Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 12, No. 4, (October 2016), pp. 517–526
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Some notions on convex soft sets

S. N. Majeed

Received 5 March 2016; Accepted 11 April 2016

Abstract. Soft theory, formulated by Molodtsov, is considered as one of the best effective tool to deal with uncertainties. So far, it has been applied to different mathematical concepts such as set operations, algebraic structure (e.g., group and ring theory) and topological spaces. Recently, Deli has introduced convex and concave soft sets which are essential concepts for optimization and related topics. In this paper, we investigate further convex soft sets and study some of its properties. The convex hull and the cone of a soft set are also introduced with their characterizations. Finally, affine soft sets and affine hull of a soft sets are studied with some of their properties.

2010 AMS Classification: 52A20, 52A07, 52A99

Keywords: Soft Sets, Convex Soft Sets, Cone Soft Sets, Affine Soft Sets.

Corresponding Author: S. N. Majeed (majsy001@mymail.unisa.edu.au)

1. Introduction

Classical mathematics methods have proved to be successful in solving problems of exact nature. However, they are many situations in which the problems are complicated enough and do not have exact solutions. For instance, the uncertainty of data rise when trying to solve complicated problems in engineering, economic, physics, computer sciences and many other fields can not be handled using traditional mathematical methods. Many well-known theories have been introduced as an alternative mathematical tools to deal with uncertainties such as probability theory, fuzzy set theory [36], intuitionistic fuzzy sets theory [4, 5], vague sets theory [16], interval mathematics theory [5, 17], and rough sets theory [29]. However, former theories may not be suitable to apply to all problems due to possibly, the inadequacy of the parametrization tool of the theory [27]. Alternatively, Molodtsov in 1999 formulated the so-called soft theory (see [27]) to deal with uncertainties and to overcome the difficulties associated with the former theories. It is worth noting that fuzzy sets and rough sets can be considered as special types of soft sets.

Soft theory has been widely studied and involved in different mathematical directions. Maji et al. [26] presented some operations on soft sets, followed by Ali et al. [3] and Sezgin and Atagün [32]. Babitha and Sunil [6] introduced relations on soft set and studied soft functions. The application of soft theory in algebraic structures was first introduced by Aktaş and Çağman [2] in which they defined soft groups and derived some of its basic properties. The research in this direction continued and extended to include soft semigroups, soft rings, soft semirings, soft ideal etc (cf. [1, 10, 15, 20, 21, 33, 37]). Soft topological space was introduced in different ways [9, 18, 34]. Since then many researchers have further studied soft topological spaces and constructed its important notions (see e.g. [19, 28, 38]). Applications of soft sets in decision making problems have presented by Maji et al [25] and Çağman et al. [7, 8]. Chen [11] presented a new definition of soft set parametrization reduction and Pei and Miao [30] showed that soft sets are a class of special information systems.

In the field of convex analysis, convex and concave soft sets are recently introduced by Deli [12] as a soft version of Zadeh's definition of fuzzy set [36]. Deli in [12] defined convex and concave soft sets and studied some of their properties. Note that convex sets and its related notions are extended to fuzzy sets by Zadeh [36] and by Lowen [24]. Studying different properties of convex fuzzy sets are continued by many researchers, see for example [13, 14, 22, 23, 35] and reference therein.

In this paper, we continue the study of convex soft sets and we introduce affine soft sets. The organization of the paper is as follows. In section 2, we review the necessary definitions and preliminary notions of soft sets used throughout the paper. In section 3, we introduce some properties of convex soft sets, define convex hull of a soft set and study its characterizations, and define a convex cone of a soft set with some of its properties. Finally, section 4 devoted to the study of an affine soft set, an affine hull of a soft set and studying their properties. Note that the results shown in this paper are satisfied for ordinary convex and affine sets and for convex and affine fuzzy sets (see [24, 31]).

2. Preliminaries

In this section, we gather some definitions and notions related to soft set theory needed in this paper. Throughout this paper, $X = \mathbb{R}$ denotes the universe set, $P(X)$ is the power set of X, and $E = \mathbb{R}^n$ the n-dimensional parameter space.

Definition 2.1 ([27])**.** A soft set over the universe set X is defined as a pair (F, E) such that E is a set of parameters and $F : E \to P(X)$ is a mapping. Mathematically, the soft set (F, E) is a parameterized family of subsets of the set X which can be written as a set of ordered pairs

$$(F, E) = \{(e, F(e)) : e \in E \text{ and } F(e) \subseteq X\}.$$

Following [7], the set of all soft sets over X will be denoted by $S(X)$.

Definition 2.2 ([26])**.** A soft set $(F, E) \in S(X)$ is said to be null soft set if $F(e) = \phi$ for all $e \in E$. The null soft set is denoted by Φ.

Some operations on soft sets are given below.

Definition 2.3 ([7, 26]). Let (F, E) and (G, E) are two soft sets in $S(X)$. Then (F, E) is a soft subset of (G, E), denoted by $(F, E)\widetilde{\subseteq}(G, E)$, if and only if $F(e) \subseteq G(e)$ for all $e \in E$.

Definition 2.4 ([28]). Let $\{(F_i, E) : i \in I\} \subseteq S(X)$. Then

(i) The union of $\{(F_i, E) : i \in I\}$ is defined by $\widetilde{\bigcup}_{i\in I}(F_i, E) = (\widetilde{\bigcup}_{i\in I}F_i, E)$ such that

$$(\widetilde{\bigcup}_{i\in I}F_i, E) = \bigcup_{i\in I} F_i(e)\,, \quad \forall e \in E.$$

(ii) The intersection of $\{(F_i, E) : i \in I\}$ is defined by $\widetilde{\bigcap}_{i\in I}(F_i, E) = (\widetilde{\bigcap}_{i\in I}F_i, E)$ such that

$$(\widetilde{\bigcap}_{i\in I}F_i, E) = \bigcap_{i\in I} F_i(e)\,, \quad \forall e \in E.$$

Definition 2.5 ([7]). Let $(F, E) \in S(X)$ and $A \in P(X)$. Then, A-inclusion of the soft set (F, E), denoted by F^A, is defined as

$$F^A = \{e \in E : F(e) \supseteq A\}.$$

3. Convex soft sets

Convexity in soft theory is very recently discussed by Deli in [12]. Deli was the first to define convex and concave soft sets and study some of their basic properties. In this section, we give more properties related to convex soft set. We shall also introduce the concept of the convex hull of a soft set and a cone of a soft set. First, let us recall the definition of a convex soft set.

Definition 3.1 ([12]). The soft set $(F, E) \in S(X)$ is called convex if and only if

$$F(\lambda e_1 + (1 - \lambda)e_2) \supseteq F(e_1) \cap F(e_2)\,, \tag{3.1}$$

for all $e_1, e_2 \in E$ and $\lambda \in [0, 1]$.

Proposition 3.2 ([12]). *$(F, E) \in S(X)$ is a convex soft set if and only if F^A is a convex set for $A \in P(X)$.*

Proof. See the proof of Theorem 3.5 in [12]. □

In a convex soft set, the inclusion (3.1) holds for any convex combinations of finite elements in the parameter set E as we show next.

Proposition 3.3. A soft set $(F, E) \subseteq S(X)$ is a convex soft set if and only if for all $e_1, ..., e_n \in E$ and $\lambda_1, ..., \lambda_n \in [0, 1]$ such that $\sum_{i=1}^n \lambda_i = 1$ we have

$$F(\sum_{i=1}^{n} \lambda_i e_i) \supseteq \bigcap_{i=1}^{n} F(e_i). \tag{3.2}$$

Proof. Assume that the inclusion (3.2) holds, for all $e_1, ..., e_n \in E$ and $\lambda_1, ..., \lambda_n \in [0, 1]$ such that $\sum_{i=1}^n \lambda_i = 1$. In particular, for any two elements $e_1, e_2 \in E$ and $\lambda \in [0, 1]$, it yields

$$F(\lambda e_1 + (1 - \lambda)e_2) \supseteq F(e_1) \cap F(e_2).$$

From Definition 3.1, (F, E) is a convex soft set over X. To show the reverse direction and to make the proof easier, let us define, for any $n \in N$, the set of all finite subsets of E which form convex combinations of E. Namely, let

$$C(e,n) = \big\{\{e_1, ..., e_n\} \subset E : \exists \lambda_i \in [0,1] \quad \text{with} \ \sum_{i=1}^{n} \lambda_i = 1,\ e = \sum_{i=1}^{n} \lambda_i e_i\big\}. \tag{3.3}$$

Using the induction, we show the inclusion in (3.2) $\forall n \in N; n > 1$. Let $n = 2$. Then $\exists \{e_1, e_2\} \in C(e, 2)$ and $\lambda \in [0, 1]$ such that $e = \lambda e_1 + (1 - \lambda)e_2$. Since (F, E) is a convex soft set, $F(e) \supseteq F(e_1) \cap F(e_2)$ for all $e = \lambda e_1 + (1 - \lambda)e_2$. Assume now (3.2) holds for $n = k$. i.e.,

$$F(e) \supseteq \bigcap_{i=1}^{k} F(e_i)\,, \tag{3.4}$$

for $\{e_1, ..., e_k\} \in C(e, k)$ such that $e = \sum_{i=1}^{k} \lambda_i e_i$ and $\sum_{i=1}^{k} \lambda_i = 1$. We must prove (3.2) is true for $n = k + 1$. Given $\{e_1, ..., e_{k+1}\} \in C(e, k + 1)$ and $\lambda_1, ..., \lambda_{k+1} \in [0, 1]$ such that $\sum_{i=1}^{k+1} \lambda_i = 1$. Assume that at least one $\lambda_i \in [0, 1]$, say $\lambda_1 \neq 1$. Let z be a convex combination of k elements of E. Thus

$$z = \lambda'_2 e_2 + ... + \lambda'_{k+1} e_{k+1}\,,$$

where $\lambda'_i = \frac{\lambda_i}{1-\lambda_1} \geq 0$ for $i = 2, ..., k + 1$. So

$$\lambda'_2 + ... + \lambda'_{k+1} = \frac{\lambda_2 + ... + \lambda_{k+1}}{1 - \lambda_1} = \frac{1 - \lambda_1}{1 - \lambda_1} = 1.$$

Hence $\{e_2, ..., e_{k+1}\} \in C(z, k)$. From (3.4), $F(z) \supseteq \bigcap_{i=2}^{k+1} F(e_i)$. Now,$e_1, z \in E$ and (F, E) is a convex soft set. Then from induction

$$\begin{aligned} F(\lambda_1 e_1 + (1 - \lambda_1)z) &\supseteq F(e_1) \cap F(z)\,, \\ &\supseteq F(e_1) \cap F\Big(\sum_{i=2}^{k+1} \frac{\lambda_i}{1-\lambda_1} e_i\Big)\,, \\ &\supseteq \bigcap_{i=1}^{k+1} F(e_i). \end{aligned}$$

Therefore we obtain (3.2) as required. □

Proposition 3.4 ([12]). *The intersection of an arbitrary collection of convex soft sets* $\{(F_i, E) : i \in I\} \subseteq S(X)$ *is a convex soft set.*

We use the above fact to define the smallest convex soft set in $S(X)$ containing a fixed soft set.

Definition 3.5. The convex hull of a soft set (F, E), denoted by $\widetilde{\text{conv}}(F, E)$, is the smallest convex soft set over X containing (F, E). In other word,

$$\widetilde{\text{conv}}(F, E) = \widetilde{\bigcap}_{(G,E) \widetilde{\supseteq} (F,E)} \{(G, E) : (G, E) \in S(X) \text{ are convex soft sets}\}.$$

Another characterization of the convex hull of a soft set is given next.

Proposition 3.6. The convex hull of a soft set $(F,E)\in S(X)$ is given by

$$\mathrm{conv}F(e)=\bigcup_{n\in N}\bigcup_{B\in C(e,n)}\bigcap\{F(e'):e'\in B\}\,, \tag{3.5}$$

where $C(e,n)$ is defined as in (3.3).

Proof. From the definition of the convex hull of (F,E), for each $e\in E$

$$\mathrm{conv}F(e)=\bigcap G(e);\;\; F(e)\subseteq G(e)\in S(X) \text{ is a convex soft set.}$$

Define the soft set

$$\widetilde{F}(e)=\bigcup_{n\in N}\bigcup_{B\in C(e,n)}\bigcap\{F(e'):e'\in B\}. \tag{3.6}$$

We need to show that $\mathrm{conv}F(e)=\widetilde{F}(e)$. Let us start with the inclusion "$\supseteq$". Since every (G,E) is a convex soft set, using Proposition 3.3,

$$G(e)\supseteq\bigcup_{n\in N}\bigcup_{B\in C(e,n)}\bigcap\{G(e'):e'\in B\}\supseteq\widetilde{F}(e).$$

Taking the intersection, to the most left side of the above expression, over all convex soft sets $G(e)$ containing $F(e)$ yields

$$\bigcap_{G(e)\supseteq F(e)}G(e)\supseteq\widetilde{F}(e)\,,$$

which implies $\mathrm{conv}F(e)\supseteq\widetilde{F}(e)$. To obtain the inclusion "$\subseteq$", it is enough to prove that $(\widetilde{F},E)$ is a convex soft set $\big($i.e., $\widetilde{F}\big(\lambda e+(1-\lambda)e'\big)\supseteq\widetilde{F}(e)\cap\widetilde{F}(e')\;\;\forall e,e'\in E,\lambda\in[0,1]\big)$. Indeed, since $\widetilde{\mathrm{conv}}(F,E)$ is a smallest convex soft set containing (F,E), $\widetilde{\mathrm{conv}}(F,E)\subseteq(\widetilde{F},E)$. Let $e=\sum_{i=1}^{r}\alpha_ie_i$ and $e'=\sum_{i=1}^{s}\beta_ie'_i$ where $\{e_1,...,e_r\}\in C(e,r)$, $\{e'_1,...,e'_s\}\in C(e',s)$, $\sum_{i=1}^{r}\alpha_i=\sum_{i=1}^{s}\beta_i=1$, respectively. Therefore,

$$\widetilde{F}(\lambda e+(1-\lambda)e')=\widetilde{F}\big(\lambda\sum_{i=1}^{r}\alpha_ie_i+(1-\lambda)\sum_{i=1}^{s}\beta_ie'_i\big)\,,$$

where $\lambda\sum_{i=1}^{r}\alpha_i+(1-\lambda)\sum_{i=1}^{s}\beta_i=1$ and $\{e_1,...,e_r,e'_1,...,e'_s\}\in C(\lambda e+(1-\lambda)e',r+s)$. Notice that, since $E=\mathbb{R}^n$ then $\lambda e+(1-\lambda)e'\in E=\mathbb{R}^n\;\;\forall e,e'\in E$. Applying the definition of the soft set in (3.6) to e,e' and to $z:=\lambda e+(1-\lambda)e'$, we get

$$\begin{aligned}&\bigcup_{r+s\in N}\bigcup_{A\cup B\in C(z,r+s)}\bigcap_{i=1}^{r+s}\{F(z_i):z_i\in A\cup B\}\\ \supseteq&\bigcup_{r\in N}\bigcup_{A\in C(e,r)}\bigcap_{i=1}^{r}\{F(e_i):e_i\in A\}\bigcap\bigcup_{s\in N}\bigcup_{B\in C(e',s)}\bigcap_{i=1}^{s}\{F(e'_i):e'_i\in B\}\,,\end{aligned}$$

i.e.,

$$\widetilde{F}\big(z=\lambda e+(1-\lambda)e'\big)\supseteq\widetilde{F}(e)\cap\widetilde{F}(e').$$

as required. □

Next we define a convex cone soft set and deduce some of its properties.

Definition 3.7. A soft set $(F, E) \in S(X)$ is said to be cone if for all $e \in E$ and $\lambda > 0$ we have

$$F(\lambda e) \supseteq F(e).$$

If (F, E) is convex then it is called convex cone soft set.

Proposition 3.8. $(F, E) \in S(X)$ is a cone soft set if and only if F^A is a cone set for $A \in P(X)$.

Proof. The proof proceeds as that of Theorem 3.5 in [12]. □

Proposition 3.9. A soft set $(F, E) \in S(X)$ is a convex cone if and only if for each $e, e' \in E$ and $\lambda > 0$

(i) $F(\lambda e) \supseteq F(e)$,
(ii) $F(e + e') \supseteq F(e) \cap F(e')$.

Proof. Assume that (F, E) is a convex cone soft set over X. This yields (i). Take now arbitrary $e, e' \in E$ and choose $\lambda = \frac{1}{2}$. Since (F, E) is a convex cone soft set over X

$$F(\frac{1}{2}e + \frac{1}{2}e') \supseteq F(e) \cap F(e') \tag{3.7}$$

and

$$F(2(\frac{1}{2}e + \frac{1}{2}e')) \supseteq F(\frac{1}{2}e + \frac{1}{2}e')\,. \tag{3.8}$$

From (3.7) and (3.8) we obtain (ii). Indeed,

$$F(e + e') = F(2(\frac{1}{2}e + \frac{1}{2}e')) \supseteq F(\frac{1}{2}e + \frac{1}{2}e') \supseteq F(e) \cap F(e').$$

To show the other direction (i.e., (F, E) is a convex cone soft set). From part (i), (F, E) is a cone. Using (i) and (ii) with $\lambda \in [0, 1]$, we conclude that (F, E) is a convex soft set. i.e.,

$$F(\lambda e + (1 - \lambda)e') \supseteq F(\lambda e) \cap F((1 - \lambda)e') \supseteq F(e) \cap F(e').$$

□

A direct consequence from the preceding proposition is given next

Corollary 3.10. $(F, E) \in S(X)$ is a convex cone if and only if for all $e_1, ..., e_n \in E$ and $\lambda_i > 0$

$$F(\sum_{i=1}^{n} \lambda_i e_i) \supseteq \bigcap_{i=1}^{n} F(e_i).$$

Proposition 3.11. The intersection of an arbitrary collection of convex cone soft sets $\{(F_i, E) : i \in I\} \subseteq S(X)$ is a convex cone soft set.

Proof. The proof is analogous to that of Proposition 3.4. □

Inspired by the preceding proposition, one can define the smallest convex cone of an arbitrary soft set.

Definition 3.12. The convex cone of a soft set $(F, E) \in S(X)$, denoted by $\widetilde{\text{cone}}(F, E)$, is the smallest convex cone soft set containing (F, E). i.e,

$$\widetilde{\text{cone}}(F, E) = \widetilde{\bigcap}_{(G,E)\tilde{\supseteq}(F,E)} \big\{(G, E) : (G, E) \in S(X) \text{ is a convex cone soft set}\big\}.$$

A relation between the convex hull and the convex cone of two soft sets over X is given next.

Proposition 3.13. Let (F, E) and $(\overline{F}, E)$ are two soft sets over X such that

$$(\overline{F}, E) = \bigcup_{\beta>0} (F, \beta E)\,,$$

then $\widetilde{\text{cone}}(F, E) = \widetilde{\text{conv}}(\overline{F}, E)$.

Proof. To show $\widetilde{\text{cone}}(F, E) = \widetilde{\text{conv}}(\overline{F}, E)$, it is enough to prove that $\widetilde{\text{conv}}(\overline{F}, E)$ is a smallest cone soft set contains (F, E). We proceed first to prove $\widetilde{\text{conv}}(\overline{F}, E)$ is a cone soft set over X contains (F, E). From (3.5),

$$\text{conv}\overline{F}(e) = \bigcup_{n\in N} \bigcup_{B\in C(e,n)} \bigcap\{\overline{F}(e') : e' \in B\}.$$

Note that $\forall e \in E, n \in N$ and $\beta > 0$, $\{e_1, ..., e_n\} \in C(e, n)$ if and only if $\{\beta e_1, ..., \beta e_n\} \in C(\beta e, n)$ and from the definition of $(\overline{F}, E)$, we have

$$\overline{F}(e') = \overline{F}(\beta e') \;\; \forall e' \in E. \tag{3.9}$$

This yields, $\text{conv}\overline{F}(\beta e) = \text{conv}\overline{F}(e)$ $\forall e \in E$. Thus, $\widetilde{\text{conv}}(\overline{F}, E)$ is a convex cone soft set over X. Since $\widetilde{\text{conv}}(\overline{F}, E)\tilde{\supseteq}(\overline{F}, E)\tilde{\supseteq}(F, E)$, then $\widetilde{\text{conv}}(\overline{F}, E)$ is a convex cone soft set contains (F, E). It only remains to prove $\widetilde{\text{conv}}(\overline{F}, E)$ is the smallest convex cone soft set contains (F, E). Let $(G, E)\tilde{\supseteq}(F, E)$ be a convex cone soft set over X we must show $(G, E)\tilde{\supseteq}\widetilde{\text{conv}}(\overline{F}, E)$. Since (G, E) is a cone soft set, then from (3.9) $G(e) = G(\beta e)$ $\forall e \in E, \forall \beta > 0$. Thus, from the definition of $(\overline{F}, E)$,

$$G(e) = G(\beta e) \supseteq \bigcup_{\beta>0} F(\beta e) = \overline{F}(e).$$

Since (G, E) is a convex soft set,

$$(G, E) = \widetilde{\text{conv}}(\overline{G}, E)\tilde{\supseteq}(\overline{F}, E)\tilde{\supseteq}\widetilde{\text{conv}}(\overline{F}, E)\,,$$

i.e., $\widetilde{\text{conv}}(\overline{G}, E)\tilde{\supseteq}\widetilde{\text{conv}}(\overline{F}, E)$ as required. □

4. Affine soft set

Affine set and its related notions have been already defined and studied in each of ordinary sets and fuzzy sets [24, 31]. In a similar manner, we define affine soft set and study some of its properties.

Definition 4.1. A soft set (F, E) is said to be affine over X if for all $e_1, e_2 \in E$ and $\lambda \in \mathbb{R}$ we have

$$F(\lambda e_1 + (1 - \lambda)e_2) \supseteq F(e_1) \cap F(e_2).$$

Proposition 4.2. $(F, E) \in S(X)$ is an affine soft set if and only if F^A is an affine set for $A \in P(X)$.

Proof. Assume that (F, E) is an affine soft set over X. Let $e_1, e_2 \in F^A$. Then $F(e_1) \supseteq A$ and $F(e_2) \supseteq A$. Since (F, E) is an affine soft set, for each $\lambda \in \mathbb{R}$,

$$F(\lambda e_1 + (1-\lambda)e_2) \supseteq F(e_1) \cap F(e_2) \supseteq A \cap A = A.$$

Thus $\lambda e_1 + (1-\lambda)e_2 \in F^A$ which yields that F^A is an affine set.

Conversely, assume now F^A is an affine set for all $\lambda \in \mathbb{R}$. Let $A \in P(X)$ such that $A = F(e_1) \cap F(e_2)$. From last assertion we obtain $F(e_1) \supseteq A$ and $F(e_2) \supseteq A$, $e_1, e_2 \in F^A$. Thus $\lambda e_1 + (1-\lambda)e_2 \in F^A$. So, from the definition of A-inclusion,

$$F(\lambda e_1 + (1-\lambda)e_2) \supseteq A = F(e_1) \cap F(e_2).$$

This indicates (F, E) is an affine soft set on X. □

Proposition 4.3. An arbitrary intersection of affine soft sets is an affine soft set.

Proof. The proof is similar to the one of Theorem 3.2 in [12] for the case of convex soft set. □

In analogy with the definition of the convex hull of a convex soft set, we introduce the affine hull of a soft set.

Definition 4.4. The affine hull of a soft set (F, E), denoted by $\widetilde{\mathrm{aff}}(F, E)$ is the intersection of all affine soft sets containing (F, E) (or smallest affine soft set over X that contains (F, E)), i.e.,

$$\widetilde{\mathrm{aff}}(F, E) = \widetilde{\bigcap}_{(G,E) \widetilde{\supseteq} (F,E)} \{(G, E) : (G, E) \in S(X) \text{ is affine soft set}\}.$$

Proposition 4.5. $(F, E) \in S(X)$ is an affine soft set if and only if for all $e_1, ..., e_n \in E$ and $\lambda_1, ..., \lambda_n \in \mathbb{R}$ such that $\sum_{i=1}^{n} \lambda_i = 1$ we have

$$F(\sum_{i=1}^{n} \lambda_i e_i) \supseteq \bigcap_{i=1}^{n} F(e_i).$$

Proof. The proof is similar to the proof of Proposition 3.3. □

We show next an alternative way to characterize an affine soft set through a soft subspace. First let us define a soft subspace.

Definition 4.6. A soft set $(F, E) \in S(X)$ is a subspace if

(i) $F(\lambda e) \supseteq F(e) \quad \forall e \in E$ and $\lambda \in \mathbb{R}$,
(ii) $F(e_1 + e_2) \supseteq F(e_1) \cap F(e_2) \quad \forall e_1, e_2 \in E$.

Proposition 4.7. Let $(F, E) \in S(X)$. Then the followings are equivalent :

(1) (F, E) is a soft subspace.
(2) $F(\alpha e_1 + \beta e_2) \supseteq F(e_1) \cap F(e_2) \quad \forall e_1, e_2 \in E$ and $\alpha, \beta \in \mathbb{R}$.
(3) for each $e_1, ..., e_n \in E$ and $\lambda_1, ..., \lambda_n \in \mathbb{R}$

$$F(\sum_{i=1}^{n} \lambda_i e_i) \supseteq \bigcap_{i=1}^{n} F(e_i).$$

(4) (F, E) is an affine soft set such that $F(0) = \bigcup_{e \in E} F(e)$.

Proof. The proof of $\big[(1) \Longrightarrow (2) \Longrightarrow (3) \Longrightarrow (4)\big]$ can be easily verified. So, we only show $\big[(4) \Longrightarrow (1)\big]$. Let $e_1, e_2 \in E = \mathbb{R}^n$ such that $e_2 = 0$ and $\lambda \in \mathbb{R}$. Since (F, E) is an affine soft set, $F(\lambda e_1 + (1-\lambda)e_2) = F(\lambda e_1) \supseteq F(e_1) \cap F(0) = F(e_1)$. i.e.,

$$F(\lambda e_1) \supseteq F(e_1)\,. \tag{4.10}$$

From the assumption and the fact that $\frac{1}{2}(e_1 + e_2) \in E$,

$$F\big(\frac{1}{2}(e_1+e_2)\big) = F\big(\frac{1}{2}e_1 + (1-\frac{1}{2})e_2\big) \supseteq F(e_1) \cap F(e_2)\,,$$

From (4.10) and the last inclusion, we get

$$F(e_1+e_2) = F\big(2(\frac{1}{2}(e_1+e_2)\big) \supseteq F(\frac{1}{2}(e_1+e_2)) \supseteq F(e_1) \cap F(e_2). \tag{4.11}$$

From (4.10) and (4.11), it follows (F, E) is a soft subspace. □

References

[1] U. Acar, F. Koyuncu and B. Tanay, Soft sets and soft rings, Comput. Math. Appl. 59 (2010) 3458–3463.

[2] H. Aktaş and N. Çağman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2726–2735.

[3] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57 (2009) 1547–1553.

[4] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87–96.

[5] K. Atanassov, Operations over interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 64 (1994) 159–174.

[6] K. V. Babitha and J. J. Sunil, Soft set relations and functions, Comput. Math. Appl. 60 (2010) 1840–1849.

[7] N. Çağman and S. Enginoğlu, Soft set theory and uni-int decision making, Eur. J. Oper. Res. 207 (2010) 848–855.

[8] N. Çağman and S. Enginoğlu, Soft matrix theory and its decision makings, Comput. Math. Appl. 59 (2010) 3308–3314.

[9] N. Çağman and S. Karataş, S. Enginoğlu, Soft topology, Comput. Math. Appl. 62 (2011) 351–358.

[10] Y. Çelik, C. Ekiz and S. Yamak, A new view on soft rings, Hacet. J. Math. Stat. 40 (2011) 273–286.

[11] D. Chen, E. E. C. Tsang and D. S. Yeung and X. Wang, The parametrization reduction of soft sets and its applications, Comput. Math. Appl. 49 (2005) 757–763.

[12] I. Deli, Convex and concave soft sets and some properties, arXiv:1307.4960v1 [math.GM] 17 Jul 2013.

[13] J. Drewniak, Convex and strongly convex fuzzy sets, J. Math. Anal. Appl. 126 (1987) 292–300.

[14] E. E. Ammar, Some properties of convex fuzzy sets and convex fuzzy cones, Fuzzy Sets and Systems 16 (1999) 381–386.

[15] F. Feng, Y. B. Jun and X. Zhao, Soft semirings, Comput. Math. Appl. 56 (2008) 2621–2628.

[16] W. L. Gau and D. J. Buehrer, Vague sets, IEEE Trans. System, Man, Cybernet 23(1993) 610–614.

[17] M. B. Gorzalzany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy sets and systems 21 (1987) 1–17.

[18] H. Hazra, P. Majumdar and S. K. Samanta, Soft topology, Fuzzy Inf. Eng. 4 (2012) 105–115.

[19] S. Hussain and B. Ahmad, Some properties of soft topological spaces, Comput. Math. Appl. 62 (2011) 4058–4067.

[20] Y. B. Jun, Soft BCK/BCI-algebras, Comput. Math. Appl. 56 (2008) 1408–1413.

[21] Y. B. Jun and C. H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Inform. Sci. 178 (2008) 2466–2475.

[22] A. K. Katsaras and D. B. Lin, Fuzzy vector spaces and fuzzy vector topological vector spaces, J. Math. Anal. Appl. 58 (1977) 135–146.
[23] Y. Lin, Some properties of convex fuzzy sets, J. Math. Anal. Appl. 111 (1985) 119–129.
[24] R. Lowen, Convex Fuzzy Sets, Fuzzy Sets and Systems 3 (1980) 291–310.
[25] P. K. Maji, R. Biswas and A. R. Roy, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (2002) 1077–1083.
[26] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.
[27] D. Molodtsov, Soft set theory first results, Comput. Math. Appl. 37 (1999) 19–31.
[28] SK. Nazmul and S. K. Samanta, Neighbourhood properties of soft topological spaces, Ann. Fuzzy Math. Inform. 6 (2012) 1–15.
[29] Z. Pawlak, Rough sets, International Journal of Computer and Information Sciences 11 (1982) 341–356.
[30] D. Pei and D. Miao, From soft sets to information systems,in: Granular Computing, IEEE International Conference 2 (2005) 617–621.
[31] R. T. Rockafellar, Convex Analysis, Princeton University Press, Princeton 1970.
[32] A. Sezgin and A. O. Atagün, On operations of soft sets, Comput. Math. Appl. 61 (2011) 1457–1467.
[33] M. Shabir and M. Ali, Soft ideals and generalized fuzzy ideals in semigroups, New Math. Nat. Comput. 5 (2009) 599–615.
[34] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61 (2011) 1786–1799.
[35] X. Yang, Some properties of convex fuzzy sets, Fuzzy Sets and Systems 72 (1995) 129–132.
[36] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338–353.
[37] J. Zhan and Y.B. Jun, Soft BL-algebras based on fuzzy sets, Comput. Math. Appl. 59 (2010) 2037–2046.
[38] I. Zorlutuna, M. Akdag and W. K. Min, S. Atmaca, Remarks on soft topologoical spaces, Ann. Fuzzy Math. Inform. 3 (2012) 171–185.

S. N. Majeed (majsy001@mymail.unisa.edu.au)
Department of Mathematics, College of Education for Pure Sciences Ibn-AL-Haitham, University of Baghdad, Baghdad, Iraq