Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 12, No. 4, (October 2016), pp. 547–558
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

A new view on intuitionistic cokernal compact spaces

T. Ramya, M. K. Uma, E. Roja

Received 5 November 2015; Revised 14 December 2015; Accepted 4 January 2016

Abstract. In this paper, we introduce the concepts of intuitionistic $\mathcal{C}$-compact set, intuitionistic cokernal compact space, intuitionistic $\mathcal{C}$-compact continuous function and intuitionistic $\mathfrak{R}$-compact spaces are studied.

2010 AMS Classification: 54A40, 54E55

Keywords: Intuitionistic $\mathcal{C}$-compact set, Intuitionistic cokernal compact space, Intuitionistic $\mathcal{C}$-compact continuous function, Intuitionistic $\mathfrak{R}$-compact space.

Corresponding Author: T. Ramya (`tramyamath@gmail.com`)

1. Introduction

The concept of an intuitionistic set was introduced by D.Coker in [2]. The intuitionistic set is the discrete form of intuitionistic fuzzy set. The concept of extremally disconnected spaces was introduced by C. Duraisamy, M. Dhavamani and N. Rajesh [3]. The concept of extremally disconnectedness is used to solve social problems through data structures.

Motivated by these applications, the concepts of intuitionistic cokernel compact spaces, intuitionistic $\mathcal{C}$-compact set and intuitionistic $\mathcal{C}$-compact continuous function and intuitionistic $\mathfrak{R}$-compact spaces are introduced and studied. In this connection, some interesting properties and characterizations are established.

2. Preliminaries

Definition 2.1 ([1, 2])**.** Let X be a nonempty fixed set.An intuitionistic set(IS, for short) A is an object having the form $A = \langle X, A^1, A^2\rangle$, for all $x \in X$ where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \phi$.The set A^1 is called the set of members of A while A^2 is called the set of nonmembers of A.Every crisp set A on a non empty set X is obviously an intuitionistic set having the form $A = \langle x, A, A^c\rangle$.

Definition 2.2 ([1, 2])**.** Let X be a nonempty set and the Intuitionistic sets A and B in the form $A = \langle x, A^1, A^2\rangle$, $B = \langle x, B^1, B^2\rangle$. Then

(i) $A \subseteq B$ iff $A^1 \subseteq B^1$ and $B^2 \subseteq A^2$;
(ii) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
(iii) $A \subseteq B$ iff $A^1 \cup A^2 \supseteq B^1 \cup B^2$;
(iv) $\overline{A} = \langle x, A^2, A^1\rangle$;
(v) $\cup A_i = \langle x, \cup A_{i1}, \cap A_{i2}\rangle$;
(vi) $\cap A_i = \langle x, \cap A_{i1}, \cup A_{i2}\rangle$;
(vii) $A - B = A \cap \overline{B}$
(viii) $\phi_\sim = \langle x, \phi, X\rangle$ and $X_\sim = \langle x, X, \phi\rangle$.

Definition 2.3 ([1, 2])**.** let X and Y be two nonempty sets and $f : X \to Y$ a function

(i) If $B = \langle x, B^1, B^2\rangle$ is an intuitionistic set in Y, then the preimage of B under f,denoted by $f^{-1}(B)$,is the intuitionistic set in X defined by
$$f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2)\rangle.$$
(ii) If $A = \langle x, A^1, A^2\rangle$ is an intuitionistic set in X, then the image of A under f,denoted by $f(A)$,is the intuitionistic set in Y defined by
$$f(A) = \langle y, f(A_1), f_-(A_2)\rangle, \text{ where } f_-(A^2) = (f(A^{2c}))^c.$$

Definition 2.4 ([2])**.** An intuitionistic topology(IT, for short)on a nonempty set X is a family T of ISs in X satisfying the following axioms:

(i) $\phi_\sim$ and $X_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case the ordered pair (X, T) is called an intuitionistic topological space (ITS, for short) on X and any intuitionistic set in T is known as an intuitionistic open set in X.The complement $\overline{A}$ of an intuitionistic open set A is called an intuitionistic closed set(ICS for short) in X.

Definition 2.5 ([1])**.** Let (X, T) be an intuitionistic topological space. If a family $\{G : G = \langle x, G^1, G^2\rangle : i \in J\}$ of IOS's in X satisfies the condition $\cup\{G : G = \langle x, G^1, G^2\rangle : i \in I\} = X_\sim$, then it is called an intuitionistic open cover of X. A finite subfamily of an intuitionistic open cover $\{G : G = \langle x, G^1, G^2\rangle : i \in J\}$ of X, which is also an intuitionistic open cover of X, is called a finite intuitionistic subcover of $\{G : G = \langle x, G^1, G^2\rangle : i \in J\}$.

Definition 2.6 ([3])**.** An intuitionistic topological space (X, T) is said to be intuitionistic extremally disconnected if and only if $cl(U) \in \tau$ for every $U \in \tau$.

3. Intuitionistic cokernal compact spaces

Definition 3.1. Let (X, T) be an intuitionistic topological space. Then $A = \langle x, A^1, A^2\rangle \in T$ is said to be intuitionistic $\mathcal{C}$-compact set if every $A \subseteq \cup_{i\in\tau} A_i^c$ where A_i^c is an intuitionistic closed set in (X, T). The complement of an intuitionistic $\mathcal{C}$-compact set is an intuitionistic $\mathcal{C}$-cocompact set.

Definition 3.2. Let (X,T) be an intuitionistic topological space and $A = \langle x, A^1, A^2\rangle$ be an intuitionistic set in (X,T). Then the intuitionistic $\mathcal{C}$-compact kernal of A and intuitionistic $\mathcal{C}$-compact cokernal of A are denoted and defined by

$$I\mathcal{K}_{\mathcal{C}}{}^{\circ}(A) = \cup\{K = \langle x, K^1, K^2\rangle : K \text{ is an intuitionistic } \mathcal{C}\text{-compact set in } (X,T) \text{ and } K \subseteq A\}$$

and

$$I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A) = \cap\{K = \langle(x, K^1, K^2\rangle : K \text{ is an intuitionistic } \mathcal{C}\text{-co compact set in } (X,T) \text{ and } A \subseteq K\}.$$

Remark 3.3. Let (X,T) be an intuitionistic topological space and $A = \langle x, A^1, A^2\rangle$ be an intuitionistic set of X. Then

(i) $I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A) = A$ if and only if A is an intuitionistic $\mathcal{C}$-cocompact set.
(ii) $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(A) = A$ if and only if A is an intuitionistic $\mathcal{C}$-compact set.

Definition 3.4. An intuitionistic topological space (X,T) is said to be an intuitionistic cokernal compact space if the intuitionistic $\mathcal{C}$-compact cokernal of every intuitionistic $\mathcal{C}$-compact set is an intuitionistic $\mathcal{C}$-compact set.

Example 3.5. Let $X = \{a,b,c\}$. Then the intuitionistic sets A,B, C, D, E, F, G,H and I of X are defined by $A = \langle\{a\},\{b\}\rangle$, $B = \langle\{c\},\{a,b\}\rangle$, $C = \langle\{a,b\},\{c\}\rangle\}$, $D = \langle\{a,c\},\{b\}\rangle$, $E = \langle\{a,b\},\{\phi\}\rangle$, $F = \langle\{\phi\},\{a,b\}\rangle$, $G = \langle\{b,c\},\{a\}\rangle$, $H = \langle\{b\},\{a\}\rangle$, and $I = \langle\{a\},\{b,c\}\rangle$. Then the family $T = \{\phi_\sim, X_\sim, A, B, C, D, E, F, G, H, I\}$ is an intuitionistic topology on X. Clearly, (X,T) is an intuitionistic cokernal compact space.

Proposition 3.6. *Let (X,T) be any intuitionistic topological space. Let $A = \langle x, A^1, A^2\rangle$ be an intuitionistic $\mathcal{C}$-compact set in X. Then the following conditions hold:*

(i) $\overline{I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)} = I\mathcal{K}_{\mathcal{C}}{}^{\circ}(\overline{A})$.
(ii) $\overline{I\mathcal{K}_{\mathcal{C}}{}^{\circ}(A)} = I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(\overline{A})$.

Proof. (i) $I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A) = \cap\{K = \langle x, K^1, K^2\rangle : K$ is an intuitionistic $\mathcal{C}$-cocompact set in (X,T) and $K \supseteq A\}$.
Taking complements on both sides,

$$\overline{I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)} = \cup\{\overline{K} : \overline{K} \text{ is an intuitionistic } \mathcal{C}\text{-compact set in } (X,T) \text{ and } \overline{K} \subseteq \overline{A}\} = I\mathcal{K}_{\mathcal{C}}{}^{\circ}\overline{(A)}.$$

(ii) $I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\natural}(A) = \cup\{K = \langle x, K^1, K^2\rangle : K$ is an intuitionistic $\mathcal{C}$-compact set in (X,T) and $K \subseteq A\}$.
Taking complements on both sides,

$$\overline{I\mathcal{K}_{\mathcal{C}}{}^{\circ}(A)} = \cap\{\overline{K} : \overline{K} \text{ is an intuitionistic } \mathcal{C}\text{-cocompact set in } (X,T) \text{ and } \overline{K} \supseteq \overline{A}\} = I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}\overline{(A)}.$$

□

Proposition 3.7. *Let (X,T) be an intuitionistic topological space. Then the following statements are equivalent:*

(i) *(X,T) is an intuitionistic cokernal compact space.*
(ii) *For each intuitionistic $\mathcal{C}$-cocompact set A, $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(A)$ is an intuitionistic $\mathcal{C}$-cocompact set.*

(iii) *For each intuitionistic $\mathcal{C}$-compact set A, we have* $IC\mathcal{K}_{\mathcal{C}}{}^{\neg}\overline{(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))} = \overline{(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))}$.

(iv) *For every pair of intuitionistic $\mathcal{C}$-compact sets A and B with $\overline{B} = IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)$, we have* $IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(B) = \overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)}$

Proof. $(i) \Rightarrow (ii)$

Let A be an intuitionistic $\mathcal{C}$-cocompact set in (X,T). Then $\overline{A}$ is an intuitionistic $\mathcal{C}$-compact set in (X,T). Then by assumption, $\overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(\overline{A})}$ is an intuitionistic $\mathcal{C}$-compact set in (X,T). Now, $IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(\overline{A}) = \overline{I\mathcal{K}_{\mathcal{C}}{}^{\circ}(A)}$. Therefore, $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(A)$ is an intuitionistic $\mathcal{C}$-cocompact set in (X,T). Hence, $(i) \Rightarrow (ii)$.

$(ii) \Rightarrow (iii)$

Let A be an intuitionistic $\mathcal{C}$-compact set in (X,T). Then $\overline{A}$ is an intuitionistic $\mathcal{C}$-cocompact set in (X,T). By assumption $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(\overline{A}) = \overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)}$ is an intuitionistic $\mathcal{C}$-cocompact set. Now,
$IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(\overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)}) = \overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)}$. Hence, $(ii) \Rightarrow (iii)$.

$(iii) \Rightarrow (iv)$

Let A and B be any two intuitionistic $\mathcal{C}$-compact sets in (X,T) such that $\overline{B} = IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)$. By (iii), $IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(\overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)}) = \overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)}$. This implies that $IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(B) = \overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)}$. Hence, $(iii) \Rightarrow (iv)$.

$(iv) \Rightarrow (i)$

Let A and B be any two intuitionistic $\mathcal{C}$-compact sets in (X,T) such that $B = \overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)}$. By (iv), it follows that, $IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(B) = \overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)}$. That is, $\overline{IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)}$ is an intuitionistic $\mathcal{C}$-cocompact set in (X,T).

This implies that $IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)$ is an intuitionistic $\mathcal{C}$-compact set in (X,T). Thus, (X,T) is an intuitionistic $\mathcal{C}$-compact cokernal compact space. Hence, $(iv) \Rightarrow (i)$. Hence the proof. $\square$

Proposition 3.8. *Let (X,T) be an intuitionistic topological space. Then (X,T) is an intuitionistic cokernal compact space if and only if for each intuitionistic $\mathcal{C}$-compact set A and intuitionistic $\mathcal{C}$-cocompact set B such that $A \subseteq B$, $IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A) \subseteq I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)$.*

Proof. Let (X,T) be an intuitionistic cokernal compact space. Let A be an intuitionistic $\mathcal{C}$-compact set and B is an intuitionistic $\mathcal{C}$-cocompact set in (X,T) such that $A \subseteq B$.

Then by (ii) of Proposition 3.7, $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)$ is an intuitionistic $\mathcal{C}$-cocompact set in (X,T). Therefore,
$IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)) = I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)$. Since A is an intuitionistic $\mathcal{C}$-compact set and $A \subseteq B$, $A \subseteq I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)$.
Now, $IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A) \subseteq IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)) = I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)$. This implies that, $IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)) \subseteq I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)$.

Conversely, let B be an intuitionistic $\mathcal{C}$-cocompact set in (X,T). Then, $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)$ is an intuitionistic $\mathcal{C}$-compact set and $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B) \subseteq B$. Then by assumption,
$IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)) \subseteq I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B))$. Also, $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B) \subseteq IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B))$. This implies that, $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B) = IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B))$.

Therefore, $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)$ is an intuitionistic closed set in (X,T). By (ii), of Proposition 3.2, (X,T) is an intuitionistic cokernal compact space.. $\square$

Definition 3.9. Let (X, T) and (Y, S) be any two intuitionistic cokernal compact spaces. A function $f : (X, T) \rightarrow (Y, S)$ is called an intuitionistic $\mathcal{C}$-compact open function if $f(A)$ is an intuitionistic $\mathcal{C}$-compact set in (Y, S), for each intuitionistic $\mathcal{C}$-compact set A in (X, T).

Proposition 3.10. *Let (X, T) and (Y, S) be any two intuitionistic cokernal compact spaces. Let $f : (X, T) \rightarrow (Y, S)$ be an intuitionistic $\mathcal{C}$-compact open and surjective function. Then $f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)) \subseteq I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(A))$, for each intuitionistic set A in (Y, S).*

Proof. Let A be an intuitionistic set in (Y, S) and $B = f^{-1}(\overline{A})$. Then,
$I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f^{-1}\overline{(A)}) = I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)$ is an intuitionistic $\mathcal{C}$-compact set in (X, T). Now, $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B) \subseteq B$. Hence, $f(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B)) \subseteq f(B)$.

That is, $I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B))) \subseteq I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f(B))$. Since f is an intuitionistic $\mathcal{C}$-compact open function, $f(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B))$ is an intuitionistic $\mathcal{C}$-compact set in (Y, S).
Therefore, $f(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B))) \subseteq I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f(B)) = I\mathcal{K}_{\mathcal{C}}{}^{\circ}(\overline{A})$. Hence
$I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f^{-1}(\overline{A})) \subseteq f^{-1}(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(\overline{A}))$. This implies that,
$\overline{I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f^{-1}(\overline{A}))} \supseteq \overline{f^{-1}(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(\overline{A}))}$ implies
$I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(A)) \supseteq f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(\overline{A}))$. Therefore, $f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A) \subseteq I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(A))$.
Hence the proof. □

Definition 3.11. Let (X, T) and (Y, S) be any two intuitionistic cokernal compact spaces. A function $f : (X, T) \rightarrow (Y, S)$ is called an intuitionistic $\mathcal{C}$-compact continuous function if $f^{-1}(A)$ is intuitionistic $\mathcal{C}$-compact set in (X, T) for every intuitionistic $\mathcal{C}$-compact set A in (Y, S).

Remark 3.12. Let (X, T) and (Y, S) be any two intuitionistic cokernal compact spaces. Let $f : (X, T) \rightarrow (Y, S)$ be any function. Then the following statements are equivalent:

(i) $f : (X, T) \rightarrow (Y, S)$ is an intuitionistic $\mathcal{C}$-compact continuous function.
(ii) $I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(A)) \subseteq f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))$ for each intuitionistic $\mathcal{C}$-compact set $A = \langle x, A^1, A^2 \rangle$ in (Y, S).

Proof. $(i) \Rightarrow (ii)$
Given $f : (X, T) \rightarrow (Y, S)$ is an intuitionistic $\mathcal{C}$-compact continuous function. Let $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic $\mathcal{C}$-compact set in (Y, S). Let $I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)$ is an intuitionistic $\mathcal{C}$-cocompact set in (Y, S) and hence $f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))$ is an intuitionistic $\mathcal{C}$-compact set in (X, T). Therefore, $I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))) = f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))$.
Since, $A \subseteq I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)$, $f^{-1}(A) = f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))$. Therefore,

$$I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(A)) \subseteq I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))) = f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))$$

That is,$I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(A)) \subseteq f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))$.
$(ii) \Rightarrow (i)$
Given that $I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(A)) \subseteq f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))$, for each intuitionistic $\mathcal{C}$-compact set in (Y, S). Let A be an intuitionistic $\mathcal{C}$-cocompact set in (Y, S). It is enough to show that $f^{-1}(V)$ is an intuitionistic $\mathcal{C}$-compact set in (X, T). Since $V = I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A)$, $f^{-1}(A) = f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))$ but it is given that $I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(A)) \subseteq f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))$, hence $I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(A)) \subseteq f^{-1}(A)) \subseteq f^{-1}(I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(A))$. Thus $f^{-1}(A)) = I\mathcal{C}\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(A))$,

that is, $f^{-1}(A)$ is an intuitionistic $\mathcal{C}$-cocompact set in (X,T). This proves that f is an intuitionistic $\mathcal{C}$-compact continuous function. □

Proposition 3.13. *Let (X,T) and (Y,S) be any two intuitionistic cokernal compact spaces. Let $f:(X,T)\to(Y,S)$ be a bijective function. Then f is an intuitionistic $\mathcal{C}$-compact continuous function if for each intuitionistic set A in (X,T),*
$f(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A))\subseteq I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A))$.

Proof. Assume that f is an intuitionistic $\mathcal{C}$-compact continuous function and A be an intuitionistic set in (X,T). Hence, $f^{-1}(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A)))$ is an intuitionistic $\mathcal{C}$-cocompact set in (X,T). By Remark 3.12, $I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f^{-1}(f(A)))\subseteq f^{-1}(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A)))$. Since f is an injective function, $I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A)\subseteq f^{-1}(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A)))$. Taking f on both sides $f(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A))\subseteq f(f^{-1}(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A))))$. Since f is a surjective function,
$f(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A))\subseteq I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A))$. □

Proposition 3.14. *Let (X,T) and (Y,S) be any two intuitionistic cokernal compact spaces. Let $f:(X,T)\to(Y,S)$ be any function. Then the following statements are equivalent:*

(i) *$f:(X,T)\to(Y,S)$ is an intuitionistic $\mathcal{C}$-cocompact continuous function.*
(ii) *$I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A))\subseteq f(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A))$, for each intuitionistic $\mathcal{C}$-compact set $A=\langle x,A^1,A^2\rangle$ in (X,T).*

Proof. $(i)\Rightarrow(ii)$
Let $A=\langle x,A^1,A^2\rangle$ be an intuitionistic $\mathcal{C}$-compact set in (X,T). Clearly, $I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A)$ is an intuitionistic $\mathcal{C}$-cocompact set in (X,T).
Since f is an intuitionistic $\mathcal{C}$-cocompact function, $f(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A))$ is an intuitionistic $\mathcal{C}$-cocompact set in (Y,S). Thus

$$I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A))\subseteq I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A)))=f(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A))$$

Hence, $(i)\Rightarrow(ii)$.
$(ii)\Rightarrow(i)$
Let A be any intuitionistic $\mathcal{C}$-cocompact set in (X,T). Then $A=I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A)$. By (ii),

$$I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A))\subseteq f(I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A))$$
$$=f(A)\subseteq I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A)).$$

Thus $f(A)=I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(f(A))$ and hence $f(A)$ is an intuitionistic $\mathcal{C}$-cocompact set in (Y,S).
Therefore f is an intuitionistic $\mathcal{C}$-cocompact function. Hence, $(ii)\Rightarrow(i)$. □

Definition 3.15. Let (X,T) be an intuitionistic topological space. An intuitionistic set $A=\langle x,A^1,A^2\rangle$ is said to be intuitionistic $\mathcal{C}$-cpt set if it is both intuitionistic $\mathcal{C}$-compact set and intuitionistic $\mathcal{C}$-cocompact set.

Remark 3.16. Let (X,T) be an intuitionistic cokernal compact space.
Let $\{A_i,\overline{B_i}/i\in N\}$ be a collection such that A_i's are intuitionistic $\mathcal{C}$-compact sets and B_j's are intuitionistic $\mathcal{C}$-cocompact sets. Let A and B be any two intuitionistic $\mathcal{C}$-cpt sets. If $A_i\subseteq A\subseteq B_j$ and $A_i\subseteq B\subseteq B_j$ for all $i,j\in N$, then there exists an intuitionistic $\mathcal{C}$-cpt set C such that $I\mathcal{CK}_{\mathcal{C}}{}^{\neg}(A_i)\subseteq C\subseteq I\mathcal{K}_{\mathcal{C}}{}^{\circ}(B_j)$ for all $i,j\in N$.

Proof. By Proposition 3.8, $I\mathcal{CK_C}^{\neg}(A_i) \subseteq I\mathcal{CK_C}^{\neg}(A) \cap I\mathcal{K_C}^{\circ}(B) \subseteq I\mathcal{K_C}^{\natural}(B_j)$ for all $i, j \in N$.
Therefore, $C = I\mathcal{CK_C}^{\neg} \cap I\mathcal{K_C}^{\circ}(B)$ is an intuitionistic $\mathcal{C}$-cpt set satisfying the required conditions. □

Notation 3.17. ICS denotes the collection of all intuitionistic $\mathcal{C}$-cpt sets

Proposition 3.18. *Let (X, T) be an intuitionistic cokernal compact space. Let $\{(A_q)\}_{q\in Q}$ and $\{(B_q)\}_{q\in Q}$ be monotone increasing collections of intuitionistic $\mathcal{C}$-compact sets and intuitionistic $\mathcal{C}$-cocompact sets of (X, T) respectively. Suppose that $A_{q_1} \subseteq B_{q_2}$ whenever $q_1 < q_2$ (Q is the set of all rational numbers). Then there exists a monotone increasing collection $\{C_q\}_{q\in Q}$ of intuitionistic $\mathcal{C}$-cpt sets such that $I\mathcal{CK_C}^{\neg}(A_{q_1}) \subseteq C_{q_2}$ and $C_{q_1} \subseteq I\mathcal{K_C}^{\circ}(B_{q_2})$ whenever $q_1 \leq q_2$.*

Proof. Let us arrange all rational numbers into a sequence $\{q_n\}$ (without repetitions). For every $n \geq 2$, we shall define inductively a collection
$\{C_{q_i} \mid 1 \leq i \leq n\} \subseteq ICS$ of intuitionistic $\mathcal{C}$-cpt sets such that

$I\mathcal{CK_C}^{\neg}(A_q) \subseteq C_{q_i}$ if $q \leq q_i$, $C_{q_i} \subseteq I\mathcal{K_C}^{\circ}(B_q)$ if $q_i \leq q$ for all $i < n$..... (S_n)

.
By Proposition 3.8, the countable collections $\{I\mathcal{CK_C}^{\neg}(A_q)\}$ and $\{I\mathcal{K_C}^{\circ}(B_q)\}$ satisfy $I\mathcal{CK_C}^{\neg}(A_{q_1}) \subseteq I\mathcal{K_C}^{\circ}(B_{q_2})$ if $q_1 < q_2$. By Remark 3.3, there exists an intuitionistic $\mathcal{C}$-cpt set D_1 such that

$$I\mathcal{CK_C}^{\neg}(A_{q_1}) \subseteq D_1 \subseteq I\mathcal{K_C}^{\circ}(B_{q_2}).$$

Letting $C_{q_1} = D_1$, we get (S_2). Assume that C_{q_i} are already defined for $i < n$ and satisfy (S_n). Define $E = \cup\{C_{q_i} \mid i < n, q_i < q_n\} \cup (A_{q_n})$ and $F = \cap\{C_{q_j}/j < n, q_j > q_n\} \cap B_{q_n}$. Then,

$$I\mathcal{CK_C}^{\neg}(C_{q_i}) \subseteq I\mathcal{CK_C}^{\neg}(E) \subseteq I\mathcal{K_C}^{\circ}(C_{q_j})$$

and

$$I\mathcal{CK_C}^{\neg}(C_{q_i}) \subseteq I\mathcal{CK_C}^{\neg}(F) \subseteq I\mathcal{K_C}^{\circ}(C_{q_j}),$$

whenever $q_i < q_n < q_j (i, j < n)$. Now,

$$A_q \subseteq I\mathcal{CK_C}^{\neg}(E) \subseteq B_{q'} \text{ and } A_q \subseteq I\mathcal{CK_C}^{\neg}(F) \subseteq B_{q'}$$

whenever $q_i < q_n < q'$.

This shows that the countable collections,

$\{C_{q_i} \mid i < n, q_i < q_n\} \cup \{A_q/q < q_n\}$ and $\{C_{q_j}/j < n, q_j < q_n\} \cup \{B_q/q > q_n\}$

together with E and F fulfill all the conditions of Remark 3.3. Hence there exists an intuitionistic $\mathcal{C}$-cpt set D_n such that

$$I\mathcal{CK_C}^{\neg}(D_n) \subseteq (B_q) \text{ if } q_n < q,\ (A_q) \subseteq I\mathcal{K_C}^{\circ}(D_n) \text{ if } q < q_n$$

,
$I\mathcal{CK_C}^{\neg}(C_{q_i}) \subseteq I\mathcal{K_C}^{\circ}(D_n)$ if $q_i < q_n$, $I\mathcal{CK_C}^{\neg}(D_n) \subseteq I\mathcal{K_C}^{\circ}(C_{q_j})$ if $q_n < q_j$ where $1 \leq i, j \leq n - 1$.

Now, setting $C_{q_n} = D_n$ we obtain an intuitionistic $\mathcal{C}$-compact sets $C_{q_1}, C_{q_2}, ..., C_{q_n}$ that satisfy (S_{n+1}). Therefore the collection $\{C_{q_i} \mid i = 1, 2, ...\}$ has the required Property. □

Definition 3.19. Let (X, T) and (Y, S)be any two intuitionistic topological spaces. A function $f : (X, T) \to (Y, S)$ is called an intuitionistic $\mathcal{C}$-compact irresolute function if $f^{-1}(A)$ is intuitionistic $\mathcal{C}$-compact set in (X, T), for each intuitionistic $\mathcal{C}$-compact set in (Y, S).

Proposition 3.20. *Let (X, T) and (Y, S) be any two intuitionistic topological spaces. A function $f : (X, T) \to (Y, S)$ is an intuitionistic $\mathcal{C}$-compact irresolute function if and only if $f(I\mathcal{CK_C}^{\neg}(A)) \subseteq I\mathcal{CK_C}^{\neg}(f(A))$, for every intuitionistic $\mathcal{C}$-compact set in (X, T).*

Proof. Suppose that f is an intuitionistic $\mathcal{C}$-compact irresolute function and let A be an intuitionistic $\mathcal{C}$-compact set in (X, T). Then $I\mathcal{CK_C}^{\neg}(f(A))$ is an intuitionistic $\mathcal{C}$-cocompact set in (Y, S). By assumption, $f^{-1}(I\mathcal{CK_C}^{\neg}(f(A)))$ is an intuitionistic $\mathcal{C}$-cocompact set in (X, T). Now, $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(I\mathcal{CK_C}^{\neg}(f(A)))$.
Now,

$$\begin{aligned} A &\subseteq f^{-1}(I\mathcal{CK_C}^{\neg}(A)) \\ I\mathcal{CK_C}^{\neg}(A)) &\subseteq I\mathcal{CK_C}^{\neg}(f^{-1}(I\mathcal{CK_C}^{\neg}(f(A)))) \\ I\mathcal{CK_C}^{\neg}(A)) &\subseteq f^{-1}(I\mathcal{CK_C}^{\neg}(f(A))). \end{aligned}$$

That is, $f(I\mathcal{CK_C}^{\neg}(A)) \subseteq I\mathcal{CK_C}^{\neg}(f(A))$.

Conversely, suppose that A is an intuitionistic $\mathcal{C}$-cocompact set in (Y, S). Then, $I\mathcal{CK_C}^{\neg}(A) = A$
Now by assumption,
$f(I\mathcal{CK_C}^{\neg}(f^{-1}(A))) \subseteq I\mathcal{CK_C}^{\neg}(f(f^{-1}(A))) = I\mathcal{CK_C}^{\neg}(A) = A$.
This implies that,

$$I\mathcal{CK_C}^{\neg}(f^{-1}(A)) \subseteq f^{-1}(A).$$

But,

$$I\mathcal{CK_C}^{\neg}(f^{-1}(A)) \supseteq f^{-1}(A).$$

Hence

$$I\mathcal{CK_C}^{\neg}(f^{-1}(A)) = f^{-1}(A).$$

That is, $f^{-1}(A)$ is an intuitionistic $\mathcal{C}$-cocompact set in (X, T). Hence, f is an intuitionistic compact irresolute function. □

4. PROPERTIES OF INTUITIONISTIC $\mathfrak{R}$-COMPACT SPACES

Definition 4.1. Let (X, T) be an intuitionistic cokernal compact space and let $A = \langle x, A^1, A^2 \rangle$ be any intuitionistic set in (X, T). Then A is said to be an intuitionistic $\mathfrak{R}\mathcal{C}$-compact if $A = I\mathcal{K_C}^{\circ}(I\mathcal{CK_C}^{\neg}(A))$

Definition 4.2. Let (X, T) be an intuitionistic cokernal compact space and let $A = \langle x, A^1, A^2 \rangle$ be any intuitionistic set in (X, T). Then A is said to be an intuitionistic $\mathfrak{R}\mathcal{C}$-cocompact if $A = I\mathcal{CK_C}^{\neg}(I\mathcal{K_C}^{\circ}(A))$

Remark 4.3. Every intuitionistic $\mathfrak{R}\mathcal{C}$-compact is an intuitionistic $\mathcal{C}$-compact.

Proposition 4.4. *Let (X,T) and (Y,S) be any two intuitionistic topological spaces. If $f:(X,T)\to(Y,S)$ is an intuitionistic $\mathcal{C}$-compact continuous function of (X,T) into an intuitionistic cokernal compact space (Y,S), and if $V=\langle x,V^1,V^2\rangle$ is an intuitionistic $\mathfrak{RC}$-compact in (Y,S), then $f^{-1}(V)$ is an intuitionistic $\mathfrak{R}$-$\mathcal{C}$-compact in (X,T).*

Proof. Since V is an intuitionistic $\mathfrak{RC}$-compact in (Y,S),from Remark 5.2.1, it follows that V is an intuitionistic $\mathcal{C}$-compact in (Y,S). Since f is intuitionistic continuous, $f^{-1}(V)$ is an intuitionistic $\mathcal{C}$-compact in (X,T). That is,

$$I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f^{-1}(V))=f^{-1}(V) \tag{4.1}$$

Since (Y,S) is an intuitionistic cokernal compact space and since V is an intuitionistic $\mathfrak{RC}$-compact in (Y,S), $V=I\mathcal{K}_{\mathcal{C}}{}^{\circ}(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(V)=IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(I\mathcal{K}_{\mathcal{C}}{}^{\circ}(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(V)=IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(V))$. That is,

$$V=IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(V) \tag{4.2}$$

$IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(V))\subseteq f^{-1}(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(V))$ because f is an intuitionistic $\mathcal{C}$-compact continuous function. Therefore,

$$I\mathcal{K}_{\mathcal{C}}{}^{\circ}(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(V)))\subseteq I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f^{-1}(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(V))).$$

From (4.2), it follows that

$$I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f^{-1}(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(V)))=I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f^{-1}(V)) \tag{4.3}$$

From (4.1) and (4.3), it follows that

$$I\mathcal{K}_{\mathcal{C}}{}^{\circ}(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(V)))\subseteq f^{-1}(V) \tag{4.4}$$

Since $f^{-1}(V)\subseteq I\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(V))$. Then,

$$I\mathcal{K}_{\mathcal{C}}{}^{\circ}(f^{-1}(V))\subseteq I\mathcal{K}_{\mathcal{C}}{}^{\circ}(I\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(V))).$$

From (4.1), it follows that

$$f^{-1}(V)\subseteq I\mathcal{K}_{\mathcal{C}}{}^{\circ}(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(V))) \tag{4.5}$$

Therefore, from (4.4) and (4.5) it follows that

$$f^{-1}(V)=I\mathcal{K}_{\mathcal{C}}{}^{\circ}(IC\mathcal{K}_{\mathcal{C}}{}^{\neg}(f^{-1}(V))).$$

Hence, $f^{-1}(V)$ is an intuitionistic $\mathfrak{RC}$-compact in (X,T). □

Definition 4.5. Let (X,T) and (Y,S) be any two intuitionistic topological spaces. If $f:(X,T)\to(Y,S)$ be a function. Then f is said to be intuitionistic $\mathcal{C}$-compact function if the image of each intuitionistic $\mathcal{C}$-compact set in (X,T) is an intuitionistic $\mathcal{C}$-compact set in (Y,S).

Definition 4.6. Let (X,T) and (Y,S) be any two intuitionistic topological spaces. If $f:(X,T)\to(Y,S)$ be a function. Then f is said to be intuitionistic $\mathcal{C}$-cocompact function if the image of each intuitionistic $\mathcal{C}$-cocompact set in (X,T) is an intuitionistic $\mathcal{C}$-cocompact set in (Y,S).

Proposition 4.7. *Let (X,T) and (Y,S) be any two intuitionistic topological spaces. If $f:(X,T)\to(Y,S)$ is an intuitionistic continuous bijective function of an intuitionistic cokernal compact space (X,T) into a space (Y,S). If $V=\langle x,V^1,V^2\rangle$ is an intuitionistic $\mathfrak{RC}$-compact set in (X,T), then $f(V)$ is an intuitionistic $\mathfrak{RC}$-compact set in (Y,S).*

Proof. Since V is an intuitionistic $\mathfrak{RC}$-compact set in (X,T) and since (X,T) is an intuitionistic cokernal compact space,

$$V = I\mathcal{K_C}^{\circ}(I\mathcal{K_C}^{\neg}(V)) = I\mathcal{K_C}^{\neg}(V).$$

That is, $V=I\mathcal{K_C}^{\neg}(V)$. Since f is an intuitionistic $\mathcal{C}$-compact continuous bijective function,

$$f(V) = f(I\mathcal{K_C}^{\neg}(V)) \subseteq I\mathcal{K_C}^{\neg}(f(V)).$$

Since f is an intuitionisti continuous function,

$$f(V) = I\mathcal{K_C}^{\circ}(f(V)) \subseteq I\mathcal{K_C}^{\circ}(I\mathcal{K_C}^{\neg}(f(V))).$$

That is,

$$f(V) \subseteq I\mathcal{K_C}^{\circ}(I\mathcal{K_C}^{\neg}(f(V))) \tag{4.6}$$

Now, $I\mathcal{K_C}^{\circ}(I\mathcal{K_C}^{\neg}(f(V))) \subseteq I\mathcal{K_C}^{\neg}(f(V))$. Since f is an intuitionistic $\mathcal{C}$-compact bijective function, f is an intuitionistic $\mathcal{C}$ cocompact function. Hence

$$\begin{aligned} I\mathcal{K_C}^{\neg}(f(V)) &\subseteq f(I\mathcal{K_C}^{\neg}(V)) \\ &= f(V). \end{aligned}$$

Then,

$$I\mathcal{K_C}^{\circ}(I\mathcal{K_C}^{\neg}(f(V))) \subseteq f(V) \tag{4.7}$$

From (4.5) and (4.6), it follows that $I\mathcal{K_C}^{\circ}(I\mathcal{K_C}^{\neg}(f(V))) = f(V)$. That is, $f(V)$ is an intuitionistic $\mathfrak{R}$-$\mathcal{C}$-compact set in (Y,S).

□

Definition 4.8. Let (X,T) be an intuitionistic topological space. If a family $\{G_i =< x, G_{i^1}, G_{i^2} >: j\in J\}$ of intuitionistic $\mathfrak{R}$-$\mathcal{C}$-compact in (X,T) satisfies the condition $\cup\{G_i : j\in J\}=\widetilde{X}$, then it is called an intuitionistic $\mathfrak{RC}$ compact cover of (X,T).

Definition 4.9. An intuitionistic topological space (X,T) is said to be intuitionistic $\mathfrak{RC}$ compact space if and only if every intuitionistic $\mathfrak{RC}$-compact cover of (X,T) has a finite subfamily the intuitionistic $\mathcal{C}$-compact cokernals of whose members cover the space (X,T).

Proposition 4.10. *Let (X,T) and (Y,S) be any two intuitionistic topological spaces. Let $f:(X,T)\to(Y,S)$ be an intuitionistic $\mathcal{C}$-compact function of an intuitionistic $\mathfrak{RC}$-compact space (X,T) onto an intuitionistic cokernal compact space (Y,S), then (Y,S) is an intuitionistic $\mathfrak{R}$-compact space.*

Proof. Let $\{V_j = \{\langle, x, V_j^1, V_j^2\rangle\}$ be an intuitionistic $\mathfrak{RC}$-compact cover of (Y, S). Since f is an intuitionistic $\mathcal{C}$-compact continuous function and (Y, S) is an intuitionistic cokernal compact space, from Proposition 4.1, $\{f^{-1}(V_j) : j \in J\}$ is an intuitionistic $\mathfrak{RC}$-compact cover of (X, T). Since (X, T) is an intuitionistic $\mathfrak{R}$-compact space, there exists a finite subfamily $f^{-1}(V_{j_1}), ...f^{-1}(V_{j_n})$ such that

$$\widetilde{X} = \cup_{i=1}^{n} I\mathcal{K_C}^{\neg}(f_{-1}(V_{j_i})))$$

Thus,

$$\widetilde{Y} = f(\cup_{i=1}^{n} I\mathcal{K_C}^{\neg}(f^{-1}(V_{j_i}))) \subseteq \cup_{i=1}^{n} I\mathcal{K_C}^{\neg}(V_{j_i})$$

Hence $\widetilde{Y} = \cup_{i=1}^{n} I\mathcal{K_C}^{\neg}(V_{j_i})$.

□

Proposition 4.11. *Let (X, T) and (Y, S) be any two intuitionistic topological spaces. Let $f : (X, T) \to (Y, S)$ be an intuitionistic $\mathcal{C}$-compact continuous bijective function of an intuitionistic cokernal compact space (X, T) onto an intuitionistic $\mathfrak{R}$-compact space (Y, S), then (X, T) is an intuitionistic $\mathfrak{R}$-compact space.*

Proof. Let $\{V_\alpha = \{\langle, x, V_\alpha^1, V_\alpha^2\rangle : \alpha \in J\}$ be an intuitionistic $\mathfrak{RC}$-compact cover of (X, T). From Proposition 4.7, $\{f^{-1}(V_\alpha) : j \in J\}$ is an intuitionistic $\mathfrak{RC}$-compact cover of (Y, S). Since (Y, S) is an intuitionistic $\mathfrak{R}$-compact space, there exists $f^{-1}(V_{\alpha_1}), ...f^{-1}(V_{\alpha_n})$ such that

$$\widetilde{Y} = \cup_{i=1}^{n} I\mathcal{K_C}^{\neg}(f^{-1}(V_{\alpha_i})))$$

Then, $\widetilde{X} = f^{-1}(\cup_{i=1}^{n} I\mathcal{K_C}^{\neg}(f(V_{\alpha_i})))$. Since f is an intuitionistic $\mathcal{C}$-cocompact function, $I\mathcal{K_C}^{\neg}(f(V_{\alpha_i})) = f(I\mathcal{K_C}^{\neg}(V_{\alpha_i}))$. Thus,

$$\widetilde{X} = \cup_{i=1}^{n} f^{-1}(f(I\mathcal{K_C}^{\neg}(V_{\alpha_i}))) = \cup_{i=1}^{n} I\mathcal{K_C}^{\neg}(V_{\alpha_i}).$$

Therefore, (X, T) is an intuitionistic $\mathfrak{R}$-compact space. □

5. Conclusions

In this paper, the concepts of intuitionistic cokernel compact spaces, intuitionistic $\mathcal{C}$-compact set and intuitionistic $\mathcal{C}$-compact continuous function are introduced and studied. Some interesting properties are also established.

Acknowledgements. The authors thanks to the referee's for their valuable comments and suggestions for supporting to publish this paper.

References

[1] D. Coker, A note on intuitionistic sets and intuitionistic points, TURKISH. J. math.20(3)(1996). 343–351.
[2] D. Coker, An Introduction to Intuitionistic Topological Spaces, Preliminary Report, Ankara,1995.
[3] C. Duraisamy, M. Dhavamani and N. Rajesh, On Intuitionistic Weakly Open (Closed) Functions, European Journal of Scientific Research, 57(2011), 646-651.

RAMYA. T (tramyamath@gmail.com) – Department of mathematics, Sri Sarada College for Women, Salem - 16, Tamil Nadu , India

UMA. M.K (mkuma70@yahoo.com) – Department of mathematics, Sri Sarada College for Women, Salem - 16, Tamil Nadu , India

ROJA. E (AMV-1982@rediffmail.com) – Department of mathematics, Sri Sarada College for Women, Salem - 16, Tamil Nadu , India