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A generalized statistical convergence in intuitionistic fuzzy n-normed linear spaces

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Abstract. In the current paper we have given a new definition of convergence of sequences in an intuitionistic fuzzy n-normed linear space (IFnNLS) which removes any ambiguity that could arise from the previously defined notion of the same. Relying on this new definition, in this paper, we introduce and study a new kind of generalized convergence called the λ-statistical convergence and λ-statistically Cauchy sequences in IFnNLSs. We characterize λ-statistically convergent sequences and show that this convergence is stronger than the usual convergence on intuitionistic fuzzy n-normed linear spaces.

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1. Introduction

It has been seen that many problems that we study in analysis are concerned with large classes of objects and most of such interesting classes turn out to be vector spaces or linear spaces. Also these classes are generally supplied with metrics or topologies. By introducing a norm in linear spaces, in many cases, we can get a structure of the space which is compatible with that metric or topology. The resulting structure is called a normed linear space.

The theory of 2-norm and n-norm on a linear space was introduced by S. Gähler [12, 13], which was developed by S. S. Kim and Y. J. Cho [18], R. Malceski [22], A. Misiak [23], H. Gunawan and M. Mashadi [14]. Zadeh [44] introduced the fuzzy set theory in 1965. Later on the fuzzy logic became an important area of research in various branches of mathematics like metric and topological spaces, theory of functions etc. Situations where crisp norm is unable to measure the length of a vector accurately, the notion of fuzzy norm happens to be useful. The idea of a fuzzy norm on a linear space was initiated by Katsaras [19] in 1984. In 1992, Felbin [11] introduced an alternative idea of a fuzzy norm whose associated metric is of Kaleva and Seikkala [15] type. Further in 1994, Cheng and Mordeson [4] introduced another notion of fuzzy norm on a linear space whose associated metric is of Kramosil and Michalek [20] type. Again in 2003, following Cheng and Mordeson, one more notion of fuzzy normed linear space was given by Bag and Samanta [2] from a different perspective. In 2006, using the ideas of Park [33], Saadati and Park [34, 35] introduced the concept of intuitionistic fuzzy normed linear spaces.

Vijayabalaji and Narayanan [42] extended n-normed linear space to fuzzy n-normed linear space. The theory of intuitionistic fuzzy sets occurs in the celebrated paper of Atanassov [1]. Vijayabalaji et al. [43] introduced the notion of intuitionistic fuzzy n-normed linear space (IFnNLS). The idea of statistical convergence was first introduced by Steinhaus [38] and Fast [10]. Karakus [16] studied statistical convergence on probabilistic normed spaces. Then Karakus et al. [17] generalized it on intuitionistic fuzzy normed spaces.

In this paper we have given a new definition of the notion of convergence of a sequence in an IFnNLS which is different from the one defined in [43] and other related works. This definition removes any ambiguity that could arise from previous definitions. We have developed all our results based on this new definition. Some recent work in this direction may be found in [6, 7, 8, 9, 25, 27, 37]. Also, some motivating work in the current context worth mentioning are [3, 5, 21, 24, 26, 36, 39, 40, 41]. A few more papers related to the central theme of the paper are [28, 29, 30, 31, 32].

The aim of the present paper is to introduce and investigate the λ-statistical convergence and λ-statistically Cauchy sequences on intuitionistic fuzzy n-normed linear spaces and obtain some important results on them.

2. Preliminaries

First we recall some notations and basic definitions which will be used in this paper.

Definition 2.1 ([34])**.** A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous $t-$norm if

(i) $*$ is associative and commutative,
(ii) $*$ is continuous,
(iii) $a * 1 = a$ for all $a \in [0,1]$,
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 2.2 ([34])**.** A binary operation $\circ : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous $t-$conorm if

(i) $\circ$ is associative and commutative,
(ii) $\circ$ is continuous,
(iii) $a \circ 0 = a$ for all $a \in [0,1]$,
(iv) $a \circ b \leq c \circ d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 2.3 ([14]). Let $n \in \mathbb{N}$ and X be a real linear space of dimension $d \geq n$ (d may be infinite). A real valued function $\|.\|$ on $\underbrace{X \times X \times \cdots \times X}_{n} = X^n$ is called an n-norm on X if it satisfies the following properties:
(i) $\|x_1, x_2, \ldots, x_n\| = 0$ if and only if $x_1, x_2, \ldots, x_n$ are linearly dependent,
(ii) $\|x_1, x_2, \ldots, x_n\|$ is invariant under any permutation,
(iii) $\|x_1, x_2, \ldots, \alpha x_n\| = |\alpha| \|x_1, x_2, \ldots, x_n\|$ for any $\alpha \in \mathbb{R}$,
(iv) $\|x_1, x_2, \ldots, x_{n-1}, y + z\| \leq \|x_1, x_2, \ldots, x_{n-1}, y\| + \|x_1, x_2, \ldots, x_{n-1}, z\|$,
and the pair $(X, \|.\|)$ is called an n-normed linear space.

Saadati and Park [34] introduced the concept of intuitionistic fuzzy normed space while the concept of intuitionistic fuzzy 2-normed space was introduced by Mursaleen and Lohani [27]. Vijayabalaji et al. [43] introduced the concept of intuitionistic fuzzy n-normed linear space.

Definition 2.4 ([43]). An intuitionistic fuzzy n-normed linear space or in short IFnNLS is the five-tuple $(X, \mu, \nu, *, \circ)$, where X is a linear space over a field F, $*$ is a continuous t-norm, $\circ$ is a continuous t-conorm, μ, ν are fuzzy sets on $X^n \times (0, \infty)$, μ denotes the degree of membership and ν denotes the degree of non-membership of $(x_1, x_2, \ldots, x_n, t) \in X^n \times (0, 1)$ satisfying the following conditions for every $(x_1, x_2, \ldots, x_n) \in X^n$ and $s, t > 0$:
(i) $\mu(x_1, x_2, \ldots, x_n, t) + \nu(x_1, x_2, \ldots, x_n, t) \leq 1$,
(ii) $\mu(x_1, x_2, \ldots, x_n, t) > 0$, (iii) $\mu(x_1, x_2, \ldots, x_n, t) = 1$ if and only if $x_1, x_2, \ldots, x_n$ are linearly dependent,
(iv) $\mu(x_1, x_2, \ldots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots, x_n$,
(v) $\mu(x_1, x_2, \ldots, cx_n, t) = \mu(x_1, x_2, \ldots, x_n, \frac{t}{|c|})$ if $c \neq 0, c \in F$,
(vi) $\mu(x_1, x_2, \ldots, x_n, s) * \mu(x_1, x_2, \ldots, x_n^{'}, t) \leq \mu(x_1, x_2, \ldots, x_n + x_n^{'}, s + t)$,
(vii) $\mu(x_1, x_2, \ldots, x_n, t) : (0, \infty) \to [0, 1]$ is continuous in t,
(viii) $\lim_{t\to\infty} \mu(x_1, x_2, \ldots, x_n, t) = 1$ and $\lim_{t\to 0} \mu(x_1, x_2, \ldots, x_n, t) = 0$,
(ix) $\nu(x_1, x_2, \ldots, x_n, t) > 0$,
(x) $\nu(x_1, x_2, \ldots, x_n, t) = 0$ if and only if $x_1, x_2, \ldots, x_n$ are linearly dependent,
(xi) $\nu(x_1, x_2, \ldots, x_n, t)$ is invariant under any permutation of $x_1, x_2, \ldots, x_n$,
(xii) $\nu(x_1, x_2, \ldots, cx_n, t) = \nu(x_1, x_2, \ldots, x_n, \frac{t}{|c|})$ if $c \neq 0, c \in F$,
(xiii) $\nu(x_1, x_2, \ldots, x_n, s) \circ \nu(x_1, x_2, \ldots, x_n^{'}, t) \geq \nu(x_1, x_2, \ldots, x_n + x_n^{'}, s + t)$,
(xiv) $\nu(x_1, x_2, \ldots, x_n, t) : (0, \infty) \to [0, 1]$ is continuous in t,
(xv) $\lim_{t\to\infty} \nu(x_1, x_2, \ldots, x_n, t) = 0$ and $\lim_{t\to 0} \nu(x_1, x_2, \ldots, x_n, t) = 1$.

Example 2.5 ([43]). Let $(X, \|.\|)$ be an n-normed linear space. Also let $a * b = ab$ and $a \circ b = \min\{a + b, 1\}$ for all $a, b \in [0, 1]$, $\mu(x_1, x_2, \ldots, x_n, t) = \frac{t}{t+\|x_1,x_2,\ldots,x_n\|}$ and $\nu(x_1, x_2, \ldots, x_n, t) = \frac{\|x_1,x_2,\ldots,x_n\|}{t+\|x_1,x_2,\ldots,x_n\|}$. Then $(X, \mu, \nu, *, \circ)$ is an IFnNLS.

Definition 2.6 ([38]). If K is a subset of $\mathbb{N}$, the set of natural numbers, then the natural density of K, denoted by $\delta(K)$, is given by

$$\delta(K) = \lim_{n\to\infty} \frac{1}{n} |\{k \leq n : k \in K\}|$$

whenever the limit exists, where $|A|$ denotes the cardinality of the set A.

Definition 2.7 ([10]). A sequence $x = \{x_k\}$ of numbers is statistically convergent to L if $\delta(\{k \in \mathbb{N} : |x_k - L| \geq \epsilon\}) = 0$, for every $\epsilon > 0$. We denote it by $St - \lim x = L$.

Definition 2.8. [25] Let $\lambda = \{\lambda_n\}$ be a non-decreasing sequence of positive numbers tending to ∞ such that $\lambda_{n+1} \leq \lambda_n + 1$, $\lambda_1 = 1$. Let $K \subseteq \mathbb{N}$. Then the λ-density of K is given by

$$\delta_\lambda(K) = \lim_n \frac{1}{\lambda_n} |\{n - \lambda_n + 1 \leq k \leq 1 : k \in K\}|.$$

If $\lambda_n = n$ for every $n \in \mathbb{N}$, then λ-density reduces to natural density.

A sequence $x = \{x_k\}$ of numbers is said to be λ-statistically convergent to the number l if for every $\epsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} |\{k \in I_n : |x_k - l| \geq \epsilon\}| = 0,$$

where $I_n = \{[n - \lambda_n + 1, n]\}$. We denote it by $St^\lambda - \lim x = l$.

Definition 2.9 ([37]). Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS. We say that a sequence $x = \{x_k\}$ in X is convergent to $L \in X$ with respect to the intuitionistic fuzzy n-norm $(\mu, \nu)^n$ if, for every $\epsilon > 0$, $t > 0$ and $y_1, y_2, \ldots, y_{n-1} \in X$, there exists $k_0 \in \mathbb{N}$ such that $\mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \epsilon$ and $\nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \epsilon$ for all $k \geq k_0$. It is denoted by $(\mu, \nu)^n - \lim x = L$ or $x_k \overset{(\mu,\nu)^n}{\to} L$ as $k \to \infty$.

Definition 2.10 ([37]). Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS. Then the sequence $x = \{x_k\}$ in X is called a Cauchy sequence with respect to the intuitionistic fuzzy n-norm $(\mu, \nu)^n$ if, for every $\epsilon > 0$, $t > 0$ and $y_1, y_2, \ldots, y_{n-1} \in X$, there exists $k_0 \in \mathbb{N}$ such that $\mu(y_1, y_2, \ldots, y_{n-1}, x_k - x_m, t) > 1 - \epsilon$ and $\nu(y_1, y_2, \ldots, y_{n-1}, x_k - x_m, t) < \epsilon$ for all $k, m \geq k_0$.

3. λ-statistical convergence in IFnNLS

We now obtain our main results.

Definition 3.1. Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS. A sequence $x = \{x_k\}$ in X is said to be λ- statistically convergent to $L \in X$ with respect to the intuitionistic fuzzy n-norm $(\mu, \nu)^n$ if, for every $\epsilon > 0$, $t > 0$ and $y_1, y_2, \ldots, y_{n-1} \in X$,

$$\delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \leq 1 - \epsilon \text{ or } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \geq \epsilon\}) = 0.$$

It is denoted by $St_\lambda^{(\mu,\nu)^n} - \lim x = L$.

Using Definition 3.1 and properties of λ-density, we obtain the following lemma.

Lemma 3.2. *Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS and θ be a lacunary sequence. Then for every $\epsilon > 0$, $t > 0$ and $y_1, y_2, \ldots, y_{n-1} \in X$, the following statements are equivalent:*

(i) $St_\lambda^{(\mu,\nu)^n} - \lim x = L$.

(ii) $\delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \leq 1 - \epsilon\}) = \delta_\lambda(\{k \in \mathbb{N} : \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \geq \epsilon\}) = 0$.

(iii) $\delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \epsilon$ *and* $\nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \epsilon\}) = 1$.

(iv) $\delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \epsilon\}) = \delta_\lambda(\{k \in \mathbb{N} : \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \epsilon\}) = 1$.

(v) $St^\lambda - \lim \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) = 1$ *and* $St^\lambda - \lim \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) = 0$.

Theorem 3.3. *Let* $(X, \mu, \nu, *, \circ)$ *be an IFnNLS. If a sequence* $x = \{x_k\}$ *in* X *is* λ*-statistically convergent with respect to the intuitionistic fuzzy n-norm* $(\mu, \nu)^n$*, then* $St_\lambda^{(\mu,\nu)^n} - \lim x$ *is unique.*

Proof. Let us assume that $St_\lambda^{(\mu,\nu)^n} - \lim x = L_1$ and $St_\lambda^{(\mu,\nu)^n} - \lim x = L_2$. Given $\epsilon > 0$, choose $r \in (0, 1)$ such that $(1 - r) * (1 - r) > 1 - \epsilon$ and $r \circ r < \epsilon$. Then for any $t > 0$ and $y_1, y_2, \ldots, y_{n-1} \in X$, define the following sets:

$$K_{\mu,1}(r, t) = \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L_1, t) > 1 - r\},$$

$$K_{\mu,2}(r, t) = \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L_2, t) > 1 - r\},$$

$$K_{\nu,1}(r, t) = \{k \in \mathbb{N} : \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L_1, t) < r\},$$

$$K_{\nu,2}(r, t) = \{k \in \mathbb{N} : \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L_2, t) < r\}.$$

Since $St_\lambda^{(\mu,\nu)^n} - \lim x = L_1$, using Lemma 3.2, we have

$$\delta_\lambda(K_{\mu,1}(r, t)) = \delta_\lambda(K_{\nu,1}(r, t)) = 1 \text{ for all } t > 0.$$

Also using $St_\lambda^{(\mu,\nu)^n} - \lim x = L_2$, we get

$$\delta_\lambda(K_{\mu,2}(r, t)) = \delta_\lambda(K_{\nu,2}(r, t)) = 1 \text{ for all } t > 0.$$

Now let $K_{\mu,\nu}(r, t) = (K_{\mu,1}(r, t) \cap K_{\mu,2}(r, t)) \cup (K_{\nu,1}(r, t) \cap K_{\nu,2}(r, t))$. Then $\delta_\lambda(K_{\mu,\nu}(r, t)) = 1$. Now if $k \in K_{\mu,\nu}(r, t)$, first let us consider the case $k \in (K_{\mu,1}(r, t) \cap K_{\mu,2}(r, t))$. Then we have

$$\begin{aligned} &\mu(y_1, y_2, \ldots, y_{n-1}, L_1 - L_2, t) \\ &\geq \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L_1, \frac{t}{2}) * \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L_2, \frac{t}{2}) \\ &> (1 - r) * (1 - r). \end{aligned} \tag{3.1}$$

Since $(1 - r) * (1 - r) > 1 - \epsilon$, we have $\mu(y_1, y_2, \ldots, y_{n-1}, L_1 - L_2, t) > 1 - \epsilon$ and since $\epsilon > 0$ was arbitrary, we get by 3.2, $\mu(y_1, y_2, \ldots, y_{n-1}, L_1 - L_2, t) = 1$ for all $t > 0$ and $y_1, y_2, \ldots, y_{n-1} \in X$, which implies that $L_1 = L_2$.

On the other hand, if $k \in (K_{\nu,1}(r, t) \cap K_{\nu,2}(r, t))$, then using similar technique we can prove that

$$\nu(y_1, y_2, \ldots, y_{n-1}, L_1 - L_2, t) < \epsilon.$$

Again since $\epsilon > 0$ was arbitrary, we get $\nu(y_1, y_2, \ldots, y_{n-1}, L_1 - L_2, t) = 0$ for all $t > 0$ and $y_1, y_2, \ldots, y_{n-1} \in X$, which implies that $L_1 = L_2$. Therefore, we conclude that $St_\lambda^{(\mu,\nu)^n} - \lim x$ is unique. □

Theorem 3.4. *Let $(X,\mu,\nu,*,\circ)$ be an IFnNLS. If $(\mu,\nu)^n-\lim x=L$, then $St_\lambda^{(\mu,\nu)^n}-\lim x=L$.*

Proof. Let $(\mu,\nu)^n-\lim x=L$. Thus for every $r>0$, $t>0$ and $y_1,y_2,\dots,y_{n-1}\in X$, there exists $k_0\in\mathbb{N}$ such that $\mu(y_1,y_2,\dots,y_{n-1},x_k-L,t)>1-r$ and $\nu(y_1,y_2,\dots,y_{n-1},x_k-L,t)<r$ for all $k\geq k_0$. So the set $\{k\in\mathbb{N}:\mu(y_1,y_2,\dots,y_{n-1},x_k-L,t)\leq 1-r$ or $\nu(y_1,y_2,\dots,y_{n-1},x_k-L,t)\geq r\}$ has at most finitely many terms. Thus it follows that

$$\delta_\lambda(\{k\in\mathbb{N}:\mu(y_1,y_2,\dots,y_{n-1},x_k-L,t)\leq 1-r$$

or

$$\nu(y_1,y_2,\dots,y_{n-1},x_k-L,t)\geq r\})=0.$$

Thus $St_\lambda^{(\mu,\nu)^n}-\lim x=L$. □

It follows from the following example that the converse of Theorem 3.4 is not true in general.

Example 3.5. Consider $X=\mathbb{R}^n$ with

$$\|x_1,x_2,\dots,x_n\|=abs\left(\begin{vmatrix} x_{11} & \cdots & x_{1n}\\ \vdots & \ddots & \vdots\\ x_{n1} & \cdots & x_{nn}\end{vmatrix}\right),$$

where $x_i=(x_{i1},x_{i2},\dots,x_{in})\in\mathbb{R}^n$ for each $i=1,2,\dots,n$ and let $a*b=ab$, $a\circ b=\min\{a+b,1\}$ for all $a,b\in[0,1]$. Now for all $y_1,y_2,\dots,y_{n-1},x\in\mathbb{R}^n$ and $t>0$, let us define $\mu(y_1,y_2,\dots,y_{n-1},x,t)=\frac{t}{t+\|y_1,y_2,\dots,y_{n-1},x\|}$ and $\nu(y_1,y_2,\dots,y_{n-1},x,t)=\frac{\|y_1,y_2,\dots,y_{n-1},x\|}{t+\|y_1,y_2,\dots,y_{n-1},x\|}$. Then $(\mathbb{R}^n,\mu,\nu,*,\circ)$ is an IFnNLS.

Define a sequence $x=\{x_k\}$ whose terms are given by

$$x_k=\begin{cases}(k,0,\dots,0)\in\mathbb{R}^n & \text{if } n-[\sqrt{\lambda_n}]+1\leq k\leq n\\ (0,0,\dots,0)\in\mathbb{R}^n & \text{otherwise.}\end{cases}$$

For every $0<\epsilon<1$ and for any $y_1,y_2,\dots,y_{n-1}\in X$, $t>0$, let $K_n(\epsilon,t)=\{k\in\mathbb{N}:\mu(y_1,y_2,\dots,y_{n-1},x_k,t)\leq 1-\epsilon$ or $\nu(y_1,y_2,\dots,y_{n-1},x_k,t)\geq\epsilon\}$.
Now

$$\begin{aligned}K_n(\epsilon,t)&=\{k\in\mathbb{N}:\|y_1,y_2,\dots,y_{n-1},x_k\|\geq\frac{\epsilon t}{1-\epsilon}>0\}\\ &\subseteq\{k\in\mathbb{N}:x_k=(k,0,\dots,0)\in\mathbb{R}^n\}.\end{aligned}\tag{3.2}$$

Thus we have $\frac{1}{\lambda_n}|\{k\in I_n:k\in K_n(\epsilon,t)\}|\leq\frac{[\sqrt{\lambda_n}]}{\lambda_n}\to 0$ as $n\to\infty$.
Hence $St_\lambda^{(\mu,\nu)^n}-\lim x=0$. However the sequence $x=\{x_k\}$ is not convergent in $(\mathbb{R}^n,\mu,\nu,*,\circ)$ with respect to the intuitionistic fuzzy n-norm $(\mu,\nu)^n$.

Theorem 3.6. *Let $(X,\mu,\nu,*,\circ)$ be an IFnNLS. Then $St_\lambda^{(\mu,\nu)^n}-\lim x=L$ if and only if there exists an increasing index sequence $K=\{k_n\}$ of the natural numbers such that $\delta_\lambda(K)=1$ and $(\mu,\nu)^n-\lim_n x_{k_n}=L$.*

Proof. Necessity. Let $St_{\lambda}^{(\mu,\nu)^n} - \lim x = L$. Now for any $t > 0$, $j \in \mathbb{N}$ and $y_1, y_2, \ldots, y_{n-1} \in X$, let

$$K_{\mu,\nu}(j,t) = \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \frac{1}{j} \text{ and } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \frac{1}{j}\}.$$

Then we have, for $t > 0$ and $j \in \mathbb{N}$,

$$K_{\mu,\nu}(j+1,t) \subseteq K_{\mu,\nu}(j,t). \tag{3.3}$$

Since $St_{\lambda}^{(\mu,\nu)^n} - \lim x = L$, obviously,

$$\delta_{\lambda}(\{K_{\mu,\nu}(j,t)\}) = 1. \tag{3.4}$$

Let p_1 be an arbitrary number of $K_{\mu,\nu}(1,t)$. Then by equation 3.4, there exists a number $p_2 \in K_{\mu,\nu}(2,t), (p_2 > p_1)$ such that for all $r \geq p_2$,

$$\frac{1}{\lambda_n}|\{k \in I_r : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \frac{1}{2} \text{ and } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \frac{1}{2}\}| > \frac{1}{2}.$$

Again by equation 3.4 there is a number $p \in K_{\mu,\nu}(3,t), p_3 > p_2$, such that for all $r \geq p_3$,

$$\frac{1}{\lambda_n}|\{k \in I_r : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \frac{1}{3} \text{ and } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \frac{1}{3}\}| > \frac{2}{3}.$$

and so on.
Thus by induction, we can construct an increasing index sequence $\{p_j\}_{j \in \mathbb{N}}$ of the natural numbers such that $p_j \in K_{\mu,\nu}(j,t)$ and that the following statement holds for all $r \geq p_j, (j \in \mathbb{N})$:

$$\frac{1}{\lambda_n}|\{k \in I_r : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \frac{1}{j} \text{ and } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \frac{1}{j}\}| > \frac{j-1}{j}. \tag{3.5}$$

Now we construct an increasing index sequence K as follows:

$$K = \{k \in \mathbb{N} : 1 < k < p_1\} \cup [\cup_{j \in \mathbb{N}}\{k \in K_{\mu,\nu}(j,t) : p_j \leq k < p_{j+1}\}]. \tag{3.6}$$

Now by equations 3.3, 3.5, 3.6 we have, for all $r \in \mathbb{N}, (p_j \leq r < p_{j+1})$, that

$$\begin{aligned}
&\frac{1}{\lambda_n}|\{k \in I_r : k \in K\}| \\
&\quad \geq \frac{1}{\lambda_n}|\{k \in I_r : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \frac{1}{j} \\
&\qquad \text{and } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \frac{1}{j}\}| \\
&\quad > \frac{j-1}{j}.
\end{aligned}$$

Thus it follows that $\delta_\lambda(K) = 1$. Let $\epsilon > 0$ and choose a number $j \in \mathbb{N}$ such that $\frac{1}{j} < \epsilon$. Assume that $k \geq v_j$ and $k \in K$. Then by the definition of K, there exists a number $m \geq j$ such that $v_m \leq k < v_{m+1}$ and $k \in K_{\mu,\nu}(j,t)$. Hence for every $\epsilon > 0$,

$$\mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \frac{1}{j} > 1 - \epsilon$$

and

$$\nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \frac{1}{j} < \epsilon$$

for all $k \geq v_j$ and $k \in K$. Hence $(\mu,\nu)^n - \lim_{k\in K} x_k = L$.

Sufficiency. Now we assume that there is an increasing index sequence $K = \{k_n\}$ of the natural numbers such that $\delta_\lambda(K) = 1$ and $(\mu,\nu)^n - \lim_{k\in K} x_k = L$. Then, for every $\epsilon > 0$, $t > 0$, $y_1, y_2, \ldots, y_{n-1} \in X$, there is a number n_0 such that for each $k \geq n_0$ (where $k \in K$), we have

$$\mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \epsilon \text{ and } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \epsilon.$$

Let $H_{\mu,\nu}(\epsilon,t) = \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) > 1 - \epsilon \text{ and } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) < \epsilon\}$. Then we have $H_{\mu,\nu}(\epsilon,t) \subseteq \mathbb{N} \setminus \{k_{n_0}, k_{n_0+1}, k_{n_0+2}, \ldots\}$. Since $\delta_\lambda(K) = 1$, we get $\delta_\lambda(\mathbb{N}\setminus\{k_{n_0}, k_{n_0+1}, k_{n_0+2}, \ldots\}) = 0$, which gives $\delta_\lambda(H_{\mu,\nu}(\epsilon,t)) = 0$. Therefore $S_\lambda^{(\mu,\nu)^n} - \lim x = L$. □

Theorem 3.7. *Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS. Then $St_\lambda^{(\mu,\nu)^n} - \lim x = L$ if and only if there exists a sequence y such that $(\mu,\nu)^n - \lim y = L$ and $\delta_\lambda(\{n \in \mathbb{N} : x_n = y_n\}) = 1$.*

Proof. Let $St_\lambda^{(\mu,\nu)^n} - \lim x = L$. By Theorem 3.6, we get an increasing index sequence $K = \{k_n\}$ of the natural numbers such that $\delta_\lambda(K) = 1$ and $(\mu,\nu)^n - \lim_{n\in K} x_n = L$. Then clearly the sequence y defined by

$$y = \begin{cases} x_k & \text{if } k \in K \\ L & \text{otherwise} \end{cases}$$

serves the purpose.

To prove the converse, suppose x and y be two sequences such that $(\mu,\nu)^n - \lim y = L$ and $\delta_\lambda(\{n \in \mathbb{N} : x_n = y_n\}) = 1$. Then for every $t > 0$, $\epsilon \in (0,1)$ and

$y_1, y_2, ..., y_{n-1} \in X$, we have

$$\begin{aligned}&\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \leq 1 - \epsilon \\ &\qquad\qquad\qquad \text{ or } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \geq \epsilon\} \\ &\subseteq \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \leq 1 - \epsilon \\ &\qquad \text{ or } \nu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \geq \epsilon\} \cup \{k \in \mathbb{N} : x_k \neq y_k\}.\end{aligned}$$

Thus

$$\begin{aligned}&\delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \leq 1 - \epsilon \\ &\qquad\qquad\qquad \text{ or } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \geq \epsilon\}) \\ &\leq \delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \leq 1 - \epsilon \\ &\qquad \text{ or } \nu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \geq \epsilon\}) + \delta_\lambda(\{k \in \mathbb{N} : x_k \neq y_k\}).\end{aligned}$$

Since $(\mu, \nu)^n - \lim y = L$, the set

$$\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \leq 1 - \epsilon \text{ or } \nu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \geq \epsilon\}$$

contains at most finitely many terms. So

$$\begin{aligned}&\delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \leq 1 - \epsilon \\ &\qquad\qquad\qquad \text{ or } \nu(y_1, y_2, \ldots, y_{n-1}, y_k - L, t) \geq \epsilon\}) \\ &= 0.\end{aligned}$$

Also by hypothesis, $\delta_\lambda(\{k \in \mathbb{N} : x_k \neq y_k\}) = 0$. Hence

$$\begin{aligned}&\delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \leq 1 - \epsilon \\ &\qquad\qquad\qquad \text{ or } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, t) \geq \epsilon\}) \\ &= 0.\end{aligned}$$

and consequently $S_\lambda^{(\mu,\nu)^n} - \lim x = L$. □

Theorem 3.8. *Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS. If $St_\lambda^{(\mu,\nu)^n} - \lim x = L$, then there is a convergent sequence y and a λ-statistically null sequence z such that y is convergent to L, $x = y + z$ and $\delta_\lambda(\{k \in \mathbb{N} : z_k = 0\}) = 1$*

Proof. By Theorem 3.6, we get an increasing index sequence $K = \{k_n\}$ of the natural numbers such that $\delta_\lambda\{K\} = 1$ and $(\mu, \nu)^n - \lim_{n \in K} x_n = L$. Then it is easy to see that the sequences y and z, defined by

$$y = \begin{cases} x_k & \text{if } k \in K \\ L & \text{otherwise} \end{cases}$$

and

$$z = \begin{cases} 0 & \text{if } k \in K \\ x_k - L & \text{otherwise} \end{cases}$$

satisfy the required properties. □

Lemma 3.9. *Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS. Then*

(i) If $S_\lambda^{(\mu,\nu)^n} - \lim x = \xi$ and $S_\lambda^{(\mu,\nu)^n} - \lim y = \eta$, then $S_\lambda^{(\mu,\nu)^n} - \lim(x + y) = \xi + \eta$.
(ii) If $S_\lambda^{(\mu,\nu)^n} - \lim x = \xi$ and $\alpha \in \mathbb{R}$, then $S_\lambda^{(\mu,\nu)^n} - \lim \alpha x = \alpha\xi$.
(iii) If $S_\lambda^{(\mu,\nu)^n} - \lim x = \xi$ and $S_\lambda^{(\mu,\nu)^n} - \lim y = \eta$, then $S_\lambda^{(\mu,\nu)^n} - \lim(x - y) = \xi - \eta$.

Proof. (i) For a given $\epsilon > 0$, choose $\gamma > 0$ such that $(1-\gamma) * (1-\gamma) > 1-\epsilon$. Now for any $t > 0$, $y_1, y_2, \ldots, y_{n-1} \in X$, we define the following sets:

$$K_{\mu,1}(\gamma, t) = \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - \xi, \frac{t}{2}) > 1-\gamma\}$$

and

$$K_{\mu,2}(\gamma, t) = \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, y_k - \eta, \frac{t}{2}) > 1-\gamma\}.$$

Since $S_\lambda^{(\mu,\nu)^n} - \lim x = \xi$, clearly $\delta_\lambda(K_{\mu,1}(\gamma, t)) = 1$. Similarly, $\delta_\lambda(K_{\mu,2}(\gamma, t)) = 1$. Let $K_\mu(\gamma, t) = K_{\mu,1}(\gamma, t) \cap K_{\mu,2}(\gamma, t)$. Then $\delta_\lambda(K_\mu(\gamma, t)) = 1$. Now if $k \in K_\mu(\gamma, t)$, we have

$$\begin{aligned}
&\mu(y_1, y_2, \ldots, y_{n-1}, x_k + y_k - (\xi + \eta), t) \\
&\quad \geq \mu(y_1, y_2, \ldots, y_{n-1}, x_k - \xi, \frac{t}{2}) * \mu(y_1, y_2, \ldots, y_{n-1}, y_k - \eta, \frac{t}{2}) \\
&\quad > (1-\gamma) * (1-\gamma) \\
&\quad > 1-\epsilon.
\end{aligned}$$

This shows that $\delta_\lambda(\{k \in I_r : \mu(y_1, y_2, \ldots, y_{n-1}, x_k + y_k - (\xi + \eta), t) \leq 1-\epsilon\}) = 0$. Hence $S_\lambda^{(\mu,\nu)^n} - \lim(x+y) = \xi + \eta$.

(ii) Let $\alpha = 0$. Then for every $r > 0$, $t > 0$, $y_1, y_2, \ldots, y_{n-1} \in X$, there exists $n_0 = 1$ such that $\mu(y_1, y_2, \ldots, y_{n-1}, 0x_k - 0\xi, t) = 1 > 1-r$ and $\nu(y_1, y_2, \ldots, y_{n-1}, 0x_k - 0\xi, t) = 0 < r$ for all $k \geq n_0$. So $(\mu,\nu)^n - \lim 0x = 0\xi$. This implies that $S_\lambda^{(\mu,\nu)^n} - \lim 0x = 0\xi$.

Now let $\alpha(\neq 0) \in \mathbb{R}$. Let $K_n(\epsilon, t) = \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - \xi, \frac{t}{|\alpha|}) > 1-\epsilon\}$. Since $S_\lambda^{(\mu,\nu)^n} - \lim x = \xi$, we have $\delta_\lambda(K_n(\epsilon, t)) = 1$. Now if $k \in K_n(\epsilon, t)$, then $\mu(y_1, y_2, \ldots, y_{n-1}, \alpha x_k - \alpha\xi, t) = \mu(y_1, y_2, \ldots, y_{n-1}, x_k - \xi, \frac{t}{|\alpha|}) > 1-\epsilon$. Thus $\delta_\lambda(\{k \in I_n : \mu(y_1, y_2, \ldots, y_{n-1}, \alpha x_k - \alpha\xi, t) \leq 1-\epsilon\}) = 0$. Hence $S_\lambda^{(\mu,\nu)^n} - \lim \alpha x = \alpha\xi$.

(iii) The result follows from (i) and (ii). □

Definition 3.10. Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS. For $x \in X$, $t > 0$ and $0 < r < 1$, the ball centered at x with radius r is defined by

$$\begin{aligned}
B(x, r, t) = \{y \in X : &\ \mu(y_1, y_2, \ldots, y_{n-1}, x - y, t) > 1-r \text{ and} \\
&\ \nu(y_1, y_2, \ldots, y_{n-1}, x - y, t) < r \text{ for all } y_1, y_2, \ldots, y_{n-1} \in X\}.
\end{aligned}$$

Definition 3.11. Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS. A subset Y of X is said to be bounded if for every $0 < r < 1$, $y_1, y_2, \ldots, y_{n-1} \in X$, there exists $t_0 > 0$ such that $\mu(y_1, y_2, \ldots, y_{n-1}, x, t_0) < 1-r$ and $\nu(y_1, y_2, \ldots, y_{n-1}, x, t_0) < r$ for all $x \in Y$.

From Lemma 3.9 it follows that the set of all bounded λ-statistically convergent sequences on intuitionistic fuzzy n-normed linear spaces is a linear subspace of the space $l_\infty^{(\mu,\nu)^n}$ of all bounded sequences on intuitionistic fuzzy n-normed linear spaces.

Theorem 3.12. *Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS and $S_{b\lambda}^{(\mu,\nu)^n}(X)$ the space of bounded λ-statistically convergent sequences on IFnNLSs. Then the set $S_{b\lambda}^{(\mu,\nu)^n}(X)$ is a closed linear subspace of the space $l_\infty^{(\mu,\nu)^n}(X)$.*

Proof. It suffices to show that $\overline{S_{b\lambda}^{(\mu,\nu)^n}(X)} \subset S_{b\lambda}^{(\mu,\nu)^n}(X)$. Let $y \in \overline{S_{b\lambda}^{(\mu,\nu)^n}(X)}$. Since $B(y,r,t) \cap S_{b\lambda}^{(\mu,\nu)^n}(X) \neq \Phi$, there is an $x \in B(y,r,t) \cap S_{b\lambda}^{(\mu,\nu)^n}(X)$.

Let $t > 0$ and $\epsilon \in (0,1)$. We choose $r \in (0,1)$ such that $(1-r) * (1-r) > 1-\epsilon$ and $r \circ r < \epsilon$. Since $x \in B(y,r,t) \cap S_{b\lambda}^{(\mu,\nu)^n}(X)$, let $St_{\lambda}^{(\mu,\nu)^n} - \lim x = L$.
Then for every $z_1, z_2, \ldots, z_{n-1} \in X$, there is a set $K \subseteq \mathbb{N}$ with $\delta_\lambda(K) = 1$ such that

$$\mu(z_1, z_2, \ldots, z_{n-1}, y_k - x_k, \frac{t}{2}) > 1-r, \nu(z_1, z_2, \ldots, z_{n-1}, y_k - x_k, \frac{t}{2}) < r$$

and

$$\mu(z_1, z_2, \ldots, z_{n-1}, x_k - L, \frac{t}{2}) > 1-r, \nu(z_1, z_2, \ldots, z_{n-1}, x_k - L, \frac{t}{2}) < r$$

for all $k \in K$. Thus for any $z_1, z_2, \ldots, z_{n-1} \in X$, we have

$$\begin{aligned}\mu(z_1, z_2, \ldots, z_{n-1}, y_k - L, t) &\geq \mu(z_1, z_2, \ldots, z_{n-1}, y_k - x_k, \frac{t}{2}) * \mu(z_1, z_2, \ldots, z_{n-1}, x_k, \frac{t}{2}) \\ &> (1-r) * (1-r) \\ &> 1-\epsilon,\end{aligned}$$

and

$$\begin{aligned}\nu(z_1, z_2, \ldots, z_{n-1}, y_k - L, t) &\leq \nu(z_1, z_2, \ldots, z_{n-1}, y_k - x_k, \frac{t}{2}) \circ \nu(z_1, z_2, \ldots, z_{n-1}, x_k, \frac{t}{2}) \\ &< r \circ r \\ &< \epsilon,\end{aligned}$$

for all $k \in K$.
So

$$\begin{aligned}&\delta_\lambda(\{k \in K : \mu(z_1, z_2, \ldots, z_{n-1}, y_k - L, t) > 1-\epsilon \\ &\qquad \text{and } \nu(z_1, z_2, \ldots, z_{n-1}, y_k - L, t) < \epsilon\}) \\ &= 1.\end{aligned}$$

Hence $y \in S_{b\lambda}^{(\mu,\nu)^n}(X)$. □

4. λ-statistically Cauchy sequences in IFnNLS

Here we introduce the notion of λ-statistically Cauchy sequences on an IFnNLS and discuss some properties.

Definition 4.1. Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS. We say that a sequence $x = \{x_k\}$ in X is λ-statistically Cauchy with respect to the intuitionistic fuzzy n-norm $(\mu,\nu)^n$ if, for every $\epsilon > 0$, $t > 0$ and $y_1, y_2, ..., y_{n-1} \in X$, there exists a number $m \in \mathbb{N}$ satisfying

$$\begin{aligned}&\delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - x_m, t) \leq 1-\epsilon \\ &\qquad \text{or } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - x_m, t) \geq \epsilon\}) \\ &= 0.\end{aligned}$$

Theorem 4.2. *Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS. If a sequence $x = \{x_k\}$ is λ-statistically convergent with respect to the intuitionistic fuzzy n-norm $(\mu,\nu)^n$, then it is λ-statistically Cauchy with respect to the intuitionistic fuzzy n-norm $(\mu,\nu)^n$.*

Proof. Let $x = \{x_k\}$ be a λ-statistically convergent sequence which converges to L. For a given $\epsilon > 0$, choose $r > 0$ such that $(1-r) * (1-r) > 1-\epsilon$ and $r \circ r < \epsilon$. Then for any $t > 0$, $y_1, y_2, \ldots, y_{n-1} \in X$, define

$$K_\mu(r,t) = \{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, \frac{t}{2} > 1 - r\}$$

and

$$K_\nu(r,t) = \{k \in \mathbb{N} : \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, \frac{t}{2} < r\}.$$

Then $\delta_\lambda(K_\mu(r,t)) = \delta_\lambda(K_\nu(r,t)) = 1$. Let $K(r,t) = K_\mu(r,t) \cap K_\nu(r,t)$. Then $\delta_\lambda(K(r,t)) = 1$. If $k \in K(r,t)$ and we choose a fixed $N \in K(r,t)$, then

$$\begin{aligned}
&\mu(y_1,y_2, \ldots, y_{n-1}, x_k - x_N, t) \\
&\quad \geq \mu(y_1, y_2, \ldots, y_{n-1}, x_k - L, \frac{t}{2}) * \mu(y_1, y_2, \ldots, y_{n-1}, x_N - L, \frac{t}{2}) \\
&\quad > (1-r) * (1-r) \\
&\quad > 1 - \epsilon
\end{aligned}$$

and

$$\begin{aligned}
&\nu(y_1,y_2, \ldots, y_{n-1}, x_k - x_N, t) \\
&\quad \leq \nu(y_1, y_2, \ldots, y_{n-1}, x_k - L, \frac{t}{2}) \circ \nu(y_1, y_2, \ldots, y_{n-1}, x_N - L, \frac{t}{2}) \\
&\quad < r \circ r \\
&\quad < \epsilon.
\end{aligned}$$

Thus

$$\begin{aligned}
&\delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - x_N, t) > 1 - \epsilon \\
&\qquad\qquad \text{and } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - x_N, t) < \epsilon\}) \\
&= 1.
\end{aligned}$$

So

$$\begin{aligned}
&\delta_\lambda(\{k \in \mathbb{N} : \mu(y_1, y_2, \ldots, y_{n-1}, x_k - x_N, t) \leq 1 - \epsilon \\
&\qquad\qquad \text{or } \nu(y_1, y_2, \ldots, y_{n-1}, x_k - x_N, t) \geq \epsilon\}) \\
&= 0.
\end{aligned}$$

Hence $x = \{x_k\}$ is lacunary statistically Cauchy. □

Using similar technique in the proof of Theorem 3.6 we can state the following:

Theorem 4.3. *Let $(X, \mu, \nu, *, \circ)$ be an IFnNLS and $x = \{x_k\}$ be a sequence in X. Then the following conditions are equivalent:*

(i) x is λ-statistically Cauchy with respect to the intuitionistic fuzzy n-norm $(\mu, \nu)^n$.

(ii) There exists an increasing index sequence $K = \{k_n\}$ of the natural numbers such that $\delta_\lambda(K) = 1$ and the subsequence $\{x_{k_n}\}_{n \in \mathbb{N}}$ is a Cauchy sequence with respect to the intuitionistic fuzzy n-norm $(\mu, \nu)^n$.

5. Conclusions

In this paper, we introduced a new definition of convergence of sequence in an IFnNLS and based on this new definition we studied a generalized convergence in the same space. As every crisp n-norm can induce an intuitionistic fuzzy n-norm, the results obtained in this paper are more general than the corresponding results for n-normed spaces.

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