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Decision Making in an Information system via new topology

Nirmala Rebecca Paul

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Abstract. This paper introduces a topology called rough topology in terms of the boundary condition and proves that the nano topology is finer than the rough topology. The rough topology consists of a maximum of three elements. A new closure operator is defined and it is proved that it satisfies the Kuratowski closure operator. The new closure contains the nano closure of any subset of an universe with respect to the same equivalence relation. An algorithm is developed to find the core for any incomplete or complete Information system. This algorithm is defined in terms of the rough topology and it is applied to analyse the real life problems. The deciding factors for cardiovascular disease, breast cancer and the environmental factors that facilitates the yield of plants are derived.

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Corresponding Author: Nirmala Rebecca Paul (nimmi_rebecca@yahoo.com)

1. Introduction

The Rough set theory was introduced by Pawlak[6] and it is used as an useful tool for representing, reasoning and decision making. Rough set theory deals with approximations of sets in terms of the equivalence relations defined on the universe. Xibei et.al.[7] has introduced the improved version of the the classical Rough set theory called multi granulation.Rough set theory is used for many practical problems to select the set of attributes necessary for classification of objects in the considered universe.Rough set theory paved the way for many researchers to introduce new concepts. Feng[2] has introduced soft rough approximations and soft rough sets and has applied this new concept for house purchase problem. Naveed[5] has introduced the notion of generalized rough set. Anja[1] has introduced the notion of Rough interval valued intuitionistic fuzzy sets. The knowledge reduction problems are highly involved in Information system. Any Information system consists of several attributes and it is necessary to pick the minimal attributes for the classification of objects. A new topology called rough topology consisting of a maximum of three elements is defined and it is contained in the nano topology defined by Thivagar et.al. [4]. Rough open sets and rough closed sets are defined and a new closure is defined such that it satisfies the Kuratowski closure operator. This new closure of any subset of the universe contains the nano closure of any set with the same equivalence relation. An useful algorithm is developed to find the minimum number of attributes for the classification of objects in the considered universe called core. This algorithm is defined in terms of the rough topology and it is applied to analyse the real life problems. The deciding factors for cardiovascular, breast cancer and the environmental factors that facilitates the yield of plants are derived.

2. Preliminaries

Definition 2.1 ([6])**.** Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with with respect to R and it is denoted by $U_R(X)$. $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 ([4])**.** Let U be a non-empty, finite universe of objects and R be an equivalence relation on U. Let $X \subseteq U$. Let $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$. Then $\tau_R(X)$ is a topology on U called as the nano topology with respect to X.

Definition 2.3 ([3])**.** An information system is of the form $(U, A, \{V_a\}, f_a)$ where U is a non-empty finite set of objects, called the universe, A is a finite non-empty set of attributes, V_a is the attribute value set of an attribute $a \in A$ and $f_a : U \to V_a$ is called the information function. If $f_a(x)$ is equal to a missing value for some $x \in U$ and $a \in A$, then the information system is called an incomplete information system (IIS) Otherwise it is a complete information system(CIS). A missing value is denoted by "*". That is an IIS is of the form $(U, A, \{V_a\}, f_a)$ where $a \in A$ and $* \in \bigcup V_a$. An IIS can also be denoted by (U, A).

Definition 2.4 ([3])**.** Let U be an universe and A be a finite set of attributes. For any subset B of A, there is a binary relation on U corresponding to B given by $R(B) = \{(x, y\} \in UXU : f_a(x) = f_a(y) or f_a(x) = * or f_a(y) = * for any a \in B\}$. Then R(B) is a tolerance relation on U(reflexive and symmetric). $S_B(x)$ denotes the maximal set of objects which are possibly indiscrenible with x by the tolerance relation on U. That is $S_B(x) = \{y \in U : (x, y) \in R(B)\}, x \in U$

Definition 2.5 ([3])**.** If (U, A) is an IIS and $B \subseteq A$, then a subset X of U is said to be the tolerance class with respect to B, if $(x, y) \in R(B)$ for any $x, y \in X$. $U/R(B)$ denotes the set of all maximal tolerance classes with respect to B and is called a full cover of U.

Definition 2.6 ([8])**.** A set valued information system is a quadruple $S = (U, A, V, f)$ where U is a non-empty finite set of objects, A is a finite set of attributes , $V = \bigcup V_a$ where V_a is a domain of the attribute 'a', $f : UXA \to P(V)$ is a function such that for every $x \in U$ and $a \in A, f(x, a) \subseteq V_a$. Also $f(x, a) \geqslant 1$. The attribute set A is divided into two subsets, namely C of condition attributes and a decision attribute d.

3. Rough topology

A new topology called rough topology is defined in terms of the boundary condition defined on an universe. The nano topology is finer than the rough topology.

Definition 3.1. Let U be an universe and R be an equivalence relation on U and $\tau_R(X) = \{\phi, U, B_R(X)\}$, where $X \subseteq U$. Then $\tau_R(X)$ is called the rough topology on U and it satisfies conditions:

(i) $\phi, U \in \tau_R(X)$,

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

In this case, $(U, \tau_R(X))$ is called the rough topological space on U with respect to X.

It is obvious that $\tau_R(X)$ forms a topology on U with respect to the equivalence relation R. Furthermore, every rough open set is nano open but not conversely.

Remark 3.2. The rough topology is contained in the nano topology defined on U with respect to the equivalence relation R.

Example 3.3. Let $U = \{a, b, c, d, e\}, U/R = \{\{a, c\}, \{b, e\}, \{d\}\}, X = \{a, b, d, e\}$, $L_R(X) = \{b, d, e\}, U_R(X) = \{a, b, c, d, e\}, B_R(X) = \{a, c\}$. Then

The nano topology on X is $\tau_R(X) = \{\phi, U, \{b, d, e\}, \{a, c\}\}$.

The rough topology on X is $\tau_R(X) = \{\phi, U, \{a, c\}\}$.

Thus every nano topology is finer than the rough topology.

Remark 3.4. The rough topology will have two elements ϕ,U when $B_R(X) = \phi$ otherwise it will have the three elements $\phi, U, B_R(X)$ when $B_R(X) \neq \phi$.

Remark 3.5. Rough closed sets form a rough topology.

Definition 3.6. If $(U, \tau_R(X))$is a rough topological space with respect to X, where $X \subseteq U$.Let $A \subseteq U$,then the rough interior of A is defined as the union of all rough open sets contained in A and it is denoted by Rint(A) which is the largest rough open subset of A. The rough closure of A is defined as the intersection of all rough closed sets containing A and it is denoted by Rcl(A) and it is the smallest rough closed set containing A.

Theorem 3.7. *Let $(U, \tau_R(X))$be a rough topological space with respect to X, $X \subseteq U, A, B \subseteq U$. Then*
(1) $A \subseteq Rcl(A)$,
(2)] *A is rough closed iff* $Rcl(A) = A$,
(3) $Rcl(\phi) = \phi, Rcl(U) = U$,
(4) *if*$A \subseteq B$ *then* $Rcl(A) \subseteq Rcl(B)$,
(5) $Rcl(A \cup B) = Rcl(A) \cup Rcl(B)$,
(6) $Rcl(A \cap B) \subseteq Rcl(A) \cap Rcl(B)$,
(7) $Rcl(Rcl(A)) = Rcl(A)$

Proof. (1) By definition, It is clear that $A \subseteq Rcl(A)$.
(2) If A is rough closed, then A is the smallest rough closed set containing A itself. Thus Rcl(A)=A. Conversely if Rcl(A)=A, then A is the smallest rough closed set containing itself. Thus A is rough closed.
(3) Since ϕ and U are rough closed, $Rcl(\phi) = \phi$ and $Rcl(U) = U$.
(4) Since $A \subseteq B$, $A \subseteq Rcl(A)$ and $B \subseteq Rcl(B)$. Then $A \subseteq Rcl(B)$. Thus Rcl(B) is the rough closed set containing A. Since Rcl(A) is the smallest rough closed containing A, $Rcl(A) \subseteq Rcl(B)$.
(5) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, $Rcl(A) \subseteq Rcl(A \cup B)$ and $Rcl(B) \subseteq Rcl(A \cup B)$ by(4). Then $Rcl(A) \cup Rcl(B) \subseteq Rcl(A \cup B)$. Since $A \subseteq Rcl(A)$ and $B \subseteq Rcl(B), A \cup B \subseteq Rcl(A) \cup Rcl(B)$. Since $Rcl(A \cup B)$ is the smallest rough closed set containing $A \cup B, Rcl(A \cup B) \subseteq Rcl(A) \cup Rcl(B)$. Thus $Rcl(A \cup B) = Rcl(A) \cup Rcl(B)$.
(6) Since $A \cap B \subseteq A, A \cap B \subseteq B, Rcl(A \cap B) \subseteq Rcl(A) \cap Rcl(B)$.
(7) Since Rcl(A) is rough closed, $Rcl(Rcl(A)) = Rcl(A)$ □

Theorem 3.8. *The Rough closure in a rough topological space is the Kuratowski closure operator.*

Proof. From Theorem 3.7, it follows that $Rcl(\phi) = \phi$, $A \subseteq Rcl(A)$, $Rcl(A \cup B) = Rcl(A) \cup Rcl(B)$ and $Rcl(Rcl(A)) = Rcl(A)$. □

4. Algorithm

In this section an algorithm is developed to find the deciding factors or core to pick the minimum number of attributes necessary for the classification of objects.

Step 1. Given a finite universe U, a finite set A of attributes which is divided into two classes C of condition attributes and D of decision attributes. An equivalence relation R on U corresponds to C and a subset X of U which represents the data as an information table, columns of which are labeled by attributes and rows by objects.The entries of the table are attribute values.

Step 2. Find the boundary region of X with respect to R.

Step 3. Generate the rough topology $\tau_{R(C)}(X)$ on U corresponding to the set C of all conditional attributes,where $\tau_{R(C)}(X)$ is the topology corresponding to the equivalence relation R(C).

Step 4. Remove an attribute x from C and find the equivalence relation without x. That is $C - \{x\}$. Find the boundary region of X with respect to the new R.

Step 5. Generate the rough topology $\tau_{R(C-\{x\})}(X)$ on U.

Step 6. Repeat steps 3 and 4 for all attributes in C.

Step 7. Find the attributes in C for which $\tau_{R(C-\{x\})}(X) = \tau_{R(C)}(X)$.

Step 8. This set $C - \{x\}$ form the core. If more than one $C - \{x\}$ for which $\tau_{R(C-\{x\})}(X) = \tau_{R(C)}(X)$, then the intersection of the sets $C - \{x\}$ form the CORE.

Definition 4.1. Let (U, A) be an information system. Where U is an universe, A is the set of attributes is divided into a set of C of condition attributes and and a set of D of decision attribute. A subset B of C is said to be a CORE if $\tau_{R(B)}(X) = \tau_{R(C)}(X)$ and $\tau_{R(B)}(X) \neq \tau_{R(B-\{r\})}(X)$ for all $r \in B$ where $X \subseteq U$, $\tau_{R(B)}(X)$ is the rough topology corresponding to $B \subseteq C$. It is not necessary that all condition attributes in an information system depict the decision attribute. That is the decision attribute depends not on the whole set of condition attributes but on a subset of it is called the CORE.

Example 4.2. In this example Rough topology concepts is applied to find the key environmental factors for the yield of plants in a complete information system.The yield of plants depend on the the environment in which they are planted. The environmental factors which influence the growth of the plants are Temperature,Soil, Sun Light and Water content.The following table gives the information about the yield of 8 plants Ground nut, Rose,Banana, Paddy, Sugarcane,Chilli, Cotton and Corn and they are represented by $P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8$ respectively.

Plants	Temperature(T)	Soil(S)	Water(W)	Sun light(Su)	Yield
P_1	Moderate	Red	Medium	Moderate	High
P_2	High	Red	Medium	High	Low
P_3	Moderate	Sand	Medium	Moderate	High
P_4	High	Loose Soil	Medium	High	High
P_5	Moderate	Red	Medium	Low	Low
P_6	Moderate	Hard	Large	Low	High
P_7	High	Loose Soil	Large	High	Low
P_8	High	Loose Soil	Medium	High	Low

Here $U = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$. The columns of the table represent the environmental factors and the rows represents the 8 varieties of plants.The entries in the table are attribute values. Temperature, Soil, Sun light and Water are the condition attributes and yield is the decision attribute.

Case1: Plants with high yield are taken as X. Here $X = \{P_1, P_3, P_4, P_6\}$. Let R be the equivalence relation on U with respect to the condition attributes $C = \{Temperature, Soil, Sunlight, Water\} = \{T, S, Su, W\}$ the equivalence classes determined by R corresponding to C is given by $U/R(C) = \{\{P_1\}, \{P_2\}, \{P_3\}, \{P_4, P_8\}, \{P_5\}, \{P_6\}, \{P_7\}\}$. The boundary region with respect to R is given by $B_{R(C)}(X) = \{P_4, P_8\}$. The rough topology with respect to R is given by

$$\tau_{R(C)}(X) = \{\phi, U, \{P_4, P_8\}\}.$$

Step 1. If Temperature is removed from the set of condition attributes then the equivalence classes corresponding to $C_1 = \{S, Su, W\}$ is given by $U/R(C_1) = \{\{P_1\}, \{P_2\}, \{P_3\}, \{P_4, P_8\}, \{P_5\}, \{P_6\}, \{P_7\}\}$. The boundary region corresponding to this equivalence relation is given by $B_{R(C_1)}(X) = \{P_4, P_8\}$ and $\tau_{R(C)}(X) = \tau_{R(C_1)}(X)$. If Soil is removed from the condition attributes then the equivalence class corresponding to $C_2 = \{T, Su, W\}$ is given by $U/R(C_2) = \{\{P_1, P_3\}, \{P_2, P_4, P_8\},$

$\{P_5\}, \{P_6\}, \{P_7\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_2)}(X) = \{P_2, P_4, P_8\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_2)}(X) = \{\phi, U, \{P_2, P_4, P_8\}\}$. If water is removed from the set of condition attributes then the equivalence classes corresponding to $C_3 = \{T, S, Su\}$ is given by $U/R(C_3) = \{\{P_1\}, \{P_2\}, \{P_4, P_7, P_8\}, \{P_3\}, \{P_5\}, \{P_6\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_3)}(X) = \{P_4, P_7, P_8\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_2)}(X) = \{\phi, U, \{P_4, P_7, P_8\}\}$. If sun light is removed from the set of condition attributes then the equivalence classes corresponding to $C_4 = \{T, S, W\}$ is given by $U/R(C_4) = \{\{P_1, P_5\}, \{P_2\}, \{P_3\}, \{P_4, P_8\}, \{P_6\}, \{P_7\}\}$. Then the boundary region corresponding to the equivalence is given by

$$B_{R(C_4)}(X) = \{P_1, P_4, P_5, P_8\}$$

and

$$\tau_{R(C)}(X) \neq \tau_{R(C_4)}(X) = \{\phi, U, \{P_1, P_4, P_5, P_8\}\}.$$

Thus $CORE = \{S, W, Su\}$.

Case 2: Plants with low yield. Let $X = \{P_2, P_5, P_7, P_8\}$ represents the set of plants with low yield. Let R be the equivalence relation on U with respect to the condition attributes $C = \{Temperature, Soil, Sunlight, Water\} = \{T, S, Su, W\}$ is given by $U/R(C) = \{\{P_1\}, \{P_2\}, \{P_3\}, \{P_4, P_8\}, \{P_5\}, \{P_6\}, \{P_7\}\}$. The boundary region with respect to R is given by $B_{R(C)}(X) = \{P_4, P_8\}$. The rough topology with respect to R is given by $\tau_{R(C)}(X) = \{\phi, U, \{P_4, P_8\}\}$.

Step 1. If Temperature is removed from the set of condition attributes then the equivalence classes corresponding to $C_1 = \{S, Su, W\}$ is given by $U/R(C_1) = \{\{P_1\}, \{P_2\}, \{P_3\}, \{P_4, P_8\}, \{P_5\}, \{P_6\}, \{P_7\}\} = U/R(C)$. The boundary region corresponding to this equivalence relation is given by $B_{R(C_1)}(X) = \{P_4, P_8\}$ and $\tau_{R(C)}(X) = \tau_{R(C_1)}(X)$. If Soil is removed from the condition attributes then the equivalence class corresponding to $C_2 = \{T, Su, W\}$ is given by $U/R(C_2) = \{\{P_1, P_3\}, \{P_2, P_4, P_8\}, \{P_5\}, \{P_6\}, \{P_7\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_2)}(X) = \{P_2, P_4, P_8\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_2)}(X) = \{\phi, U, \{P_2, P_4, P_8\}\}$. If water is removed from the set of condition attributes then the equivalence classes corresponding to $C_3 = \{T, S, Su\}$ is given by $U/R(C_3) = \{\{P_1\}, \{P_2\}, \{P_4, P_7, P_8\}, \{P_3\}, \{P_5\}, \{P_6\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_3)}(X) = \{P_4, P_7, P_8\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_3)}(X) = \{\phi, U, \{P_4, P_7, P_8\}\}$. If sun light is removed from the set of condition attributes then the equivalence classes corresponding to $C_4 = \{T, S, W\}$ is given by $U/R(C_4) = \{\{P_1, P_5\}, \{P_2\}, \{P_3\}, \{P_4, P_8\}, \{P_6\}, \{P_7\}\}$. Subsequently the boundary region corresponding to the equivalence relation is given by

$$B_{R(C_4)}(X) = \{P_1, P_4, P_5, P_8\}$$

and

$$\tau_{R(C)}(X) \neq \tau_{R(C_4)}(X) = \{\phi, U, \{P_1, P_4, P_5, P_8\}\}.$$

Thus $CORE = \{S, W, Su\}$.

Observation 4.3. From both cases the CORE is found to be Soil, Water and Sun Light which have close connection with the yield of a plant.

Example 4.4. This example gives the set of patients with different symptoms. Using the algorithm and the rough topology concept the key attributes for Cardiovascular disease are obtained.

Patients	Pain in the shoulder(P)	Unusual fatigue(F)	Breathlessness(B)	Sweating(S)	Cardiovascular
P_1	Severe	Yes	Yes	No	Yes
P_2	Mild	Yes	No	No	No
P_3	Mild	Yes	Yes	Yes	Yes
P_4	No	No	Yes	Yes	No
P_5	Severe	Yes	No	No	Yes
P_6	Severe	Yes	No	No	No

The columns of the table represent the different symptoms of patients suffering from cardiovascular disease and the rows represent 6 different patients.The entries in the table are attribute values. Pain in shoulder,Unusual Fatigue, Breathlessness and Sweating are the condition attributes and cardiovascular disease is the decision attribute.

Case1: Patients with Cardiovascular disease are taken as X. Here $X = \{P_1, P_3, P_5\}$. Let R be the equivalence relation on U with respect to the condition attributes $C = \{Paininshoulder, UnusualFatigue, Breadhlessnes, Sweating\} = \{P, F, B, S\}$ and the equivalence classes determined by R corresponding to C is given by $U/R(C) = \{\{P_1\}, \{P_2\}, \{P_3\}, \{P_5, P_6\}, \{P_4\}\}$. The boundary region with respect to R is given by $B_{R(C)}(X) = \{P_5, P_6\}$. The rough topology with respect to R is given by $\tau_{R(C)}(X) = \{\phi, U, \{P_5, P_6\}\}$.

Step 1. If Pain in shoulder is removed from the set of condition attributes, then the equivalence classes corresponding to $C_1 = \{F, B, S\}$ is given by $U/R(C_1) = \{\{P_1\}, \{P_2, P_5, P_6\}, \{P_3\}, \{P_4\}\} \neq U/R(C)$. The boundary region corresponding to this equivalence relation is given by $B_{R(C_1)}(X) = \{P_2, P_5, P_6\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_1)}(X) = \{\phi, U, \{P_2, P_5, P_6\}\}$. If Unusual Fatigue is removed from the condition attributes, then the equivalence class corresponding to $C_2 = \{P, B, S\}$ is given by $U/R(C_2) = \{\{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5, P_6\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_2)}(X) = \{P_5, P_6\}$ and $\tau_{R(C)}(X) = \tau_{R(C_2)}(X) = \{\phi, U, \{P_5, P_6\}\}$. If Breathlessness is removed from the set of condition attributes, then the equivalence classes corresponding to $C_3 = \{P, F, S\}$ is given by $U/R(C_3) = \{\{P_1, P_5, P_6\}, \{P_2\}, \{P_3\}, \{P_4\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_3)}(X) = \{P_1, P_5, P_6\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_3)}(X) = \{\phi, U, \{P_1, P_5, P_6\}\}$. If Sweating is removed from the set of condition attributes, then the equivalence classes corresponding to $C_4 = \{P, F, B\}$ is given by $U/R(C_4) = \{\{P_1\}, \{P_5, P_6\}, \{P_2\}, \{P_3\}, \{P_4\}\}$. Then the boundary region corresponding to the equivalence classes is given by $B_{R(C_4)}(X) = \{P_5, P_6\}$ and $\tau_{R(C)}(X) = \tau_{R(C_4)}(X) = \{\phi, U, \{P_5, P_6\}\}$. Thus $CORE = \{P, B, S\} \cap \{P, F, B\} = \{P, B\}$.

Case 2: Patients with no cardiovascular disease. Let $X = \{P_2, P_4, P_6\}$ represents the patients with no cardiovascular disease. Let R be the equivalence relation on U with respect to the condition attributes $C = \{P, F, B, S\}$ and the equivalence classes determined by R corresponding to C is given by

$$U/R(C) = \{\{P_1\}, \{P_2\}, \{P_3\}, \{P_5, P_6\}, \{P_4\}\}.$$

The boundary region with respect to R is given by $B_{R(C)}(X) = \{P_5, P_6\}$. The rough topology with respect to R is given by $\tau_{R(C)}(X) = \{\phi, U, \{P_5, P_6\}\}$.

Step 1. If Pain is removed from is removed from the set of condition attributes then the equivalence classes corresponding to $C_1 = \{F, B, S\}$ is given by $U/R(C_1) = \{\{P_1\}, \{P_2, P_5, P_6\}, \{P_3\}, \{P_4\}\}$. The boundary region corresponding to this equivalence classes is given by $B_{R(C_1)}(X) = \{P_2, P_5, P_6\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_1)}(X)$. If Fatigue is removed form the condition attributes, then the equivalence class corresponding to $C_2 = \{P, B, S\}$ is given by $U/R(C_2) = \{\{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5, P_6\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_2)}(X) = \{P_5, P_6\}$ and $\tau_{R(C)}(X) = \tau_{R(C_2)}(X) = \{\phi, U, \{P_5, P_6\}\}$. If Breathlessness is removed from the set of condition attributes then the equivalence classes corresponding to $C_3 = \{P, F, S\}$ is given by $U/R(C_3) = \{\{P_1, P_5, P_6\}, \{P_2\}, \{P_3\}, \{P_4\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_3)}(X) = \{P_1, P_5, P_6\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_3)}(X) = \{\phi, U, \{P_1, P_5, P_6\}\}$. If Sweating is removed from the set of condition attributes then the equivalence classes corresponding to $C_4 = \{P, F, B\}$ is given by $U/R(C_4) = \{\{P_1\}, \{P_5, P_6\}, \{P_2\}, \{P_3\}, \{P_4\}\}$. Then the boundary region corresponding to the equivalence is given by $B_{R(C_4)}(X) = \{P_5, P_6\}$ and $\tau_{R(C)}(X) = \tau_{R(C_4)}(X) = \{\phi, U, \{P_5, P_6\}\}$.
Thus $CORE = \{P, B, S\} \cap \{P, F, B\} = \{P, B\}$.

Observation 4.5. From both the cases it is found that Pain in the shoulder and Breathlessness are the symptoms which are closely connected with the disease Cardiovascular disease.

Example 4.6. This example gives information about patients with different symptoms of breast cancer namely,lump in breast, inverted nipple,rashes, nipple discharge and swelling in the armpit and they are represented shortly by L,I,R D and S

Patients	L	I	R	D	S	Breast Cancer
P_1	Yes	Yes	*	Yes	No	Yes
P_2	Yes	Yes	Yes	*	*	Yes
P_3	No	Yes	No	*	Yes	No
P_4	Yes	No	*	No	*	No
P_5	No	Yes	*	Yes	Yes	No
P_6	Yes	*	No	Yes	*	Yes

The columns of the table represent the symptoms for breast cancer and the rows represent the patients. The entries in the table are the attribute values. The given information system is incomplete and is given by (U, A) where $U = \{P_1, P_2, P_3, P_4, P_5, P_6\}$ and $A = \{L, I, R, D, S, Breastcancer\}$ which is divided into a set of C of condition attributes given by $C = \{L, I, R, D, S\}$ and $D = \{Breastcancer\}$. The attribute Inverted Nipple generates the tolerance classes $\{P_1, P_2, P_3, P_5, P_6\}$ and $\{P_4, P_6\}$, since the missing attribute value for P_6 can be 'Yes' or 'No'. Similarly, the maximal tolerance classes for other combination of attributes can be formed.. Considering all condition attributes together, the maximal tolerance classes are $\{P_1, P_2\}, \{P_1, P_6\}, \{P_3, P_5\}, \{P_4\}$ and $U/R(C) = \{\{P_1, P_2\}, \{P_1, P_6\}, \{P_3, P_5\}, \{P_4\}\}$.

Step 1. X be the set of patients diagnosed with Breast cancer. $X = \{P_1, P_2, P_6\}$. The boundary region is given by $B_{R(C)}(X) = \phi$ and the rough topology $\tau_{R(C)}(X) = \{\phi, U\}$. If the attribute Lump in breast is removed from the condition attributes, then the equivalence classes corresponding to $C_1 = \{I, R, D, S\}$ is given by $U/R(C_1) = \{\{P_1, P_2\}, \{P_3, P_5, P_6\}, \{P_4\}, \{P_2, P_5\}, \{P_1, P_6\}\}$. The boundary region corresponding to this equivalence relation is given by $B_{R(C_1)}(X) = \{P_3, P_5\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_1)}(X)$. If Inverted Nipple is removed from the condition attributes, then the equivalence class corresponding to $C_2 = \{L, R, D, S\}$ is given by $U/R(C_2) = \{\{P_1, P_2\}, \{P_1, P_6\}, \{P_2, P_4\}, \{P_3, P_5\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_2)}(X) = \{P_4\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_2)}(X) = \{\phi, U, \{P_4\}\}$. If Rashes is removed from the set of condition attributes, then the equivalence classes corresponding to $C_3 = \{L, I, D, S\}$ is given by $U/R(C_3) = \{\{P_1, P_2, P_6\}, \{P_3, P_5\}, \{P_4\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_3)}(X) = \phi$ and $\tau_{R(C)}(X) = \tau_{R(C_3)}(X) = \{\phi, U\}$. If Nipple discharge is removed from the set of condition attributes then the equivalence classes corresponding to $C_4 = \{L, I, R, S\}$ is given by $U/R(C_4) = \{\{P_1, P_2\}, \{P_1, P_6\}, \{P_3, P_5\}, \{P_4, P_6\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_4)}(X) = \{P_4\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_4)}(X) = \{\phi, U, \{P_4\}\}$. If Swelling in the arm pit s removed from the set of condition attributes, then the equivalence classes corresponding to $C_5 = \{L, I, R, D\}$ is given by $U/R(C_5) = \{\{P_1, P_2\}, \{P_1, P_6\}, \{P_3, P_5\}, \{P_4\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_5)}(X) = \phi$ and $\tau_{R(C)}(X) = \tau_{R(C_5)}(X) = \{\phi, U\}$. Thus the $CORE = \{L, I, D, S\} \cap \{L, I, R, D\} = \{L, I, D\}$.

Step 2. Patients without Breast cancer.$X = \{P_3, P_4, P_5\}$. The boundary region is given by $B_{R(C)}(X) = \phi$ and the rough topology $\tau_{R(C)}(X) = \{\phi, U\}$.If the attribute Lump in breast is removed from the condition attributes, then the equivalence classes corresponding to $C_1 = \{I, R, D, S\}$ is given by $U/R(C_1) = \{\{P_1, P_2\}, \{P_3, P_5, P_6\}, \{P_4\}, \{P_2, P_5\}, \{P_1, P_6\}\}$. The boundary region corresponding to this equivalence relation is given by $B_{R(C_1)}(X) = \{P_2, P_3, P_5, P_6\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_1)}(X)$. If Inverted Nipple is removed form the condition attributes, then the equivalence class corresponding to $C_2 = \{L, R, D, S\}$ is given by $U/R(C_2) = \{\{P_1, P_2\}, \{P_1, P_6\}, \{P_2, P_4\}, \{P_3, P_5\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_2)}(X) = \{P_2, P_4\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_2)}(X) = \{\phi, U, \{P_2, P_4\}\}$. If Rashes is removed from the set of condition attributes then the equivalence classes corresponding to $C_3 = \{L, I, D, S\}$ is given by $U/R(C_3) = \{\{P_1, P_2, P_6\}, \{P_3, P_5\}, \{P_4\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_3)}(X) = \phi$ and $\tau_{R(C)}(X) = \tau_{R(C_2)}(X) = \{\phi, U\}$. If Nipple discharge is removed from the set of condition attributes, then the equivalence classes corresponding to $C_4 = \{L, I, R, S\}$ is given by $U/R(C_4) = \{\{P_1, P_2\}, \{P_1, P_6\}, \{P_3, P_5\}, \{P_4, P_6\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_4)}(X) = \{P_4, P_6\}$ and $\tau_{R(C)}(X) \neq \tau_{R(C_4)}(X) = \{\phi, U, \{P_4, P_6\}\}$. If Swelling in the arm pit is removed from the set of condition attributes then the equivalence classes corresponding to $C_5 = \{L, I, R, D\}$ is given by $U/R(C_5) = \{\{P_1, P_2\}, \{P_1, P_6\}, \{P_3, P_5\}, \{P_4\}\}$. The boundary region corresponding to the equivalence relation is given by $B_{R(C_5)}(X) = \phi$ and $\tau_{R(C)}(X) = \tau_{R(C_5)}(X) = \{\phi, U\}$. Thus the $CORE = \{L, I, D, S\} \cap \{L, I, R, D\} = \{L, I, D\}$.

Observation 4.7. Thus the CORE is Lump in Breast, Inverted Nipple and Nipple Discharge are the key symptoms which are closely connected with the disease Breast Cancer.

5. Conclusion

This paper introduces a rough topology which is coarser than the nano topology. An algorithm is developed to find the CORE in an information system. Three examples are discussed to find the CORE of the systems. The paper concludes that Soil,Sun Light and Water are the the key factors that have close connection with yield of a plant. Also Lump in Breast, Inverted Nipple and Nipple Discharge are the key symptoms which are closely connected to the disease Breast Cancer. Similarly Pain in the Shoulder and Breathlessness are the key factors connected to cardiovascular disease.

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Nirmala Rebecca Paul (nimmi_rebecca@yahoo.com)
Associate Professor of Mathematics, Lady Doak College, Madurai-625002, Tamilnadu, India