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Soft single point space and soft metrizable

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Abstract. Firstly, we introduced soft metric space which is defined over an initial universe with fixed set of parameter. And we gave some basic properties about it. Then we introduced soft metrizable. Hereafter we sayed that soft discrete space is soft non-metrizable while soft single point space is soft metrizable. Finally, we indicated some properties of soft metrizable. For example we showed that every soft metrizable space is soft n-T_4.

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1. Introduction

None mathematical tools can successfully deal with the several kinds of uncertainties in complicated problems in engineerig, economics, environment, sociology, medical science, etc, so Molodtsov [11] introduced the concept of a soft set in order to solve these problems in 1999. However, there are some theories such as theory of probability, theory of fuzzy sets [16], theory of intuitionistic fuzzy sets [1], theory of vague sets [4], theory of interval mathematics [6] and the theory of rough sets [12], which can be taken into account as mathematical tools for dealing with uncertainties. But these theories have their own difficulties. Maji et al. [9] introduced a few operators for soft set theory and made a more detailed theoretical study of the soft set theory. Recently, study on the soft set theory and its applications in different fields has been making progress rapidly [2, 13, 15, 8, 7]. Shabir and Naz [14] introduced the concept of soft topological spaces which are defined over an inital universe with fixed set of parameter. They indicated that a soft topological space gives a parameterized family of topological spaces and introduced the concept of soft open sets, soft closed sets, soft interior point, soft closure, soft discrete topological spaces and soft separation axioms. They indicated that if a soft topological space

(X, τ, E) is a soft T_i space then (X, τ, E) is a soft T_{i-1} space for $i = 1, 2$. In this case, Won Keun Min indicated that if a soft topological space (X, τ, E) is a soft T_3 space then (X, τ, E) is a soft T_2 space. Göçür and Kopuzlu [5] showed that if a soft topological space (X, τ, E) is a soft T_4 space, then (X, τ, E) may not be a soft T_2 space (also may not be a soft T_3 space from Theorem 2.26) . In this case, They described a new soft separation axiom which is called soft $n - T_4$ space. Then they indicated that if a soft topological space (X, τ, E) is a soft $n - T_4$ space, then (X, τ, E) is a soft T_3 space. Finally, they showed that a soft discrete topological space (X, τ, E) may not be a soft T_3 space. And so they introduced that soft single point space, which is also soft subspace of soft discrete space. Then they indicated that any soft single point space is soft n-T_4 in [5].

In this paper, we introduce soft metric space which is defined over an initial universe set with fixed set of parameter. And we give some basic properties about it. And then we introduce soft metrizable. Hereafter we say that soft discrete space is soft non-metrizable while soft single point space is soft metrizable. Finally, we indicate some properties of soft metrizable. For example we showed that every soft metrizable space is soft n-T_4.

2. Preliminaries

Definition 2.1 ([11])**.** Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parametrized family of subsets of the universe U. For $e \in A, F(e)$ may be considered as the set of $e-$approximate elements of the soft set (F, A). Clearly, a soft set is not a set

Definition 2.2 ([9])**.** For two soft sets (F, A) and (G, B) over a common universe U, (F, A) is a soft subset of (G, B), denoted by $(F, A)\widetilde{\subseteq}(G, B)$, if $A \subset B$ and $e \in A$, $F(e) \subseteq G(e)$. (F, A) is said to be a soft superset of (G, B), if (G, B) is a soft subset of (F, A), $(F, A)\widetilde{\supseteq}(G, B)$.

Definition 2.3 ([9])**.** Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4 ([9])**.** A soft set (F, A) over U is said to be a NULL soft set denoted by $\tilde{\varnothing}$ if for all $e \in A$, $F(e) = \varnothing$ (null set).

Definition 2.5 ([9])**.** A soft set (F, A) over U is said to be an absolute soft set denoted by $\tilde{A}$ if for all $e \in A, F(e) = U$. Clearly $\tilde{A}^c = \widetilde{\varnothing}$ and $\tilde{\varnothing}^c = \tilde{A}$

Definition 2.6 ([9])**.** The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A)\widetilde{\cup}(G, B) = (H, C)$.

Definition 2.7 ([3])**.** The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted $(F, A)\widetilde{\cap}(G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.8 ([14])**.** Let (F, E) be a soft set over X and $x \in X$. We say that $x\widetilde{\in}(F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$. Note that for any $x \in X$, $x\widetilde{\notin}(F, E)$,if $x \notin F(\alpha)$ for some $\alpha \in E$

Definition 2.9 ([14])**.** Let $x \in X$. Then (x, E) denotes the soft set over X for which $x(e) = \{x\}$, for all $e \in E$.

Definition 2.10 ([14])**.** Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X, if it satisfies the following axioms:

(i) $\widetilde{\varnothing}, \widetilde{X}$ belong to τ.
(ii) the union of any number of soft sets in τ belongs to τ,
(iii) the intersection of any two soft sets in τ belongs to τ.
The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.11 ([14])**.** Let (X, τ, E) be a soft space over X. Then the members of τ are said to be soft open sets in X.

Definition 2.12 ([14])**.** Let X be an initial universe set, E be the set of parameters and τ be the collection of all soft sets which can be defined over X. Then τ is called the soft discrete topology on X and (X, τ, E) is said to be a soft discrete space over X.

Proposition 2.13 ([14])**.** *Let (X, τ, E) be a soft space over X. Then the collection $\tau_\alpha = \{F(\alpha)|(F, E) \in \tau\}$ for each $\alpha \in E$, defines a topology on X.*

Definition 2.14 ([14])**.** Let (X, τ, E) be a soft space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)'$ belongs to τ

Proposition 2.15 ([14])**.** *Let (X, τ, E) be a soft space over X. Then*

(1) $\tilde{\varnothing}$, $\tilde{X}$ are closed soft sets over X,
(2) the intersection of any number of soft closed sets is a soft closed set over X,
(3) the union of any two soft closed sets is a soft closed set over X.

Theorem 2.16. [14] *Let (Y, τ_Y, E) be a soft subspace of soft topological space (X, τ, E) and (F, E) be a soft set over X. Then*

(1) (F, E) is soft open in Y if and only if $(F, E) = \widetilde{Y}\widetilde{\cap}(G, E)$ for some $(G, E) \in \tau$,

(2) (F, E) is soft closed in Y if and only if $(F, E) = \widetilde{Y}\widetilde{\cap}(G, E)$ for some soft closed set (G, E) in X.

Definition 2.17 ([14])**.** Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$. If there exist soft open sets (F, E) and (G, E) such that $x\widetilde{\in}(F, E)$, $y\widetilde{\notin}(F, E)$ or $y\widetilde{\in}(G, E)$, $x\widetilde{\notin}(G, E)$, then (X, τ, E) is called a soft T_0 space.

Definition 2.18 ([14])**.** Let (X, τ, E) be a soft topological space over X and $x, y \in X$ such that $x \neq y$. If there exist soft open sets (F, E) and (G, E) such that $x\widetilde{\in}(F, E)$, $y\widetilde{\notin}(F, E)$ and $y\widetilde{\in}(G, E)$, $x\widetilde{\notin}(G, E)$, then (X, τ, E) is called a soft T_1 space.

Definition 2.19 ([14]). Let (X, τ, E) be a soft toplogical space over X and $x, y \in X$ such that $x \neq y$. If there exist soft open sets (F, E) and (G, E) such that $x \widetilde{\in} (F, E)$, $y \widetilde{\in} (G, E)$ and $(F, E) \widetilde{\cap} (G, E) = \widetilde{\varnothing}$, then (X, τ, E) is called a soft T_2 space.

Definition 2.20 ([14]). Let (X, τ, E) be a soft toplogical space over X, (G, E) be a soft closed set in X and $x \in X$ such that $x \widetilde{\notin} (G, E)$. If there exist soft open sets (F_1, E) and (F_2, E) such that $x \widetilde{\in} (F_1, E)$, $(G, E) \widetilde{\subset} (F_2, E)$ and $(F_1, E) \widetilde{\cap} (F_2, E) = \widetilde{\varnothing}$, then (X, τ, E) is called a soft regular space.

Definition 2.21 ([14]). Let (X, τ, E) be a soft topological space over X. Then (X, τ, E) is said to be a soft T_3-space, if it is soft regular and soft T_1-space.

Definition 2.22 ([14]). Let (X, τ, E) be a soft topological space over X, (F, E) and (G, E) soft closed sets such that $(F, E) \widetilde{\cap} (G, E) = \tilde{\varnothing}$. If there exist soft open sets (F_1, E) and (F_2, E) such that $(F, E) \tilde{\subset} (F_1, E)$, $(G, E) \tilde{\subset} (F_2, E)$ and $(F_1, E) \widetilde{\cap} (F_2, E) = \tilde{\varnothing}$, then (X, τ, E) is called a soft normal space.

Definition 2.23 ([14]). Let (X, τ, E) be a soft topological space over X. Then (X, τ, E) is said to be a soft T_4 space, if it is soft normal and soft T_1 space.

Theorem 2.24 ([14]). *If (X, τ, E) is a soft T_1-space, then (X, τ, E) is a soft T_0 space.*

Theorem 2.25 ([14]). *If (X, τ, E) is a soft T_2-space then (X, τ, E) is a soft T_1 space.*

Theorem 2.26 ([10]). *A soft T_3 space is soft T_2.*

Remark 2.27 ([5], Example 3.12). Let (X, τ, E) be a soft discrete space over X. Then (X, τ, E) is not a soft T_3 space.

Theorem 2.28 ([5]). *Let (X, τ, E) be a soft discrete space over X. Then (X, τ, E) is a soft T_2 space.*

Remark 2.29 ([5], Remark 3.18). A soft T_4 space need not be a soft T_3 space.

Remark 2.30 ([5]). A soft T_4 space need not be a soft T_2 space.

Definition 2.31 ([5]). Let (X, τ, E) be a soft topological space over X and $x, y \in X$. Let (F, E) and (G, E) soft closed sets such that $x \widetilde{\in} (F, E)$ and $(F, E) \widetilde{\cap} (G, E) = \tilde{\varnothing}$. If there exist soft open sets (F_1, E) and (F_2, E) such that $y \widetilde{\in} (F_2, E)$, $(F, E) \tilde{\subset} (F_1, E)$, $(G, E) \tilde{\subset} (F_2, E)$ and $(F_1, E) \widetilde{\cap} (F_2, E) = \tilde{\varnothing}$, then (X, τ, E) is a soft n-normal space.

Definition 2.32 ([5]). Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is a soft n-normal space and soft T_1 space, then (X, τ, E) is a soft $n - T_4$ space.

Remark 2.33 ([5], Example 3.24). Let $(X,\ \tau,\ E)$ be a soft discrete space over X. Then (X, τ, E) is not a soft n-T_4 space.

Theorem 2.34 ([5]). *Soft $n - T_4$ space is soft T_3 space.*

Corollary 2.35 ([5]). *Soft $n - T_4$ space $\Longrightarrow$ soft T_3 space. $\Longrightarrow$ Soft T_2 space $\Longrightarrow$ soft T_1 space $\Longrightarrow$ soft T_0 space.*

2.1. **Soft single point space.**

Definition 2.36. Let X be an initial universe set, E be the set of parameters, $x \in X$ and A be a subset of X. Let (A, E) be defined as $A(e) = A$, for all $e \in E$. Then $\tau = \{(A, E) | \forall A \subset X\}$ is a soft topology over X. In this case, τ is called soft single point topology over X and (X, τ, E) is said to be a soft single point space over X.

Example 2.37. Let $X = \{x, y, z\}$, $E = \{e_1, e_2\}$ and

$$\tau = \{\widetilde{\varnothing}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\},$$

where

$$\begin{array}{ll} F_1(e_1) = \{x\}, & F_1(e_2) = \{x\}, \\ F_2(e_1) = \{y\}, & F_2(e_2) = \{y\}, \\ F_3(e_1) = \{z\}, & F_3(e_2) = \{z\}, \\ F_4(e_1) = \{x, y\}, & F_4(e_2) = \{x, y\}, \\ F_5(e_1) = \{x, z\}, & F_5(e_2) = \{x, z\}, \\ F_6(e_1) = \{y, z\}, & F_6(e_2) = \{y, z\}. \end{array}$$

Then (X, τ, E) is a soft single point space over X.

Theorem 2.38. *Let X be an initial universe set, E be the set of parameters. If (X, τ, E) is a soft single point space, then each soft element of (X, τ, E) is both soft open and soft closed set.*

Proof. Let X be an initial universe set, E be the set of parameters and (X, τ, E) be a soft single point space. Let (A, E) be defined as $A(e) = A$, for all $e \in E$. $\tau = \{(A, E) | \forall A \subset X\}$ from Definition 2.36. Since $\tau' = \{(A, E)' | \forall A' \subset X\}$, $(A, E)'$ is a soft open set for all $A' \subset X$. Thus (A, E) is soft open and soft closed set in X for all $A \subset X$. □

Theorem 2.39. *Let X be an initial universe set, E be the set of parameters. If (X, τ, E) is a soft single point space, then (X, τ_e) is a discrete space for all $e \in E$.*

Proof. Let X be an initial universe set, E be the set of parameters and (X, τ, E) be a soft single point space. (A, E) is defined as $A(e) = A$, for all $e \in E$. Then $\tau = \{(A, E) | \forall A \subset X\}$is a soft topology over X from Definition 2.36. Here A is open set in (X, τ_e) for all $A \subset X$ and for all $e \in E$. Thus (X, τ_e) is a discrete space for all $e \in E$. □

Theorem 2.40. *Let X be an initial universe set, E be the set of parameters. Then soft single point space (X, τ_1, E) is soft subspace of soft discrete space (X, τ_2, E).*

Proof. It is obvious from Definition 2.12 and Definition 2.36. □

Example 2.41. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ and

$$\tau_1 = \{\widetilde{\varnothing}, \widetilde{X}, (F_1, E), (F_2, E)\},$$

where

$$\begin{array}{ll} F_1(e_1) = \{x\}, & F_1(e_2) = \{x\}, \\ F_2(e_1) = \{y\}, & F_2(e_2) = \{y\}. \end{array}$$

Then (X, τ_1, E) is a soft single point space over X.

Now let

$$\begin{aligned}\tau_2 \;=\; &\{\widetilde{\varnothing}, \widetilde{X}, (F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E),\\ &(F_8,E),(F_9,E),(F_{10},E),(F_{11},E),(F_{12},E),(F_{13},E),(F_{14},E)\},\end{aligned}$$

where

$$\begin{array}{ll}
F_1(e_1)=\{x\}, & F_1(e_2)=\{x\},\\
F_2(e_1)=\{x\}, & F_2(e_2)=\{\ \},\\
F_3(e_1)=\{\ \}, & F_3(e_2)=\{x\},\\
F_4(e_1)=\{y\}, & F_4(e_2)=\{y\},\\
F_5(e_1)=\{\ \}, & F_5(e_2)=\{y\},\\
F_6(e_1)=\{y\}, & F_6(e_2)=\{\ \},\\
F_7(e_1)=\{x\}, & F_7(e_2)=\{y\},\\
F_8(e_1)=\{y\}, & F_8(e_2)=\{x\},\\
F_9(e_1)=X, & F_9(e_2)=\{x\},\\
F_{10}(e_1)=\{x\}, & F_{10}(e_2)=X,\\
F_{11}(e_1)=\{y\}, & F_{11}(e_2)=X,\\
F_{12}(e_1)=X, & F_{12}(e_2)=\{y\},\\
F_{13}(e_1)=X, & F_{13}(e_2)=\{\ \},\\
F_{14}(e_1)=\{\ \}, & F_{14}(e_2)=X.
\end{array}$$

Then (X,τ_2,E) is a soft discrete space over X.

Note that (X,τ_1,E) is soft subspace of soft discrete space (X,τ_2,E).

Theorem 2.42. *Let (X,τ,E) be a soft single point space over X and $x,y\in X$. Then (X,τ,E) is a soft T_1 space over X.*

Proof. Let (X,τ,E) be a soft single point space over X and let $x,y\in X$ such that $x\neq y$. Then there exist soft open sets $(x,E),(y,E)$ such that $x\widetilde{\in}(x,E)\in\tau$, $y\widetilde{\notin}(x,E)$ and $y\widetilde{\in}(y,E)\in\tau$, $x\widetilde{\notin}(y,E)$. Thus (X,τ,E) is a soft T_1 space over X. □

Theorem 2.43. *Let (X,τ,E) be a soft single point space over X and $x,y\in X$. Then (X,τ,E) is a soft n-T_4 space over X.*

Proof. Let (X,τ,E) be a soft single point space over X and $x,y\in X$. And let (A,E) and (B,E) be soft closed sets such that $x\widetilde{\in}(A,E)$ and $(A,E)\widetilde{\cap}(B,E)=\widetilde{\varnothing}$. Then there exist soft open sets (A,E) and (B,E) such that $y\widetilde{\in}(B,E)$, $(A,E)\tilde{\subset}(A,E)$, $(B,E)\tilde{\subset}(B,E)$ and $(A,E)\widetilde{\cap}(B,E)=\tilde{\varnothing}$ from Theorem 2.38. Thus (X,τ,E) is a soft n-normal space. Also from theorem 2.42, (X,τ,E) is a soft T_1 space. So (X,τ,E) is a soft n-T_4 space over X, from Definition 2.32. □

3. Main results

3.1. **Soft metric and soft metrizable space.** In this section, let X be an initial universe set and E be the non-empty set of parameters. Let $\widetilde{X}$ be the absolute soft set, i.e., $F(e)=X$, for all $e\in E$, where $(F,E)=\widetilde{X}$. Let $\widetilde{\mathbb{R}}$ denote the set of all soft real numbers. We use notations $\widetilde{r}$ to denote soft real numbers such that $F(e)=r$, for all $e\in E$, where $(F,E)=\widetilde{r}$. For instance, $\widetilde{0}$ is the soft real number where $F(e)=0$, for all $e\in E$, where $(F,E)=(0,E)$, for $0\in\mathbb{R}$. Also for shortly, we use $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z},\widetilde{a},\widetilde{b}$ instead of (x,E), (y,E), $(z,E),(a,E),(b,E)$ respectively for all $x,y,z,a,b\in X$ and for all $e\in E$.

Definition 3.1. A mapping $d : \widetilde{X} \times \widetilde{X} \to \widetilde{\mathbb{R}}$ is said to be a soft metric on the soft set $\widetilde{X}$ if d satisfies the following conditions:

(i) $d(\widetilde{x}, \widetilde{y}) \geq \widetilde{0}$, for all $x, y \in X$,

(ii) $d(\widetilde{x}, \widetilde{y}) = \widetilde{0}$ if and only if $\widetilde{x} = \widetilde{y}$,

(iii) $d(\widetilde{x}, \widetilde{y}) = d(\widetilde{y}, \widetilde{x})$ for all $x, y \in X$,

(iv) $d(\widetilde{x}, \widetilde{z}) \leq d(\widetilde{x}, \widetilde{y}) + d(\widetilde{y}, \widetilde{z})$, for all $x, y, z \in X$.

The soft set $\widetilde{X}$ with a soft metric d on $\widetilde{X}$ is called a soft metric space and denoted by $(\widetilde{X}, d, E)$.

Definition 3.2. Let $(\widetilde{X}, d, E)$ be a soft metric space and $\widetilde{r}$ be a non-negative soft real number. Then for any $a \in X$, the soft open ball with centre $\widetilde{a}$ and radius $\widetilde{r}$ is defined, denoted by $B(\widetilde{a}, \widetilde{r})$, as follows:

$$B(\widetilde{a}, \widetilde{r}) = \{x \in X; d(\widetilde{x}, \widetilde{a}) < \widetilde{r}\}.$$

Definition 3.3. Let $(\widetilde{X}, d, E)$ be a soft metric space and $\widetilde{r}$ be a non-negative soft real number. Then for any $a \in X$, by a soft closed ball with centre $\widetilde{a}$ and radius $\widetilde{r}$ satisfy $d(\widetilde{x}, \widetilde{a}) \leq \widetilde{r}$, the soft closed ball with centre $\widetilde{a}$ and radius $\widetilde{r}$ is defined, denoted by $B[\widetilde{a}, \widetilde{r}]$, as follows:

$$B[\widetilde{a}, \widetilde{r}] = \{x \in X; d(\widetilde{x}, \widetilde{a}) \leq \widetilde{r}\}.$$

Definition 3.4. A soft topological space (X, τ, E) is said to be a soft metrizable, if a soft metric d can be defined on $\widetilde{X}$ such that the soft topology induced by d is τ. Otherwise, the soft space X is called soft non-metrizable.

Example 3.5. Let (X, τ, E) be a soft single point space over X. Let $d : \widetilde{X} \times \widetilde{X} \to \widetilde{\mathbb{R}}$ be defined as follows:

$$d(\widetilde{x}, \widetilde{y}) = \begin{cases} \widetilde{0}, & \widetilde{x} = \widetilde{y} \\ \widetilde{1}, & \widetilde{x} \neq \widetilde{y}, \end{cases}$$

for any $x, y \in X$. And for any $a \in X$, let $B(\widetilde{a}, \widetilde{r})$ be defined by

$$B(\widetilde{a}, \widetilde{r}) = \begin{cases} \widetilde{a}, & \widetilde{r} \leq \widetilde{1} \\ \widetilde{X}, & \widetilde{r} > \widetilde{1}. \end{cases}$$

Then clearly, d induce τ. Thus soft single point space (X, τ, E) is soft metrizable space.

Theorem 3.6. *Any soft metrizable space is soft T_1.*

Proof. Let X be a soft metrizable space with soft metric d. Let $a, b \in X$, $\widetilde{\epsilon}$ be a non-negative soft real number, $d(\widetilde{a}, \widetilde{b}) = \widetilde{\epsilon}$ and let (A, E) and (B, E) be soft open sets in (X, τ, E) such that $a \widetilde{\in} (A, E)$ and $b \widetilde{\in} (B, E)$, respectively. Then for each $a \widetilde{\in} (A, E)$, there exist the soft open ball $B(\widetilde{a}, \widetilde{\epsilon/3})$ such that $b \widetilde{\notin} B(\widetilde{a}, \widetilde{\epsilon/3})$.

Similarly, there exist the soft open ball $B(\widetilde{b}, \widetilde{\epsilon/3})$ such that $a \widetilde{\notin} B(\widetilde{a}, \widetilde{\epsilon/3})$. Thus X is soft T_1. □

Theorem 3.7. *Any soft metrizable space is soft n-normal.*

Proof. Let X be a soft metrizable space with soft metric d. Let (A, E) and (B, E) be disjoint soft closed soft subsets of X. For each $a\widetilde{\in}(A, E)$, let us choose $\widetilde{\epsilon_a}$, which is non-negative soft real number, so that the soft ball $B(\widetilde{a}, \widetilde{\epsilon_a})$ does not intersect (B, E).

Similarly, for each $b\widetilde{\in}(B, E)$, let us choose $\widetilde{\epsilon_b}$, which is non-negative soft real number, so that the soft ball $B(\widetilde{b}, \widetilde{\epsilon_b})$ does not intersect (A, E). Define

$$(U, E) = \widetilde{\bigcup_{a\widetilde{\in}(A,E)}} B(\widetilde{a}, \widetilde{\epsilon_a/2})$$

and

$$(V, E) = \widetilde{\bigcup_{b\widetilde{\in}(B,E)}} B(\widetilde{b}, \widetilde{\epsilon_b/2}).$$

Then (U, E) and (V, E) are soft open sets containing (A, E) and (B, E), respectively. Moreover $b\widetilde{\in}(V, E)$.

Now we assert they are disjoint. Assume that $z\widetilde{\in}(U, E)\widetilde{\cap}(V, E)$. Then

$$z\widetilde{\in}B(\widetilde{a}, \widetilde{\epsilon_a/2})\widetilde{\cap}B(\widetilde{b}, \widetilde{\epsilon_b/2}),$$

for some $a\widetilde{\in}(A, E)$ and $b\widetilde{\in}(B, E)$. Thus by the triangle inaquality,

$$d(\widetilde{a}, \widetilde{b}) < (\widetilde{\epsilon_a + \epsilon_b})/2.$$

If $\epsilon_a \leq \epsilon_b$, then $d(\widetilde{a}, \widetilde{b}) < \widetilde{\epsilon_b}$. Thus the soft ball $B(\widetilde{b}, \widetilde{\epsilon_b})$ contains $\widetilde{a}$.

If $\widetilde{\epsilon_b} \leq \widetilde{\epsilon_a}$, then $d(\widetilde{a}, \widetilde{b}) < \widetilde{\epsilon_a}$. Thus the soft ball $B(\widetilde{a}, \widetilde{\epsilon_a})$ contains $\widetilde{b}$.
Neither situation is possible. So (U, E) and (V, E) are disjoint. □

Theorem 3.8. *Any soft metrizable space is soft n-T_4.*

Proof. It is obvious that Definition 2.32, Theorem 3.6 and Theorem 3.7. □

Proposition 3.9. *Any soft discrete space is soft non-metrizable.*

Proof. It is obvious that Remark 2.33, Definition 2.32 and Theorem 3.7. □

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