Novel fuzzy based model on analysis of invasiveness due to dispersal related traits of plants

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Abstract. Invasive Alien Species (IAS) can be considered as a serious threat to the existence of the biodiversity as they alter physical, chemical and biological components of the environment. Invasive potential of species can be recognized by their biological traits. Therefore, it is very important if a model could be developed to measure the biological risk of plant species before introducing to a new environment. This work aims to incorporate dispersal related biological traits into a mathematical model to evaluate the risk of plant species. To build up the model, four factors of dispersal traits have been considered and grade of important weights of these traits towards invasiveness was assigned as explained in Chang’s extent analysis and Buckly’s methods (Column geometric mean method). The present model is found to be a better tracking system for identifying potential invaders compared to the conventional manually conducted risk assessment methods when incorporated with important weights as per Chang’s extent analysis.

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Keywords: Invasive Alien Species, Biological traits, Fuzzy Membership Functions (FMF), Grade of Important weights.

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1. Introduction

Invasive Alien Species (IAS) can be considered as a serious threat to the existence of the biodiversity of the environment as they alter physical, chemical and biological components of the environment. Therefore the spread of these species has been recognized as a global environmental problem and article 8(h) of the Biodiversity Convention highlights the measures to prevent the introduction of, control or eradicate those alien species which threaten ecosystems, habitats or species [3].

Risk assessment is a key tool to identify the potential Invasive Alien Species (IAS). Many countries in all over the world are now using this tool as a vital part of
a comprehensive prevention strategy \cite{6, 15}. These risk assessments are in the form of questionnaires based on risk factors of IAS with predefined answers. The outcome of the assessment will be the risk value of a particular IAS, which is the sum of scores that have been given to each question by the domain expert. Risk assessment, however, it is not a clear cut process and also context dependent. On the other hand, most of the risk factors which affect the invasiveness of species are accompanied with imprecision and uncertainty. Data for some risk factors have been gathered from the knowledge of experts in plant sciences due to the unavailability or lack of a proper mechanism to measure data \cite{13}. Therefore, it is very important to develop a mathematical model which incorporates uncertainty and imprecision of input data to evaluate the risk of IAS efficiently than in the conventional manually conducted risk assessments.

One of the major concerns in determining invasiveness of IAS is their biological traits \cite{12} Invasive plants usually possess higher ability adaptation, reproduction and dispersal, and thus make them establish in a great diversity of habitats. The seed dispersal has been identified as a key trait of invasiveness hence considered as one major factor in identifying the potential invasive plants. The efficiency of dispersal depends on the contribution of few direct and indirectly related plant traits such as annual seed rain, number of seeds in a fruit, seeds viability and dispersal strength. However, some other plant traits can also contribute to the efficiency of dispersal in minor scales. To the best of the author's knowledge, the article provide for the first time an evaluation on dispersal related risks in seed plants through a Fuzzy model to generate more quick and precise decisions compared to the conventional risk assessment methods manually conducted for IAS.

The present work is an effort to evaluate the invasive potential of plants with regard to their efficiency of dispersal using fuzzy approach in Mathematics. Fuzzy logic concepts provide opportunities to quantify or convert parameters into a measurable scale. Aggregation on those fuzzified parameters could be developed into a model to meet the situations described as vague or imprecise terms, or situations that are too complex or ill defined to be analyzed by conventional mathematical or statistical tools. This work aims to convert plant traits related to seed dispersal into a measurable scale using Fuzzy Membership Functions (FMF) and proposed a method to generate invasion risk of particular plant species based on its strengths to disperse. In this task, the fuzzy pairwise comparison matrix has been used to evaluate the grade of importance of each parameter in dispersal category. These weights have been determined by using Fuzzy Analytic Hierarchy Process (FAHP) \cite{5, 14}. In the FAHP calculations, two different methods have been adopted, namely Chang’s extent analysis \cite{2} and Buckly’s method (Column geometric mean method) \cite{Huang}. Here, two different models have been developed by incorporating importance weights and normalized fuzzy membership values of each dispersal trait. The difference between these two models is reflected by weighting method which has been used to determine the grade of importance weights of the model parameters using FAHP. The models have been validated by testing a set of well known invasive plant species and non invasive species in Sri Lanka.
2. Methodology

The data set of known 21 invasive alien species and four non invasive species was obtained from the invasive species specialist group of the Ministry of Environment and Renewable Resources, Sri Lanka. It contains single-valued observations of four dispersal related traits such as annual seed rain/$m^2$, viability of seeds, number of seeds per fruit and long distance dispersal strength and invasion risk scores which were used to conduct National Risk Assessment (NRA).

2.1. Fuzzy membership functions (FMF) for dispersal traits. In this subsection we focused on developing FMF for the four biological traits related to dispersal as mentioned above. These four traits were the parameters of the model which have been developed in this study. In order to develop membership functions the lower and upper boundary points have been determined for the risk factors by considering their behavior with regards to invasive potential [4, 7, 8, 9, 10, 11, 16]. When determining these boundary points the following assumptions have been made according to the experts’ suggestions. Assumptions:

- lower boundary point is the lowest possible value which has the minimum effect to the invasive potential of a plant species.
- upper boundary point is the extreme value which has the maximum effect to the invasive potential of a plant species. For future tasks, in order to compatible to any invasive plant other than in the database defined the upper boundary as an unrealistic value.
- the invasive potential of plant species will increase when the values of risk factors increase from lower boundary point to upper boundary point.

The functions between lower and upper boundary points have been predicted in order to compatible with the actual impacts of biological traits as below.

2.1.1. FMF $U_A(x)$ for Seeds per fruit ($SF$).

$$U_A(x) = \begin{cases} 
1 & \text{for } x \leq 1 \\
1 - 2[(x - 1)/1000]^2 & \text{for } 1 < x \leq 501 \\
2[(x - 1001)/1000]^2 & \text{for } 501 < x \leq 1001 \\
0 & \text{for } x > 1001.
\end{cases}$$

2.1.2. FMF $U_B(x)$ for annual seed production per $m^2$ ($ASR$).

$$U_B(x) = \begin{cases} 
2 \frac{(10000 - x)^2}{8 \times 10^8} + 0.75 & \text{for } 0 \leq x \leq 10000 \\
2 \frac{(100000 - x)^2}{5.4 \times 10^{10}} + 0.45 & \text{for } 10000 < x \leq 100000 \\
2 \frac{(10 \times 10^6 - x)^2}{4.356 \times 10^{14}} & \text{for } 100000 < x \leq 10 \times 10^6 \\
0 & \text{for } x > 10 \times 10^6.
\end{cases}$$
### Table 1. Linguistic scale for Importance

<table>
<thead>
<tr>
<th>Linguistic scale</th>
<th>Triangular Fuzzy Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutely more important</td>
<td>(5/2,3,7/2)</td>
</tr>
<tr>
<td>Very strongly more important</td>
<td>(2,5/2,3)</td>
</tr>
<tr>
<td>Strongly more important</td>
<td>(3/2,2,5/2)</td>
</tr>
<tr>
<td>Weakly more important</td>
<td>(1,3/2,2)</td>
</tr>
<tr>
<td>Equally important</td>
<td>(1/2,1,3/2)</td>
</tr>
<tr>
<td>Just equal</td>
<td>(1,1,1)</td>
</tr>
</tbody>
</table>

2.1.3. FMF $U_C(x)$ for viability of seeds in months ($VIA$).

$$U_C(x) = \begin{cases} 
1 & \text{for } x \leq 3 \\
1 - 2[(x - 3)^2/237606] & \text{for } 3 < x \leq 602 \\
2[(1200 - x)^2/1028572] & \text{for } 602 < x \leq 1200 \\
0 & \text{for } x > 1200.
\end{cases}$$

2.1.4. FMF $U_D(x)$ for Long distance dispersal strength ($LDD$).

$$U_D(x) = \begin{cases} 
1 - 2[x^2/160] & \text{for } 0 \leq x \leq 2 \\
0.95 - 2[(x - 2)^2/60] & \text{for } 2 < x \leq 5 \\
2[(10 - x)^2/77] & \text{for } 5 < x \leq 10.
\end{cases}$$

Data for the long distance dispersal strength of plant species were represented in the form of a ten point scale which represents point 0 as the lowest dispersal strength while point 10 as the highest dispersal strength.

2.2. Grade of importance weights of parameters.

2.2.1. Fuzzy pairwise comparison. In order to find the importance weights, the pairwise comparisons of four parameters related to dispersal was conducted through a questionnaire distributed among the related field of Plant Science. They indicated the grade of importance in a linguistic scale for pair-wise comparison using triangular fuzzy numbers (Table 1) [2].

2.2.2. Evaluating important weights. Steps for evaluating grade of important weights using FAHP adopted with Chang’s Extent analysis method and Buckly’s method were explained as below [2, 5].

Step 1. Evaluate the fuzzy reciprocal matrix

$$P = [\tilde{q}_{ij}],$$

where $\tilde{q}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, $l_{ij}$, $m_{ij}$ and $u_{ij}$ are the lower limit, peak and upper limit of the triangular fuzzy number. $\tilde{q}_{ij} = \frac{1}{q_{ji}} = \left(\frac{1}{u_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}}\right), \forall i, j = 1, 2, ..., n.$

Step 2. Aggregate the experts’ responds using geometric mean method.

$$\tilde{q}_{ij} = (\tilde{q}_{ij}^1 \otimes \tilde{q}_{ij}^2 \otimes ... \otimes \tilde{q}_{ij}^n)^{1/n},$$

where $\tilde{q}_{ij}$ was the triangular fuzzy number in the $i^{th}$ column and $j^{th}$ row of the fuzzy positive reciprocal matrix and $\tilde{q}_{ij}^n$ was the respond value of the $n^{th}$ expert.
Step 3. Calculate the fuzzy important weights.

Method I: Chang’s extent analysis [2].

\[
S_i = \sum_{j=1}^{m} M_{gi}^{j} \bigotimes \left( \sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right)^{-1},
\]

\[
\sum_{j=1}^{m} M_{gi}^{j} = \left[ \sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right],
\]

\[
\left( \sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^{j} \right)^{-1} = \left[ \frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right],
\]

where \( S_i \) is the \( i \)-th fuzzy weight, in matrix \( m \) and \( M_{gi}^{j} \) is the triangular fuzzy number calculated after comparing the questionnaires. After comparing each parameter, a minimum has been generated for each group as in Eq. 2.11.

\[ V(M \geq M_1, M_2, \ldots, M_k) = \min V(M \geq M_i), i = 1, 2, \ldots, k. \]

Assume that \( d(A_i) = \min V(S_t \geq S_k) \) for \( k = 1, 2, \ldots, n; k \neq i \). Then the weight vector is given by

\[ \hat{W} = (d(A_1), (d(A_2)), \ldots, (d(A_n)))^T, \]

where \( A_i, i = 1, 2, \ldots, n \).

Via normalization, the normalized weight vectors are

\[ W = (d(A_1), (d(A_2)), \ldots, (d(A_n)))^T. \]

Method II: Buckly’s method (Column geometric mean method [5]).

\[ \hat{w}_i = r_i \bigotimes (r_1 \bigoplus r_2 \bigoplus \ldots \bigoplus r_n)^{-1}, \]

\[ r_i = \left( \frac{1}{q_{ij}^1} \times \frac{1}{q_{ij}^2} \times \ldots \times \frac{1}{q_{ij}^n} \right)^{\frac{1}{n}}, \]

where \( \hat{w}_i \) is the fuzzy weight value of each column in the fuzzy positive reciprocal matrix and \( r_i \) is the geometric mean of the triangular fuzzy number.

Step 4. Defuzzification of fuzzy weights into non fuzzy values using Center of Gravity method [1].

2.3. Proposed method. Let \( X \) be the collection of species denoted generically by \( x \). Let \( U_A(x), U_B(x), U_C(x) \) and \( U_D(x) \) be the grades of membership of \( x \) with respect to the factors; number of seeds/fruit, annual seed rain/m², viability of seeds in months, long distance dispersal strength respectively. These grades of membership have been calculated from Eqs. 2.1 to 2.4. Let us denote \( w_i, i = 1, \ldots, 4 \) be the normalized grade of importance weights obtained from either Chang’s method or Buckly’s method for the factors number of seeds/fruit, annual seed rain/m², viability of seeds in months, long distance dispersal strength respectively. Therefore the proposed method for evaluating invasive potential (\( Inv(x) \)) of invasive species due to the dispersal related factor was:

\[ Inv(x) = w_1 U_A^p + w_2 U_B^p + w_3 U_C^p + w_4 U_D^p, x \in X. \]
Table 2. Grade of importance weights for Dispersal related factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>Weights (Chang’s method)</th>
<th>Weights (Buckly’s method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of seeds per fruit</td>
<td>0.2884</td>
<td>0.23924</td>
</tr>
<tr>
<td>Annual seed production per m²</td>
<td>0.2785</td>
<td>0.26698</td>
</tr>
<tr>
<td>Viability of seeds</td>
<td>0.2504</td>
<td>0.29315</td>
</tr>
<tr>
<td>Long distance dispersal strength</td>
<td>0.1827</td>
<td>0.20062</td>
</tr>
</tbody>
</table>

where \( p_j, j = 1, ..., 4 \) are the unknown weights for the grade of membership of each factor. Here \( U_C(x) \) stands for the grade of membership of the complement of a normalized fuzzy set for each factor. In the process of finding the values for \( p_j \) in Eq. 2.15, data of 19 invasive species which show invasiveness mainly through dispersal related factors has been used. At the beginning, the initial weights for \( p_j \)'s were changed until the model generates satisfactory risk values for the selected invasive plant species. In this task two different models which fitted well with two categories of plant species have been identified. Models for each plant category were given below:

Category I: A plant whose \( ASR \leq 20000 \) and \( VIA \leq 3\) yrs.
Here the model was given as,

\[
P(x) = w_1 U_A^1 + w_2 U_B^1 + w_3 U_C^1 + w_4 U_D^1, x \in X.
\]

Category II: A plant does not belong to category I.
In this case, the model could be written as,

\[
P(x) = w_1 U_A^1 + w_2 U_B^1 + w_3 U_C^1 + w_4 U_D^1, x \in X.
\]

Here we named the above model as Model I and Model II, respectively which incorporated weights obtained from Chang’s extent analysis method and Buckly’s method respectively. In the next section we provide results which have been obtained from two different weighting methods.

3. Results

3.1. Grade of important weights. The grade of important weights for each seed dispersal related factor which have been obtained using the column geometric mean method (Buckley’s method) and Chang’s extent analysis method are shown in Table 2.

3.2. Test results. Table 3 compares the risk values of 19 invasive plant species obtained from the proposed model in each grade of importance weighing method and NRA. Note that the data of four model parameters related to these species have been used as the test data in the development process of the model (section 2.3.)

3.3. Validation results. The model has been validated using two different plant groups. Note that the first group consists of some invasive alien species which show invasiveness mainly through invasive attributes other than seed dispersal attributes and the second group consists of non invasive plant species.

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### Table 3. Test results

<table>
<thead>
<tr>
<th>Invasive species</th>
<th>NRA score</th>
<th>Model I (Chang’s Method)</th>
<th>Model II (Buckly’s Method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternanthera philoxeroides</td>
<td>36</td>
<td>23</td>
<td>37</td>
</tr>
<tr>
<td>Clidemia hirta</td>
<td>71</td>
<td>69</td>
<td>73</td>
</tr>
<tr>
<td>Miconia calvescens</td>
<td>86</td>
<td>70</td>
<td>73</td>
</tr>
<tr>
<td>Alstonia macrophylla</td>
<td>50</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>Annona glabra</td>
<td>57</td>
<td>35</td>
<td>38</td>
</tr>
<tr>
<td>Clusia rosea</td>
<td>50</td>
<td>44</td>
<td>47</td>
</tr>
<tr>
<td>Dillenia suffructicosa</td>
<td>50</td>
<td>34</td>
<td>37</td>
</tr>
<tr>
<td>Ageratina riparia</td>
<td>43</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Mimosa invisa</td>
<td>86</td>
<td>66</td>
<td>70</td>
</tr>
<tr>
<td>Myroxylon balsamum</td>
<td>43</td>
<td>42</td>
<td>45</td>
</tr>
<tr>
<td>Tithonia diversiflora</td>
<td>43</td>
<td>27</td>
<td>27</td>
</tr>
<tr>
<td>Mikania micrantha</td>
<td>57</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Prosopis juliflora</td>
<td>71</td>
<td>70</td>
<td>74</td>
</tr>
<tr>
<td>Ulex europaeus</td>
<td>71</td>
<td>63</td>
<td>68</td>
</tr>
<tr>
<td>Mimosa pigna</td>
<td>86</td>
<td>67</td>
<td>72</td>
</tr>
<tr>
<td>Chromolaena odorata</td>
<td>64</td>
<td>64</td>
<td>68</td>
</tr>
<tr>
<td>Parthenium hysterophorus</td>
<td>57</td>
<td>41</td>
<td>41</td>
</tr>
<tr>
<td>Lantana camara</td>
<td>50</td>
<td>63</td>
<td>67</td>
</tr>
<tr>
<td>Imperata cylindrical</td>
<td>64</td>
<td>75</td>
<td>77</td>
</tr>
</tbody>
</table>

### Table 4. Validation results

<table>
<thead>
<tr>
<th>Category of species</th>
<th>Species</th>
<th>NRA Score</th>
<th>Model I (Chang’s method)</th>
<th>Model II (Buckly’s method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Invasive</td>
<td>Sphagneticola trilobata</td>
<td>50</td>
<td>41</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Cuscuta campestris</td>
<td>43</td>
<td>41</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Pueraria Montana</td>
<td>36</td>
<td>27</td>
<td>27</td>
</tr>
<tr>
<td>Non Invasive</td>
<td>Cassia fistula</td>
<td>36</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Cissus rotundifolia</td>
<td>36</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Hedychium gardnerianum</td>
<td>21</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Magnefera indica</td>
<td>36</td>
<td>18</td>
<td>19</td>
</tr>
</tbody>
</table>

### 3.4. Discussion.

Table 2 shows the grade of important weights of dispersal related factors using Chang’s and Buckly’s methods. One may see that the factor number of seeds per fruit takes the highest important weight among the factors and long distance dispersal strength takes the lowest in Chang’s method. In Buckly’s method viability of seeds factor takes highest important weight and long distance dispersal factor takes the lowest important weight. According to Table 3, it may clearly be seen that, most of the plant species’ risk scores obtained from the model using each weighting method were compatible with the NRA risk level. One may also see that some invasive species take same NRA score. But in reality the actual impact should
differ from species to species and the model may give a specific impact value to a particular species. For example, species *Prosopis juliflora* and *Ulex europaeus* take the same NRA score, but in the model it clearly discriminates their impact level.

The validation results in Table 4 showed that the species risk scores in each weighting method have been reduced to that of the NRA score. In reality we expect low NRA scores for these two plant groups, but some species take considerably high scores. But in the model one may see that it tries to keep the species scores behind their NRA scores. This model gives a better prediction using Chang’s weighting method compared to Buckly’s method.

4. Conclusion

In this paper, a model has been constructed for the first time to assess the risk of IAS using two different important weighting methods. We have used FAHP accompanied with Chang’s extent analysis method and Buckly’s methods to find the grade of important weights of the model parameters with respect to the invasiveness. The proposed model gives a better prediction of risks of invasive alien species if its invasion is dominated by seed dispersal relates factors. It is also worth mentioning that model with weights obtained from Chang’s extent analysis method has produced significant improved results in comparison to Buckley’s method and also it gives a better prediction compared to the National Risk Assessment method (NRA). The model needs to be modified by incorporating the risk factors other than dispersal, e.g. growth, ecology, establishment, management aspects etc to evaluate overall invasion risk. But the limited amount of available data on those factors set serious constraints to the evaluation of overall risk of invasive alien plants. The authors are investigating to extend this study with other parameters presently and hope to communicate the results in the future.

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