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Soft digraphs, operations and applications

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Abstract. In this paper, the notion of soft digraph has been introduced. The basic terminologies and operations of soft digraphs have been defined. Later matrix representation of a soft digraph has been shown. Finally applications of soft digraph in solving a decision making problem, entropy calculation, medical diagnosis problem have also been given at the end.

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1. Introduction

Nowadays, many fields like engineering, medical science, sociology, economics etc deal with the complexities of modeling with uncertain data. Researchers cannot always successfully use the classical methods due to the presence of uncertainties in these type of problems. Some well known theories viz. fuzzy set theory [24], probability theory, intutionistic fuzzy sets theory [2, 3], vague sets theory [7], theory of rough sets [22] can be considered as a mathematical tool for dealing with uncertainties. But still these theories have certain limitations. Therefore research is still going for finding better theories which can model the natural phenomena more realistically. Molodtsov [20]] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which traditional mathematical tools can not handle. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. Other authors like Maji, Roy and Biswas [14, 15] have further studied the theory of soft sets and used this theory to solve some decision making problems. They have also introduced the concept of fuzzy soft set and intutionistic fuzzy soft set [11, 12, 13], a more generalized concept, which is a combination of fuzzy set and soft set and have studied its properties. In 2009, Ali et al [1] has defined some new operations on soft sets. Research in soft set theory (SST) has been done in many areas like algebra, entropy calculation, applications([6, 9, 10, 23]) etc (see [17, 18], for example). In 2010, Majumdar and Samanta [18] has introduced the notion of generalized fuzzy soft set, where a degree has attached to each fuzzy set that corresponds a parameter. On the other hand, the digraph theory plays an important role in study of mathematics and other subjects like networking, image processing etc.

In this paper we have introduced a new representation of soft sets in light of digraph theory. In other words, the theory of soft digraphs has been introduced to study the properties and applications of soft sets using diagraph theory. The organization of the rest of the paper as follows: In Section 2, some preliminary definitions and results regarding soft set and digraph theory are given which will be used in the rest of the paper. Section 3 introduces the notion of soft digraph and studies some of its important properties. Section 4 is devoted to the application of soft digraphs in solving a decision making problem, soft entropy calculation and medical diagnosis problems.

2. Preliminaries

2.1. **Soft set theory.** In this subsection some definitions, results and examples regarding soft sets are given which will be used in the rest of this paper. The idea of soft sets was first given by Molodtsov [20]. Later Maji and Roy [15] have defined operations on soft set and studied their properties.

Definition 2.1 ([11])**.** Suppose U be an initial universal set and let E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a *soft set* over U if and only if F is a mapping given by $F : A \to P(U)$.

Throughout this paper, we consider U and E to be a finite set.

Example 2.2. As an illustration, consider the following example. Suppose a soft set (F, E) describes choice of places which the authors are going to visit with his family.

U = the set of places under consideration = $\{x_1, x_2, x_3, x_4, x_5\}$. E = {desert, forest, mountain, sea beach} = $\{e_1, e_2, e_3, e_4\}$. Let $F(e_1) = \{x_1, x_2\}$, $F(e_2) = \{x_1, x_2, x_3\}$, $F(e_3) = \{x_4\}$, $F(e_4) = \{x_2, x_5\}$.

So, the soft set (F, E) is a family $\{F(e_i); i = 1, \dots, 4\}$ of U.

Definition 2.3 ([15])**.** For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if
(i) $A \subseteq B$,
(ii) $\forall e \in A,\ F(e) \subseteq G(e)$.

Definition 2.4 ([15])**.** Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

In 2008, Majumdar and Samanta [16] have defined the complement of a soft set as follows:

Definition 2.5 ([16])**.** The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to P(U)$ is a mapping given by

$$F^c(\alpha) = U - F(\alpha), \forall \alpha \in A.$$

Definition 2.6 ([15]). A soft set (F, A) over U is said to be null soft set denoted by $\widetilde{\Phi}$, if $\forall e \in A, F(e) = \phi$.

Definition 2.7 ([15]). A soft set (F, A) over U is said to be absolute soft set denoted by $\widetilde{A}$, if $\forall e \in A, F(e) = U$.

Definition 2.8 ([15]). Union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C) , where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write, $(H, C) = (F, A)\widetilde{\bigcup}(G, B)$.

Definition 2.9 ([15]). Intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cap B$, and $e \in C$, $H(e) = F(e) \cap G(e)$. We write $(H, C) = (F, A)\widetilde{\bigcap}(G, B)$.

Definition 2.10 ([15]). If (F, A) and (G, B) are two soft sets, then *(F,A) AND (G,B)* denoted by $(F, A) \wedge (G, B)$ is defined by

$$(F, A) \wedge (G, B) = (H, A \times B),$$

where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$, $\forall\ (\alpha, \beta) \in A \times B$.

Definition 2.11 ([15]). If (F, A) and (G, B) are two soft sets, then *(F,A) OR (G,B)* denoted by $(F, A) \vee (G, B)$ is defined by

$$(F, A) \vee (G, B) = (O, A \times B),$$

where $O(\alpha, \beta) = F(\alpha) \cup G(\beta)$, $\forall\ (\alpha, \beta) \in A \times B$.

Definition 2.12 ([16]). If (F, E) and (G, E) are two soft sets over an universal set U, the similarity measure between (F, E) and (G, E) is defined by,

$$S(F, G) = \frac{\sum_i F(e_i).G(e_i)}{\sum_i [F(e_i)^2 \vee G(e_i)^2]}.$$

Two soft sets (F, E) and (G, E) are called significantly similar if $S(F, G) > \frac{1}{2}$.

Definition 2.13 ([19]). A soft set (F, A) is said to be a deterministic soft set over U if the following holds:
(i) $\bigcup_{e \in A} F(e) = U$,
(ii) $F(e) \bigcap F(f) = \phi$, where $e, f \in A$.

Definition 2.14. Let (F, A) be any soft set. Then another soft set (F^*, A) is said to be equivalent to (F, A) if there exists a bijective mapping $\sigma : A \to A$ defined as: $\sigma(F_x) = F^*_x$, where $F_x = \{e : x \in F(e)\}$ and $F^*_x = \{\acute{e} : x \in F^*(e)\}$. Let $C(F)$ denote the collection of all soft sets which are equivalent with (F, A).

Definition 2.15 ([19]). Let $X^*(U)$ be the collection of all soft sets over U. A mapping $S : X^*(U) \to [0,1]$ is said to be soft set entropy or softness measure if S satisfies the following properties:

(S1) $S(\widetilde{\Phi}) = 1, S(\widetilde{A}) = 1$.
(S2) $S(F) = 0$, if F is deterministic soft set.
(S3) $S(F) \leq S(G)$, if $F(\neq \widetilde{\Phi}) \subseteq G$.
(S4) $S(F^*) = S(F)$, where $F^* \in C(F)$.

We have the following theorem in [19] about Soft Set Entropy.

Theorem 2.16 ([19]). *The function $S : X^*(U) \to [0,1]$ defined below is an entropy (or measure of softness) of a soft set:*

$$S(F) = \begin{cases} 1 - \frac{|U|}{\sum_{x\in U} |\{e : x \in F(e)\}|}, & \text{if } F \neq \widetilde{\Phi} \text{ or } \widetilde{A} \\ 1, \text{ if } F = \widetilde{\Phi} \text{ or } \widetilde{A}. \end{cases}$$

2.2. **Digraphs.** Most of the theory regarding graphs and digraphs can be found in any standard reference, for example, in [4] and [8]. A directed graph or digraph D is a pair (V, A) where V is a finite nonempty set of objects, called vertices, and A a (possibly empty) set of ordered pairs of vertices, called arcs or directed edges. We denote the vertex set and the arc set of D by V_D and A_D, respectively. Sometimes, we simply write $v \in D$ (resp. $(u, v) \in D$) to mean $v \in V_D$ (resp. $(u, v) \in A_D$). The order of D, denoted by $|D|$, is the number of vertices of D.

An arc $x = (u, u)$ in D is called a loop in D. The outdegree (resp. indegree) of a vertex v in D is the number of vertices of D adjacent from (resp. to) v. It is customary to represent a digraph by a diagram with nodes representing the vertices and directed line segments (arcs) representing the arcs of the digraph. If two arcs of D have the same end vertices then the arcs are called parallel arcs. A digraph with no parallel arcs is called a simple digraph, otherwise that digraph is called a multi digraph.

A digraph D_1 is a subdigraph of the digraph D if $V_{D_1} \subseteq V_D,\ A_{D_1} \subseteq A_D$. The complement of a simple digraph D is the simple digraph $\overline{D}$, where $V_{\overline{D}} = V_D$ and $(v, w) \in A_{\overline{D}}$ if and only if $(v, w) \notin A_D$.

Given two subdigraphs D_1 and D_2 of D, the union of $D_1 \cup D_2$ is the subdigraph of D with vertex set containing of all these vertices which are in either D_1 or D_2 (or both) and with arc set containing of all those arcs which are in either D_1 and D_2 (or both), i.e.,

$$V_{D_1\cup D_2} = V_{D_1} \cup V_{D_2},$$
$$A_{D_1\cup D_2} = A_{D_1} \cup A_{D_2}.$$

If D_1 and D_2 are two subdigraphs of D with at least one vertex in common then the intersection $D_1 \cap D_2$ is given by

$$V_{D_1\cap D_2} = V_{D_1} \cap V_{D_2},$$
$$A_{D_1\cap D_2} = A_{D_1} \cap A_{D_2}.$$

Let $D = (V_D, A_D)$ and $E = (V_E, A_E)$ be digraphs such that $|V_D| = n, |V_E| = m$. The cartesian product $D \times E = (V, A)$ of D and E is defined as $V = V_D \times V_E$ and

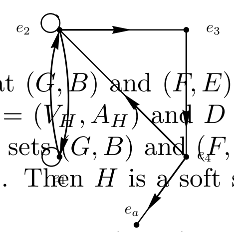
$A = \{((u_1, v_1), (u_2, v_2)) | ((u_1, u_2) \in A_D \text{ and } v_1 = v_2) \text{ or } (u_1 = u_2 \text{ and } (v_1, v_2) \in A_E)\}$.

3. Soft digraph

Recall that, $U = \{h_i; i \in \Delta\}$ be an initial universal set and let $E = \{e_i; i = 1, \ldots, n\}$ be a set of parameters. Let $P(U)$ denote the power set of U. Then a pair (F, E) is called a soft set over U if and only if F is a mapping given by $F : E \to P(U)$. Now we associate a digraph corresponding to every soft set, called a soft digraph, as follows:

Definition 3.1. Let (F, E) be a soft set over a universal set U. Here we introduce a dummy parameter e_a such that $F(e_a) = \phi$. Consider $D = (V_D, A_D)$ be any digraph with vertex set V_D and arc set A_D such that, $V_D = E \cup \{e_a\}$, and
$A_D = \{(e_i, e_j) : h_j \in F(e_i) \text{ and } j \leq |E|\} \cup \{(e_i, e_a) : h_j \in F(e_i) \text{ and } j > |E|\}$. Then D is called a soft digraph corresponding to the soft set (F, E). The vertex e_a is called the universal vertex for any soft digraph D.

Example 3.2. Consider the digraph D corresponding to the soft set (F, E) in Example 2.2. It is clear that D is a soft digraph by Definition 3.1.



Definition 3.3. Suppose that (G, B) and (F, E) are two soft sets over a common universal set U. Consider $H = (V_H, A_H)$ and $D = (V_D, A_D)$ be two soft digraphs corresponding to the two soft sets (G, B) and (F, E) respectively. Therefore, $V_H = B \cup \{e_a\}$ and $V_D = E \cup \{e_a\}$. Then H is a soft subdigraph of D if $V_H \subseteq V_D$ and $A_H \subseteq A_D$.

Note that $V_H \subseteq V_D$ implies $B \subseteq E$. Also $A_H \subseteq A_D$ implies $\forall e \in B, G(e) \subseteq F(e)$. Thus, if H is a soft subdigraph of D, then (G, B) is a soft subset of (F, E).

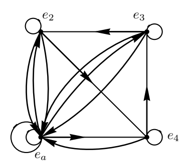
Example 3.4. Suppose $B = \{e_2, e_3, e_4\} \subseteq E$. Let (G, B) be soft set over the same universe $U = \{x_1, x_2, x_3, x_4, x_5\}$ in Example 2.2 as considered such that, $G(e_2) = \{x_3\}$, $G(e_3) = \{x_4\}$, $G(e_4) = \{x_2, x_5\}$. Therefore, $(G, B) \subseteq (F, E)$. Now we draw the digraph D_1 which is a soft subdigraph of D.

Definition 3.5. Suppose $H = (V_H, A_H)$ and $D = (V_D, A_D)$ be two soft digraphs corresponding to two equal soft sets (G, B) and (F, E) respectively. Then H and D are said to be the equal soft digraph if $V_H = V_D$ and $A_H = A_D$ respectively.

In this case also, we see that $B = E$, and $G(e) = F(e)$ $\forall e \in B = E$

Definition 3.6. Suppose (F, E) be a soft set over a universal set U and $D = (A_D, V_D)$ be a soft digraph corresponding to it. Consider a digraph $\overline{D}$ such that $V_D = V_{\overline{D}}, A_{\overline{D}} = \{(e_i, e_j) : h_j \notin F(e_i) \text{ and } j \leq |E|\} \cup \{(e_i, e_a) : h_j \notin F(e_i) \text{ and } j > |E|\}$. Then the soft digraph $\overline{D}$ is called a complement of the soft digraph D. It can be easily seen that the soft set corresponding to $\overline{D}$ is the complement soft set (F^c, E) of (F, E), where $F^c : E \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\alpha), \forall \alpha \in E$.

Example 3.7. We take the complement soft set (G^c, B) of the soft set (G, B) defined in the Example 3.4 over the universal set $U = \{x_1, x_2, x_3, x_4, x_5\}$. Then we have $G^c(e_2) = \{x_1, x_2, x_4, x_5\}$, $G^c(e_3) = \{x_1, x_2, x_3, x_5\}$, $G^c(e_4) = \{x_1, x_3, x_4\}$. Also by definition of soft digraph, $G(e_a) = \phi$ for the universal vertex e_a in D_1. Therefore, we have $G^c(e_a) = \{x_1, x_2, x_3, x_4, x_5\}$. Now we draw the complement digraph $\overline{D_1}$ of D_1. Clearly $\overline{D_1}$ is the soft digraph corresponding to the soft set (G^c, B).



$\overline{D_1}$

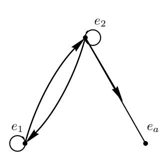
Definition 3.8. Suppose (F, A) and (G, B) are two soft set over a common universe U. Let H_1 and H_2 are two soft digraphs corresponding to the soft sets (F, A) and (G, B) respectively. Consider a digraph D by taking the union of two soft digraph H_1 and H_2 respectively as follows:

$V_D = V_{H_1} \cup V_{H_2} = A \cup B \cup \{e_a\}$, where e_a is the universal vertex for a soft digraph and $A_D = A_{H_1} \cup A_{H_2} \cup S$ where $S = \big\{\{(e_i, e_j)\} \backslash \{(e_i, e_a)\}$, if $h_j \in F(e_i)$, $e_i \in A \backslash B, e_j \in B \backslash A\big\}$ or $S = \big\{\{(e_i, e_j)\} \backslash \{(e_i, e_a)\}$, if $h_j \in G(e_i), e_i \in B \backslash A, e_j \in A \backslash B\big\}$ or $S = \phi$ in all other cases.

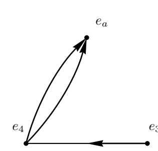
Then the soft digraph D is called the *union of two soft digraphs* H_1 and H_2. One can see that the soft digraph D corresponds the soft set which is the union of two soft sets (F, A) and (G, B) respectively.

Example 3.9. Consider (H, A) be a soft set over the universal set U in Example 2.2 as follows:

$A = \{e_1, e_2\}$, $H(e_1) = \{x_1, x_2\}, H(e_2) = \{x_1, x_2, x_3\}$. We draw a soft digraph D_2 specifying the soft set (H, A). Again we take the soft set (G, B) over the same



D_2



D_3

universal set U as follows:

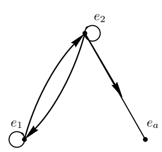
$B = \{e_3, e_4\}$, $G(e_3) = \{x_4\}, G(e_4) = \{x_2, x_5\}$. We draw the soft digraph D_3 which specifies the soft set (G, B) and take the union of two soft digraphs D_2 and D_3 respectively. Clearly D in Example 3.2 is the union of two soft digraph D_2 and D_3. Also D represents the soft set which is the union of soft sets (H, A) and (G, B) respectively.

Definition 3.10. Suppose (F, A) and (G, B) are two soft set over a common universe U. Let H_1 and H_2 are two soft digraphs corresponding to the soft set (F, A) and (G, B) respectively. Then the soft digraph D is called the intersection of two soft digraphs H_1 and H_2 if the following holds: $V_D = V_{H_1} \cap V_{H_2}$ and $A_D = \{(e_i, e_j) : (e_i, e_j) \in A_{H_1} \cap A_{H_2}\} \cup \{(e_i, e_a); h_j \in F(e_i) \cap G(e_i), e_i \in A \cap B \text{ and } e_j \notin A \cap B\}$.

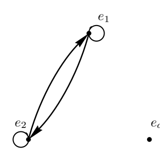
Also, one can verify that the soft digraph D represents the soft set which is the intersection of two soft sets (F, A) and (G, B) respectively.

Example 3.11. Consider (P, A) be a soft set over the universal set U in Example 2.2 as follows:

$A = \{e_1, e_2\}$, $P(e_1) = \{x_1, x_2\}, P(e_2) = \{x_1, x_2, x_3\}$. We draw a soft digraph D_4 specifying the soft set (P, A). Again we consider the soft set (Q, B) over the



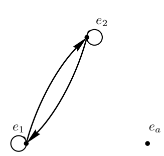
D_4



D_5

same universal set U where $B = \{e_1, e_2\}$, $Q(e_1) = \{x_1, x_2\}, Q(e_2) = \{x_1, x_2\}$. It can be easily seen that the soft digraph D_5 specifies the soft set (Q, B). We take the intersection soft digraph D_6 of two soft digraphs D_4 and D_5 respectively. It is clear that $D_6 = (V_{D_6}, A_{D_6})$ is the intersection of two soft digraph D_4 and D_5, where,

$$\begin{aligned} V_{D_6} &= \{e_1, e_2, e_a\}, \\ A_{D_6} &= \{(e_1, e_1), (e_1, e_2), (e_2, e_1), (e_2, e_2)\} \end{aligned}$$

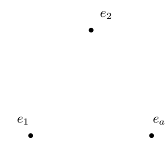


D_6

The soft digraph D_6 represents the intersection of two soft sets (P, A) and (Q, B) respectively.

Definition 3.12. Let (F, A) be a null soft set over a common universe U. Then, we have $\forall e \in A$, $F(e) = \phi$. A soft digraph $D = (V_D, A_D)$ is said to be a null soft digraph corresponding to the soft set (F, A) if the following holds: $V_D = A \cup \{e_a\}$, where e_a is the universal vertex for the soft digraph D, and $A_D = \phi$. A null digraph corresponding to a null soft set is a digraph without any arcs i.e. a point digraph.

Example 3.13. To see a null soft digraph, we consider the soft set (Q, C) over the universal set U in Example 2.2 where $B = \{e_1, e_2\}$, $Q(e_1) = \phi = Q(e_2)$. It can be easily seen that the soft digraph D_7 is a null soft digraph of the soft set (Q, C).

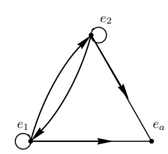


D_7

Definition 3.14. Let (F, A) be a absolute soft set over a common universe U. A soft absolute digraph $D = (V_D, A_D)$, where $V_D = A \cup \{e_a\}$, corresponding to a absolute soft set (F, A) is a digraph with all possible arcs between the vertices $\{e_1, e_2, \ldots, e_n\}$ in D.

For an absolute soft set (F, A), if $|A| \geq |U|$, then the soft subdigraph D corresponding to the soft set (F, A) is a complete digraph of order $|A|+1$ with an isolated vertex e_a.

Example 3.15. We consider the soft set (Q, E) over the universal set
$U = \{x_1, x_2, x_3\}$ where $E = \{e_1, e_2\}$, $Q(e_1) = U = Q(e_2)$. Clearly, the soft absolute digraph D_8
is a soft digraph of the absolute soft set (Q, E).



D_8

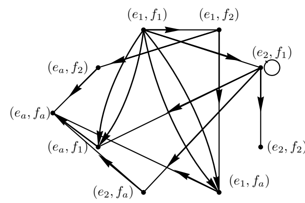
Definition 3.16. Suppose $H_1 = (V_{H_1}, A_{H_1})$ and $H_2 = (V_{H_2}, A_{H_2})$ be two soft digraphs corresponding two soft sets (F, A) and (G, B) over a universal set $U = \{h_1, h_2, \ldots, h_r\}$. Let $A = \{e_i; i \in I\}$ and $B = \{f_i; i \in I\}$ be two parameter sets of order m and n respectively. Consider a soft digraph $D = (V_D, A_D)$ of $H_1 \wedge H_2$ as follows:

$V_D = V_{H_1} \times V_{H_2} = \big((A \cup \{e_a\}) \times (B \cup \{f_a\})\big)$,

$A_D = \big\{\big((e_i, f_k), (e_j, f_l)\big) | \big(f_k = f_l \text{ and } h_j \in F(e_i), j < m\big)$
or $\big(e_i = e_j \text{ and } h_l \in G(f_k), l < n\big)\big\} \cup \big\{\big((e_i, f_k), (e_a, f_l)\big) | f_k = f_l$
and $h_j \in F(e_i), j > m\big\} \cup \big\{\big((e_i, f_k), (e_j, f_a)\big)$
$|e_i = e_j \text{ and } h_l \in G(f_k), l > n\big\}$.

Clearly, the soft digraph D corresponds the soft set $(F, A) \wedge (G, B)$.

Example 3.17. Consider two soft sets (F, A) and (G, B) over the universal set $U = \{x_1, x_2, \ldots, x_8\}$. We define $A = \{e_1, e_2\}$, $F(e_1) = \{x_1, x_2, x_3, x_5, x_7\}$ and $F(e_2) = \{x_1, x_2, x_3\}$. Also we define $B = \{f_1, f_2\}, G(f_1) = \{x_2, x_3, x_7\}$ and $G(f_2) = \{x_5, x_6, x_8\}$.



D_9

We now take the *AND* product of two soft sets (F, A) and (G, B) i.e. $(F, A) \wedge (G, B)$ as defined below:

$$\begin{aligned} H(e_1, f_1) &= \{x_2, x_3, x_7\}, \\ H(e_1, f_2) &= \{x_5\}, \\ H(e_2, f_1) &= \{x_2, x_3\}, \\ H(e_2, f_2) &= \widetilde{\phi} \end{aligned}$$

Now we draw the soft digraph D_9, where $V_{D_9} = \{(e_1, f_1), (e_1, f_2), (e_2, f_1), (e_2, f_2), (e_1, f_a), (e_2, f_a), (e_a, f_1), (e_a, f_2), (e_a, f_a)\}$ and

$A_{D_9} = \big\{\big((e_1, f_1), (e_1, f_2)\big), \big((e_1, f_1), (e_2, f_1)\big), \big((e_1, f_1), (e_a, f_1)\big), \big((e_1, f_1), (e_a, f_1)\big), \big((e_1, f_1), (e_1, f_a)\big), \big((e_1, f_1), (e_1, f_a)\big), \big((e_2, f_1), (e_2, f_a)\big), \big((e_2, f_1), (e_a, f_1)\big), \big((e_1, f_2), (e_1, f_a)\big), \big((e_1, f_2), (e_a, f_2)\big), \big((e_2, f_1), (e_2, f_1)\big), \big((e_2, f_1), (e_2, f_2)\big), \big((e_1, f_a), (e_a, f_a)\big), \big((e_2, f_a), (e_a, f_a)\big), \big((e_a, f_1), (e_a, f_a)\big), \big((e_a, f_2), (e_a, f_a)\big)\big\}$.

It is clear that the soft digraph D_9 corresponds the soft set $(F, A) \wedge (G, B)$.

3.1. **Matrix representation of a soft digraph.** Several researchers have solved decision making problems on soft sets by using the tabular representation ([5],[14],[21],[25]). In this paper, we will made a similar tabular representation of soft digraphs. We say that an $n \times n$ matrix $B = [b_{ij}]$ specifies a soft digraph D of order n if $D = (V_D, A_D)$ with $|V_D| = n$, and for $1 \leq i, j \leq n$, $(i, j) \in A_D$ if and only if the entry b_{ij} of B is specified. In this case, we write,

$$b_{ij} = \begin{cases} 1, & \text{if } (i, j) \in A_D \\ 0, & \text{if } (i, j) \notin A_D. \end{cases}$$

In case, D is a multi soft digraph then b_{ij}, the (i, j)-th entry of B is the number of parallel arcs between the vertex i and vertex j.

Example 3.18. Consider a matrix $B = [b_{ij}]$ specifying the soft digraph D of Example 3.2 as follows:

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Here, we take the first rows as $\{e_1, e_2, \ldots, e_a\}$ and columns as $\{e_1, e_2, \ldots, e_a\}$. It can be easily seen that B is the matrix representation of the soft digraph D.

4. Applications of soft digraph theory

Molodstov initiated [20] soft set theory with its applications in real life situations and different branches of mathematics. In this section, we will use the soft digraph theory in decision making problem,entropy calculation, medical diagnosis problems.

4.1. **A decision making problem.** In any decision making problem we first represent the problem into a soft set. Then corresponding to the soft set we draw its soft digraph. Then we select our choice parameters and represent the soft digraph to a matrix. Ultimately we take maximum or minimum of the column sums (C_i) of this matrix and take our decision. The algorithm is described below:

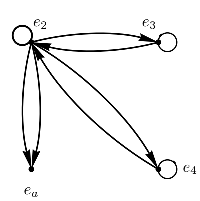
(1) Input the soft set (F, E).
(2) Input the set Q of choice of parameters of the author which is a subset of E.
(3) Draw the soft digraph D corresponding to the soft set (F, Q).
(4) Find out the matrix M specifying the soft digraph D.
(5) Find out the column sum C_i of each column of the matrix M.
(6) Choose k, for which $x_k = max\ C_i, i \neq a$ or $min\ C_i, i \neq a$ (according to the problem).

Then x_k is the optimal selection. If there exists more than one optimal solution, then any solution can be taken. To illustrate the algorithm we have solved a decision making problem using this method.

Example 4.1. Consider the following problem: The author wish to visit a place. Suppose a soft set (F, E) describes choice of places.

U = the set of all places under consideration = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$. E = {desert, ice land, forest, foreign, mountain, metro city, sea beach, village} be a set of parameters = $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$.

Let $F(e_1) = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, $F(e_2) = \{x_1, x_2, x_3, x_4, x_6\}$, $F(e_3) = \{x_2, x_3\}$, $F(e_4) = \{x_2, x_4\}$, $F(e_5) = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, $F(e_6) = \{x_1, x_3, x_6\}$, $F(e_7) = \{x_1, x_2, x_3, x_4, x_6\}$, $F(e_8) = \phi$.



D_{10}

Suppose the author is interested to visit a place with his family on the basis of his choice of parameters which constitute a subset $Q = \{e_2, e_3, e_4\}$ of the set E. Please note that the choice of parameters for places is dependent on the author i.e. the choice of parameters vary from person to person. So the solution will vary according to the choice of the person. Now by definition of soft subset, for any set $Q \subseteq E$, (F,Q) is a soft set over U since (F,E) is a soft set over U. Consider a digraph $D_{10} = (V_{D_{10}}, A_{D_{10}})$ corresponding to the soft set (F,Q). Here $V_{D_{10}} = \{e_2, e_3, e_4, e_a\}$. It is clear that D_{10} is the soft digraph corresponding to the soft set (F,Q). Now consider a matrix M specifying the digraph D_{10} as follows:

$$M = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In matrix M, the column sum of each column i.e. the sum of elements of a column are as follows; $C_1 = 3,\ C_2 = 2,\ C_3 = 2,\ C_4 = 2$. Therefore $max\ C_i = C_1 = 3, \forall i$, which are the sum of the corresponding in-degree of the vertices among e_2, e_3, e_4, e_a. Decision: The author can visit the place x_2.

4.2. **Soft entropy calculation.** If we re-write the Theorem 2.16 in view of soft digraph theory, the soft entropy of a soft set can be easily calculated using a soft-digraph. In this case, we have the following:

$$S(F) = \begin{cases} 1 - \frac{|U|}{\text{size of the digraph}}, & \text{if } F \neq \widetilde{\Phi} \text{ or } \widetilde{A} \\ 1, \text{ if } F = \widetilde{\Phi} \text{ or } \widetilde{A}. \end{cases}$$

Here, $\widetilde{\Phi}, \widetilde{A}$ corresponds to null soft digraph and absolute digraph respectively, $|U| =$ no of elements is the universe U and size of the digraph=total no of arcs in the digraph. Now, if we calculate the soft entropy of the soft digraph D_{10}, then we have $E(D_{10}) = 1 - \frac{6}{9} = 0.33$.

4.3. **A medical diagnosis problem using similarity measure of two soft digraphs.** This technique of similarity measure of two soft digraphs corresponding to the two soft sets can be applied to detect whether an ill person is suffering from certain disease or not.

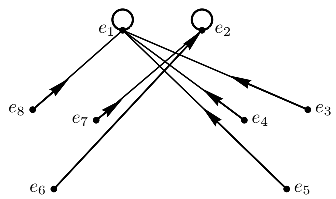
In this section we will try to find out the possibility that an ill person with certain visible symptoms is suffering from dengue. For this we first construct a model soft digraph for dengue and then the soft digraph for the ill person. Next we will find the similarity measure of their two soft digraphs. If they are significantly similar then we conclude that the person is suffering from dengue.

Suppose our universal set contains only two elements say x_1, x_2, where x_1 and x_2 stands for yes and no respectively. Here the set of parameters E is the set of certain visible symptoms. Let

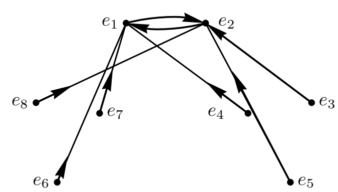
$$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\},$$

where e_1= high body temperature, e_2 = low body temperature, e_3 = high blood pressure, e_4 =cramps on muscles , e_5 = body ache, e_6 = head ache, e_7 =red bubbles in hand, e_8 = low blood pressure. Now we consider a model soft set (F,E) of dengue as follows: $F(e_1) = \{x_1\}, F(e_2) = \{x_2\}, F(e_3) = \{x_1\}, F(e_4) = \{x_1\}, F(e_5) = \{x_1\}, F(e_6) = \{x_2\}, F(e_7) = \{x_2\}, F(e_8) = \{x_1\}$.

Again we take the ill person who is having fever, cramps on muscles, head ache, red bubbles in hand. After taking to him we can construct his soft set (G, E) as follows: $G(e_1) = \{x_2\}, G(e_2) = \{x_1\}, G(e_3) = \{x_2\}, G(e_4) = \{x_1\}, G(e_5) = \{x_2\}, G(e_6) = \{x_1\}, G(e_7) = \{x_1\}, G(e_8) = \{x_2\}$.



H_1

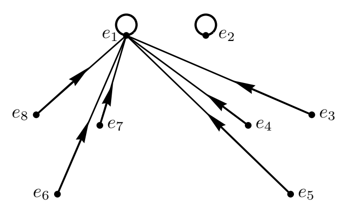


H_2

We draw the soft digraphs H_1 and H_2 corresponding to the soft sets (F, E) and (G, E) respectively. Now we consider the similarity measure using the soft digraphs as follows:

$$S(F, G) = \frac{|A_{H_1} \cap A_{H_2}|}{|A_{H_1} \cup A_{H_2}|} = \frac{1}{15} < \frac{1}{2}.$$

Hence the two soft sets (F, E) and (G, E) are not significantly similar. Therefore we conclude that the person is not possibly suffering from dengue. Whereas a person suffering from the following symptoms whose corresponding soft set (K, E) is given below: $K(e_1) = \{x_1\}, K(e_2) = \{x_2\}, K(e_3) = \{x_1\}, K(e_4) = \{x_1\}, K(e_5) = \{x_1\}, K(e_6) = \{x_1\}, K(e_7) = \{x_1\}, K(e_8) = \{x_1\}$. Now we draw the soft digraph H_3 corresponding to the soft set (K, E) as follows:



H_3

Then the similarity measure of the two soft sets (F, E) and (K, E) are given by:

$$S(F, K) = \frac{|A_{H_1} \cap A_{H_3}|}{|A_{H_1} \cup A_{H_3}|} = \frac{6}{10} > \frac{1}{2}.$$

Here the two soft sets (F, E) and (K, E) are significantly similar. Therefore we conclude that the person is possibly suffering from dengue.

Soft digraph theory is useful for solving the decision making problems for many reasons. For instance, firstly, a digraph corresponding to each soft set can be easily obtained. For each parameter of a soft set, we get a vertex of the soft digraph. After that one can calculate the matrix representation of the soft digraph. Then the maximum column sum of that matrix can easily calculated and the optimal solution is obtained. Otherwise, we can also calculate the in-degree of each vertex and obtain the optimal solution by observing the maximum in-degree among the vertices. Secondly, for pictorial representation of each soft set, the calculation is much easier than previous techniques. Also, we can calculate the soft entropy of soft digraph by calculating the number of elements in the universe and the size of the soft digraph.

For these reasons, one may use soft digraph theory for solving the decision making problems in soft set theory.

5. Conclusion

Molodtstov introduced the soft set theory in his paper [20] to deal with the uncertainties. Later on several authors have studied SST in detail and have applied this theory in solving many practical problems ([11], [15] ,[18]). In this paper we have developed the notion of soft digraphs and studied some of its important properties and applied in many problems. In future, one may study the decision making problems using soft digraphs. Also the important properties of soft digraphs are still open.

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