] defined the soft int-rings. Sezgin et al. [20] defined soft intersection near-rings. Zhan and Jun [23] defined the soft BL-algebras and then Zhan and Xu [24] introduced the soft lattice implication algebras based on fuzzy sets. Lee et al. [16] studied the generalized int-soft subsemigroups. Jana and Pal [12] worked the applications of new soft intersection set on groups.

In this work, we first recall semigroup, semiring, k-ideals of semiring and basically introduced the soft set theory in Section 2. We then define concept of soft int-ideal of a semiring in Section 3. We also introduce the concept of soft k-int-ideal of a semiring by using intersection operation of set and investigate the basic properties of soft k-int-ideal. Moreover, we define a soft k-product of two soft left k-int-ideals and work on their algebraic structures in detail. We finally conclude the paper in Section 4.

2. Preliminaries

In this section, we remind the semigroup, semiring, k-ideals of semiring and soft sets with their basic properties.

Throughout this work, U is a universal set, E is a set of parameters, $A \subseteq E$ and $P(U)$ is the power set of U.

Definition 2.1 ([11])**.** A nonempty set S together with a binary operation * is a semigroup, if * is associative in S, that is, $\forall a, b, c \in S, a * (b * c) = (a * b) * c$.

A semigroup is commutative, if * is commutative in S, that is, $\forall a, b \in S, a{*}b = b{*}a$.

Definition 2.2 ([11])**.** A semiring is a nonempty set S together with two binary operations addition and multiplication denoted by + , . respectively, satisfying

(i) $(S, +)$ is a commutative semigroup,

(ii) $(S, .)$ is a semigroup,

(iii) distributive low holds, that is,

$$a.(b + c) = a.b + a.c \text{ and } (a + b).c = a.c + b.c,$$

$\forall a, b, c \in S$.

From now on, a semiring is symbolized as S or R.

Definition 2.3 ([11])**.** A subset I of S is a left (right) ideal of S, if

(i) $a + b \in I$, for all $a, b \in I$

(ii) $b.a \in I (a.b \in I)$, for any $a \in I$ and $b \in S$.

If I is both a left and right ideal, then I is an ideal.

Definition 2.4 ([15])**.** Let I be a left (right) ideal of S. A left (right) ideal of S is a left (right) k-ideal of S, if $b, c \in I$, $a \in S$, $a + b = c$ implies $a \in I$.

Definition 2.5 ([18])**.** (ϑ, A) is a soft set over U where $\vartheta : E \to P(U)$ is a function such that $\vartheta(a) = \varnothing$, if $a \notin A$.

Definition 2.6 ([17])**.** Let (ϑ, A) be a soft set over U. Then (ϑ, A) is an empty soft set, if $\vartheta(a) = \varnothing$, for each $a \in A$. An empty soft set is symbolized by $\widetilde{\Phi}$.

(ϑ, A) is said to be a universal soft set, if $\vartheta(a) = U$, for each $a \in A$. A universal soft set is symbolized by $\widetilde{A}$.

Definition 2.7 ([6])**.** Let (ϑ, A) and (χ, A) be two soft sets over U. (ϑ, A) is said to be a soft subset of (χ, A), if $\vartheta(a) \subseteq \chi(a)$, for each $a \in A$. A soft subset is symbolized by $\vartheta \widetilde{\subseteq} \chi$.

(ϑ, A) and (χ, A) said to equal soft sets, if $\vartheta(a) = \chi(a)$, for each $a \in A$. Equal soft sets are symbolized by $\vartheta \widetilde{=} \chi$.

Definition 2.8 ([6])**.** Let (ϑ, A) and (χ, A) be two soft sets over U.

(i) The union of ϑ and χ, denoted by $\vartheta\widetilde{\cup}\chi$, is defined as
$(\vartheta \cup \chi)(a) = \vartheta(a) \cup \chi(a)$, for each $a \in A$.

(ii) The intersection of ϑ and χ, denoted by $\vartheta\widetilde{\cap}\chi$, is defined as
$(\vartheta \cap \chi)(a) = \vartheta(a) \cap \chi(a)$, for each $a \in A$.

Definition 2.9 ([6])**.** Let (ϑ, A) and (χ, A) be two soft sets over U. $\wedge$-product and $\vee$-product of ϑ and χ are defined by

$$(\vartheta \wedge \chi)(a, b) = \vartheta(a) \cap \chi(b), \quad (\vartheta \vee \chi)(a, b) = \vartheta(a) \cup \chi(b),$$

for each $a, b \in A$, respectively. They are symbolized by $\vartheta \wedge \chi$ and $\vartheta \vee \chi$, respectively.

Definition 2.10 ([7])**.** Let (ϑ, A) be a soft set over U and ψ be a function from a set A to a set B. For each $b \in B$,

$$\psi(\vartheta, A) : B \to P(U),\ \psi(\vartheta, A)(b) = \begin{cases} \cup\{\vartheta(a) : a \in A, \psi(a) = b\}, & \text{if } b \in \psi(A) \\ \varnothing, & \text{if } b \notin \psi(A) \end{cases}$$

is a soft image of (ϑ, A) under ψ.

For all $a \in A$,

$$\psi^{-1}(\vartheta) : A \to P(U),\ \psi^{-1}(\vartheta)(a) = \vartheta(\psi(a))$$

is a soft preimage (or soft inverse image) of (ϑ, A) under ψ.

Definition 2.11 ([7])**.** Let (ϑ, A) be a soft set over U and $\alpha \in P(U)$. A set

$$\vartheta_\alpha = \{a \in A : \vartheta(a) \supseteq \alpha\}$$

is an α-inclusion of (ϑ, A).

3. Soft k-int-ideal

In this section, we first define concept of soft int-ideal of a semiring. We then introduce the concept of soft k-int-ideal of a semiring by using intersection operation of set and investigate the basic properties of soft k-int-ideal. We finally define a soft k-product of two soft left k-int-ideals and study their algebraic structures.

Definition 3.1. Let (ϑ, S) be a soft set over U. (ϑ, S) is said to be a soft left (right) int-ideal over U, if it satisfies the following axioms:

(i) $\vartheta(a + b) \supseteq \vartheta(a) \cap \vartheta(b)$,

(ii) $\vartheta(a.b) \supseteq \vartheta(b)$ $(\vartheta(a.b) \supseteq \vartheta(a))$,

for each $a, b \in S$.

(ϑ, S) is a soft int-ideal over U, if it is both soft left int-ideal and soft right int-ideal over U.

Example 3.2. Suppose that set of the real numbers $\mathcal{R}$ is the universal set and set of the positive integers $\mathcal{Z}^+$ is the subset of set of parameters. A soft set $(\vartheta, \mathcal{Z}^+)$ over $\mathcal{R}$ is defined by

$$\vartheta(a) = (-a, a),$$

where $(-a, a)$ is an open interval for each $a \in \mathcal{Z}^+$. It shows that $(\vartheta, \mathcal{Z}^+)$ is a soft int-ideal of $\mathcal{Z}^+$ over $\mathcal{R}$.

Definition 3.3. Let A be a subset of S. The function

$$\lambda_A(a) = \begin{cases} U, & \text{if } a \in A \\ \varnothing, & \text{if } a \notin A \end{cases}$$

is said to be a soft characteristic function of A.

Definition 3.4. Let (ϑ, S) be a soft left int-ideal over U. (ϑ, S) is said to be a soft left k-int-ideal over U, if for each $a, b, c \in S, a + b = c$ implies $\vartheta(a) \supseteq \vartheta(b) \cap \vartheta(c)$.

A soft right k-int-ideal can be defined similarly.

Theorem 3.5. *Let I be a subset of S. I is a left(right) k-ideal of S iff (λ_I, S) is a soft left(right) k-int-ideal over U.*

Proof. Suppose that I is a left k-ideal of S. Let $a, b, c \in S$ such that $a + b = c$. If $b, c \in I$, then $a \in I$ as I is a left k-ideal of S. Thus,

$$\lambda_I(a) = U = \lambda_I(b) \cap \lambda_I(c).$$

In the contrary case,

$$\lambda_I(b) \cap \lambda_I(c) = \varnothing \subseteq \lambda_I(a).$$

So, (λ_I, S) is a soft left k-int-ideal over U.

Conversely, suppose that (λ_I, S) is a soft left k-int-ideal over U. It is clear that I is a left ideal of S. Let $a \in S$, $b, c \in I$ such that $a + b = c$. Then,

$$\lambda_I(a) \supseteq \lambda_I(b) \cap \lambda_I(c) = U.$$

Thus, $\lambda_I(a) = U$. So $a \in I$. Hence, I is a left k-ideal of S. □

Theorem 3.6. *Let (ϑ, S) and (χ, S) be two soft left(right) k-int-ideals over U. Then, $(\vartheta \tilde{\cap} \chi, S)$ is a soft left(right) k-int-ideal over U.*

Proof. Let $a, b, c \in S$ such that $a + b = c$. Then,

$$\begin{aligned} (\vartheta \cap \chi)(a) &= \vartheta(a) \cap \chi(a) \\ &\supseteq [\vartheta(b) \cap \vartheta(c)] \cap [\chi(b) \cap \chi(c)] \\ &= [\vartheta(b) \cap \chi(b)] \cap [\vartheta(c) \cap \chi(c)] \\ &= (\vartheta \cap \chi)(b) \cap (\vartheta \cap \chi)(c). \end{aligned}$$

Thus, $(\vartheta \tilde{\cap} \chi, S)$ is a soft left k-int-ideal over U. □

Remark 3.7. The following example shows that $(\vartheta \tilde{\cup} \chi, S)$ is not a soft k-int-ideal over U.

Example 3.8. Suppose that set of the positive integers $\mathcal{Z}^+$ is the universal set and $S = \{a, b, c, d\}$ is the subset of set of parameters. The soft k-int-ideals (ϑ, S) and (χ, S) over $\mathcal{Z}^+$ are defined by:

+	a	b	c
a	a	b	b
b	b	a	a
c	b	a	a

and

·	a	b	c
a	a	a	a
b	a	a	a
c	a	a	a

$\vartheta(a) = \{1, 2, 4, 7\}$, $\vartheta(b) = \{1, 2, 4, 7\}$, $\vartheta(c) = \{3, 4\}$ and $\chi(a) = \{4, 5, 8, 9\}$, $\chi(b) = \{4, 5, 8, 9\}$, $\chi(c) = \{5\}$. It shows that $(\vartheta \cup \chi)(ac) \not\supseteq (\vartheta \cup \chi)(c)$. Then, $(\vartheta \tilde{\cup} \chi, S)$ is not a soft int-ideal over $\mathcal{Z}^+$. Thus, $(\vartheta \tilde{\cup} \chi, S)$ is not a soft k-int-ideal over $\mathcal{Z}^+$.

Theorem 3.9. *Let (ϑ, S) be a soft left(right) k-int-ideal over U and (χ, R) be a soft left(right) k-int-ideal over U. Then, $(\vartheta \wedge \chi, S \times R)$ is a soft left(right) k-int-ideal over U.*

Proof. Let $(a_1, a_2), (b_1, b_2), (c_1, c_2) \in S \times R$ such that $(a_1, a_2) + (b_1, b_2) = (c_1, c_2)$. Then,

$$\begin{array}{rcl} (\vartheta \wedge \chi)(a_1, a_2) & = & \vartheta(a_1) \cap \chi(a_2) \\ & \supseteq & [\vartheta(b_1) \cap \vartheta(c_1)] \cap [\chi(b_2) \cap \chi(c_2)] \\ & = & [\vartheta(b_1) \cap \chi(b_2)] \cap [\vartheta(c_1) \cap \chi(c_2)] \\ & = & (\vartheta \wedge \chi)(b_1, b_2) \cap (\vartheta \wedge \chi)(c_1, c_2). \end{array}$$

Thus, $(\vartheta \wedge \chi, S \times R)$ is a soft left k-int-ideal over U. □

Remark 3.10. The following example shows that $(\vartheta \vee \chi, S \times R)$ is not a soft k-int-ideal over U.

Example 3.11. Suppose that set of the positive integers $\mathcal{Z}^+$ is the universal set, $S = \{a, b, c, d\}$ and Z_4 are the subsets of set of parameters. The soft k-int-ideals (ϑ, S) and (χ, Z_4) over $\mathcal{Z}^+$ are defined by:

+	a	b	c
a	a	b	b
b	b	a	a
c	b	a	a

and

·	a	b	c
a	a	a	a
b	a	a	a
c	a	a	a

$\vartheta(a) = \{1, 2, 4, 7\}$, $\vartheta(b) = \{1, 2, 4, 7\}$, $\vartheta(c) = \{3, 4\}$ and $\chi(a) = \{b \in \mathcal{Z}^+ : ab \equiv 0(mod4)\}$ for all $a \in Z_4$. Then, $(\vartheta \vee \chi)(a, 2) \not\supseteq (\vartheta \vee \chi)(c, 2) \cap (\vartheta \vee \chi)(b, 0)$. Thus, $(\vartheta \vee \chi, S \times Z_4)$ is not a soft k-int-ideal over $\mathcal{Z}^+$.

Definition 3.12. Let (ϑ, S) and (χ, S) be two soft sets over U. Then $\vartheta \tilde{\circ_k} \chi$ is said to be a soft k-product, wif

$$(\vartheta \circ_k \chi)(a) = \begin{cases} \bigcap\{\vartheta(a_i) \cap \chi(b_i) : i = 1, 2\}, \text{ if } a + a_1 b_1 = a_2 b_2 \\ \varnothing, \text{ if } a \text{ cannot be predicated as } a + a_1 b_1 = a_2 b_2. \end{cases}$$

Theorem 3.13. *Let (ϑ, S) be a soft right k-int-ideal and (χ, S) be a soft left k-int-ideal over U. Then, $\vartheta \tilde{\circ_k} \chi \tilde{\subseteq} \vartheta \tilde{\cap} \chi$.*

Proof. Let $a \in S$. If $(\vartheta \circ_k \chi)(a) = \varnothing$, then it is a clear proof. Let $(\vartheta \circ_k \chi)(a) \neq \varnothing$. Since (ϑ, S) is a soft right k-int-ideal over U,

$$\begin{array}{rcl} \vartheta(a) & \supseteq & \vartheta(a_1 b_1) \cap \vartheta(a_2 b_2) \\ & \supseteq & \vartheta(a_1) \cap \vartheta(a_2) \end{array}$$

for each $a_i, b_i \in S, i = 1, 2$, satisfying $a + a_1 b_1 = a_2 b_2$. Similarly,

$$\chi(a) \supseteq \chi(b_1) \cap \chi(b_2).$$

Then,

$$\begin{array}{rcl} (\vartheta \circ_k \chi)(a) & = & \bigcap\{\vartheta(a_i) \cap \chi(b_i) : i = 1, 2, a + a_1 b_1 = a_2 b_2\} \\ & \subseteq & \vartheta(a) \cap \chi(a) \\ & = & (\vartheta \cap \chi)(a). \end{array}$$

Thus, $\vartheta \tilde{\circ_k} \chi \tilde{\subseteq} \vartheta \tilde{\cap} \chi$. □

Lemma 3.14. *Let (ϑ, S) be a soft set over U. (ϑ, S) is a soft left(right) int-ideal over U iff ϑ_α is a left(right) ideal of S, for any $\alpha \in P(U)$ such that $\vartheta_\alpha \neq \varnothing$.*

Proof. Assume that (ϑ, S) is a soft left int-ideal over U. Let $a, b \in \vartheta_\alpha$. Then, $\vartheta(a) \supseteq \alpha$ and $\vartheta(b) \supseteq \alpha$. It follows that

$$\begin{aligned} \vartheta(a+b) &\supseteq \vartheta(a) \cap \vartheta(b) \\ &\supseteq \alpha. \end{aligned}$$

Thus, $a + b \in \vartheta_\alpha$. Let $a \in \vartheta_\alpha$ and $s \in S$. It shows that $\vartheta(a) \supseteq \alpha$. So,

$$\begin{aligned} \vartheta(sa) &\supseteq \vartheta(a) \\ &\supseteq \alpha. \end{aligned}$$

Hence, $sa \in \vartheta_\alpha$. Therefore, ϑ_α is a left ideal of S, for any $\alpha \in P(U)$ such that $\vartheta_\alpha \neq \varnothing$.

Conversely, let ϑ_α be a left ideal of S, for any $\alpha \in P(U)$ such that $\vartheta_\alpha \neq \varnothing$. Let $a, b \in \vartheta_\beta$ such that $\beta = \vartheta(a) \cap \vartheta(b)$, for each $a, b \in S$. Then, $a + b \in \vartheta_\beta$. Thus,

$$\begin{aligned} \vartheta(a+b) &\supseteq \beta \\ &= \vartheta(a) \cap \vartheta(b). \end{aligned}$$

Also, for $a \in \vartheta_\gamma$ such that $\gamma = \vartheta(a)$, we obtain $sa \in \vartheta_\gamma$, for each $s \in S$. So, $\vartheta(sa) \supseteq \vartheta(a)$. Hence, (ϑ, S) is a soft left int-ideal over U. □

Theorem 3.15. *Let (ϑ, S) be a soft set over U. (ϑ, S) is a soft left(right) k-int-ideal over U iff ϑ_α is a left(right) k-ideal of S for any $\alpha \in P(U)$ such that $\vartheta_\alpha \neq \varnothing$.*

Proof. Following Lemma 3.14, it was proved that a soft set (ϑ, S) is a soft left int-ideal iff ϑ_α is a left ideal of S, for any $\alpha \in P(U)$ such that $\vartheta_\alpha \neq \varnothing$. Suppose that (ϑ, S) is a soft left k-int ideal over U. Let $a, k \in \vartheta_\alpha$, $s \in S$, $s + a = k$. Since $a, k \in \vartheta_\alpha$, we have $\vartheta(a) \supseteq \alpha$, $\vartheta(k) \supseteq \alpha$. Also, $\vartheta(s) \supseteq \vartheta(a) \cap \vartheta(k)$. Then, $\vartheta(s) \supseteq \alpha$, and so $s \in \vartheta_\alpha$. Thus, ϑ_α is a left k-ideal of S.

Conversely, let ϑ_α be a left k-ideal of S, for any $\alpha \in P(U)$ such that $\vartheta_\alpha \neq \varnothing$. Let $a, s, k \in S$ such that $a + s = k$. Suppose that $\vartheta(s) = \alpha_1$, $\vartheta(k) = \alpha_2$ ($\alpha_i \in P(U)$). Let $\alpha_1 \cap \alpha_2 = \alpha$. Then, $s \in \vartheta_\alpha$ and $k \in \vartheta_\alpha$. Since ϑ_α is a left k-ideal of S, we have $a \in \vartheta_\alpha$, i.e., $\vartheta(a) \supseteq \vartheta(s) \cap \vartheta(k)$. Thus, (ϑ, S) is a soft left k-int-ideal over U. □

Lemma 3.16. *Let ψ be a function from A to B and let (ϑ, A) be a soft set over U. For each $\varnothing \neq \alpha \in P(U)$,*

$$(\psi(\vartheta))_\alpha = \bigcap \{\psi(\vartheta_{\alpha \setminus \beta}) : \varnothing \subset \beta \subset \alpha\}.$$

Proof. Let $\varnothing \neq \alpha \in P(U)$. If $b \in (\psi(\vartheta))_\alpha$, then

$$\alpha \subseteq \psi(\vartheta)(b) = \bigcup \{\vartheta(c) : c \in A, \psi(c) = b\}.$$

This means that there exists $a_0 \in \psi^{-1}(b)$ such that $\vartheta(a_0) \supset \alpha \setminus \beta$, for all $\beta \in P(U)$ with $\varnothing \subset \beta \subset \alpha$. Thus, $b = \psi(a_0) \in \psi(\vartheta_{\alpha \setminus \beta})$. So, $b \in \bigcap \{\psi(\vartheta_{\alpha \setminus \beta}) : \varnothing \subset \beta \subset \alpha\}$.

Conversely, let $b \in \bigcap \{\psi(\vartheta_{\alpha \setminus \beta}) : \varnothing \subset \beta \subset \alpha\}$. Then, $b \in \psi(\vartheta_{\alpha \setminus \beta})$, for all $\beta \in P(U)$ with $\varnothing \subset \beta \subset \alpha$, which implies that there exists $a_0 \in \vartheta_{\alpha \setminus \beta}$ such that $b = \psi(a_0)$. It follows that $\vartheta(a_0) \supseteq \alpha \setminus \beta$ and $a_0 \in \psi^{-1}(b)$. Thus

$$(\psi(\vartheta))(b) = \bigcup \{\vartheta(c) : c \in \vartheta^{-1}(b)\} \supseteq \bigcup \{\alpha \setminus \beta : \varnothing \subset \beta \subset \alpha\} = \alpha.$$

So, $b \in (\psi(\vartheta))_\alpha$. □

Theorem 3.17. *Let ψ be a surjective homomorphism from R to S. Let (ϑ, R) be a soft left(right) k-int-ideal over U. Then $(\psi(\vartheta, R), S)$ is a soft left(right) k-int-ideal over U.*

Proof. We show that each nonempty α-inclusion of $(\psi(\vartheta, R), S)$ is a left(right) k-ideal of S. Let $(\psi(\vartheta, R))_\alpha$ be a nonempty α-inclusion of $(\psi(\vartheta, R), S)$, for all $\alpha \in P(U)$.

If $\alpha = \varnothing$, then $(\psi(\vartheta, R))_\alpha = S$.

Suppose that $\alpha \neq \varnothing$. From Lemma 3.16, $(\psi(\vartheta, R))_\alpha = \bigcap\{\psi(\vartheta_{\alpha\setminus\beta}) : \varnothing \subset \beta \subset \alpha\}$. Then, $\psi(\vartheta_{\alpha\setminus\beta})$ is a nonempty, for all $\varnothing \subset \beta \subset \alpha$. Thus, $\vartheta_{\alpha\setminus\beta}$ is a nonempty α-inclusion of $(\psi(\vartheta, R), S)$, for each $\varnothing \subset \beta \subset \alpha$. Since (ϑ, R) is a soft left(right) k-int-ideal over U, it is shown from Proposition 3.14, that $\vartheta_{\alpha\setminus\beta}$ is a left(right) k-ideal of R. Since ψ is a surjective homomorphism, $\psi(\vartheta_{\alpha\setminus\beta})$ is a left(right) k-ideal of S. So, $(\psi(\vartheta, R))_\alpha$ being an intersection of a family of left(right) k-ideals is also a left(right) k-ideal of S. □

Theorem 3.18. *Let ψ be a surjective homomorphism from R to S. Let (ϑ, S) be a soft left(right) k-int-ideal over U. Then $(\psi^{-1}(\vartheta), R)$ is a soft left(right) k-int-ideal over U.*

Proof. For any $a, b, c \in S$ such that $a + b = c(\psi(a) + \psi(b) = \psi(c))$,

$$\begin{aligned} \psi^{-1}(\vartheta)(a) &= \vartheta(\psi(a)) \\ &\supseteq \vartheta(\psi(b)) \cap \vartheta(\psi(c)) \\ &= \psi^{-1}(\vartheta)(b) \cap \psi^{-1}(\vartheta)(c). \end{aligned}$$

Then $(\psi^{-1}(\vartheta), R)$ is a soft left(right) k-int-ideal over U. □

4. Conclusions

In this study, we defined the notion of soft k-int-ideal and investigated their related properties. We then focused on the concept of soft k-product and obtain related properties. To extend our work, for further research someone can define soft h-int-ideal of a hemiring and study on their properties.

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