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A note on soft G-metric spaces about fixed point theorems

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Abstract. In this paper, we introduce the notion of soft G-Cauchy sequences and soft-G-complete spaces. Also we investigate some properties of such spaces. Then, we obtain three fixed point theorems for mappings satisfying sufficient conditions on such spaces. Moreover, we prove fixed point results of mappings defined on soft G-metric spaces.

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1. Introduction

The concept of soft set was introduced by Molodtsov [8] as a new mathematical tool for dealing with uncertainties. Then, soft set theory was studied in detail by Maji et al.[7]. Also, some algebraic operations on soft sets were obtained by Ali et. al.[1]. Shabir and Naz [13] introduced the soft topological spaces and investigated their fundamental properties. Zorlutuna et. al. [15] also studied on soft topological spaces. In addition, the notions of soft real set and soft real number were defined and their properties were given in [2].

Fixed point theory plays an important role and has many applications in mathematics. Many researchers obtained fixed point theorems for various mappings in different metric spaces. G-metric spaces [10] which are a generalization of metric spaces is one such space. Also, a lot of fixed point theorems were obtained in this structure [6, 9, 11, 12]. In addition, the concept of soft mapping and its fixed points were introduced by Wardowski [14]. Besides, the notion of soft metric spaces and Banach fixed point theorem were given in these spaces [3].

Guler et. al. [5] introduced the concept of soft G-metric space according to a soft element and obtained some of its properties. Then, they defined soft G-convergence and soft G-continuity. Moreover, they proved existence and uniqueness of fixed points in soft G-metric spaces.

In this paper, the notion of soft G-complete space is introduced and some properties of such spaces are investigated. Then, three general fixed point theorems for mappings satisfying sufficient conditions are proved on soft G-metric spaces.

2. Preliminaries

Throughout this paper, X will be a nonempty initial universal set and E will be a nonempty parameter set. Let $\mathcal{P}(X)$ denote the power set of X.

Definition 2.1 ([8])**.** A pair (F, E), where F is a mapping from E to $\mathcal{P}(X)$, is called a soft set over X.

Definition 2.2 ([7])**.** Let (F_1, E) and (F_2, E) be two soft sets over a common universe X. Then, (F_1, E) is said to be a soft subset of (F_2, E), if $F_1(\lambda) \subseteq F_2(\lambda)$, for all $\lambda \in E$. This is denoted by $(F_1, E)\widetilde{\subseteq}(F_2, E)$.

(F_1, E) is said to be soft equal to (F_2, E), if $F_1(\lambda) = F_2(\lambda)$, for all $\lambda \in E$. This is denoted by $(F_1, E) = (F_2, E)$.

Definition 2.3 ([1])**.** The complement of a soft set (F, E) is defined as $(F, E)^c = (F^c, E)$, where $F^c : E \to \mathcal{P}(X)$ is a mapping given by $F^c(\lambda) = X\backslash F(\lambda)$, for all $\lambda \in E$.

Definition 2.4 ([7])**.** Let (F, E) be a soft set over X.

(i) (F, E) is said to be a null soft set, if $F(\lambda) = \varnothing$, for all $\lambda \in E$. This is denoted by $\widetilde{\varnothing}$.

(ii) (F, E) is said to be an absolute soft set, if $F(\lambda) = X$, for all $\lambda \in E$. This is denoted by $\widetilde{X}$.

Clearly, we have $(\widetilde{X})^c = \widetilde{\varnothing}$ and $(\widetilde{\varnothing})^c = \widetilde{X}$.

Definition 2.5 ([13])**.** The difference (H, E) of two soft sets (F_1, E) and (F_2, E) over X, denoted by $(F_1, E)\backslash(F_2, E)$, is defined as $H(\lambda) = F_1(\lambda)\backslash F_2(\lambda)$, for all $\lambda \in E$.

Definition 2.6 ([7])**.** The union (H, E) of two soft sets (F, E) and (G, E) over a common universe X, denoted by $(F, E)\widetilde{\cup}(G, E)$ is defined as $H(\lambda) = F(\lambda) \cup G(\lambda)$, for all $\lambda \in E$.

The following definition of intersection of two soft sets is given as that of the bi-intersection in [4].

Definition 2.7 ([4])**.** The intersection (H, E) of two soft sets (F, E) and (G, E) over a common universe X, denoted by $(F, E)\widetilde{\cap}(G, E)$, is defined as $H(\lambda) = F(\lambda) \cap G(\lambda)$, for all $\lambda \in E$.

Definition 2.8 ([2])**.** A function $\varepsilon : E \to X$ is said to be a soft element of X. A soft element ε of X is said to belong to a soft set (F, E) of X, denoted by $\varepsilon\widehat{\in}(F, E)$, if $\varepsilon(e) \in F(e)$ for each $e \in E$.

In that case, ε is also said to be a soft element of the soft set (F, E). Thus, every singleton soft set (a soft set (F, E) for which $F(e)$ is a singleton set for each $e \in E$) can be identified with a soft element by simply identifying the singleton set with the element that contains each $e \in E$.

Definition 2.9 ([2])**.** Let $\mathbf{R}$ be the set of real numbers, $\mathcal{B}(\mathbf{R})$ be the collection of all nonempty bounded subsets of $\mathbf{R}$ and E be a set of parameters. Then a mapping $F : E \to \mathcal{B}(\mathbf{R})$ is called a soft real set.
It is denoted by (F, E) and $\mathbf{R}(E)$ denotes the set of all soft real sets.

Also, $\mathbf{R}(E)^*$ denotes the set of all non-negative soft real sets ((F, E) is said to be a non-negative soft real set, if $F(\lambda)$ is a subset of the set of non-negative real numbers for each $\lambda \in E$).

In particular, if (F, E) is a singleton soft set, then it is called a soft real number, by identifying (F, E) with the corresponding soft element.
$\mathbb{R}(E)$ denotes the set of all soft real numbers. Also, $\mathbb{R}(E)^*$ denotes the set of all non-negative soft real numbers.

Definition 2.10 ([2])**.** Let $(F, E), (G, E) \in \mathbf{R}(E)$.

(i) $(F, E) = (G, E)$, if $F(\lambda) = G(\lambda)$, for each $\lambda \in E$.
(ii) $(F + G)(\lambda) = \{a + b : a \in F(\lambda), b \in G(\lambda)\}$, for each $\lambda \in E$.
(iii)$(F - G)(\lambda) = \{a - b : a \in F(\lambda), b \in G(\lambda)\}$, for each $\lambda \in E$.
(iv) $(F.G)(\lambda) = \{a.b : a \in F(\lambda), b \in G(\lambda)\}$, for each $\lambda \in E$.
(v)$(F/G)(\lambda) = \{a/b : a \in F(\lambda), b \in G(\lambda)\backslash\{0\}\}$, provided $0 \notin G(\lambda)$, for each $\lambda \in E$.

In this paper, as demonstrated in [3], $S(\widetilde{X})$ denotes the set of soft sets (F, E) over X for which $F(\lambda) \neq \varnothing$ for all $\lambda \in E$ and $SE((F, E))$ denotes the collection of all soft elements of (F, E) for any soft set $(F, E) \in S(\widetilde{X})$.

Also, $\widetilde{x}, \widetilde{y}, \widetilde{z}$ denote soft elements of a soft set and $\widetilde{r}, \widetilde{s}, \widetilde{t}$ denote soft real numbers, whereas $\overline{r}, \overline{s}, \overline{t}$ denote a particular type of soft real numbers such that $\overline{r}(\lambda) = r$, for all $\lambda \in E$.

Definition 2.11 ([3])**.** For two soft real numbers $\widetilde{r}, \widetilde{s}$,

(i) $\widetilde{r} \widetilde{\leq} \widetilde{s}$, if $\widetilde{r}(\lambda) \leq \widetilde{s}(\lambda)$, for all $\lambda \in E$,
(ii) $\widetilde{r} \widetilde{\geq} \widetilde{s}$, if $\widetilde{r}(\lambda) \geq \widetilde{s}(\lambda)$, for all $\lambda \in E$,
(iii) $\widetilde{r} \widetilde{<} \widetilde{s}$, if $\widetilde{r}(\lambda) < \widetilde{s}(\lambda)$, for all $\lambda \in E$,
(iv) $\widetilde{r} \widetilde{>} \widetilde{s}$, if $\widetilde{r}(\lambda) > \widetilde{s}(\lambda)$, for all $\lambda \in E$.

Definition 2.12 ([3])**.** A mapping $d : SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ is said to be a soft metric on $\widetilde{X}$ if d satisfies the following conditions:

(M1) $d(\widetilde{x}, \widetilde{y}) \widetilde{\geq} \overline{0}$, for all $\widetilde{x}, \widetilde{y} \widetilde{\in} \widetilde{X}$,
(M2) $d(\widetilde{x}, \widetilde{y}) = \overline{0}$ if and only if $\widetilde{x} = \widetilde{y}$,
(M3) $d(\widetilde{x}, \widetilde{y}) = d(\widetilde{y}, \widetilde{x})$, for all $\widetilde{x}, \widetilde{y} \widetilde{\in} \widetilde{X}$,
(M4) $d(\widetilde{x}, \widetilde{z}) \widetilde{\leq} d(\widetilde{x}, \widetilde{y}) + d(\widetilde{y}, \widetilde{z})$, for all $\widetilde{x}, \widetilde{y}, \widetilde{z} \widetilde{\in} \widetilde{X}$.

The soft set $\widetilde{X}$ with a soft metric d on $\widetilde{X}$ is said to be a soft metric space and is denoted by $(\widetilde{X}, d)$.

Definition 2.13 ([3])**.** Let $(\widetilde{x_n})$ be a sequence of soft elements in $(\widetilde{X}, d)$.

The sequences $(\widetilde{x_n})$ is said to be convergent in $(\widetilde{X}, d)$, if there is a soft element $\widetilde{x} \widetilde{\in} \widetilde{X}$ such that $d(\widetilde{x_n}, \widetilde{x}) \to \overline{0}$ as n$\to \infty$.

A sequence $(\widetilde{x_n})$ of soft elements in $(\widetilde{X}, d)$ is said to be Cauchy sequences in $\widetilde{X}$, if for every $\widetilde{\epsilon} \widetilde{\geq} \overline{0}$, there is a natural number m such that $d(\widetilde{x_i}, \widetilde{x_j}) \widetilde{\leq} \widetilde{\epsilon}$, whenever i, j$\geq$ m.

Definition 2.14 ([3])**.** A soft metric space $(\widetilde{X}, d)$ is said to be complete if every Cauchy sequence in $\widetilde{X}$ converges to some soft element of $\widetilde{X}$.

Definition 2.15 ([5])**.** Let X be a nonempty set and E be the nonempty set of parameters. A mapping $\widetilde{G} : SE(\widetilde{X}) \times SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ is said to be a soft generalized metric or soft G-metric on $\widetilde{X}$, if $\widetilde{G}$ satisfies the following conditions:

$(\widetilde{G}_1)$ $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z}) = \overline{0}$, if $\widetilde{x} = \widetilde{y} = \widetilde{z}$,

$(\widetilde{G}_2)$ $\overline{0} \widetilde{<} \widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y})$, for all $\widetilde{x}, \widetilde{y} \in SE(\widetilde{X})$ with $\widetilde{x} \neq \widetilde{y}$,

$(\widetilde{G}_3)$ $\widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y}) \widetilde{\leq} \widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z})$, for all $\widetilde{x}, \widetilde{y}, \widetilde{z} \in SE(\widetilde{X})$ with $\widetilde{y} \neq \widetilde{z}$,

$(\widetilde{G}_4)$ $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z}) = \widetilde{G}(\widetilde{x}, \widetilde{z}, \widetilde{y}) = \widetilde{G}(\widetilde{y}, \widetilde{z}, \widetilde{x}) = ...$,

$(\widetilde{G}_5)$ $\widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{z}) \widetilde{\leq} \widetilde{G}(\widetilde{x}, \widetilde{a}, \widetilde{a}) + \widetilde{G}(\widetilde{a}, \widetilde{y}, \widetilde{z})$, for all $\widetilde{x}, \widetilde{y}, \widetilde{z}, \widetilde{a} \in SE(\widetilde{X})$.

The soft set $\widetilde{X}$ with a soft G-metric $\widetilde{G}$ on $\widetilde{X}$ is said to be a soft G-metric space and is denoted by $(\widetilde{X}, \widetilde{G}, E)$.

Proposition 2.16 ([5])**.** *For any soft metric d on $\widetilde{X}$, we can construct a soft G-metric by the following mappings $\widetilde{G}_s$ and $\widetilde{G}_m$:*

(1) $\widetilde{G}_s(d)(\widetilde{x}, \widetilde{y}, \widetilde{z}) = \frac{1}{3}(d(\widetilde{x}, \widetilde{y}) + d(\widetilde{y}, \widetilde{z}) + d(\widetilde{x}, \widetilde{z}))$,

(2) $\widetilde{G}_m(d)(\widetilde{x}, \widetilde{y}, \widetilde{z}) = \max\{d(\widetilde{x}, \widetilde{y}), d(\widetilde{y}, \widetilde{z}), d(\widetilde{x}, \widetilde{z})\}$.

Proposition 2.17 ([5])**.** *For any soft G-metric $\widetilde{G}$ on $\widetilde{X}$, we can construct a soft metric $d_{\widetilde{G}}$ on $\widetilde{X}$ defined by*

$$d_{\widetilde{G}}(\widetilde{x}, \widetilde{y}) = \widetilde{G}(\widetilde{x}, \widetilde{y}, \widetilde{y}) + \widetilde{G}(\widetilde{x}, \widetilde{x}, \widetilde{y}).$$

Definition 2.18 ([5])**.** Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space and $(\widetilde{x_n})$ be a sequence of soft elements in $\widetilde{X}$. The sequence $(\widetilde{x_n})$ is said to be soft G-convergent at $\widetilde{x}$ in $\widetilde{X}$, if for every $\widetilde{\epsilon} \widetilde{\geq} \overline{0}$, chosen arbitrarily, there exists a natural number N=N($\widetilde{\epsilon}$) such that $\overline{0} \widetilde{\leq}$ $\widetilde{G}(\widetilde{x_n}, \widetilde{x_n}, \widetilde{x})$ $\widetilde{<} \widetilde{\epsilon}$, whenever n$\geq$ N, i.e., n$\geq$ N $\Rightarrow$ $(\widetilde{x_n}) \in B_{\widetilde{G}}(\widetilde{x}, \widetilde{\epsilon})$.
We denote this by $\widetilde{x_n} \to \widetilde{x}$ as n$\to \infty$ or by $\lim_{n\to\infty}(\widetilde{x_n}) = \widetilde{x}$.

Proposition 2.19 ([5])**.** *Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space, for a sequence $(\widetilde{x_n})$ in $\widetilde{X}$ and soft element $\widetilde{x}$, then the followings are equivalent:*

(1) *$(\widetilde{x_n})$ is soft G-convergent to $\widetilde{x}$,*

(2) *$d_{\widetilde{G}}(\widetilde{x_n}, \widetilde{x}) \to \overline{0}$ as n$\to \infty$,*

(3) *$\widetilde{G}(\widetilde{x_n}, \widetilde{x_n}, \widetilde{x}) \to \overline{0}$ as n$\to \infty$,*

(4) *$\widetilde{G}(\widetilde{x_n}, \widetilde{x}, \widetilde{x}) \to \overline{0}$ as n$\to \infty$,*

(5) *$\widetilde{G}(\widetilde{x_n}, \widetilde{x_m}, \widetilde{x}) \to \overline{0}$ as n, m$\to \infty$.*

Definition 2.20 ([5])**.** Let $(\widetilde{X}, \widetilde{G}, E)$, $(\widetilde{X'}, \widetilde{G'}, E')$ be two soft G-metric spaces. Then a function f: $\widetilde{X} \to \widetilde{X'}$ defined by $f(\widetilde{a}) = \widetilde{f(a)}$ is soft G-continuous at a soft element $\widetilde{a} \in SE(\widetilde{X})$ if and only if for every $\widetilde{\epsilon} \widetilde{>} \overline{0}$, there exists $\widetilde{\delta} \widetilde{\geq} \overline{0}$ such that $\widetilde{x}, \widetilde{y} \widetilde{\in} \widetilde{X}$ and $\widetilde{G}(\widetilde{a}, \widetilde{x}, \widetilde{y}) \widetilde{<} \widetilde{\delta}$ implies that $\widetilde{G'}(\widetilde{f(a)}, \widetilde{f(x)}, \widetilde{f(y)}) \widetilde{<} \widetilde{\epsilon}$.

A function f is soft G-continuous if and only if it is soft G-continuous at all $\widetilde{a} \in SE(\widetilde{X})$.

Proposition 2.21 ([5])**.** *Let $(\widetilde{X}, \widetilde{G}, E)$, $(\widetilde{X'}, \widetilde{G'}, E')$ be two soft G-metric spaces. Then a function f: $\widetilde{X} \to \widetilde{X'}$ is soft G-continuous at a soft element $\widetilde{a} \in SE(\widetilde{X})$ if* *and only if it is soft G-sequentially continuous at a soft element* $\widetilde{a} \in SE(\widetilde{X})$, *i.e., whenever* $(\widetilde{x_n})$ *is soft G-convergent to* $\widetilde{a}$, $(\widetilde{f(x_n)})$ *is soft* G'*-convergent to* $\widetilde{f(a)}$.

Definition 2.22 ([5])**.** Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space. Let $T : (\widetilde{X}, \widetilde{G}, E) \rightarrow (\widetilde{X}, \widetilde{G}, E)$ be a mapping. If there exists a soft element $\widetilde{x_0} \in SE(\widetilde{X})$ such that T$(\widetilde{x_0})$ $= \widetilde{x_0}$, then $\widetilde{x_0}$ is called a fixed point of T.

Definition 2.23 ([5])**.** Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space. Let $T : (\widetilde{X}, \widetilde{G}, E) \rightarrow (\widetilde{X}, \widetilde{G}, E)$ be a mapping. For every $x_0 \in SE(\widetilde{X})$, we can write the sequence of soft elements by applying T on $\widetilde{x_0}$; $\widetilde{x_1}=T(\widetilde{x_0})$, $\widetilde{x_2}=T(\widetilde{x_1}) = T^2(\widetilde{x_0})$,...,$\widetilde{x_n}=T(\widetilde{x_{n-1}}) = T^n(\widetilde{x_0})$. We say that the sequence has been constructed by iteration method.

Theorem 2.24 ([5])**.** *Let* $(\widetilde{X}, \widetilde{G}, E)$ *be a soft G-metric space. Let* $T : (\widetilde{X}, \widetilde{G}, E) \rightarrow (\widetilde{X}, \widetilde{G}, E)$ *be a mapping such that T satisfies the followings:*

(i) $\widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{z}) \widetilde{\leq} \overline{a}\widetilde{G}(\widetilde{x}, T\widetilde{x}, T\widetilde{x}) + \overline{b}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}) + \overline{c}\widetilde{G}(\widetilde{z}, T\widetilde{z}, T\widetilde{z})$ *for all* $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z} \in SE(\widetilde{X})$ *where* $\overline{0} \widetilde{<} \overline{a} + \overline{b} + \overline{c} \widetilde{<} \overline{1}$,

(ii) *T is soft G-continuous at a soft element* $\widetilde{u} \in SE(\widetilde{X})$,

(iii) *There is* $\widetilde{x} \in SE(\widetilde{X})$; $T^n(\widetilde{x})$ *has a subsequence* $T^{ni}(\widetilde{x})$ *soft G-converges to* $\widetilde{u}$. *Then* $\widetilde{u}$ *is a unique fixed point. (i.e* $T\widetilde{u} = \widetilde{u}$*).*

3. Soft G-completeness

Definition 3.1. Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space and $(\widetilde{x_n})$ be a sequence of soft elements in $\widetilde{X}$.

The sequence $(\widetilde{x_n})$ is said to be soft G-Cauchy, if for every $\widetilde{\epsilon} \widetilde{\geq} \overline{0}$, chosen arbitrarily, there exists a natural number k such that $\widetilde{G}(\widetilde{x_n}, \widetilde{x_m}, \widetilde{x_l}) \widetilde{<} \widetilde{\epsilon}$, whenever n, m, l $\geq$ k.

A soft G-metric space $(\widetilde{X}, \widetilde{G}, E)$ is said to be soft G-complete, if every soft G-Cauchy sequence in $(\widetilde{X}, \widetilde{G}, E)$ is soft G-convergent in $(\widetilde{X}, \widetilde{G}, E)$.

Proposition 3.2. *Let* $(\widetilde{X}, \widetilde{G}, E)$ *be a soft G-metric space and* $(\widetilde{x_n})$ *be a sequence of soft elements in* $\widetilde{X}$. *Then the followings are equivalent:*

(1) *the sequence* $(\widetilde{x_n})$ *is soft G-Cauchy,*

(2) *for every* $\widetilde{\epsilon} \widetilde{\geq} \overline{0}$, *there exists a natural number k such that* $\widetilde{G}(\widetilde{x_n}, \widetilde{x_m}, \widetilde{x_m}) \widetilde{<} \widetilde{\epsilon}$ *for any n,m* $\geq$ *k,*

(3) $(\widetilde{x_n})$ *is a Cauchy sequence in the soft metric space* $(\widetilde{X}, d_{\widetilde{G}}, E)$.

Proof. (1) $\Rightarrow$ (2): It is obvious by axiom $(\widetilde{G}_3)$.

(2) $\Leftrightarrow$ (3): It is clear by the definition of $d_{\widetilde{G}}$.

(2) $\Rightarrow$ (1): If we set $\widetilde{a} = \widetilde{x_m}$, then it is obvious by axiom $(\widetilde{G}_5)$. □

Corollary 3.3. *Every soft G-convergent sequence in any soft G-metric space* $(\widetilde{X}, \widetilde{G}, E)$ *is soft G-Cauchy.*

Proposition 3.4. *A soft G-metric space* $(\widetilde{X}, \widetilde{G}, E)$ *is soft G-complete if and only if* $(X, d_{\widetilde{G}}, E)$ *is complete soft metric space.*

Proof. It follows from Propositions 3.2 and 2.17. □

Theorem 3.5. *Let $(\widetilde{X},\widetilde{G},E)$ be a soft G-complete space and $T:(\widetilde{X},\widetilde{G},E)\rightarrow(\widetilde{X},\widetilde{G},E)$ be a mapping that satisfies the following condition for all $\widetilde{x},\widetilde{y},\widetilde{z}\in SE(\widetilde{X})$,*

$$\widetilde{G}(T\widetilde{x},T\widetilde{y},T\widetilde{z})\,\widetilde{\leq}\,\overline{a}\widetilde{G}(\widetilde{x},T\widetilde{x},T\widetilde{x})+\overline{b}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y})+\overline{c}\widetilde{G}(\widetilde{z},T\widetilde{z},T\widetilde{z})+\overline{d}\widetilde{G}(\widetilde{x},\widetilde{y},\widetilde{z}) \quad (3.1)$$

where $\overline{0}\widetilde{\leq}\overline{a}+\overline{b}+\overline{c}+\overline{d}\widetilde{<}\overline{1}$.
Then T has a unique fixed point, say $\widetilde{u}$, and T is soft G-continuous at $\widetilde{u}$.

Proof. Let $\widetilde{x_0}\in SE(\widetilde{X})$ be an arbitrary soft element and define the sequence $(\widetilde{x_n})_{n\in N}$ by $\widetilde{x_n}=T^n(\widetilde{x_0})$. From (3.1), we get

$$\begin{aligned}\widetilde{G}(\widetilde{x_n},\widetilde{x_{n+1}},\widetilde{x_{n+1}})\widetilde{\leq}\,&\overline{a}\widetilde{G}(\widetilde{x_{n-1}},\widetilde{x_n},\widetilde{x_n})+\overline{b}\widetilde{G}(\widetilde{x_n},\widetilde{x_{n+1}},\widetilde{x_{n+1}})+\overline{c}\widetilde{G}(\widetilde{x_n},\widetilde{x_{n+1}},\widetilde{x_{n+1}})\\&+\overline{d}\widetilde{G}(\widetilde{x_{n-1}},\widetilde{x_n},\widetilde{x_n}).\end{aligned} \quad (3.2)$$

Then

$$\widetilde{G}(\widetilde{x_n},\widetilde{x_{n+1}},\widetilde{x_{n+1}})\widetilde{\leq}(\overline{a}+\overline{d})\widetilde{G}(\widetilde{x_{n-1}},\widetilde{x_n},\widetilde{x_n})+(\overline{b}+\overline{c})\widetilde{G}(\widetilde{x_n},\widetilde{x_{n+1}},\widetilde{x_{n+1}}). \quad (3.3)$$

Thus we have

$$\widetilde{G}(\widetilde{x_n},\widetilde{x_{n+1}},\widetilde{x_{n+1}})\widetilde{\leq}\frac{(\overline{a}+\overline{d})}{\overline{1}-(\overline{b}+\overline{c})}\widetilde{G}(\widetilde{x_{n-1}},\widetilde{x_n},\widetilde{x_n}). \quad (3.4)$$

Let $\overline{k}=\frac{(\overline{a}+\overline{d})}{\overline{1}-(\overline{b}+\overline{c})}$. Since $\overline{0}\widetilde{\leq}\overline{a}+\overline{b}+\overline{c}+\overline{d}\widetilde{<}\overline{1}$, $\overline{0}\widetilde{\leq}\overline{k}\widetilde{<}\overline{1}$. So we get

$$\widetilde{G}(\widetilde{x_n},\widetilde{x_{n+1}},\widetilde{x_{n+1}})\widetilde{\leq}\overline{k}\widetilde{G}(\widetilde{x_{n-1}},\widetilde{x_n},\widetilde{x_n}). \quad (3.5)$$

Hence we have the following inequalities :

$$\begin{aligned}&\widetilde{G}(\widetilde{x_{n-1}},\widetilde{x_n},\widetilde{x_n})\widetilde{\leq}\overline{k}\widetilde{G}(\widetilde{x_{n-2}},\widetilde{x_{n-1}},\widetilde{x_{n-1}}),\\&\widetilde{G}(\widetilde{x_{n-2}},\widetilde{x_{n-1}},\widetilde{x_{n-1}})\widetilde{\leq}\overline{k}\widetilde{G}(\widetilde{x_{n-3}},\widetilde{x_{n-2}},\widetilde{x_{n-2}}),\\&\quad\vdots\end{aligned} \quad (3.6)$$

Combining (3.5) and (3.6), we obtain

$$\widetilde{G}(\widetilde{x_n},\widetilde{x_{n+1}},\widetilde{x_{n+1}})\widetilde{\leq}(\overline{k})^n\widetilde{G}(\widetilde{x_0},\widetilde{x_1},\widetilde{x_1}). \quad (3.7)$$

For all $m,n\in N$ such that $n<m$, we have

$$\begin{aligned}\widetilde{G}(\widetilde{x_n},\widetilde{x_m},\widetilde{x_m})&\widetilde{\leq}\widetilde{G}(\widetilde{x_n},\widetilde{x_{n+1}},\widetilde{x_{n+1}})+\widetilde{G}(\widetilde{x_{n+1}},\widetilde{x_{n+1}},\widetilde{x_{n+2}})+...+\widetilde{G}(\widetilde{x_{m-1}},\widetilde{x_m},\widetilde{x_m})\\&\widetilde{\leq}((\overline{k})^n+(\overline{k})^{n+1}+...+(\overline{k})^{m-1})\widetilde{G}(\widetilde{x_0},\widetilde{x_1},\widetilde{x_1}) \quad (3.8)\\&\widetilde{\leq}\frac{(\overline{k})^n}{\overline{1}-\overline{k}}\widetilde{G}(\widetilde{x_0},\widetilde{x_1},\widetilde{x_1}),\end{aligned}$$

by ($\widetilde{G_5}$) and (3.7).
Thus $\widetilde{G}(\widetilde{x_n},\widetilde{x_m},\widetilde{x_m})\rightarrow 0$ as $m,n\rightarrow\infty$. So $(\widetilde{x_n})$ is a soft G-Cauchy sequence. Since $(\widetilde{X},\widetilde{G},E)$ is soft G-complete, there exists $\widetilde{u}\in SE(\widetilde{X})$ such that $(\widetilde{x_n})$ soft G-converges to $\widetilde{u}$.

Assume that $T(\widetilde{u}) \neq \widetilde{u}$, i.e., $T(\widetilde{u}(\lambda_0)) \neq \widetilde{u}(\lambda_0)$ for some $\lambda_0 \in E$. Then by (3.1), we have

$$\widetilde{G}(\widetilde{x_n}, T\widetilde{u}, T\widetilde{u}) \widetilde{\leq} \overline{a}\widetilde{G}(\widetilde{x_{n-1}}, \widetilde{x_n}, \widetilde{x_n}) + \overline{b}\widetilde{G}(\widetilde{u}, T\widetilde{u}, T\widetilde{u}) + \overline{c}\widetilde{G}(\widetilde{u}, T\widetilde{u}, T\widetilde{u}) + \overline{d}\widetilde{G}(\widetilde{x_{n-1}}, \widetilde{u}, \widetilde{u}). \tag{3.9}$$

Thus

$$\widetilde{G}(\widetilde{x_n}, T\widetilde{u}, T\widetilde{u}) \widetilde{\leq} \overline{a}\widetilde{G}(\widetilde{x_{n-1}}, \widetilde{x_n}, \widetilde{x_n}) + (\overline{b} + \overline{c})\widetilde{G}(\widetilde{u}, T\widetilde{u}, T\widetilde{u}) + \overline{d}\widetilde{G}(\widetilde{x_{n-1}}, \widetilde{u}, \widetilde{u}). \tag{3.10}$$

By taking the limit as $n \to \infty$, we get

$$\widetilde{G}(\widetilde{u}, T\widetilde{u}, T\widetilde{u}) \widetilde{\leq} (\overline{b} + \overline{c})\widetilde{G}(\widetilde{u}, T\widetilde{u}, T\widetilde{u}), \tag{3.11}$$

since $(\widetilde{x_n}) \to \widetilde{u}$. This is a contradiction. Hence $T\widetilde{u} = \widetilde{u}$.

Let us prove uniqueness. Suppose there exists a soft element $\widetilde{v}$ such that $\widetilde{u} \neq \widetilde{v}$ and $T\widetilde{v} = \widetilde{v}$. Then by (3.1), we get

$$\begin{aligned}\widetilde{G}(\widetilde{u}, \widetilde{v}, \widetilde{v}) = \widetilde{G}(T\widetilde{u}, T\widetilde{v}, T\widetilde{v}) &\widetilde{\leq} \overline{a}\widetilde{G}(\widetilde{u}, T\widetilde{u}, T\widetilde{u}) \\ &+ (\overline{b} + \overline{c})\widetilde{G}(\widetilde{v}, T\widetilde{v}, T\widetilde{v}) + \overline{d}\widetilde{G}(\widetilde{u}, \widetilde{v}, \widetilde{v}) \\ &= \overline{d}\widetilde{G}(\widetilde{u}, \widetilde{v}, \widetilde{v}).\end{aligned} \tag{3.12}$$

Thus we find that $\widetilde{u} = \widetilde{v}$.

Let us prove that T is soft G-continuous at $\widetilde{u}$. Let $(\widetilde{y_n})$ be a sequence of soft elements in $\widetilde{X}$ such that $(\widetilde{y_n}) \to \widetilde{u}$. Then by (3.1), we have

$$\begin{aligned}\widetilde{G}(\widetilde{u}, T\widetilde{y_n}, T\widetilde{y_n}) &\widetilde{\leq} \overline{a}\widetilde{G}(\widetilde{u}, T\widetilde{u}, T\widetilde{u}) + (\overline{b} + \overline{c})\widetilde{G}(\widetilde{y_n}, T\widetilde{y_n}, T\widetilde{y_n}) + \overline{d}\widetilde{G}(\widetilde{u}, \widetilde{y_n}, \widetilde{y_n}) \\ &= (\overline{b} + \overline{c})\widetilde{G}(\widetilde{y_n}, T\widetilde{y_n}, T\widetilde{y_n}) + \overline{d}\widetilde{G}(\widetilde{u}, \widetilde{y_n}, \widetilde{y_n}).\end{aligned} \tag{3.13}$$

Also, by $(\widetilde{G}_5)$, we have

$$\widetilde{G}(\widetilde{y_n}, T\widetilde{y_n}, T\widetilde{y_n}) \widetilde{\leq} \widetilde{G}(\widetilde{y_n}, \widetilde{u}, \widetilde{u}) + \widetilde{G}(\widetilde{u}, T\widetilde{y_n}, T\widetilde{y_n}). \tag{3.14}$$

Then, we combine (3.13) and (3.14), to get

$$\widetilde{G}(\widetilde{u}, T\widetilde{y_n}, T\widetilde{y_n}) \widetilde{\leq} (\overline{b} + \overline{c})\widetilde{G}(\widetilde{y_n}, \widetilde{u}, \widetilde{u}) + (\overline{b} + \overline{c})\widetilde{G}(\widetilde{u}, T\widetilde{y_n}, T\widetilde{y_n}) + \overline{d}\widetilde{G}(\widetilde{u}, \widetilde{y_n}, \widetilde{y_n}).$$

Thus

$$\widetilde{G}(\widetilde{u}, T\widetilde{y_n}, T\widetilde{y_n}) \widetilde{\leq} \frac{(\overline{b} + \overline{c})}{\overline{1} - (\overline{b} + \overline{c})}\widetilde{G}(\widetilde{y_n}, \widetilde{u}, \widetilde{u}) + \frac{\overline{d}}{\overline{1} - (\overline{b} + \overline{c})}\widetilde{G}(\widetilde{u}, \widetilde{y_n}, \widetilde{y_n}). \tag{3.15}$$

By taking the limit as $n \to \infty$, we obtain $\widetilde{G}(\widetilde{u}, T\widetilde{y_n}, T\widetilde{y_n}) \to \overline{0}$, since $(\widetilde{y_n}) \to \widetilde{u}$. So $T(\widetilde{y_n}) \to \widetilde{u} = T\widetilde{u}$, from Proposition 2.19. Hence T is soft G-continuous at $\widetilde{u}$, by Proposition 2.21. □

Corollary 3.6. *Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-complete space and let $T : (\widetilde{X}, \widetilde{G}, E) \to (\widetilde{X}, \widetilde{G}, E)$ be a mapping that satisfies the following condition for all $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z} \in SE(\widetilde{X})$,*

$$\widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{z}) \widetilde{\leq} \overline{a}\widetilde{G}(\widetilde{x}, T\widetilde{x}, T\widetilde{x}) + \overline{b}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}) + \overline{c}\widetilde{G}(\widetilde{z}, T\widetilde{z}, T\widetilde{z}) \tag{3.16}$$

where $\overline{0} \widetilde{\leq} \overline{a} + \overline{b} + \overline{c} \widetilde{<} \overline{1}$. Then T has a unique fixed point, say $\widetilde{u}$, and T is soft G-continuous at $\widetilde{u}$.

Proof. If we take $\overline{d} = \overline{0}$ in Theorem 3.5, it is obvious. □

The following example shows that (ii) and (iii) in Theorem 2.24 do not guarantee the soft G-completeness of soft G-metric space.

Example 3.7. Consider the soft G-metric space $(\widetilde{X}, \widetilde{G}_m(d), E)$ where $\widetilde{X}(\lambda) = (0, 1]$ in the real line and $\widetilde{d}(\widetilde{x}, \widetilde{y})(\lambda) = |\widetilde{x}(\lambda) - \widetilde{y}(\lambda)|$ for each $\lambda \in E$ and $\widetilde{x}, \widetilde{y} \in SE(\widetilde{X})$. Let $T(\widetilde{x}) = \widetilde{x}/\overline{4}$. Although the sequence $(\widetilde{x_n})$ of soft elements in $\widetilde{X}$ where $\widetilde{x}_n(\lambda) = 1/n$ for each $n \in N$ and for each $\lambda \in E$ is a soft G- Cauchy, it is not soft G-convergent. Then $(\widetilde{X}, \widetilde{G}_m(d), E)$ is not soft G-complete space. But the conditions (ii) and (iii) in Theorem 2.24 are satisfied.

Theorem 3.8. *Let $(\widetilde{X}, \widetilde{G}, E)$ be a soft G-metric space and let $\widetilde{M} \widetilde{\subset} \widetilde{X}$ which meets the condition " there exists a sequence $(\widetilde{x_n})$ in $\widetilde{M}$ such that $(\widetilde{x_n}) \to \widetilde{x}$ for every $\widetilde{x} \in SE(\widetilde{X})$". Let $T : (\widetilde{X}, \widetilde{G}, E) \to (\widetilde{X}, \widetilde{G}, E)$ be a mapping that satisfies the following:*

(i) *$\widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{z}) \,\widetilde{\leq}\, \overline{a}(\widetilde{G}(\widetilde{x}, T\widetilde{x}, T\widetilde{x}) + \widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}) + \widetilde{G}(\widetilde{z}, T\widetilde{z}, T\widetilde{z}))$ for all $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z} \in SE(\widetilde{M})$ and $\overline{0} \,\widetilde{<}\, \overline{a} \,\widetilde{<}\, \frac{\overline{1}}{6}$,*

(ii) *T is a soft G-continuous mapping,*

(iii) *There is $\widetilde{x} \in SE(\widetilde{X})$ such that $T^n(\widetilde{x})$ soft G-converges to $\widetilde{u}$, for each $n \in N$. Then $\widetilde{u}$ is a unique fixed point. (i.e $T\widetilde{u} = \widetilde{u}$).*

Proof. It is enough to show that condition (i) in Theorem 2.24 holds for any $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z} \in SE(\widetilde{X})$.

Case 1: For $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z} \in SE(\widetilde{X \setminus M})$, let $(\widetilde{x_n})$, $(\widetilde{y_n})$ and $(\widetilde{z_n})$ be sequences in $\widetilde{M}$ such that $(\widetilde{x_n}) \to \widetilde{x}$, $(\widetilde{y_n}) \to \widetilde{y}$ and $(\widetilde{z_n}) \to \widetilde{z}$. Then by axiom $(\widetilde{G}_5)$, we have

$$\widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{z}) \widetilde{\leq} \widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{y}) + \widetilde{G}(T\widetilde{y}, T\widetilde{y}, T\widetilde{z})$$

and

$$\widetilde{G}(T\widetilde{z}, T\widetilde{y}, T\widetilde{y}) \widetilde{\leq} \widetilde{G}(T\widetilde{z}, T\widetilde{z_n}, T\widetilde{z_n}) + \widetilde{G}(T\widetilde{z_n}, T\widetilde{y_n}, T\widetilde{y_n}) + \widetilde{G}(T\widetilde{y_n}, T\widetilde{y}, T\widetilde{y}). \tag{3.17}$$

By (i), we obtain

$$\widetilde{G}(T\widetilde{z_n}, T\widetilde{y_n}, T\widetilde{y_n}) \widetilde{\leq} \overline{a}\{\widetilde{G}(\widetilde{z_n}, T\widetilde{z_n}, T\widetilde{z_n}) + \overline{2}\widetilde{G}(\widetilde{y_n}, T\widetilde{y_n}, T\widetilde{y_n})\}. \tag{3.18}$$

Again by axiom $(\widetilde{G}_5)$, we have

$$\widetilde{G}(\widetilde{z_n}, T\widetilde{z_n}, T\widetilde{z_n}) \widetilde{\leq} \widetilde{G}(\widetilde{z_n}, \widetilde{z}, \widetilde{z}) + \widetilde{G}(\widetilde{z}, T\widetilde{z}, T\widetilde{z}) + \widetilde{G}(T\widetilde{z}, T\widetilde{z_n}, T\widetilde{z_n}) \tag{3.19}$$

and

$$\widetilde{G}(\widetilde{y_n}, T\widetilde{y_n}, T\widetilde{y_n}) \widetilde{\leq} \widetilde{G}(\widetilde{y_n}, \widetilde{y}, \widetilde{y}) + \widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}) + \widetilde{G}(T\widetilde{y}, T\widetilde{y_n}, T\widetilde{y_n}). \tag{3.20}$$

Thus, from (3.18), (3.19) and (3.20), we get

$$\begin{aligned}
\widetilde{G}(T\widetilde{z}, T\widetilde{y}, T\widetilde{y}) \widetilde{\leq} & \widetilde{G}(T\widetilde{z}, T\widetilde{z_n}, T\widetilde{z_n}) + \widetilde{G}(T\widetilde{y_n}, T\widetilde{y}, T\widetilde{y}) \\
& + \overline{a}\widetilde{G}(\widetilde{z_n}, \widetilde{z}, \widetilde{z}) + \overline{a}\widetilde{G}(T\widetilde{z}, T\widetilde{z_n}, T\widetilde{z_n}) + \overline{2}\overline{a}\widetilde{G}(\widetilde{y_n}, \widetilde{y}, \widetilde{y}) \\
& + \overline{2}\overline{a}\widetilde{G}(T\widetilde{y}, T\widetilde{y_n}, T\widetilde{y_n}) + a\widetilde{G}(\widetilde{z}, T\widetilde{z}, T\widetilde{z}) + \overline{2}\overline{a}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}) \\
\widetilde{\leq} & (\overline{1} + \overline{a})\widetilde{G}(T\widetilde{z}, T\widetilde{z_n}, T\widetilde{z_n}) + \widetilde{G}(T\widetilde{y_n}, T\widetilde{y}, T\widetilde{y}) \\
& + \overline{a}\widetilde{G}(\widetilde{z_n}, \widetilde{z}, \widetilde{z}) + \overline{2}\overline{a}\widetilde{G}(\widetilde{y_n}, \widetilde{y}, \widetilde{y}) + \overline{2}\overline{a}\widetilde{G}(T\widetilde{y}, T\widetilde{y_n}, T\widetilde{y_n}) \\
& + a\widetilde{G}(\widetilde{z}, T\widetilde{z}, T\widetilde{z}) + \overline{2}\overline{a}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}).
\end{aligned}$$

So

$$\begin{aligned}\widetilde{G}(T\widetilde{z},T\widetilde{y},T\widetilde{y})\widetilde{\leq}(\overline{1}+\overline{a})\widetilde{G}(T\widetilde{z},T\widetilde{z_n},T\widetilde{z_n})+\widetilde{G}(T\widetilde{y_n},T\widetilde{y},T\widetilde{y})\\+\overline{a}\widetilde{G}(\widetilde{z_n},\widetilde{z},\widetilde{z})+\overline{2}\overline{a}\widetilde{G}(\widetilde{y_n},\widetilde{y},\widetilde{y})+\overline{2}\overline{a}\widetilde{G}(T\widetilde{y},T\widetilde{y_n},T\widetilde{y_n})\\+a\widetilde{G}(\widetilde{z},T\widetilde{z},T\widetilde{z})+\overline{2}\overline{a}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y}).\end{aligned}\tag{3.21}$$

Similarly, we obtain

$$\begin{aligned}\widetilde{G}(T\widetilde{x},T\widetilde{y},T\widetilde{y})\widetilde{\leq}(\overline{1}+\overline{a})\widetilde{G}(T\widetilde{x},T\widetilde{x_n},T\widetilde{x_n})+\widetilde{G}(\widetilde{y_n},T\widetilde{y},T\widetilde{y})\\+\overline{a}\widetilde{G}(\widetilde{x_n},\widetilde{x},\widetilde{x})+\overline{2}\overline{a}\widetilde{G}(\widetilde{y_n},\widetilde{y},\widetilde{y})+\overline{2}\overline{a}\widetilde{G}(T\widetilde{y},T\widetilde{y_n},T\widetilde{y_n})\\+a\widetilde{G}(\widetilde{x},T\widetilde{x},T\widetilde{x})+\overline{2}\overline{a}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y}).\end{aligned}\tag{3.22}$$

Hence, from (3.21) and (3.22), we get

$$\begin{aligned}\widetilde{G}(T\widetilde{x},T\widetilde{y},T\widetilde{z})&\widetilde{\leq}\widetilde{G}(T\widetilde{x},T\widetilde{y},T\widetilde{y})+\widetilde{G}(T\widetilde{z},T\widetilde{y},T\widetilde{y})\\&\widetilde{\leq}\{(\overline{1}+\overline{a})\widetilde{G}(T\widetilde{x},T\widetilde{x_n},T\widetilde{x_n})+\widetilde{G}(\widetilde{y_n},T\widetilde{y},T\widetilde{y})\\&+\overline{a}\widetilde{G}(\widetilde{x_n},\widetilde{x},\widetilde{x})+\overline{2}\overline{a}\widetilde{G}(\widetilde{y_n},\widetilde{y},\widetilde{y})+\overline{2}\overline{a}\widetilde{G}(T\widetilde{y},T\widetilde{y_n},T\widetilde{y_n})\\&+a\widetilde{G}(\widetilde{x},T\widetilde{x},T\widetilde{x})+\overline{2}\overline{a}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y})\}\\&+\{(\overline{1}+\overline{a})\widetilde{G}(T\widetilde{z},T\widetilde{z_n},T\widetilde{z_n})+\widetilde{G}(T\widetilde{y_n},T\widetilde{y},T\widetilde{y})\\&+\overline{a}\widetilde{G}(\widetilde{z_n},\widetilde{z},\widetilde{z})+\overline{2}\overline{a}\widetilde{G}(\widetilde{y_n},\widetilde{y},\widetilde{y})+\overline{2}\overline{a}\widetilde{G}(T\widetilde{y},T\widetilde{y_n},T\widetilde{y_n})\\&+\overline{a}\widetilde{G}(\widetilde{z},T\widetilde{z},T\widetilde{z})+\overline{2}\overline{a}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y})\}.\end{aligned}$$

Taking the limit as n $\to\infty$, we obtain

$$\widetilde{G}(T\widetilde{x},T\widetilde{y},T\widetilde{z})\widetilde{\leq}\overline{a}\{\widetilde{G}(\widetilde{x},T\widetilde{x},T\widetilde{x})+\overline{4}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y})+\widetilde{G}(\widetilde{z},T\widetilde{z},T\widetilde{z})\},$$

since T is soft G-continuous.

Case 2: For $\widetilde{x}$, $\widetilde{y}\in SE(\widetilde{M})$, $\widetilde{z}\in SE(\widetilde{X\setminus M})$, let $(\widetilde{z_n})$ be a sequence in $\widetilde{M}$ such that $(\widetilde{z_n})\to\widetilde{z}$. Then by axiom ($\widetilde{G}_5$), we have

$$\widetilde{G}(T\widetilde{x},T\widetilde{y},T\widetilde{z})\widetilde{\leq}\widetilde{G}(T\widetilde{x},T\widetilde{y},T\widetilde{y})+\widetilde{G}(T\widetilde{z},T\widetilde{y},T\widetilde{y}).$$

By (i), we obtain

$$\widetilde{G}(T\widetilde{x},T\widetilde{y},T\widetilde{y})\widetilde{\leq}\overline{a}\{\widetilde{G}(\widetilde{x},T\widetilde{x},T\widetilde{x})+\overline{2}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y})\}.\tag{3.23}$$

Again by axiom ($\widetilde{G}_5$), we have

$$\widetilde{G}(T\widetilde{z},T\widetilde{y},T\widetilde{y})\widetilde{\leq}\widetilde{G}(T\widetilde{z},T\widetilde{z_n},T\widetilde{z_n})+\widetilde{G}(T\widetilde{z_n},T\widetilde{y},T\widetilde{y}).\tag{3.24}$$

By (i) and ($\widetilde{G}_5$), we get

$$\widetilde{G}(T\widetilde{z_n},T\widetilde{y},T\widetilde{y})\widetilde{\leq}\overline{a}\{\widetilde{G}(\widetilde{z_n},T\widetilde{z_n},T\widetilde{z_n})+\overline{2}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y})\}\tag{3.25}$$

and

$$\widetilde{G}(\widetilde{z_n},T\widetilde{z_n},T\widetilde{z_n})\widetilde{\leq}\widetilde{G}(\widetilde{z_n},\widetilde{z},\widetilde{z})+\widetilde{G}(\widetilde{z},T\widetilde{z},T\widetilde{z})+\widetilde{G}(T\widetilde{z},T\widetilde{z_n},T\widetilde{z_n}).\tag{3.26}$$

By inequalities (3.23), (3.24), (3.25) and (3.26), we obtain

$$\begin{aligned}\widetilde{G}(T\widetilde{x},T\widetilde{y},T\widetilde{z})\widetilde{\leq}\overline{a}\widetilde{G}(\widetilde{x},T\widetilde{x},T\widetilde{x})+\overline{2}\overline{a}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y})+\overline{a}\widetilde{G}(\widetilde{z_n},\widetilde{z},\widetilde{z})+\overline{a}\widetilde{G}(\widetilde{z},T\widetilde{z},T\widetilde{z})\\+\overline{a}\widetilde{G}(T\widetilde{z},T\widetilde{z_n},T\widetilde{z_n})+\widetilde{G}(T\widetilde{z},T\widetilde{z_n},T\widetilde{z_n})+\overline{2}\overline{a}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y}).\end{aligned}$$

Now letting n $\to\infty$ in the previous inequality, we get

$$\widetilde{G}(T\widetilde{x},T\widetilde{y},T\widetilde{z})\widetilde{\leq}\overline{a}\{\widetilde{G}(\widetilde{x},T\widetilde{x},T\widetilde{x})+\overline{4}\widetilde{G}(\widetilde{y},T\widetilde{y},T\widetilde{y})+\widetilde{G}(\widetilde{z},T\widetilde{z},T\widetilde{z})\}.$$

Case 3: For $\widetilde{y} \in SE(\widetilde{M})$, $\widetilde{x}$, $\widetilde{z} \in SE(\widetilde{X\backslash M})$, let $(\widetilde{x_n})$ and $(\widetilde{z_n})$ be sequences in $\widetilde{M}$ such that $(\widetilde{x_n}) \to \widetilde{x}$ and $(\widetilde{z_n}) \to \widetilde{z}$. Then by axiom $(\widetilde{G}_5)$, we have

$$\widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{z}) \widetilde{\leq} \widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{y}) + \widetilde{G}(T\widetilde{z}, T\widetilde{y}, T\widetilde{y}) \tag{3.27}$$

and

$$\widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{y}) \widetilde{\leq} \widetilde{G}(T\widetilde{x}, T\widetilde{x_n}, T\widetilde{x_n}) + \widetilde{G}(T\widetilde{x_n}, T\widetilde{y}, T\widetilde{y}). \tag{3.28}$$

Also

$$\widetilde{G}(T\widetilde{x_n}, T\widetilde{y}, T\widetilde{y}) \widetilde{\leq} \overline{a}\{\widetilde{G}(\widetilde{x_n}, T\widetilde{x_n}, T\widetilde{x_n}) + \overline{2}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y})\} \tag{3.29}$$

and

$$\widetilde{G}(\widetilde{x_n}, T\widetilde{x_n}, T\widetilde{x_n}) \widetilde{\leq} \widetilde{G}(\widetilde{x_n}, \widetilde{x}, \widetilde{x}) + \widetilde{G}(\widetilde{x}, T\widetilde{x}, T\widetilde{x}) + \widetilde{G}(T\widetilde{x}, T\widetilde{x_n}, T\widetilde{x_n}). \tag{3.30}$$

Thus by (3.29) and (3.30), we obtain

$$\begin{aligned} \widetilde{G}(T\widetilde{x_n}, T\widetilde{y}, T\widetilde{y}) \widetilde{\leq} & \overline{a}\widetilde{G}(\widetilde{x_n}, \widetilde{x}, \widetilde{x}) + \overline{a}\widetilde{G}(\widetilde{x}, T\widetilde{x}, T\widetilde{x}) \\ & + \overline{a}\widetilde{G}(T\widetilde{x}, T\widetilde{x_n}, T\widetilde{x_n}) + \overline{2}\overline{a}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}). \end{aligned} \tag{3.31}$$

So from (3.27) and (3.31), we have

$$\begin{aligned} \widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{y}) \widetilde{\leq} & \overline{a}\widetilde{G}(\widetilde{x_n}, \widetilde{x}, \widetilde{x}) + \overline{a}\widetilde{G}(\widetilde{x}, T\widetilde{x}, T\widetilde{x}) \\ & + (\overline{1} + \overline{a})\widetilde{G}(T\widetilde{x}, T\widetilde{x_n}, T\widetilde{x_n}) + \overline{2}\overline{a}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}). \end{aligned} \tag{3.32}$$

Similarly, we obtain

$$\begin{aligned} \widetilde{G}(T\widetilde{z}, T\widetilde{y}, T\widetilde{y}) \widetilde{\leq} & \overline{a}\widetilde{G}(\widetilde{z_n}, \widetilde{z}, \widetilde{z}) + \overline{a}\widetilde{G}(\widetilde{z}, T\widetilde{z}, T\widetilde{z}) \\ & + (\overline{1} + \overline{a})\widetilde{G}(T\widetilde{z}, T\widetilde{z_n}, T\widetilde{z_n}) + \overline{2}\overline{a}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}). \end{aligned} \tag{3.33}$$

Hence from (3.32) and (3.33), we have

$$\begin{aligned} \widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{z}) & \widetilde{\leq} \widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{y}) + \widetilde{G}(T\widetilde{z}, T\widetilde{y}, T\widetilde{y}) \\ & \widetilde{\leq} \overline{a}\widetilde{G}(\widetilde{x_n}, \widetilde{x}, \widetilde{x}) + \overline{a}\widetilde{G}(\widetilde{x}, T\widetilde{x}, T\widetilde{x}) \\ & \quad + (\overline{1} + \overline{a})\widetilde{G}(T\widetilde{x}, T\widetilde{x_n}, T\widetilde{x_n}) + \overline{2}\overline{a}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}) \\ & \quad + \overline{a}\widetilde{G}(\widetilde{z_n}, \widetilde{z}, \widetilde{z}) + \overline{a}\widetilde{G}(\widetilde{z}, T\widetilde{z}, T\widetilde{z}) \\ & \quad + (\overline{1} + \overline{a})\widetilde{G}(T\widetilde{z}, T\widetilde{z_n}, T\widetilde{z_n}) + \overline{2}\overline{a}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}). \end{aligned}$$

Now letting n $\to \infty$ in the previous inequality, we get

$$\widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{z}) \widetilde{\leq} \overline{a}\{\widetilde{G}(\widetilde{x}, T\widetilde{x}, T\widetilde{x}) + \overline{4}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}) + \widetilde{G}(\widetilde{z}, T\widetilde{z}, T\widetilde{z})\},$$

since T is soft G-continuous. Then, in all case, we have for any $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z} \in SE(\widetilde{X})$,

$$\widetilde{G}(T\widetilde{x}, T\widetilde{y}, T\widetilde{z}) \widetilde{\leq} \overline{a}\widetilde{G}(\widetilde{x}, T\widetilde{x}, T\widetilde{x}) + \overline{b}\widetilde{G}(\widetilde{y}, T\widetilde{y}, T\widetilde{y}) + \overline{c}\widetilde{G}(\widetilde{z}, T\widetilde{z}, T\widetilde{z}),$$

where $\overline{c} = \overline{a}$, $\overline{b} = \overline{4}\overline{a}$, and $\overline{0} \widetilde{<} \overline{a} + \overline{b} + \overline{c} \widetilde{<} \overline{1}$, since $\overline{0} \widetilde{<} \overline{a} \widetilde{<} \overline{\frac{1}{6}}$. Thus, T has a unique fixed point by Theorem 2.24. □

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