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An application of interval valued intuitionistic fuzzy soft matrix in medical diagnosis

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Abstract. In 2010, Y. Jiang et al introduced the concept of interval-valued intuitionistic fuzzy soft sets. Recently Rajarajeswari and Dhanalakshmi introduced the concept of interval-valued intuitionistic fuzzy soft matrix and defined different types of matrices along with examples.There are so many uncertainties in real life problems. The concept of interval-valued intuitionistic fuzzy soft matrix is one of the recent topics developed for dealing with the uncertainties present in most of our real life situation. In this work, an attempt has been made to apply the concept of interval-valued intuitionistic fuzzy soft matrix in medical diagnosis. The main objective of the paper is to study the decision making problems by using interval-valued intuitionistic fuzzy soft matrix.

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1. Introduction

Most of the real life problems in medical sciences, engineering management environment and social sciences often involves data which are not necessarily crisp. Classical mathematics is not very effective in dealing with such type of problems. Precise and deterministic in character due to various uncertainties associated with these problems. Such uncertainties are usually handled with the help of probability theory, fuzzy set theory [18], intuitionistic fuzzy sets [1], interval- valued fuzzy sets [5], Rough sets [11] etc. In fuzzy set theory [18], there were no scope to think about the hesitation in the membership degree which is arise in various real life situation. To overcome these situations, Atanassov [1] introduced theory of intuitionistic fuzzy set in 1986 as a generalization of fuzzy set. Due to the increasing complexity of the scientific environment and the lack of data about the problem domain, in the process of decision making under an intuitionistic fuzzy environment, a decision maker may provide their preferences over alternatives with interval-valued intuitionistic fuzzy values. Yang et al [16] presented the concept of the interval-valued fuzzy soft sets by combining the interval-valued fuzzy sets and soft sets. Jiang et al [6] presented the concept of interval-valued intuitionistic fuzzy soft set theory which is an extension of the intuitionistic fuzzy soft set theory[9].

Matrices play an important role in the broad area of science and engineering. The classical matrix theory cannot solve the problems involving various types of uncertainties. In [17] Yang et al, initiated a matrix representation of a fuzzy soft set and applied it in certain decision making problems. The concept of fuzzy soft matrix theory was studied by Borah et al in [3]. In [4], Chetia et al and in [15] Rajarajeswari et al defined intuitionistic fuzzy soft matrix.

Again it is well known that the matrices are important tools to model/study different mathematical problems specially in linear algebra. Due to huge applications of imprecise data in the above mentioned areas, hence are motivated to study the different matrices containing these data. Soft set is also one of the interesting and popular subject, where different types of decision making problem can be solved. So attempt has been made to study the decision making problem by using interval-valued intuitionistic fuzzy soft matrices.

The concept of interval- valued intuitionistic fuzzy soft matrix [14] is a generalization of intuitionistic fuzzy soft matrix [15]. In this article an attempt has been made to apply the concept of interval-valued intuitionistic fuzzy soft matrix in medical diagnosis. The significance of using interval- valued intuitionistic fuzzy soft matrix instead of interval-valued intuitionistic fuzzy soft set is to avoid tedious representation and get a simpler and concise form and by using these, different types of calculations can be done.

2. Preliminaries

In this section the preliminary definitions and results which will be required later part of the paper are described:

Definition 2.1 ([18])**.** Let X be a non empty set. Then a fuzzy set A is a set having the form $A = \{(x, \mu_A(x)) : x \in X\}$, where the function $\mu_A : X \to [0, 1]$ is called the membership function and $\mu_A(x)$ is called the degree of membership of each element $x \in X$.

Definition 2.2 ([1])**.** Let X be a non empty set. An intuitionistic fuzzy set A in X is an object $A = \{(x, \mu_A(x), \gamma_A(x)\}$,where the functions $\mu_A : X \to [0, 1]$ and denote the degree of membership (namely $\mu(A)(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$.

Definition 2.3 ([5])**.** An interval valued fuzzy set A over X is given by a function $\mu_A(x)$,where $\mu_A : X \to Int([0, 1])$, the set of all sub-intervals of unit interval i.e. for every $x \in X$, $\mu_A(x)$ is an interval within $[0, 1]$.

Definition 2.4 ([2, 19]). An interval valued intuitionistic fuzzy set A over universe set X is defined as $A = \{(x, \mu_A(x), \gamma_A(x) : x \in X\}$, where $\mu_A(x) : X \to Int([0,1])$ and $\gamma_A(x) : X \to Int([0,1])$ (where $Int[0,1]$ is the set of all closed intervals of $[0,1]$ are functions such that the condition: $\forall x \in X,\ 0 \leq sup\mu_A(x) + sup\gamma_A(x) \leq 1$ is satisfied.

Definition 2.5 ([10, 7]). Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes power set of U and $A \subseteq E$. Then the pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.6 ([8]). Let U be an initial universe and E be a set of parameters. Let I^U be the set of all fuzzy subsets of U and $A \subseteq E$. Then the pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by $F : A \to I^U$.

Definition 2.7 ([2, 17]). Let $U = \{c_1, c_2, c_3, ..., c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, ..., e_n\}$. Let $A \subseteq E$ and (F, A) be a fuzzy soft set in the fuzzy soft class (U, E) . Then the fuzzy soft set (F, A) can be expressed in a matrix form as $A_{m\times n} = [a_{ij}]_{m\times n}$ or $A = [a_{ij}]\quad i = 1, 2, ..., m; j = 1, 2, ..., n$

where $a_{ij}= \begin{cases} \mu_j(c_i) & if\, e_j \in A \\ 0 & if\, e_j \notin A. \end{cases}$

$\mu_j(c_i)$ represents the membership of c_i in the fuzzy set $F(e_j)$.

Definition 2.8 ([16]). Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (F, A) is called interval valued fuzzy soft set over U where F is a mapping given by $F : A \to IVFS^U$ where $IVFS^U$ denotes the collection of all interval valued fuzzy subsets of U.

Definition 2.9 ([13]). Let $U = \{c_1, c_2, c_3, ..., c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, ..., e_n\}$. Let $A \subseteq E$ and (F, A) be an interval valued fuzzy soft set over U, where F is a mapping given by $F : A \to IVFS^U$, where $IVFS^U$ denotes the collection of all interval valued fuzzy subsets of U. Then the interval valued fuzzy soft set can be expressed in a matrix form as $\widetilde{A}_{m\times n} = [a_{ij}]_{m\times n}$ or $\widetilde{A} = [a_{ij}]\quad i = 1, 2, ..., m; j = 1, 2, ..., n$

where $a_{ij}= \begin{cases} [\mu_{jL}(c_i),\, mu_{jU}(c_i)] & if\, e_j \in A \\ [0,\, 0] & if\, e_j \notin A. \end{cases}$

Definition 2.10 ([9]). Let U be an initial universe and E be a set of parameters. Let IF^U be the set of all intuitionistic fuzzy subsets of U and $A \subseteq E$. Then the pair (F, A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by $F : A \to IF^U$.

Definition 2.11 ([4, 12]). Let $U = \{c_1, c_2, c_3, ..., c_m\}$ be the Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, ..., e_n\}$. Let $A \subseteq E$ and (F, A) be a intuitionistic fuzzy soft set in the intuitionistic fuzzy soft class (U, E). Then the intuitionistic fuzzy soft set (F, A) can be expressed in a matrix form as $\widehat{A}_{m\times n} = [a_{ij}]_{m\times n}$ or $\widehat{A} = [a_{ij}]\quad i = 1, 2, ..., m; j = 1, 2, ..., n$

Where $a_{ij}= \begin{cases} (\mu_j(c_i),\, \gamma_j(c_i)) & if\, e_j \in A \\ (0,\, 1) & if\, e_j \notin A. \end{cases}$

$\mu_j(c_i)$ represents the membership of c_i in the fuzzy set $F(e_j)$
$\gamma_j(c_i)$ represents the non- membership of c_i in the fuzzy set $F(e_j)$.

Definition 2.12 ([6]). Let U be an initial universe and E be a set of parameters. Let $IVIFS^U$ be the set of all interval- valued intuitionistic fuzzy sets on U and $A \subseteq E$. Then the pair (F, A) is called an interval valued intuitionistic fuzzy soft set ($IVIFSS$ *in short*) over U, where F is a mapping given by $F : A \to IVIFS^U$.

Definition 2.13 ([14]). Let $U = \{c_1, c_2, c_3, ..., c_m\}$ be an Universal set and E be the set of parameters given by $E = \{e_1, e_2, e_3, ..., e_n\}$. Let $A \subseteq E$ and (F, A) be a interval valued intuitionistic fuzzy soft set over U, where F is a mapping given by $F : A \to I^U$, where I^U denotes the the collection of all interval -valued intuitionistic fuzzy subsets of U. Then the interval- valued intuitionistic fuzzy soft set can be expressed in a matrix form as
$\widehat{A}_{m\times n} = [a_{ij}]_{m\times n}$ or $\widehat{A} = [a_{ij}] \quad i = 1, 2, ..., m; j = 1, 2, ..., n$

$$a_{ij} = \begin{cases} ([(\mu_{jL}(c_i),\, \mu_{jU}(c_i)]\,[(\gamma_{jL}(c_i),\, \gamma_{jU}(c_i)]) & if\, e_j \in A \\ ([0,\, 0]\,[1, 1]) & if\, e_j \notin A. \end{cases}$$

$[\mu_{jL}(c_i), \mu_{jU}(c_i)]$ represents the membership of c_i in the interval valued intuitionistic fuzzy set $F(e_j)$. $[\gamma_{jL}(c_i), \gamma_{jU}(c_i)]$ represents the non-membership of c_i in the interval valued intuitionistic fuzzy set $F(e_j)$ with the condition $\mu_{jU}(c_i)+\gamma_{jU}(c_i) \leq 1$.

Note that if $\mu_{jU}(c_i) = \mu_{jL}(c_i)$ and $\gamma_{jU}(c_i) = \gamma_{jL}(c_i)$ then the $IVIFSM$ reduces to an $IFSM$.

Example 2.14 ([14]). Suppose that there are four houses under consideration, namely the universes $U = \{h_1, h_2, h_3, h_4\}$, and the parameter set $E = \{e_1, e_2, e_3, e_4\}$ where e_i stands for "beautiful", "large", "cheap", and "in green surroundings" respectively. Consider the mapping F from parameter set $A = \{e_1, e_2\} \subseteq E$ to the set of all interval valued intuitionistic fuzzy subsets of power set U. Consider an interval valued fuzzy soft set (F, A) which describes the "attractiveness of houses" that is considering for purchase. Then interval valued intuitionistic fuzzy soft set (F, A) is
$(F, A) = \{F(e_1) = (h_1, [0.6, 0.8][0.1, 0.2]), (h_2, [0.8, 0.9][0.05, 0.1]), (h_3, [0.6, 0.7][0.2, 0.25]),$
$(h_4, [0.5, 0.6][0.1, 0.2])\}F(e_2) = \{(h_1, [0.7, 0.8][0.15, 0.2]), (h_2, [0.6, 0.7][0.2, 0.25]),$
$(h_3, [0.5, 0.7][0.2, 0.25]), (h_4, [0.8, 0.9][0.1, 0.1])\}$.

We would represent this interval valued fuzzy soft set in matrix form as

$$\begin{bmatrix} ([0.6, 0.8][0.1, 0.2]) & ([0.7, 0.8][0.15, 0.2]) & ([0.0, 0.0][1.0, 1.0]) & ([0.0, 0.0][1.0, 1.0]) \\ ([0.8, 0.9][0.05, 0.1]) & ([0.6, 0.7][0.2, 0.25]) & ([0.0, 0.0][1.0, 1.0]) & ([0.0, 0.0][1.0, 1.0]) \\ ([0.6, 0.7][0.2, 0.25]) & ([0.5, 0.7][0.2, 0.25]) & ([0.0, 0.0][1.0, 1.0]) & ([0.0, 0.0][1.0, 1.0]) \\ ([0.5, 0.6][0.1, 0.2]) & ([0.8, 0.9][0.1, 0.1]) & ([0.0, 0.0][1.0, 1.0]) & ([0.0, 0.0][1.0, 1.0]) \end{bmatrix}$$

Now some important types of interval-valued intuitionistic fuzzy soft matrices and operations on them are discussed below.

Definition 2.15 ([14]). Let $\widehat{A} = [a_{ij}] \in IVIFSM_{m\times n}$, $\widehat{B} = [b_{ij}] \in IVIFSM_{m\times n}$. Then $\widehat{A}$ is a interval-valued intuitionistic fuzzy soft sub matrix of $\widehat{B}$, denoted by $\widehat{A} \subseteq \widehat{B}$ if $\mu_{AL} \leq \mu_{BL}, \mu_{AU} \leq \mu_{BU} and \gamma_{AL} \geq \gamma_{BL}, \gamma_{AU} \geq \gamma_{BU}$ for all i, j.

Definition 2.16 ([14]). An interval-valued intuitionistic fuzzy soft matrix of order $m \times n$ is called interval-valued intuitionistic fuzzy soft null(zero)matrix if all its elements are $([0, 0], [1, 1])$. It is denoted by $\widehat{\Phi}$.

Definition 2.17 ([14]). An interval-valued intuitionistic fuzzy soft matrix of order $m \times n$ is called interval-valued intuitionistic fuzzy soft universal matrix if all its elements are $([1,1],[0,0])$. It is denoted by $\widehat{U}$.

Definition 2.18 ([14]). Let $\widehat{A} = [a_{ij}] \in IVIFSM_{m\times n}, \widehat{B} = [b_{ij}] \in IVIFSM_{m\times n}$, Then $\widehat{A}$ is equal to $\widehat{B}$, denoted by $\widehat{A} = \widehat{B}$ if $\mu_{AL} = \mu_{BL}, \mu_{AU} = \mu_{BU}$ and $\gamma_{AL} = \gamma_{BL}, \gamma_{AU} = \gamma_{BU}$ for all i, j .

Definition 2.19 ([14]). Let $\widehat{A} = [a_{ij}] \in IVIFSM_{m\times n}$ where $a_{ij} = ([\mu_{jL}(c_i), \mu_{jU}(c_i)][\gamma_{jL}(c_i), \gamma_{jU}(c_i)])$. $[\mu_{jL}(c_i), \mu_{jU}(c_i)]$ represents the membership of c_i in the interval-valued intuitionistic fuzzy set $F(e_j)$. $[\gamma_{jL}(c_i), \gamma_{jU}(c_i)]$ represents the non-membership of c_i in the interval- valued intuitionistic fuzzy set $F(e_j)$ with the condition $\mu_{jU}(c_i) + \gamma_{jU}(c_i) \leq 1$. Then $\widehat{A}^T$ is interval-valued intuitionistic fuzzy soft transpose matrix of $\widehat{A}$ if $\widehat{A}^T = [a_{ji}]$.

Definition 2.20 ([14]). If $\widehat{A} = [a_{ij}] \in IVIFSM_{m\times n}, \widehat{B} = [b_{ij}] \in IVIFSM_{m\times n}$, then the sum of $\widehat{A}$ and $\widehat{B}$ denoted by $\widehat{A} + \widehat{B}$ and defined as $\widehat{A} + \widehat{B} = [c_{ij}]_{m\times n} = [max(\mu_{AL}, \mu_{BL}), max(\mu_{AU}, \mu_{BU})], [min(\gamma_{AL}, \gamma_{BL}), min(\gamma_{AU}, \gamma_{BU})]$ for all i and j.

Example 2.21 ([14]). Let us consider the two $IVIFS$ matrices

$$\widehat{A} = \begin{pmatrix} ([0.6,0.8][0.1,0.2]) & ([0.7,0.8][0.15,0.2]) \\ ([0.5,0.6][0.1,0.2]) & ([0.8,0.9][0.1,0.1]) \end{pmatrix}_{2\times 2}$$

and

$$\widehat{B} = \begin{pmatrix} ([0.8,0.9][0.05,0.1]) & ([0.6,0.7][0.2,0.25]) \\ ([0.6,0.7][0.2,0.25]) & ([0.5,0.7][0.2,0.25]) \end{pmatrix}_{2\times 2}.$$

Then sum of these two is $\widehat{A} + \widehat{B} = \begin{pmatrix} ([0.8,0.9][0.05,0.1]) & ([0.7,0.8][0.15,0.2]) \\ ([0.6,0.7][0.1,0.2]) & ([0.8,0.9][0.1,0.1]) \end{pmatrix}_{2\times 2}$.

Definition 2.22 ([14]). If $\widehat{A} = [a_{ij}] \in IVIFSM_{m\times n}, \widehat{B} = [b_{ij}] \in IVIFSM_{m\times n}$, then the subtraction of $\widehat{A}$ and $\widehat{B}$ denoted by $\widehat{A} - \widehat{B}$ and defined as $\widehat{A} - \widehat{B} = [c_{ij}]_{m\times n} = [min(\mu_{AL}, \mu_{BL}), min(\mu_{AU}, \mu_{BU})], [max(\gamma_{AL}, \gamma_{BL}), max(\gamma_{AU}, \gamma_{BU})]$ for all i and j.

Example 2.23 ([14]). Let us consider the two $IVIFS$ matrices

$$\widehat{A} = \begin{pmatrix} ([0.6,0.8][0.1,0.2]) & ([0.7,0.8][0.15,0.2]) \\ ([0.5,0.6][0.1,0.2]) & ([0.8,0.9][0.10,0.1]) \end{pmatrix}_{2\times 2}$$

and

$$\widehat{B} = \begin{pmatrix} ([0.8,0.9][0.05,0.10]) & ([0.6,0.7][0.2,0.25]) \\ ([0.6,0.7][0.20,0.25]) & ([0.5,0.7][0.2,0.25]) \end{pmatrix}_{2\times 2}.$$

Then difference of these two is $\widehat{A} - \widehat{B} = \begin{pmatrix} ([0.6,0.8][0.1,0.20]) & ([0.6,0.7][0.2,0.25]) \\ ([0.5,0.6][0.2,0.25]) & ([0.5,0.7][0.2,0.25]) \end{pmatrix}_{2\times 2}$.

Definition 2.24 ([14]). If $\widehat{A} = [a_{ij}] \in IVIFSM_{m\times n}, \widehat{B} = [b_{jk}] \in IVIFSM_{n\times p}$, then the multiplication of $\widehat{A}$ and $\widehat{B}$ denoted by $\widehat{A} * \widehat{B}$ and defined as $\widehat{A} * \widehat{B} = [c_{ij}]_{m\times p} = [max\, min(\mu_{AL_j}, \mu_{BL_j}), max\, min(\mu_{AU_j}, \mu_{BU_j})], [min\, max(\gamma_{AL_j}, \gamma_{BL_j}), min\, max(\gamma_{AU_j}, \gamma_{BU_j})]$ for all i, j, k.

Example 2.25 ([14]). Let us consider the two $IVIFS$ matrices

$$\widehat{A} = \begin{pmatrix} ([0.6, 0.8][0.1, 0.2]) & ([0.7, 0.8][0.15, 0.2]) \\ ([0.5, 0.6][0.1, 0.2]) & ([0.8, 0.9][0.10, 0.1]) \end{pmatrix}_{2\times 2}$$

and

$$\widehat{B} = \begin{pmatrix} ([0.8, 0.9][0.05, 0.10]) & ([0.6, 0.7][0.2, 0.25]) \\ ([0.6, 0.7][0.20, 0.25]) & ([0.5, 0.7][0.2, 0.25]) \end{pmatrix}_{2\times 2}.$$

Then product of these two $IVIFS$ matrices is

$$\widehat{A} * \widehat{B} = \begin{pmatrix} ([0.6, 0.8][0.1, 0.2]) & ([0.6, 0.7][0.2, 0.25]) \\ ([0.6, 0.7][0.1, 0.2]) & ([0.5, 0.7][0.2, 0.25]) \end{pmatrix}_{2\times 2}.$$

Remark 2.26. $\widehat{A} * \widehat{B} \neq \widehat{B} * \widehat{A}$.

Definition 2.27 ([14])**.** Let $\widehat{A} = [a_{ij}] \in IVIFSM_{m\times n}$,
where $a_{ij} = ([\mu_{jL}(c_i), \mu_{jU}(c_i)][\gamma_{jL}(c_i), \gamma_{jU}(c_i)])$ with $\mu_{jU}(c_i)+\gamma_{jU}(c_i) \leq 1$. Then $\widehat{A}^c$ is called interval-valued intuitionistic fuzzy soft complement matrix if $\widehat{A}^c = [b_{ij}]_{m\times n}$ where $b_{ij} = ([\gamma_{jL}(c_i), \gamma_{jU}(c_i)][\mu_{jL}(c_i), \mu_{jU}(c_i)]) \forall i, j with \mu_{jU}(c_i) + \gamma_{jU}(c_i) \leq 1$.

Example 2.28 ([14])**.** Let $\widehat{A} = \begin{pmatrix} ([0.6, 0.8][0.1, 0.2]) & ([0.7, 0.8][0.15, 0.2]) \\ ([0.5, 0.6][0.1, 0.2]) & ([0.8, 0.9][0.10, 0.1]) \end{pmatrix}$ be interval-valued intuitionistic fuzzy soft matrix then complement of this matrix is

$$\widehat{A}^c = \begin{pmatrix} ([0.1, 0.2][0.6, 0.8]) & ([0.15, 0.2][0.7, 0.8]) \\ ([0.1, 0.2][0.5, 0.6]) & ([0.10, 0.1][0.8, 0.9]) \end{pmatrix}.$$

3. Application of interval valued intuitionistic fuzzy soft matrix in medical diagnosis

Algorithm 3.1. We now construct the algorithm for applying Interval Valued Intuitionistic Fuzzy Soft matrix in medical diagnosis.

Step 1: Input the interval valued intuitionistic fuzzy soft sets (F, A) and $(F, A)^c$ over the sets of symptoms where A is the set of diseases. Also we write the soft medical knowledge ρ_1 and $\rho_2$and reassessing the relation matrices of the $IVIFSS(F, A)$ and $(F, A)^c$ respectively.

Step 2: Input the $IVIFSS(F_1, S)$ over the set of patients and write its relation matrix R.

Step 3: Compute the relation matrices $T_1 = R\cdot\rho_1$ and $T_2 = R\cdot\rho_2$. $T_{1L} = R_L\cdot\rho_{1L}$ and $T_{1U} = R_U \cdot \rho_{1U}$; $T_{2L} = R_L \cdot \rho_{2L}$ and $T_{2U} = R_U \cdot \rho_{2U}$ Where $T_1 = [T_{1L}, T_{1U}]$, $R = [R_L, R_U]$, $\rho_1 = [\rho_{1L}, \rho_{1U}]$, $T_2 = [T_{2L}, T_{2U}]$, $\rho_2 = [\rho_{2L}, \rho_{2U}]$.

Let us define the non disease matrices T_{3L}, T_{3U}, T_{4L} and T_{4U} corresponding to T_{1L}, T_{1U}, T_{2L} and T_{2U} respectively as $T_{3L} = R_L \cdot (\rho_{1L})^c$ and $T_{3U} = R_U \cdot (\rho_{1U})^c$; $T_{4L} = R_L \cdot (\rho_{2L})^c$ and $T_{4U} = R_U \cdot (\rho_{2U})^c$ Where $(\rho_{1L})^c$ is the complement of ρ_{1L}.
Now $S_{T_{1L}} = max_{i,j}[\mu_{T_{1L}}(p_i, d_j), \mu_{T_{4L}}(p_i, d_j)], min_{i,j}[\gamma_{T_{1L}}(p_i, d_j), \gamma_{T_{4L}}(p_i, d_j)]$ and $S_{T_{1U}} = max_{i,j}[\mu_{T_{1U}}(p_i, d_j), \mu_{T_{4U}}(p_i, d_j)], min_{i,j}[\gamma_{T_{1U}}(p_i, d_j), \gamma_{T_{4U}}(p_i, d_j)]$

Again $S_{T_{2L}} = max[\mu_{T_{2L}}(p_i, d_j), \mu_{T_{3L}}(p_i, d_j)], min[\gamma_{T_{2L}}(p_i, d_j), \gamma_{T_{3L}}(p_i, d_j)]$ and $S_{T_{2U}} = max[\mu_{T_{2U}}(p_i, d_j), \mu_{T_{3U}}(p_i, d_j)], min[\gamma_{T_{2U}}(p_i, d_j), \gamma_{T_{3U}}(p_i, d_j)]$ for all $i = 1, 2, 3$ and $j = 1, 2$.
Where $\mu_{T_{1L}}(p_i, d_j)$ is the membership value and $\gamma_{T_{1L}}(p_i, d_j)$ is the non membership value.

Step 4: Compute the diagnosis scores S_{T_1} and S_{T_2} where
$S_{T_1} = max[(\mu_{T_{1U}}(p_i, d_j), \mu_{T_{4U}}(p_i, d_j)), (\mu_{T_{2L}}(p_i, d_j), \mu_{T_{3L}}(p_i, d_j))], min[(\gamma_{T_{1U}}(p_i, d_j), \gamma_{T_{4U}}(p_i, d_j)), (\gamma_{T_{2L}}(p_i, d_j), \gamma_{T_{3L}}(p_i, d_j))]$
$S_{T_2} = max[(\mu_{T_{1L}}(p_i, d_j), \mu_{T_{4L}}(p_i, d_j)), (\mu_{T_{2U}}(p_i, d_j), \mu_{T_{3U}}(p_i, d_j))], min[(\gamma_{T_{1L}}(p_i, d_j), \gamma_{T_{4L}}(p_i, d_j)), (\gamma_{T_{2U}}(p_i, d_j), \gamma_{T_{3U}}(p_i, d_j))]$
for all $i = 1, 2, 3$ and $j = 1, 2$.

Step 5: Find
$S_k = S_{T_1} \widehat{\star} S_{T_2} = (\{\frac{\mu_{S_{T_1}}(p_i,d_j)+\mu_{S_{T_2}}(p_i,d_j)}{2}\}, \{\frac{\gamma_{S_{T_1}}(p_i,d_j)-\gamma_{S_{T_2}}(p_i,d_j)}{2}\})$.
Then conclude that the patient p_i is suffering from the disease d_k having maximum membership value and minimum non membership value.

Step 6: If S_k has more than one value then go to stop and repeat the process by reassessing the symptoms for the patients.

Case Study:

Suppose there are three patients p_1, p_2 and p_3 in a hospital with symptoms fever, headache, body pain (in the joint muscles) and rash problem. Let the possible diseases relating to the above symptoms be Dengue and Chikungunya. We consider the set $S = \{e_1, e_2, e_3, e_4\}$ an universal set, where e_1, e_2, e_3 and e_4 represent the symptoms- fever, headache, body pain (in the joint of muscles) and rash problem respectively and $A = \{d_1, d_2\}$ where d_1, d_2 represent parameterized Dengue and Chikungunya respectively.

The interval valued intuitionistic fuzzy soft set (F, A) is a parameterized family $\{F(d_1), F(d_2)\}$ of all interval valued intuitionistic fuzzy set over the set S and are determined from expert medical documentation. Thus the $IVIFS$ set (F, A) gives an approximate description of $IVIF$ soft medical knowledge of the two diseases and their matrices ρ_1 and ρ_2 called symptom disease matrix respectively.

Again we take $P = \{p_1, p_2, p_3\}$ as the Universal set where p_1, p_2 and p_3, represent patients respectively and $S = \{e_1, e_2, e_3, e_4\}$ as the set of parameters.

The $IVIF$ soft set (F_1, S) is another parameterized family of all $IVIF$ set and gives a collection of approximate description of patient-symptom in the hospital.This $IVIF$ soft set (F_1, S) represents a relation matrix R called patient-symptom matrix given by

Then combining the relation matrices ρ_1 and ρ_2 separately with R we get two matrices T_1 and T_2 called patient-disease and patient-non disease matrices respectively .

Suppose
$F(d_1) = \{(e_1, [0.5, 0.6], [0.3, 0.4]), (e_2, [0.4, 0.5], [0.2, 0.3]), (e_3, [0.7, 0.8], [0.1, 0.2]), (e_4, [0.3, 0.4], [0.4, 0.5])\}$
$F(d_2) = \{(e_1, [0.4, 0.5], [0.3, 0.4]), (e_2, [0.7, 0.8], [0.1, 0.2]), (e_3, [0.6, 0.7], [0.2, 0.3]), (e_4, [0.1, 0.2], [0.5, 0.6])\}$
$\rho_1 =$

$$\begin{bmatrix} & d_1 & d_2 \\ e_1 & [0.5, 0.6][0.3, 0.4] & [0.4, 0.5][0.3, 0.4] \\ e_2 & [0.4, 0.5][0.2, 0.3] & [0.7, 0.8][0.1, 0.2] \\ e_3 & [0.7, 0.8][0.1, 0.2] & [0.6, 0.7][0.2, 0.3] \\ e_4 & [0.3, 0.4][0.4, 0.5] & [0.1, 0.2][0.5, 0.6] \end{bmatrix}$$

$\rho_2 = (\rho_1)^c$, $(\rho_1)^c$ can be obtained from ρ_1 by interchanging the non-membership interval and membership interval.

$\rho_2 =$

$$\begin{bmatrix} & d_1 & d_2 \\ e_1 & [0.3,0.4][0.5,0.6] & [0.3,0.4][0.4,0.5] \\ e_2 & [0.2,0.3][0.4,0.5] & [0.1,0.2][0.7,0.8] \\ e_3 & [0.1,0.2][0.7,0.8] & [0.2,0.3][0.6,0.7] \\ e_4 & [0.4,0.5][0.3,0.4] & [0.5,0.6][0.1,0.2] \end{bmatrix}$$

$F(e_1) = \{(p_1,[0.6,0.7],[0.2,0.3]),(p_2,[0.4,0.5],[0.3,0.4]),(p_3,[0.5,0.6],[0.1,0.2])\}$
$F(e_2) = \{(p_1,[0.5,0.6],[0.1,0.2]),(p_2,[0.3,0.4],[0.4,0.5]),(p_3,[0.6,0.7],[0.2,0.3])\}$
$F(e_3) = \{(p_1,[0.3,0.4],[0.4,0.5]),(p_2,[0.6,0.7],[0.2,0.3]),(p_3,[0.2,0.3],[0.5,0.6])\}$
$F(e_4) = \{(p_1,[0.4,0.5],[0.2,0.3]),(p_2,[0.7,0.8],[0.1,0.2]),(p_3,[0.3,0.4],[0.4,0.5])\}$

$R=$

$$\begin{bmatrix} & e_1 & e_2 & e_3 & e_4 \\ p_1 & [0.6,0.7][0.2,0.3] & [0.5,0.6][0.1,0.2] & [0.3,0.4][0.4,0.5] & [0.4,0.5][0.2,0.3] \\ p_2 & [0.4,0.5][0.3,0.4] & [0.3,0.4][0.4,0.5] & [0.6,0.7][0.2,0.3] & [0.7,0.8][0.1,0.2] \\ p_3 & [0.5,0.6][0.1,0.2] & [0.6,0.7][0.2,0.3] & [0.2,0.3][0.5,0.6] & [0.3,0.4][0.4,0.5] \end{bmatrix}$$

$T_1 = R \cdot \rho_1=$

$$\begin{bmatrix} & e_1 & e_2 & e_3 & e_4 \\ p_1 & [0.6,0.7][0.2,0.3] & [0.5,0.6][0.1,0.2] & [0.3,0.4][0.4,0.5] & [0.4,0.5][0.2,0.3] \\ p_2 & [0.4,0.5][0.3,0.4] & [0.3,0.4][0.4,0.5] & [0.6,0.7][0.2,0.3] & [0.7,0.8][0.1,0.2] \\ p_3 & [0.5,0.6][0.1,0.2] & [0.6,0.7][0.2,0.3] & [0.2,0.3][0.5,0.6] & [0.3,0.4][0.4,0.5] \end{bmatrix}$$

$$\cdot \begin{bmatrix} d_1 & d_2 \\ [0.5,0.6][0.3,0.4] & [0.4,0.5][0.3,0.4] \\ [0.4,0.5][0.2,0.3] & [0.7,0.8][0.1,0.2] \\ [0.7,0.8][0.1,0.2] & [0.6,0.7][0.2,0.3] \\ [0.3,0.4][0.4,0.5] & [0.1,0.2][0.5,0.6] \end{bmatrix}$$

$$= \begin{bmatrix} & d_1 & d_2 \\ p_1 & [0.5,0.6][0.2,0.3] & [0.5,0.6][0.1,0.2] \\ p_2 & [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] \\ p_3 & [0.5,0.6][0.2,0.3] & [0.6,0.7][0.2,0.3] \end{bmatrix}$$

Thus,

$$T_{1L}= \begin{bmatrix} & d_1 & d_2 \\ p_1 & (0.5,0.2) & (0.5,0.1) \\ p_2 & (0.6,0.2) & (0.6,0.2) \\ p_3 & (0.5,0.2) & (0.6,0.2) \end{bmatrix}$$

$$T_{1U}= \begin{bmatrix} & d_1 & d_2 \\ p_1 & (0.6,0.3) & (0.6,0.2) \\ p_2 & (0.7,0.3) & (0.7,0.3) \\ p_3 & (0.6,0.3) & (0.7,0.3) \end{bmatrix}$$

$$\rho_{1L}= \begin{bmatrix} (0.5,0.3) & (0.4,0.3) \\ (0.4,0.2) & (0.7,0.1) \\ (0.7,0.1) & (0.6,0.2) \\ (0.3,0.4) & (0.1,0.5) \end{bmatrix}$$

$$\rho_{1U}=\begin{bmatrix}(0.6,0.4) & (0.5,0.4)\\(0.5,0.3) & (0.8,0.2)\\(0.8,0.2) & (0.7,0.3)\\(0.4,0.5) & (0.2,0.6)\end{bmatrix}$$

$$R_L=\begin{bmatrix}(0.6,0.2) & (0.5,0.1) & (0.3,0.4) & (0.4,0.2)\\(0.4,0.3) & (0.3,0.4) & (0.6,0.2) & (0.7,0.1)\\(0.5,0.1) & (0.6,0.2) & (0.2,0.5) & (0.3,0.4)\end{bmatrix}$$

$$R_U=\begin{bmatrix}(0.7,0.3) & (0.6,0.2) & (0.4,0.5) & (0.5,0.3)\\(0.5,0.4) & (0.4,0.5) & (0.7,0.3) & (0.8,0.2)\\(0.6,0.2) & (0.7,0.3) & (0.3,0.6) & (0.4,0.5)\end{bmatrix}$$

$$\rho_{2L}=\begin{bmatrix}(0.3,0.5) & (0.3,0.4)\\(0.2,0.4) & (0.1,0.7)\\(0.1,0.7) & (0.2,0.6)\\(0.4,0.3) & (0.5,0.1)\end{bmatrix}$$

$$\rho_{2U}=\begin{bmatrix}(0.4,0.6) & (0.4,0.5)\\(0.3,0.5) & (0.2,0.8)\\(0.2,0.8) & (0.3,0.7)\\(0.5,0.1) & (0.6,0.2)\end{bmatrix}$$

$$T_2 = R\cdot\rho_2=$$

$$\begin{bmatrix}[0.6,0.7][0.2,0.3] & [0.5,0.6][0.1,0.2] & [0.3,0.4][0.4,0.5] & [0.4,0.5][0.2,0.3]\\ [0.4,0.5][0.3,0.4] & [0.3,0.4][0.4,0.5] & [0.6,0.7][0.2,0.3] & [0.7,0.8][0.1,0.2]\\ [0.5,0.6][0.1,0.2] & [0.6,0.7][0.2,0.3] & [0.2,0.3][0.5,0.6] & [0.3,0.4][0.4,0.5]\end{bmatrix}$$

$$\cdot\begin{bmatrix}[0.3,0.4][0.5,0.6] & [0.3,0.4][0.4,0.5]\\ [0.2,0.3][0.4,0.5] & [0.1,0.2][0.7,0.8]\\ [0.1,0.2][0.7,0.8] & [0.2,0.3][0.6,0.7]\\ [0.4,0.5][0.3,0.4] & [0.5,0.6][0.1,0.2]\end{bmatrix}$$

$$=\begin{bmatrix}[0.4,0.5][0.3,0.4] & [0.4,0.5][0.2,0.3]\\ [0.4,0.5][0.3,0.4] & [0.5,0.6][0.1,0.2]\\ [0.3,0.4][0.4,0.5] & [0.3,0.4][0.4,0.5]\end{bmatrix}$$

$$T_{2L}=\begin{bmatrix}(0.4,0.3) & (0.4,0.2)\\(0.4,0.3) & (0.5,0.1)\\(0.3,0.4) & (0.3,0.4)\end{bmatrix}$$

$$T_{2U}=\begin{bmatrix}(0.5,0.4) & (0.5,0.3)\\(0.5,0.4) & (0.6,0.2)\\(0.4,0.5) & (0.4,0.5)\end{bmatrix}$$

$$T_{3L} = R_L\cdot(\rho_{1L})^c=\begin{bmatrix}(0.6,0.2) & (0.5,0.1) & (0.3,0.4) & (0.4,0.2)\\(0.4,0.3) & (0.3,0.4) & (0.6,0.2) & (0.7,0.1)\\(0.5,0.1) & (0.6,0.2) & (0.2,0.5) & (0.3,0.4)\end{bmatrix}\cdot\begin{bmatrix}(0.3,0.5) & (0.3,0.4)\\(0.2,0.4) & (0.1,0.7)\\(0.1,0.7) & (0.2,0.6)\\(0.4,0.3) & (0.5,0.1)\end{bmatrix}$$

$$=\begin{bmatrix}(0.4,0.3) & (0.4,0.2)\\(0.4,0.3) & (0.5,0.1)\\(0.3,0.4) & (0.3,0.4)\end{bmatrix}$$

$$T_{3U} = R_U{\cdot}(\rho_{1U})^c = \begin{bmatrix}(0.7,0.3) & (0.6,0.2) & (0.4,0.5) & (0.5,0.3)\\(0.5,0.4) & (0.4,0.5) & (0.7,0.3) & (0.8,0.2)\\(0.6,0.2) & (0.7,0.3) & (0.3,0.6) & (0.4,0.5)\end{bmatrix} \cdot \begin{bmatrix}(0.4,0.6) & (0.4,0.5)\\(0.3,0.5) & (0.2,0.8)\\(0.2,0.8) & (0.3,0.7)\\(0.5,0.4) & (0.6,0.2)\end{bmatrix}$$

$$= \begin{bmatrix}(0.5,0.4) & (0.5,0.3)\\(0.5,0.4) & (0.6,0.2)\\(0.4,0.5) & (0.4,0.5)\end{bmatrix}$$

$$T_{4L} = R_L{\cdot}(\rho_{2L})^c = \begin{bmatrix}(0.6,0.2) & (0.5,0.1) & (0.3,0.4) & (0.4,0.2)\\(0.4,0.3) & (0.3,0.4) & (0.6,0.2) & (0.7,0.1)\\(0.5,0.1) & (0.6,0.2) & (0.2,0.5) & (0.3,0.4)\end{bmatrix} \cdot \begin{bmatrix}(0.5,0.3) & (0.4,0.3)\\(0.4,0.2) & (0.7,0.1)\\(0.7,0.1) & (0.6,0.2)\\(0.3,0.4) & (0.1,0.5)\end{bmatrix}$$

$$= \begin{bmatrix}(0.5,0.2) & (0.5,0.1)\\(0.6,0.2) & (0.6,0.2)\\(0.5,0.2) & (0.6,0.2)\end{bmatrix}$$

$$T_{4U} = R_U{\cdot}(\rho_{2U})^c = \begin{bmatrix}(0.7,0.3) & (0.6,0.2) & (0.4,0.5) & (0.5,0.3)\\(0.5,0.4) & (0.4,0.5) & (0.7,0.3) & (0.8,0.2)\\(0.6,0.2) & (0.7,0.3) & (0.3,0.6) & (0.4,0.5)\end{bmatrix} \cdot \begin{bmatrix}(0.6,0.4) & (0.5,0.4)\\(0.5,0.3) & (0.8,0.2)\\(0.8,0.2) & (0.7,0.3)\\(0.4,0.5) & (0.2,0.6)\end{bmatrix}$$

$$= \begin{bmatrix}(0.6,0.3) & (0.6,0.2)\\(0.7,0.3) & (0.7,0.3)\\(0.6,0.3) & (0.7,0.3)\end{bmatrix}$$

$$S_{T_{1L}} = max_{i,j}(\mu_{T_{1L}}, \mu_{T_{4L}}), min_{i,j}(\gamma_{T_{1L}}, \gamma_{T_{4L}})$$

$$= \begin{bmatrix}(0.5,0.2) & (0.5,0.1)\\(0.6,0.2) & (0.6,0.2)\\(0.5,0.2) & (0.6,0.2)\end{bmatrix} = T_{1L} = T_{4L} \text{....................} (1)$$

$$S_{T_{1U}} = max_{i,j}(\mu_{T_{1U}}, \mu_{T_{4U}}), min_{i,j}(\gamma_{T_{1U}}, \gamma_{T_{4U}})$$

$$= \begin{bmatrix}(0.6,0.3) & (0.6,0.2)\\(0.7,0.3) & (0.7,0.3)\\(0.6,0.3) & (0.7,0.3)\end{bmatrix} = T_{1U} = T_{4U} \text{....................} (2)$$

$$S_{T_{2L}} = max_{i,j}(\mu_{T_{2L}}, \mu_{T_{3L}}), min_{i,j}(\gamma_{T_{2L}}, \gamma_{T_{3L}})$$

$$= \begin{bmatrix}(0.4,0.3) & (0.4,0.2)\\(0.4,0.3) & (0.5,0.1)\\(0.3,0.4) & (0.3,0.4)\end{bmatrix} = T_{2L} = T_{3L} \text{....................} (3)$$

$$S_{T_{2U}} = max_{i,j}(\mu_{T_{2U}}, \mu_{T_{3U}}), min_{i,j}(\gamma_{T_{2U}}, \gamma_{T_{3U}})$$

$$= \begin{bmatrix}(0.5,0.4) & (0.5,0.3)\\(0.5,0.4) & (0.6,0.2)\\(0.4,0.5) & (0.4,0.5)\end{bmatrix} = T_{2U} = T_{3U} \text{....................} (4)$$

$$S_{T_1} = max_{i,j}(\mu_{S_{T_{1U}}}(p_i,d_j), \mu_{S_{T_{2L}}}(p_i,d_j)), min_{i,j}(\gamma_{S_{T_{1U}}}(p_i,d_j), \gamma_{S_{T_{2L}}}(p_i,d_j))$$

$$= \begin{bmatrix}(0.6,0.3) & (0.6,0.2)\\(0.7,0.3) & (0.7,0.1)\\(0.6,0.3) & (0.7,0.3)\end{bmatrix}$$

$$S_{T_2} = max_{i,j}(\mu_{S_{T_{1L}}}(p_i,d_j), \mu_{S_{T_{2U}}}(p_i,d_j)), min_{i,j}(\gamma_{S_{T_{1L}}}(p_i,d_j), \gamma_{S_{T_{2U}}}(p_i,d_j))$$

$$= \begin{bmatrix}(0.5,0.2) & (0.5,0.1)\\(0.6,0.2) & (0.6,0.2)\\(0.5,0.2) & (0.6,0.2)\end{bmatrix}$$

$$S_k = S_{T_1} \widehat{\star} S_{T_1} = \begin{bmatrix} & d_1 & d_2 \\ p_1 & (0.55, 0.05) & (0.55, 0.05) \\ p_2 & (0.65, 0.05) & (0.65, -0.05) \\ p_3 & (0.55, 0.05) & (0.65, 0.05) \end{bmatrix}$$

It has been observed that patient p_2 is surely suffering from d_2 (maximum membership and minimum non-membership values). Comparing the values, p_2 is more effected by the d_1 than p_1 and p_3. p_3 is more effected by d_2 than p_1.

4. Conclusions

Here the concept of interval-valued intuitionistic fuzzy soft matrix in medical diagnosis is applied. A new algorithm has been developed for a case study. It can be observed in (1), (2), (3) and (4), where $T_{1L} = T_{4L}, T_{1U} = T_{4U}, T_{2L} = T_{3L}$ and $T_{2U} = T_{3U}$. These calculations can be reduced only by calculating T_{1L}, T_{1U}, T_{2L} and T_{2U}.

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