Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 12, No. 5, (November 2016), pp. 659–668
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Some properties of generalized fuzzy hyperconnected spaces

J. Chakraborty, B. Bhattacharya, A. Paul

Received 7 March 2016; Revised 29 April 2016; Accepted 20 May 2016

Abstract. The aim of this paper is to introduce the notion of generalized fuzzy hyperconnected space and to study the basic difference of it with generalized hyperconnected space due to Ekici [10]. It is proved that, 0_X and 1_X are the only fuzzy g_X-regular open sets in the new space. We also study the applications of generalized fuzzy hyperconnectedness by using some functions and establish the interrelationships among them.

2010 AMS Classification: 54A40, 54CO5, 54C08

Keywords: Generalized fuzzy hyperconnected space, Fuzzy g_X-regular open set, Fuzzy g_X-dense set, Fuzzy g_X-nowhere dense set, Fuzzy feebly (g_X, g_Y)-continuity.

Corresponding Author: J. Chakrabortyr (chakrabortyjayasree1@gmail.com)

1. Introduction

The concept of hyperconnectedness in a topological space was introduced by Steen and Seebach [16] in 1978. Then in 1992, Ajmal et al. [1] studied some of the characterizations and basic properties of the hyperconnected space. Thereafter, several authors devoted their work to study more properties of hyperconnectedness in a topological space [7, 11, 12]. In 2002, Császár [8] introduced the notions of generalized neighborhood systems and generalized topological space (in short, GTS). Moreover, Palani Chetty [13] extended the concept of generalized topological space in fuzzy setting and named it generalized fuzzy topological space (in short, GFTS), in 2008.

In 2011, Ekici first introduced the concept of generalized hyperconnectedness in GTS [10]. He also discussed some basic properties and preservation of generalized hyperconnectedness. Further, Renukadevi [14] independently introduced some more results in generalized hyperconnected spaces.

So there is a scope to define and study the fundamental properties of generalized fuzzy hyperconnectedness in GFTS. We emphasize on one result based on generalized fuzzy hyperconnectedness which indicates a comparison between GFTS and GTS. Finally, in this paper we define fuzzy feebly (g_X, g_Y)-continuous function and constitute its characterizations based on generalized fuzzy hyperconnectedness along with other functions namely fuzzy (σ, g_Y)-continuous functions and fuzzy almost (g_X, g_Y)-continuous functions.

In section 3, we establish some equivalent forms of generalized fuzzy hyperconnectedness in terms of fuzzy g_X-regular open set and fuzzy g_X-preopen set. Actually fuzzy g_X-preclosed and generalized fuzzy g_X-closed set are two generalized forms of fuzzy g_X-closed set. But here we show that a fuzzy g_X-preclosed set turns into a generalized fuzzy g_X-closed set in the perspective of generalized fuzzy hyperconnectedness even though they are independent of each other.

In section 4, we study the applications of generalized fuzzy hyperconnectedness by introducing fuzzy feebly (g_X, g_Y)-continuous function. In [6], it is shown that fuzzy almost (g_X, g_Y)-continuous function and fuzzy (σ, g_Y)-continuous function are independent of each other. But here we show that under some suitable condition in the field of hyperconnectedness fuzzy (σ, g_Y)-continuous function becomes fuzzy almost (g_X, g_Y)-continuous function.

2. Preliminaries

Let X be a nonempty set and g_X be a collection of fuzzy subsets of X. Then g_X is called a generalized fuzzy topology on X iff $0_X \in g_X$ and $G_i \in g_X$ for $i \in I \neq \phi$ implies $G = \bigvee_{i\in I} G_i \in g_X$. The pair (X, g_X) is called a GFTS on X. The elements of g_X are called fuzzy g_X-open sets and their complements are called fuzzy g_X-closed sets. The collection of all fuzzy g_X-open sets of X and the collection of all fuzzy g_X-closed sets of X are denoted by GFO(X, g_X) and GFC(X, g_X) respectively.

Here we denote the g_X-closure of a fuzzy subset λ of X, by $c_{g_X}(\lambda)$, defined to be the intersection of all fuzzy g_X-closed sets including λ and the g_X-interior of λ, denoted by $i_{g_X}(\lambda)$, defined as the union of all fuzzy g_X-open sets contained in λ. The complement of a fuzzy set λ is denoted by 1_X-λ.

We consider the following recapitulation as the ready references for the subsequent two contributing sections as our main results.

Definition 2.1 ([6]). Let (X, g_X) be a GFTS. A fuzzy set λ is called a
(i) fuzzy g_X-semiopen set, if $\lambda \leq c_{g_X} i_{g_X}(\lambda)$,
(ii) fuzzy g_X-preopen set, if $\lambda \leq i_{g_X} c_{g_X}(\lambda)$,
(iii) fuzzy g_X-regular open set, if $\lambda = i_{g_X} c_{g_X}(\lambda)$.

The complement of fuzzy g_X-semiopen, fuzzy g_X-preopen and fuzzy g_X-regular open sets are called fuzzy g_X-semiclosed, fuzzy g_X-preclosed and fuzzy g_X-regular closed sets respectively.

Definition 2.2 ([10]). Let (X, μ) be a GTS and $G \subset X$.
(i) G is called μ-dense, if $c_\mu(G) = X$,
(ii) G is called μ-nowhere dense, if $i_\mu c_\mu(G) = \phi$,
(iii) (X, μ) is called hyperconnected, if G is μ-dense, for every μ-open subset $G \neq \phi$ of (X, μ).

Theorem 2.3 ([10]). *Let (X, μ) be a GTS. The following properties are equivalent:*
(1) (X, μ) *is hyperconnected,*

(2) *G is μ-dense or μ-nowhere dense for every subset G of (X, μ),*
(3) *$G \cap H \neq \phi$, for every nonempty μ-open subsets G and H of (X, μ).*

Definition 2.4 ([15])**.** A GTS (X, μ) is said to be connected, if X cannot be written as the union of nonempty and disjoint μ-open sets G and H of (X, μ).

Definition 2.5 ([9])**.** A GTS (X, μ) is said to be an extremally disconnected (for short, ED) space, if the μ-closure of each μ-open set is μ-open.

Definition 2.6 ([2])**.** A function $f : X \to Y$ from a topological space X into a topological space Y is said to be feebly continuous, if for each open set V of Y, $f^{-1}(V) \neq \phi$ implies that $int(f^{-1}(V)) \neq \phi$.

Definition 2.7 ([5])**.** A function $f : (X, g_X) \to (Y, g_Y)$ from a GFTS to another GFTS is called fuzzy contra (g_X, g_Y)-continuous, if for each fuzzy g_Y-open set λ in Y, $f^{-1}(\lambda)$ is fuzzy g_X-closed in X.

Definition 2.8 ([6])**.** Let (X, g_X) and (Y, g_Y) be GFTS's. Then a function $f : (X, g_X) \to (Y, g_Y)$ is called a fuzzy almost (g_X, g_Y)-continuous, if $f^{-1}(\mu) \in g_X$, for each fuzzy g_Y-regular open set μ of Y.

Definition 2.9 ([6])**.** A function $f : (X, g_X) \to (Y, g_Y)$ from a GFTS (X, g_X) into another GFTS (Y, g_Y) is said to be fuzzy (σ, g_Y)-continuous, if for each fuzzy g_Y-open set μ in Y, $f^{-1}(\mu)$ is a fuzzy g_X-semiopen in X.

3. Characterizations of generalized fuzzy hyperconnectedness

Definition 3.1. A fuzzy set λ of a GFTS (X, g_X) is called fuzzy g_X-dense, if $c_{g_X}(\lambda) = 1_X$.

Definition 3.2. A GFTS (X, g_X) is called generalized fuzzy hyperconnected, if for every fuzzy g_X-open subset $\mu \neq 0_X$ of (X, g_X) is fuzzy g_X-dense.

Definition 3.3. A GFTS (X, g_X) is said to be generalized fuzzy connected, if 1_X cannot be written as the union of two nonempty disjoint fuzzy g_X-open sets of (X, g_X).

Definition 3.4. A GFTS (X, g_X) is said to be generalized fuzzy extremally disconnected, if the fuzzy g_X-closure of every fuzzy g_X-open set is fuzzy g_X-open.

Example 3.5. Let $X = \{a,b,c\}$ and $g_X = \{$*0_X,(a,0.5),(b,0.2),(c,0.7) ,(a,0.4), (b,0.6),(c,0.5),(a,0.5),(b,0.6),(c,0.7).* Then the GFTS (X, g_X) is generalized fuzzy hyperconnected.

Remark 3.6. In a GFTS (X, g_X), generalized fuzzy hyperconnectedness implies generalized fuzzy connectedness.

However the converse of the above remark may not be true in general which is demonstrated in the following example.

Example 3.7. Let $X = \{$ a,b,c$\}$ and $g_X = \{$*0_X*,$\{$*(a,0.5),(b,0.3),(c,0.2)*$\}$,$\{$*(a,0.4),(b,0.4), (c,0.1)*$\}$,$\{$*(a,0.5),(b,0.4),(c,0.2)*$\}\}$. Then one can easily verify that the GFTS (X, g_X) is generalized fuzzy connected but not generalized fuzzy hyperconnected.

It is well known that, in a general topological space, hyperconnectedness imply extremally disconnectedness. But it is not true in our context which can be easily verified with Example 3.5. However, the next theorem gives the restriction where generalized fuzzy hyperconnected space becomes generalized fuzzy extremally disconnected space.

Theorem 3.8. *Let (X, g_X) be a GFTS with $c_{g_X}(0_X) = 0_X$. If (X, g_X) is a generalized fuzzy hyperconnected space, then it is a generalized fuzzy extremally disconnected space.*

Proof. Let us suppose that (X, g_X) is a generalized fuzzy hyperconnected space with $c_{g_X}(0_X) = 0_X$. Then for each fuzzy g_X-open set μ, we have $c_{g_X}(\mu) = 1_X$ and $1_X \in g_X$. It implies that $c_{g_X}(\mu)$ is a fuzzy g_X-open set. Thus (X, g_X) is a generalized fuzzy extremally disconnected space. □

Though the converse of the above theorem may not be true in general which is verified in the following example.

Example 3.9. Let $X = \{a,b,c\}$ and $g_X = \{0_X, \{(a,0.5),(b,0.3),(c,0.4)\}, \{(a,0.5),(b,0.7),(c,0.6)\}, \{(a,0.3),(b,0.7),(c,0.2)\}, \{(a,0.5),(b,0.7),(c,0.4)\}\}$. Then the GFTS (X, g_X) is generalized fuzzy extremally disconnected space but not generalized fuzzy hyperconnected space.

Definition 3.10. A fuzzy set λ of a GFTS (X, g_X) is called fuzzy g_X-nowhere dense, if $i_{g_X} c_{g_X}(\lambda) = 0_X$.

Proposition 3.11. *In generalized fuzzy hyperconnected space (X, g_X), every fuzzy subset is either fuzzy g_X-dense set or fuzzy g_X-nowhere dense set.*

Proof. Let (X, g_X) be a generalized fuzzy hyperconnected space and $\lambda \leq 1_X$. Suppose that λ is not fuzzy g_X-nowhere dense. Then

$$c_{g_X}(1_X - c_{g_X}(\lambda)) = 1_X - i_{g_X}(c_{g_X}(\lambda)).$$

Since $i_{g_X} c_{g_X}(\lambda) \neq 0_X$, $c_{g_X}(i_{g_X} c_{g_X}(\lambda)) = 1_X$. Since $c_{g_X}(i_{g_X} c_{g_X}(\lambda)) = 1_X \leq c_{g_X}(\lambda)$, $c_{g_X}(\lambda) = 1_X$. Thus λ is a fuzzy g_X-dense. □

Ekici [10] has shown that a GTS (X, g_X) is hyperconnected iff every pair of nonempty μ-open sets has nonempty intersection. But we claim that the result given by Ekici [10] is partially true in the context of GFTS and that is being established in the next proposition.

Proposition 3.12. *In a generalized fuzzy hyperconnected space (X, g_X), every pair of nonempty fuzzy g_X-open subsets has a nonempty intersection.*

Proof. Suppose that $\lambda \wedge \mu = 0_X$, for some nonempty fuzzy g_X-open subsets λ and μ of (X, g_X). This implies that $c_{g_X}(\lambda) \wedge \mu = 0_X$ and λ is not a fuzzy g_X-dense set. Since λ is fuzzy g_X-open, $0_X \neq \lambda \leq i_{g_X}(c_{g_X}(\lambda))$ and λ is not a fuzzy g_X-nowhere dense set. This is contradicting the statement of Proposition 3.11. Thus, $\lambda \wedge \mu \neq 0_X$, for every pair of nonempty fuzzy g_X-open subsets λ and μ of (X, g_X). □

Though the condition stated above is only the necessary condition for generalized fuzzy hyperconnectedness but not sufficient, which is justified in the following example.

Example 3.13. Let $X = \{a,b\}$ and $g_X = \{0_X, \{(a,0.5),(b,0.3)\}, \{(a,0.4),(b,0.7)\}, \{(a,0.5),(b,0.7)\}\}$. Here every pair of nonempty fuzzy g_X-open subsets has nonempty intersection but the GFTS (X, g_X) is not a generalized fuzzy hyperconnected.

Theorem 3.14. *Let (X, g_X) be a GFTS with $c_{g_X}(0_X) = 0_X$. Then the following properties are equivalent:*
(1) *(X, g_X) is generalized fuzzy hyperconnected,*
(2) *1_X and 0_X are the only fuzzy g_X-regular open sets in X.*

Proof. (1) $\Rightarrow$ (2): Let (X, g_X) be a generalized fuzzy hyperconnected space. Suppose that μ is a proper fuzzy g_X-regular open set. Then we have $\mu = i_{g_X} c_{g_X}(\mu)$. This implies that $1_X - i_{g_X} c_{g_X}(\mu) = c_{g_X}(1_X - c_{g_X}(\mu)) = 1_X - \mu \neq 1_X$, where $\mu \neq 0_X$. This is a contradiction. Thus, 1_X and 0_X are the only fuzzy g_X-regular open sets in X.

(2) $\Rightarrow$ (1): Let 1_X and 0_X are the only fuzzy g_X-regular open sets in X. If possible, suppose that X is not generalized fuzzy hyperconnected. This implies that there exist a nonempty fuzzy g_X-open subset μ of X such that $c_{g_X}(\mu) \neq 1_X$. Then we have $c_{g_X} i_{g_X}(\mu) \neq 1_X$. This implies that $c_{g_X} i_{g_X}(\mu) = 0_X$, since the only fuzzy g_X-regular open sets in X are 1_X and 0_X. Thus we have $c_{g_X}(\mu) = 0_X$, where $\mu \neq 0_X$ which is not possible. So, the GFTS (X, g_X) is a generalized fuzzy hyperconnected. □

Proposition 3.15. *In a generalized fuzzy hyperconnected space (X, g_X), any fuzzy subset λ is fuzzy g_X-semiopen set if $i_{g_X}(\lambda) \neq 0_X$.*

Proof. Let X be a generalized fuzzy hyperconnected space and λ be any fuzzy subset of X, where $i_{g_X}(\lambda) \neq 0_X$. Then $c_{g_X} i_{g_X}(\lambda) = 1_X$. Thus $\lambda \leq c_{g_X} i_{g_X}(\lambda)$. So, λ is a fuzzy g_X-semiopen set. □

Azad [3] had pointed out that the collection of all fuzzy semiopen sets falls short to form the structure of fuzzy topology. Nevertheless it is easy to verify that the intersection of fuzzy semiopen sets is a fuzzy semiopen set in a fuzzy hyperconnected space.

On the other hand, the fuzzy g_X-semiopen sets resume their characterization with respect to intersection in a generalized fuzzy hyperconnected space as it is in a GFTS, which is clarified in the following example.

Example 3.16. Let $X = \{a,b,c\}$ and $g_X = \{0_X, 1_X, \{(a,0.5),(b,0.2),(c,0.6)\}, \{(a,0.4),(b,0.3),(c,0.6)\}, \{(a,0.5),(b,0.3),(c,0.6)\}\}$. Here $\{(a,0.5),(b,0.2),(c,0.6)\}$ *and* $\{(a,0.4),(b,0.3),(c,0.6)\}$ are fuzzy fuzzy g_X-semiopen sets in generalized fuzzy hyperconnected space (X, g_X). But their intersection is not a fuzzy g_X-semiopen set.

Definition 3.17. A fuzzy set λ in a GFTS (X, g_X) is called generalized fuzzy g_X-closed (briefly, gfg_X-closed), if $c_{g_X}(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and $\mu \in$ GFO (X, g_X).

Theorem 3.18. *Let (X, g_X) be a GFTS. Then the following conditions are equivalent:*
(1) *(X, g_X) is generalized fuzzy hyperconnected,*
(2) *every fuzzy g_X-preopen set is fuzzy g_X-dense.*

Proof. (1) ⇒ (2): Let us suppose that λ is any fuzzy g_X-preopen set. This implies that $\lambda \leq i_{g_X} c_{g_X}(\lambda)$. As a result, from (1), we have $c_{g_X}(\lambda) = c_{g_X}(i_{g_X} c_{g_X}(\lambda)) = 1_X$. Thus λ is fuzzy g_X-dense.

(2) ⇒ (1): Let us suppose that λ be any nonempty fuzzy g_X-preopen set. Then $\lambda \leq i_{g_X} c_{g_X}(\lambda)$. Thus from (2), we have λ is fuzzy g_X-dense. So

$$c_{g_X}(\lambda) = c_{g_X}(i_{g_X} c_{g_X}(\lambda)) = 1_X.$$

It means that the GFTS (X, g_X) is generalized fuzzy hyperconnected. □

Proposition 3.19. *Every fuzzy g_X-nowhere dense set is a gfg_X-closed set.*

Proof. Let λ be any fuzzy g$_X$-dense set in a GFTS (X, g_X). Then $i_{g_X} c_{g_X}(\lambda) = 0_X$ and there does not exist any fuzzy g_X-open set in between λ and $c_{g_X}(\lambda)$. Also let us suppose that $\lambda \leq \mu$, where μ is fuzzy g_X-open set. Then obviously $c_{g_X}(\lambda) \leq \mu$. Thus λ is gfg_X-closed set. □

But the reverse implication of the above stated proposition is not true in general which is verified in the following example.

Example 3.20. Let $X = \{a,b,c\}$ and $g_X = \{0_X, \{(a,0.5),(b,0.4),(c,0.5)\}, \{(a,0.5),(b,0.7),(c,0.5)\}\}$. Here $\{(a,0.5),(b,0.5),(c,0.5)\}$ is a gfg_X-closed set but it is not a fuzzy g_X-nowhere dense set.

Remark 3.21. Every fuzzy g_X-closed set is a fuzzy g_X-preclosed set.

On the other hand, contrary of the above remark is not true which is justified in the next example.

Example 3.22. Let $X = \{a,b,c\}$ *and* $g_X = \{0_X, \{(a,0.4),(b,0.2),(c,0.3)\}, \{(a,0.6),(b,0.4),(c,0.7)\}, \{(a,0.6),(b,0.7),(c,0.7)\}\}$. Here $\{(a,0.4),(b,0.6),(c,0.7)\}$ is a fuzzy g_X-preclosed set but not a fuzzy g_X-closed set.

Remark 3.23. Every fuzzy g_X-closed set is a gfg_X-closed set.

But the opposite implication of the above remark is not true which can be justified from Example 3.20.

We demonstrate following examples to reflect upon the independentness of fuzzy g_X-preclosed set and gfg_X-closed set in a GFTS.

Example 3.24. In Example 3.20, $\{(a,0.4),(b,0.6),(c,0.7)\}$ is fuzzy -preclosed set but not gfg_X-closed set.

Example 3.25. Let $X = \{a,b,c\}$ and $g_X = \{0_X, \{(a,0.5),(b,0.7),(c,0.4)\}, \{(a,0.5),(b,0.6),(c,0.7)\}, \{(a,0.5),(b,0.7),(c,0.7)\}\}$. Here $\{(a,0.5),(b,0.3),(c,0.2)\}$ is a gfg_X-closed set but not a fuzzy g_X-preclosed set.

Corollary 3.26. *A GFTS (X, g_X) is generalized fuzzy hyperconnected iff every fuzzy g_X-preclosed set is fuzzy g_X-nowhere dense.*

Closeness is a sufficient condition for a set to be a gfg_X-closed set and fuzzy g_X-preclosed set though there is no relation amongst this two sets. We conclude this section by the following result which establishes a relation between gfg_X-closed set and fuzzy g_X-preclosed set.

Proposition 3.27. *In a generalized fuzzy hyperconnected space, every fuzzy g_X-preclosed set is gfg_X-closed set.*

Proof. In a generalized fuzzy hyperconnected space, every fuzzy g_X-preclosed set is fuzzy g_X-nowhere dense using Theorem 3.18. Then from Proposition 3.19, we have, every fuzzy g_X-nowhere dense set is gfg_X-closed set. Thus every fuzzy g_X-preclosed set is a gfg_X-closed set. □

4. Few results on related functions and their interrelationships

In this section, we consider certain mappings between two GFTS's and study their behavior when either or both the domain and codomain spaces are replaced by generalized fuzzy hyperconnected spaces. In particular, we obtain the situation where fuzzy (σ, g_Y)-continuous function is fuzzy almost (g_X, g_Y)-continuous function though they are independent in nature.

Definition 4.1. A function $f : (X, g_X) \to (Y, g_Y)$ from a GFTS into another GFTS is said to be fuzzy feebly (g_X, g_Y)-continuous, if for every fuzzy g_X-open set λ of Y, $f^{-1}(\lambda) \neq 0_X$ implies that $i_{g_X}(f^{-1}(\lambda)) \neq 0_X$.

Theorem 4.2. *Fuzzy feebly (g_X, g_Y)-continuous surjection preserves generalized fuzzy hyperconnectedness.*

Proof. Let $f : (X, g_X) \to (Y, g_Y)$ be a fuzzy febbly (g_X, g_Y)-continuous surjection and let X be a generalized fuzzy hyperconnected space. Suppose that Y is not generalized fuzzy hyperconnected space. Then there exist two nonempty fuzzy g_Y-open sets λ_1 and λ_2 (say) such that $\lambda_1 \wedge \lambda_2 = 0_Y$. It means that $f^{-1}(\lambda_1) \wedge f^{-1}(\lambda_2) = 0_X$. By the hypothesis, if $f^{-1}(\lambda_1) \neq 0_X$ and $f^{-1}(\lambda_2) \neq 0_X$, then $i_{g_X} f^{-1}(\lambda_1) \neq 0_X$ and $i_{g_X} f^{-1}(\lambda_2) \neq 0_X$.

On the otherhand, $f^{-1}(\lambda_1) \wedge f^{-1}(\lambda_2) = 0_X$. Then $i_{g_X}(f^{-1}(\lambda_2)) \wedge i_{g_X}(f^{-1}(\lambda_2)) = 0_X$. Thus X is not generalized fuzzy hyperconnected which is a contradiction. So Y is a generalized fuzzy hyperconnected space. □

Theorem 4.3. *Every fuzzy (σ, g_Y)-continuous function $f : (X, g_X) \to (Y, g_Y)$ is a fuzzy feebly (g_X, g_Y)-continuous function, where $c_{g_X}(0_X) = 0_X$.*

Proof. Let $f : (X, g_X) \to (Y, g_Y)$ be a fuzzy (σ, g_Y)-continuous function, where $c_{g_X}(0_X) = 0_X$. Suppose that λ is a fuzzy g_Y-open subset of (Y, g_Y) such that $f^{-1}(\lambda) \neq 0_X$. This implies that $f^{-1}(\lambda)$ is a nonempty fuzzy g_X-semiopen set in (X, g_X). Since $f^{-1}(\lambda) \neq 0_X$, $i_{g_X} f^{-1}(\lambda) \neq 0_X$. Then, $f : (X, g_X) \to (Y, g_Y)$ is a fuzzy feebly (g_X, g_Y)-continuous. □

Remark 4.4. However, the converse of the above theorem may not be true always.

Example 4.5. Let $X = Y = \{a,b,c\}$, $g_X = \{0_X, 1_X, \{(a,0.5),(b,0.2),(c,0.6)\}, \{(a,0.4),(b,0.3),(c,0.6)\}, \{(a,0.5),(b,0.3),(c,0.6)\}\}$ and $g_Y = \{0_Y, \{(a,0.5),(b,0.3),(c,0.6)\}\}$. Here $f : (X, g_X) \to (Y, g_Y)$, defined by $f(a) = a$, $f(b) = b$, $f(c) = c$ is a fuzzy feebly (g_X, g_Y)-continuous function but not a fuzzy (σ, g_Y)-continuous function as $f^{-1}\{(a,0.5),(b,0.3),(c,0.6)\} = \{(a,0.5),(b,0.3),(c,0.6)\}$ is not a fuzzy g_X-semiopen set in X.

Proposition 4.6. *Let (X, g_X) be a generalized fuzzy hyperconnected space, where $c_{g_X}(0_X) = 0_X$. If $f : (X, g_X) \to (Y, g_Y)$ is a fuzzy feebly (g_X, g_Y)-continuous function, then f is fuzzy (σ, g_Y)-continuous function.*

Proof. Let $f : (X, g_X) \to (Y, g_Y)$ be a fuzzy feebly (g_X, g_Y)-continuous function and let (X, g_X) be a generalized fuzzy hyperconnected space. Suppose that λ is a fuzzy g_X-open subset of (Y, g_Y) such that $f^{-1}(\lambda) \neq 0_X$. Then $i_{g_X}(f^{-1}(\lambda)) \neq 0_X$. Since (X, g_X) is a generalized fuzzy hyperconnected space, $i_{g_X}(f^{-1}(\lambda))$ is a fuzzy g_X-semiopen set in (X, g_X). Thus, f is a fuzzy (σ, g_Y)-continuous function. □

Proposition 4.7. *Every fuzzy almost (g_X, g_Y)-continuous function is a fuzzy contra (g_X, g_Y)-continuous function.*

Proof. Let f be any fuzzy almost (g_X, g_Y)-continuous function from X to Y. Then from the definition of fuzzy almost (g_X, g_Y)-continuity, we have $f(\lambda) \leq i_{g_Y} c_{g_Y}(\mu)$, where λ and μ are fuzzy g_X-open and g_Y-open sets in X and Y, respectively. Thus $f(\lambda) \leq c_{g_Y}(\mu)$. So, f is a fuzzy contra (g_X, g_Y)-continuous function. □

Remark 4.8. But any fuzzy contra (g_X, g_Y)-continuous function may not be a fuzzy almost (g_X, g_Y)-continuous function.

Example 4.9. Let $X = Y = \{a,b,c\}$, $g_X = \{0_X, 1_X, \{(a,0.5),(b,0.8),(c,.0.7)\}, \{(a,0.5),(b,0.2),(c,0.8)\}, \{(a,0.5),(b,0.8),(c,0.8)\}\}$ *and* $g_Y = \{0_Y, \{(a,0.5),(b,0.3),(c,0.2)\}\}$. Here $f : (X, g_X) \to (Y, g_Y)$ defined by *f(a)= a, f(b)= c, f(c)= b* is a fuzzy contra *(g_X, g_Y)*-continuous function but not a fuzzy almost (g_X, g_Y)-continuous function since f^{-1} $\{(a,0.5),(b,0.3),(c,0.2)\} = \{(a,0.5),(b,0.2),(c,0.3)\}$ is not a fuzzy g_X-open set in *X*.

Proposition 4.10. *Let $f : (X, g_X) \to (Y, g_Y)$ be a fuzzy contra (g_X, g_Y)-continuous function from X to Y with $c_{g_Y}(0_Y) = 0_Y$, where Y is a generalized fuzzy hyperconnected space. Then f is a fuzzy almost (g_X, g_Y)-continuous function.*

Proof. Since f is a fuzzy contra (g_X, g_Y)-continuous function with $c_{g_Y}(0_Y) = 0_Y$, we have $f(\lambda) \leq c_{g_Y}(\mu)$, where λ and μ are fuzzy g_X-open set and fuzzy g_Y-open set in X and Y, respectively. Again by the hypothesis, Y is a generalized fuzzy hyperconnected space. Then we have

$$f(\lambda) \leq c_{g_Y}(\mu) = 1_Y \Rightarrow f(\lambda) \leq i_{g_Y} c_{g_Y}(\mu) = 1_Y.$$

Thus the proof is completed. □

Proposition 4.11. *Let $f : (X, g_X) \to (Y, g_Y)$ be a fuzzy (σ, g_Y)-continuous function with $c_{g_X}(0_X) = 0_X$ and $c_{g_Y}(0_Y) = 0_Y$ from X to Y, where Y is a generalized fuzzy hyperconnected space. Then f is a fuzzy almost (g_X, g_Y)-continuous function.*

Proof. Since f is a fuzzy (σ, g_Y)-continuous function, the inverse image of a fuzzy g_Y-open set in Y is a fuzzy g_X-semi open in X. Since Y is a generalized fuzzy hyperconnected space, 1_Y and 0_Y are the only fuzzy g_Y-regular open sets in Y. Then $f^{-1}(1_Y) = 1_X$ and $f^{-1}(0_Y) = 0_X$. Thus the inverse image of every fuzzy g_Y-regular open set in Y is a fuzzy g_X-open set in X. □

5. Conclusion

Some new properties of generalized fuzzy hyperconnectedness are found here. These new properties are very useful for studying the concepts like fuzzy g_X-preopen sets, fuzzy g_X-preclosed sets and generalized fuzzy g_X-closed sets. Fuzzy g_X-preclosed set becomes generalized fuzzy g_X-closed set under the preview of hyperconnected space and it is one of the important finding of this paper. Through fuzzy feebly (g_X, g_Y)-continuity, fuzzy almost (g_X, g_Y)-continuity and fuzzy (σ, g_Y)-continuity, we have shown some applications of generalized fuzzy hyperconnectedness. Bajpai and Thakur [4] studied Intuitionistic fuzzy WO-connectedness in intuitionistic fuzzy sets. So there is a scope to extend the above concept as Intuitionistic fuzzy WO-hyperconnectedness in intuitionistic fuzzy sets to obtain some interesting results. The findings may be extended in a L-fuzzy topological space for more generalized environment.

Acknowledgements. The authors are thankful to the learned referees for their constructive suggestions towards the improvement of the paper.

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J. Chakraborty (chakrabortyjayasree1@gmail.com)
Department of mathematics, NIT Agartala, Tripura 799046, India.

B. Bhattacharya (babybhatt75@gmail.com)
Department of mathematics, NIT Agartala, Tripura 799046, India.
A. Paul (mrarnabpaul87@gmail.com)
Department of mathematics, NIT Agartala, Tripura 799046, India.