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On T_0 objects in Q-TOP

Sheo Kumar Singh, Arun K. Srivastava

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Abstract. In this paper, we have given some characterizations of T_0-Q-topological spaces including the 'so-called' diagonal characterization which is given by using a suitable closure operator in the category of T_0-Q-topological spaces. We have further showed that the category of T_0-Q-topological spaces is the epireflective hull of the Q-Sierpinski space in the category Q-**TOP** of Q-topological spaces. We have also studied T_0-objects in the category **Str**-Q-**TOP** of stratified Q-topological spaces on the lines of Lowen and Srivastava (in 1989) by using Marny's notion of T_0-objects (in 1979).

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Corresponding Author: Sheo Kumar Singh (sheomathbhu@gmail.com)

1. Introduction

In [15], Solovyov, while introducing the Q-topological spaces, also introduced the notion of T_0-ness for Q-topological spaces.

Now it is already known that in the category **TOP** of topological spaces, T_0-topological spaces are precisely the objects of the epireflective hull of the two-point Sierpinski space. Also, it has been shown by Lowen and Srivastava in [9] that in the category **FTS** of fuzzy topological spaces, T_0-fuzzy topological spaces are precisely the objects of the epireflective hull of the fuzzy Sierpinski space I_S (of [16]). Apart from the above, in [8], Khastgir and Srivastava, gave a few characterizations of T_0-ness for fuzzy topological spaces. Also, Lowen and Srivastava in [9] have shown that T_0-fuzzy topological spaces are the T_0-objects in the category of stratified fuzzy topological spaces.

In this paper, we have proved some results for Q-topological spaces, motivated by the above-mentioned results. We have thus shown in particular that the category of T_0-Q-topological spaces is the epireflective hull of the Q-Sierpinski space in the category Q-**TOP** and have also obtained a few other characterizations of T_0-Q-topological spaces. In the last section of this paper, we have shown that within the category **Str**-Q-**TOP** of stratified Q-topological spaces T_0-objects are precisely the T_0-Q-topological spaces.

2. Preliminaries

For all undefined category-theoretic notions used in this paper, [1] may be referred. All subcategories used here are assumed to be full.

We begin by recalling the notions of Ω-algebras and their homomorphisms (most of the definitions in the preliminaries are given in [13, 14] also; we recall these here for the sake of completeness); for details, cf. [10, 15].

Definition 2.1. Let $\Omega = (n_\lambda)_{\lambda \in I}$ be a class of cardinal numbers.

- An Ω-algebra is a pair $(A, (\omega_\lambda^A)_{\lambda \in I})$ consisting of a set A and a family of maps $\omega_\lambda^A : A^{n_\lambda} \to A$. $B \subseteq A$ is called a subalgebra of $(A, (\omega_\lambda^A)_{\lambda \in I})$ if $\omega_\lambda^A((b_i)_{i \in n_\lambda}) \in B$, for every $\lambda \in I$ and every $(b_i)_{i \in n_\lambda} \in B^{n_\lambda}$. Given $S \subseteq A$, $\langle S \rangle$ denotes the subalgebra of $(A, (\omega_\lambda^A)_{\lambda \in I})$ 'generated by S', i.e., $\langle S \rangle$ is the intersection of all subalgebras of $(A, (\omega_\lambda^A)_{\lambda \in I})$ containing S.
- Given Ω-algebras $(A, (\omega_\lambda^A)_{\lambda \in I})$ and $(B, (\omega_\lambda^B)_{\lambda \in I})$, a map $f : A \to B$ is called an Ω-algebra homomorphism provided that for every $\lambda \in I$, the following diagram

$$\begin{tikzcd} A^{n_\lambda} \arrow[r, "f^{n_\lambda}"] \arrow[d, "\omega_\lambda^A"'] & B^{n_\lambda} \arrow[d, "\omega_\lambda^B"] \\ A \arrow[r, "f"'] & B \end{tikzcd}$$

  commutes.
  Let $\mathbf{Alg}(\Omega)$ denote the category of Ω-algebras and Ω-algebra homomorphisms (this category has products).
- A **variety** of Ω-algebras is a full subcategory of $\mathbf{Alg}(\Omega)$, which is closed under the formation of products, subalgebras, and homomorphic images.

Throughout this paper, $\Omega = (n_\lambda)_{\lambda \in I}$ denotes a fixed class of cardinal numbers, $\mathbf{V}$ denotes a fixed variety of Ω-algebras and Q denotes a fixed member of $\mathbf{V}$.

Each function $f : X \to Y$ between sets X and Y provides two functions $f^{\leftarrow} : 2^Y \to 2^X$ and $f^{\rightarrow} : 2^X \to 2^Y$, given by $f^{\leftarrow}(B) = \{x \in X \mid f(x) \in B\}$ and $f^{\rightarrow}(A) = \{f(x) \mid x \in A\}$, and also a function $f_Q^{\leftarrow} : Q^Y \to Q^X$, given by $f_Q^{\leftarrow}(\alpha) = \alpha \circ f$.

- Given a set X, a subset τ of Q^X is called a Q-topology on X if τ is a subalgebra of Q^X, in which case the pair (X, τ) is called a Q-topological space.
- Given two Q-topological spaces (X, τ) and (Y, η), a Q-continuous map from (X, τ) to (Y, η) is a map $f : X \to Y$ such that $f_Q^{\leftarrow}(\alpha) \in \tau$ for every $\alpha \in \eta$.

- Given a Q-topological space (X, τ) and $Y \subseteq X$, $(i_Q^{\leftarrow})^{\rightarrow}(\tau)$ $(= \{p \circ i \mid p \in \tau\})$ is called the Q-subspace topology on Y, where $i : Y \to X$ is the inclusion map. We shall denote the Q-subspace topology on Y as τ_Y.
- A Q-topological space (X, τ) is called $\mathbf{T}_0$ if for every distinct $x, y \in X$, there exists $p \in \tau$ such that $p(x) \neq p(y)$.

The meanings of homeomorphisms, embeddings, and products, etc. for Q-topological spaces are on expected lines.

Let Q-**TOP** denote the category of Q-topological spaces and Q-continuous maps between them.

Let (X, τ) be a Q-topological space, Y be a set and $q : X \to Y$ be a surjective map. Then it can be noticed that $\{\mu \in Q^Y \mid q_Q^{\leftarrow}(\mu) \in \tau\}$ turns out to be a subalgebra of the Ω-algebra Q^Y and hence a Q-topology on Y.

Definition 2.2. Let (X, τ) be a Q-topological space, Y be a set and $q : X \to Y$ be a surjective map. Then $\{\mu \in Q^Y \mid q_Q^{\leftarrow}(\mu) \in \tau\}$ is called the quotient Q-topology on Y with respect to (X, τ) and q. We shall denote it as τ/q. The pair $(Y, \tau/q)$ will be called the quotient Q-topological space with respect to (X, τ) and q.

Remark 2.3. In [15], it has been noted that Q-**TOP**, like **TOP**, has products. Also, Q-**TOP** turns out to be a topological category (as pointed out in Remark 2.1 of [14]). Using Theorems 1.2.2.9 and 1.2.3.3 of [12], it follows that Q-**TOP** is co-well-powered (epi, extremal mono)-category and well-powered (extremal epi, mono)-category. Also, Q-**TOP** is initially complete (follows from the Definition 6.1.1 (2) and Example 5.2.2 (1) of [12]). Moreover, Q-**TOP** is complete, being a topological category (follows from Theorem 1.2.1.10 of [12]); in particular, it has equalizers which are constructed, at the set-theoretical level, in the same way as in the category **SET** of sets.

3. Some characterizations of T_0-Q-topological spaces

We first present a few characterizations of T_0-Q-topological spaces which involve the role of the Q-Sierpinski space (Q, ρ) (of [13]).

Theorem 3.1. *A Q-topological space (X, τ) is T_0 if and only if the family*

$$\mathscr{F} = \{f : (X, \tau) \to (Q, \rho) \mid f \text{ is } Q\text{-continuous}\}$$

separates points of (X, τ).

Proof. By Theorem 3.1 of [13], we find that τ is just $\mathscr{F}$. The rest immediately follows from the definition of T_0-ness of Q-topological spaces. □

In [8], some characterizations of T_0-fuzzy topological spaces were given. We now proceed to give analogous characterizations of T_0-Q-topological spaces. For this, we use a closure operator (cf. [2, 3, 8]) in the category Q-**TOP**.

Let $X = (X, \tau) \in ob Q\text{-}\mathbf{TOP}$ and $M \subseteq X$. Let

$$[M] = \bigcap \{Eq(f, g) \mid f, g : X \to Y \text{ are } Q\text{-continuous maps and } Y \in ob Q\text{-}\mathbf{TOP}_0 \text{ with } f|_M = g|_M\},$$

where $Eq(f, g) = \{x \in X \mid f(x) = g(x)\}$.

Here M is said to be $[\ \ ]$-closed if $[M] = M$.

It can be easily seen that $[[M]] = [M]$.

Theorem 3.2. *Let* $(X,\tau) \in obQ$-**TOP** *and* $M \subseteq X$. *Then*

$$[M] = \bigcap\{Eq(f,g) \mid f,g \in \tau \text{ with } f|_M = g|_M\}.$$

Proof. For convenience, suppose $C_M = \bigcap\{Eq(f,g) \mid f,g \in \tau \text{ with } f|_M = g|_M\}$. It is clear that, $[M] \subseteq C_M$. To show that $C_M \subseteq [M]$, it is sufficient to show that if $x \notin [M]$, then $x \notin C_M$. Let $x \notin [M]$. Then there is some $(Y,\delta) \in obQ\text{-}\mathbf{TOP}_0$ and a pair of Q-continuous maps $f,g : (X,\tau) \to (Y,\delta)$ such that $f|_M = g|_M$ with $f(x) \neq g(x)$. Now as $f(x), g(x) \in Y$ and Y is T_0, there is some $\nu \in \delta$ such that $\nu(f(x)) \neq \nu(g(x))$, i.e., $\nu \circ f(x) \neq \nu \circ g(x)$. As $\nu : (Y,\delta) \to (Q,\rho)$ is Q-continuous, $\nu \circ f, \nu \circ g : (X,\tau) \to (Q,\rho)$ are also Q-continuous. Thus $\nu \circ f, \nu \circ g \in \tau$. Now note that $\nu \circ f|_M = \nu \circ g|_M$, while $\nu \circ f(x) \neq \nu \circ g(x)$, whereby $x \notin C_M$. So $C_M \subseteq [M]$. □

Theorem 3.3. *For any Q-topological space $X = (X,\tau)$ the following statements are equivalent:*

(1) *X is T_0.*

(2) *for every Q-topological space $Y = (Y,\delta)$ and for every Q-continuous map $f : Y \to X$, the graph $G_f = \{(y, f(y)) \mid y \in Y\}$ of f, is $[\ \ ]$-closed in $Y \times X$.*

(3) *$D_X = \{(x,x) \mid x \in X\}$ is $[\ \ ]$-closed in $X \times X$.*

Proof. (1) ⇒ (2): Let (X,τ) be a T_0-Q-topological space. It is clear that $G_f \subseteq [G_f]$. Suppose that $(y_0, x_0) \notin G_f$. Then $x_0 \neq f(y_0)$. But, by T_0-ness of X, we have some $\mu \in \tau$ such that $\mu(x_0) \neq \mu(f(y_0))$. Now $\mu : (X,\tau) \to (Q,\rho)$ is Q-continuous, by Theorem 3.1 of [13]. Note that the product Q-topology on $Y \times X$ is equal to $\langle\{p_1{}_Q^{\leftarrow}(\sigma) \mid \sigma \in \delta\} \bigcup \{p_2{}_Q^{\leftarrow}(\sigma) \mid \sigma \in \tau\}\rangle$, where $p_1 : Y \times X \to Y$ and $p_2 : Y \times X \to X$ are the two projection maps. Define $g, h : Y \times X \to (Q,\rho)$ as $g(y,x) = \mu(x)$ and $h(y,x) = \mu(f(y))$, for every $(y,x) \in Y \times X$. It can be easily verified that $g = p_2{}_Q^{\leftarrow}(\mu)$ and $h = p_1{}_Q^{\leftarrow}(f_Q^{\leftarrow}(\mu))$ (note that $f_Q^{\leftarrow}(\mu) \in \delta$), whereby it follows that g and h are Q-continuous. Now it can be easily seen that $g|_{G_f} = h|_{G_f}$, but $g(y_0,x_0) \neq h(y_0,x_0)$. Thus $(y_0,x_0) \notin [G_f]$. So $[G_f] \subseteq G_f$. Hence $[G_f] = G_f$.

(2) ⇒ (3): Suppose (2) holds. If we take $f : X \to X$ as the identity map, then we have $G_f = D_X$. Thus, by applying (2) to the identity map f, it can be seen that D_X comes out to be $[\ \ ]$-closed in $X \times X$.

(3) ⇒ (1): Suppose that D_X is $[\ \ ]$-closed in $X \times X$. If possible, suppose that (X,τ) is not T_0. Then there exist $x, y \in X$ with $x \neq y$, such that $\mu(x) = \mu(y)$, for every $\mu \in \tau$, which implies that $p_j{}_Q^{\leftarrow}(\nu)(x,x) = p_j{}_Q^{\leftarrow}(\nu)(x,y)$, for every $\nu \in \tau$ and $j = 1,2$, where $p_1, p_2 : X \times X \to X$ are the two projection maps. But then $\tilde{\mu}(x,x) = \tilde{\mu}(x,y)$, for every $\tilde{\mu} \in \langle\{p_j{}_Q^{\leftarrow}(\nu) \mid \nu \in \tau, j = 1,2\}\rangle$. Since $x \neq y$, $(x,y) \notin D_X (= [D_X])$. Thus, there is some T_0-Q-topological space (Z,δ) and Q-continuous maps $g, h : X \times X \to Z$ with the property that $g|_{D_X} = h|_{D_X}$ and $g(x,y) \neq h(x,y)$. As (Z,δ) is T_0, there is some $\sigma \in \delta$ such that $\sigma(g(x,y)) \neq \sigma(h(x,y))$. Now it is clear that, $g_Q^{\leftarrow}(\sigma), h_Q^{\leftarrow}(\sigma) \in \langle\{p_j{}_Q^{\leftarrow}(\nu) \mid \nu \in \tau, j = 1,2\}\rangle$. So $(g_Q^{\leftarrow}(\sigma))(x,y) = (g_Q^{\leftarrow}(\sigma))(x,x)$ and $(h_Q^{\leftarrow}(\sigma))(x,y) = (h_Q^{\leftarrow}(\sigma))(x,x)$. But $(g_Q^{\leftarrow}(\sigma))(x,y) = (h_Q^{\leftarrow}(\sigma))(x,y)$ (as $g|_{D_X} = h|_{D_X}$, $\sigma(g(x,x)) = \sigma(h(x,x))$). This implies that $\sigma(g(x,y)) = \sigma(h(x,y))$, a contradiction. Hence (X,τ) is T_0. □

4. Q-**TOP**$_0$ as the epireflective hull of (Q,ρ) in Q-**TOP**

As pointed out earlier, **FTS**$_0$ has been shown to be the epireflective hull of the fuzzy Sierpinski space in **FTS**. We proceed to prove an analogous result for Q-**TOP**$_0$.

Theorem 4.1. *Q-**TOP**$_0$ is an epireflective subcategory of Q-**TOP**.*

Proof. Let (X,τ) be a Q-topological space. Define a relation $\sim$ on X as follows: for every $x,y \in X$, $x \sim y$ if $\mu(x)=\mu(y)$, for every $\mu \in \tau$. It is easily verified that $\sim$ is an equivalence relation on X. Let $\tilde{X} = X/\sim$ and let $\tilde{\tau}$ be the corresponding quotient Q-topology on $\tilde{X}$ induced by the quotient map $q_X : X \to \tilde{X}$ and τ. Then $(\tilde{X},\tilde{\tau})$ turns out to be a T_0-Q-topological space and the map $q_X : (X,\tau) \to (\tilde{X},\tilde{\tau})$ an epimorphism in Q-**TOP**. Now for any T_0-Q-topological space (Y,δ) and any Q-continuous map $f : (X,\tau) \to (Y,\delta)$, define a map $\tilde{f} : (\tilde{X},\tilde{\tau}) \to (Y,\delta)$ by $\tilde{f}(\tilde{x}) = f(x)$. Then by the T_0-ness of (Y,δ), it follows that $\tilde{f}$ is well-defined. It is easily observed that $\tilde{f}$ is Q-continuous and $\tilde{f} \circ q_X = f$. □

As pointed out earlier in Remark 2.3 that Q-**TOP** is co-well-powered (epi, extremal mono)-category, so, by using Theorem 1 (of [11]) we get the following corollary.

Corollary 4.2. *Q-**TOP**$_0$ is closed under the formation of products and extremal subobjects in Q-**TOP**.*

Remark 4.3. We point out that the extremal subobjects of a Q-topological space (X,τ) are precisely the Q-subspaces of (X,τ) (cf. Proposition 21.13 of [1]).

In the following, we state a result from [15].

Theorem 4.4. (Theorem 58 of [15]) *A Q-topological space (X,τ) is T_0 if and only if (X,τ) is homeomorphic to a Q-subspace of a product of copies of (Q,ρ).*

This result can also be restated as follows.

Theorem 4.5. *(Q,ρ) $\mathscr{H}$-cogenerates Q-**TOP**$_0$, where $\mathscr{H}$ is the class of all Q-**TOP**$_0$-embeddings.*

Taking into account Remarks 2.3 and 4.3 above and Theorem 2 (of [11]), the following theorem restates Theorem 4.4 in category-theoretic terms.

Theorem 4.6. *Q-**TOP**$_0$ is the epireflective hull of (Q,ρ) in Q-**TOP**.*

5. T_0-objects in Q-**TOP**

In [11], an object A of a topological c-category (in the sence of Herrlich [7]) $\mathscr{A}$ has been called by Marny a T_0-object if and only if each $\mathscr{A}$-morphism $f : I_2 \to A$ is constant, where I_2 is an indiscrete $\mathscr{A}$-object whose underlying set has two points. In the category **TOP**, it is already known that T_0-objects are precisely the T_0-topological spaces. Also, in the category **Str-FTS**, Lowen and Srivastava [9] have studied T_0-objects. Motivated by the above results in **TOP** and **Str-FTS**, we examine if T_0-Q-topological spaces can also be viewed as 'T_0-objects' and arrive at the conclusion that these are T_0-objects in the category of stratified Q-topological spaces (defined below). We note that in [9] also, T_0-fuzzy topological spaces were shown to be T_0-objects in the category **Str-FTS** of stratified fuzzy topological spaces.

Definition 5.1 ([15])**.** A Q-topological space (X, τ) is said to be stratified if $\bar{q} \in \tau$, for each $q \in Q$, where $\bar{q} : X \to Q$ is q-valued constant map.

For general Q-topological spaces, although the collection of all Q-topologies on a set forms a complete lattice. But, unlike as in topology, we do not have a 'satisfactory' counterpart of 'indiscrete Q-topology' on a set, in the sense that no explicit description of its members is available, in general. This causes some hindrance in proving some interesting 'results'. To circumvent it, a convenient option is to work with stratified Q-topologies only, for which an explicit description of an indiscrete Q-topology is available.

From now onward in this paper, we consider only stratified Q-topological spaces.

Let **Str-Q-TOP** denote the category of all stratified Q-topological spaces.

Remark 5.2. It can be easily verified that the subcategory **Str-Q-TOP** of **Q-TOP** is also a topological category; in fact, it is a topological c-category.

For any set X, Q^X is clearly the largest Q-topology on X, which will be referred to the discrete Q-topology on X, while the indiscrete Q-topology on X is the Q-topology ι, where $\iota = \{\bar{q} \mid q \in Q\}$. Note that discrete and indiscrete objects (cf. [1], pages 120, 121) in the category **Str-Q-TOP** are respectively the discrete and indiscrete Q-topological spaces.

Proposition 5.3. *For $(X, \tau), (Y, \delta) \in ob$**Str-$Q$-TOP**, every constant map $f : (X, \tau) \to (Y, \delta)$ is Q-continuous.*

Proof. Straightforward. □

Proposition 5.4. *In the category* **Str-Q-TOP**, *T_0-objects are precisely the stratified T_0-Q-topological spaces.*

Proof. Let $(X, \tau) \in ob$**Str-Q-TOP** be a T_0-object. If possible, suppose that (X, τ) is not a stratified T_0-Q-topological space. Then there exist some $x, y \in X$, with $x \neq y$, such that $\mu(x) = \mu(y)$, for every $\mu \in \tau$. Let (D, ι) be a two-point indiscrete Q-topological space, with $D = \{a, b\}$. Consider the map $f : (D, \iota) \to (X, \tau)$, defined as $f(a) = x$ and $f(b) = y$. Then f is non-constant. Now we notice that f is Q-continuous, as, for every $\mu \in \tau$, $f_Q^{\leftarrow}(\mu) = \mu \circ f \in \iota$ (because $\mu \circ f(a) = \mu(f(a)) = \mu(x) = \mu(y) = \mu(f(b)) = \mu \circ f(b)$), which is a contradiction to the fact that (X, τ) is a T_0-object. Hence (X, τ) is T_0.

Next, let $(X, \tau) \in ob$**Str-Q-TOP** be T_0. If possible, suppose that there is some non-constant Q-continuous map $f : (D, \iota) \to (X, \tau)$. So, $f(a) \neq f(b)$. Since (X, τ) is T_0, there is some $\mu \in \tau$ such that $\mu(f(a)) \neq \mu(f(b))$, i.e., $f_Q^{\leftarrow}(\mu)(a) \neq f_Q^{\leftarrow}(\mu)(b)$. So, $f_Q^{\leftarrow}(\mu) \neq \bar{q}$, for any $q \in Q$. Hence $f_Q^{\leftarrow}(\mu) \notin \iota$, contradicting the Q-continuity of f. Hence (X, τ) is a T_0-object. □

Let **Str-Q-TOP$_0$** denote the subcategory of **Str-Q-TOP**, consisting of all T_0-objects of stratified Q-topological spaces. Then by using Proposition 1 of [11], we get the following result.

Proposition 5.5. **Str-Q-TOP$_0$** *is extremal epireflective in* **Str-Q-TOP**.

6. Conclusion

In this paper, we have given some characterizations of T_0-Q-topological spaces and have also shown that the category of T_0-Q-topological spaces is not only an epireflective subcategory of the category Q-**TOP** of Q-topological spaces but is also the epireflective hull of the Q-Sierpinski space in the category Q-**TOP**.

We point out that along with reflective subcategories, coreflective subcategories have also received much attention and have been studied extensively by many authors (cf. e.g., Herrlich and Strecker [4, 5, 6]) in the categories which occur in topology (e.g., like the categories of topological spaces, uniform spaces, etc.).

In the category **FTS** of fuzzy topological spaces, the coreflective hull of the fuzzy Sierpinski space has also been determined by V. Singh [17].

It would therefore be interesting to determine the coreflective hull of the Q-Sierpinski space in the category Q-**TOP**.

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Sheo Kumar Singh (sheomathbhu@gmail.com)
Department of Mathematics, Central University of Rajasthan, Ajmer-305817, India

Arun K. Srivastava (arunksrivastava@gmail.com)
Department of Mathematics, and Centre for Interdisciplinary Mathematical Sciences, Banaras Hindu University, Varanasi-221005, India