Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 12, No. 4, (October 2016), pp. 479–489
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Fuzzy soft Γ−semiring homomorphism

Marapureddy Murali Krishna Rao

Received 21 January 2016; Revised 13 March 2016; Accepted 4 April 2016.

Abstract. In this paper, we introduce the notion of fuzzy soft Γ−semiring homomorphism and study some properties of homomorphic image of fuzzy soft Γ−semiring.

2010 AMS Classification: 06Y60, 06B10

Keywords: fuzzy soft Γ−semiring, Fuzzy soft ideal, Fuzzy soft $k-$ideal over Γ−semiring, Fuzzy soft Γ−semiring homomorphism.

Corresponding Author: Marapureddy Murali Krishna Rao (mmarapureddy@gmail.com)

1. Introduction

Semiring is the best algebraic structure which is a common generalization of rings and distributive lattices, was first introduced by Vandiver [26] in 1934 but non trivial examples of semirings had appeared in the studies on the theory of commutative ideals of rings by Dedekind in 19th century. Semiring is an universal algebra with two binary operations called addition and multiplication where one of them distributive over the other. Bounded distributive lattices are commutative semirings which are both additively idempotent and multiplicatively idempotent. If in a ring, we do away with the requirement of having additive inverse of each element then the resulting algebraic structure becomes semiring. Most of the semirings have an order structure in addition to their algebraic structure. A natural example of semiring is the set of all natural numbers under usual addition and multiplication of numbers. In particular, if I is the unit interval on the real line, then the semiring $(I, \max, \min)$ in which 0 is the additive identity and 1 is the multiplicative identity. The theory of rings and the theory of semigroups have considerable impact on the development of the theory of semirings. In structure, semirings lie between semigroups and rings. The study of rings shows that multiplicative structure of a ring is independent of additive structure whereas in semiring multiplicative structure of a semiring is not independent of additive stricture of a semiring. Additive and multiplicative structures of a semiring play an important role in determining the structure of a semiring.

Semiring is very useful for solving problems in applied mathematics and information sciences because semiring provides an algebraic frame work for modeling. Semiring as the basic algebraic structure,was used in the areas of theoretical computer science as well as in the solutions of graph theory, optimization theory and in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches. It is well known that ideals play an important role in the study of any algebraic structures, in particular semirings. Though semiring is a generalization of a ring, ideals of semiring do not coincide with ring ideals. For example an ideal of a semiring needs not be the kernel of some semiring homomorphism. To solve this problem, Henriksen [8] defined $k-$ideals in semirings to obtain analogues of ring results for semiring.

The notion of $\Gamma-$ring was introduced by Nobusawa [20] as a generalization of ring in 1964. Sen [24] introduced the notion of $\Gamma-$semigroup in 1981. The notion of ternary algebraic system was introduced by Lehmer [11] in 1932, Lister [12] introduced ternary ring. Dutta & Kar [5] introduced the notion of ternary semiring which is a generalization of ternary ring and semiring. The notion of $\Gamma-$semiring was introduced by Murali Krishna [17] not only generalizes the notion of semiring and $\Gamma-$ring but also the notion of ternary semiring. The natural growth of gamma semiring is influenced by two things. One is the generalization of results of gamma rings and another is the generalization of results of semirings and ternary semirings. This notion provides an algebraic back ground to the non positive cones of the totally ordered rings.The set of all negative integers Z is not a semiring with respect to usual addition and multiplication but Z forms a $\Gamma-$semiring where $\Gamma = Z$. The important reason for the development of $\Gamma-$semiring is a generalization of results of rings, $\Gamma-$rings, semirings, semigroups and ternary semirings.

The theory of fuzzy sets is the most appropriate theory for dealing with uncertainty, was first introduced by Zadeh [27]. The concept of fuzzy subgroup was introduced by Rosenfeld [23]. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory etc. Uncertain data in many important applications in the areas such as economics, engineering, environment, medical sciences and business management could be caused by data randomness, information incompleteness, limitations of measuring instrument, delayed data updates etc. Molodtsov [16] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties, only partially resolves the problem is that objects in universal set often does not precisely satisfy the parameters associated to each of the elements in the set. Then Maji et al. [13] extended soft set theory to fuzzy soft set theory. Aktaş and Çağman [2] defined the soft sets and soft groups. Majumdar and Samantha [15] extended soft sets to fuzzy soft sets. İnan et al. [10] studied fuzzy soft rings and fuzzy soft ideals. Acar et al. [1], gave the basic concept of soft ring. Further more, Shah and Medhit [25] gave the concept of primary decomposition in a soft ring and a soft module. Ghosh et al. [7] initiated the study of fuzzy soft rings and fuzzy soft ideals. *Öztürk* et al. [20, 21] studied soft $\Gamma-$rings and fuzzy subnear-rings. Jun and Lee [9] studied fuzzy $\Gamma-$rings. Feng et al. [6] studied soft semirings by using the soft set theory. $\ddot{O}$. Bektaş et al. [4] studied soft $\Gamma-$semirings.

Attanassov [3] studied intuitionistic fuzzy sets. Zhou et al. [28] extended the concept of intuitionistic fuzzy soft set to semigroup theory. Maji et al. [14] introduced the concept of intuitionistic fuzzy soft set which is an extension to soft set and intuitionistic fuzzy set. Murali Krishna [18] introduced and studied fuzzy soft ideals and fuzzy soft $k-$ideals over a $\Gamma-$semiring. Murali Krishna and Venkateswarlu [19] introduced intuitionistic normal fuzzy soft $k-$ideals over a $\Gamma-$semiring and studied their properties. In continuation of paper [18], we introduce the notion of fuzzy soft $\Gamma-$semiring homomorphism and study some properties of homomorphic image of fuzzy soft $\Gamma-$semiring in this paper.

2. Preliminaries

Definition 2.1 ([17])**.** A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called a semiring provided

(i) addition is a commutative operation,

(ii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$, for all $x \in S$,

(iii) multiplication distributes over addition both from the left and from the right.

Definition 2.2 ([17])**.** Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M as a $\Gamma-$semiring, if there exists a mapping $M \times \Gamma \times M \to M$, written (x, α, y) as $x\alpha y$ such that it satisfies the following axioms, for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$,

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$,

(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$,

(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$,

(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Definition 2.3 ([17])**.** Let S be a $\Gamma-$semiring and A be a non-empty subset of S. A is called a $\Gamma-$subsemiring of S, if A is a sub-semigroup of $(S, +)$ and $A\Gamma A \subseteq A$.

Definition 2.4 ([17])**.** Let S be a $\Gamma-$semiring. A subset A of S is called a left(right) ideal of S, if A is closed under addition and $S\Gamma A \subseteq A(A\Gamma S \subseteq A)$. A is called an ideal of S if it is both a left ideal and a right ideal.

Definition 2.5 ([17])**.** Let S be a non-empty set. A mapping $f : S \to [0, 1]$ is called a fuzzy subset of S.

Definition 2.6 ([17])**.** Let f be a fuzzy subset of S. Then for $t \in [0, 1]$, the set $f_t = \{x \in S \mid f(x) \geq t\}$ is called a level subset of S with respect to f.

Definition 2.7 ([17])**.** An ideal I of a $\Gamma-$semiring S is called a $k-$ideal, if for $x, y \in S,\ x + y \in I, y \in I \Rightarrow\ x \in I$.

Definition 2.8 ([18])**.** Let f and g be fuzzy subsets of S. Then $f\cup$g, $f\cap$g are fuzzy subsets of S defined by

$$(f \cup g)(x) = max\{f(x), g(x)\}, (f \cap g)(x) = \min\{f(x), g(x)\} \text{ for all } x \in S.$$

Definition 2.9 ([18])**.** A fuzzy subset μ of S is called a non-empty fuzzy subset, if μ is not the constant function.

Definition 2.10 ([18])**.** For any two fuzzy subsets λ and μ of S, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in S$.

Definition 2.11 ([18])**.** Let f and g be fuzzy subsets of $\Gamma-$semiring S. Then $f \circ g$ is defined by

$$(f \circ g)(z) = \begin{cases} \underset{z=x\alpha y}{Sup}\ \{min\{\text{f(x)}, g(y)\}\}, \\ 0, \quad \text{otherwise} \end{cases}$$

$x, y, z \in S, \alpha \in \Gamma$.

Definition 2.12 ([18])**.** Let S and T be two sets and $\phi : S \rightarrow\ T$ be any function. A fuzzy subset μ of S is called a ϕ invariant, if $\phi(x) = \phi(y) \Rightarrow \mu(x) = \mu(y)$.

Definition 2.13 ([17])**.** A function $f : R \rightarrow\ S$ where R and S are $\Gamma-$semirings is said to be $\Gamma-$semiring homomorphism, if $f(a + b) = f(a) + f(b)$ and $f(a\alpha b) = f(a)\alpha f(b)$ for all $a, b \in R, \alpha \in \Gamma$.

Definition 2.14 ([17])**.** Let R and S be $\Gamma-$semirings and f be a function from R into S. If μ is a fuzzy subset of S, then the pre image of μ under f is the fuzzy subset of R, defined by $f^{-1}(\mu)(x) = \mu(f(x))$ for all $x \in R$.

Definition 2.15 ([17])**.** Let $\phi : R \rightarrow S$ be a homomorphism of semirings and μ be a fuzzy subset of R. We define a fuzzy subset $\phi(\mu)$ of S by

$$\phi(\mu)(x) = \begin{cases} \sup\limits_{y \in \phi^{-1}(x)} \mu(y), & \text{if } \ \phi^{-1}(x) \neq \varnothing, \\ 0, & \text{otherwise,} \end{cases} \quad \text{for all } x \in S.$$

Definition 2.16 ([18])**.** Let S be a $\Gamma-$semiring . A fuzzy subset μ of S is said to be fuzzy $\Gamma-$subsemiring of S, if it satisfies the following conditions:
(i) $\mu(x + y) \geq min\ \{\mu(x), \mu(y)\}$,
(ii) $\mu(x\alpha y) \geq min\ \{\mu(x), \mu(y)\}$, for all $x, y \in S, \alpha \in \Gamma$.

Definition 2.17 ([18])**.** A fuzzy subset μ of $\Gamma-$semiring S is called a fuzzy left(right) ideal of S, if it satisfies the following conditions:
(i) $\mu(x + y) \geq min\{\mu(x), \mu(y)\}$,
(ii) $\mu(x\alpha y) \geq \mu(y)\ (\mu(x))$, for all $x, y \in S, \alpha \in \Gamma$.

Definition 2.18 ([18])**.** A fuzzy subset μ of $\Gamma-$semiring S is called a fuzzy ideal of S, if it satisfies the following conditions:
(i) $\mu(x + y) \geq min\{\mu(x), \mu(y)\}$,
(ii) $\mu(x\alpha y) \geq max\ \{\mu(x), \mu(y)\}$, for all $x, y \in S, \alpha \in \Gamma$.

Definition 2.19 ([1])**.** Let U be an initial Universe set and E be the set of parameters. Let $P(U)$ denotes the power set of U. A pair (f, E) is called a soft set over U, where f is a mapping given by $f : E \rightarrow\ P(U)$.

Definition 2.20 ([1])**.** For a soft set (f, A), the set $\{x \in A \mid f(x) \neq \varnothing\}$ is called a support of (f, A), denoted by $Supp(f, A)$. If Supp$(f, A) \neq \varnothing$, then (f, A) is called a non null soft set.

Definition 2.21 ([10])**.** Let $(f, A), (g, B)$ be fuzzy soft sets over U. Then (f, A) is said to be fuzzy soft subset of (g, B), denoted by $(f, A) \subseteq (g, B)$, if $A \subseteq B$ and $f(a) \subseteq g(a)$, for all $a \in A$.

Definition 2.22 ([13]). Let U be an initial Universe set,E be the set of parameters and $A \subseteq E$. A pair (f, A) is called a fuzzy soft set over U where f is a mapping given by $f : A \to I^U$, where I^U denotes the collection of all fuzzy subsets of U.

Definition 2.23 ([2]). Let X be a group and (f, A) be a soft set over X. Then (f, A) is said to be soft group over X, if $f(a)$ is a subgroup of X, for each $a \in A$.

Definition 2.24 ([2]). Let X be a group and (f, A) be fuzzy soft set over X. Then (f, A) is said to be fuzzy soft group over X, if for each $a \in A, x, y \in X$,
(i) $f_a(x * y) \geq f_a(x) * f_a(y)$,
(ii) $f_a(x^{-1}) \geq f_a(x)$,
where f_a is the fuzzy subset of X corresponding to the parameter $a \in A$.

Definition 2.25 ([18]). Let S be a $\Gamma-$semiring,E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to P(S)$ where $P(S)$ is the power set of S. Then (f, A) is called a soft $\Gamma-$semiring over S, if for each $a \in A$, $f(a)$ is $\Gamma-$subsemiring of S, i.e.,
(i) $x, y \in S \Rightarrow x + y \in f(a)$,
(ii) $x, y \in S, \alpha \in \Gamma \Rightarrow x\alpha y \in f(a)$.

Definition 2.26 ([18]). Let S be a $\Gamma-$semiring, E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to [0, 1]^S$, where $[0, 1]^S$ denotes the collection of all fuzzy subsets of S. Then (f, A) is called a fuzzy soft $\Gamma-$semiring over S, if for each $a \in A, f(a) = f_a$ is the fuzzy $\Gamma-$subsemiring of S, i.e.,
(i) $f_a(x + y) \geq min\{f_a(x), f_a(y)\}$,
(ii) $f_a(x\alpha y) \geq min\{f_a(x), f_a(y)\}$, for all $x, y \in S, \alpha \in \Gamma$.

Definition 2.27 ([18]). Let S be a $\Gamma-$semiring,E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to P(S)$. Then (f, A) is called a soft left(right) ideal over S, if for each $a \in A, f(a)$ is a left(right) ideal of S, i.e.,
(i) $x, y \in f(a) \Rightarrow x + y \in f(a)$,
(ii) $x, y \in f(a), \alpha \in \Gamma, r \in S \Rightarrow r\alpha x(x\alpha r) \in f(a)$.

Definition 2.28 ([18]). Let S be a $\Gamma-$semiring, E be a parameter set, $A \subseteq E$ and $f : A \to P(S)$. Then (f, A) is called a soft ideal over S, if for each $a \in A, f(a)$ is an ideal of S, i.e.,
(i) $x, y \in f(a) \Rightarrow x + y \in f(a)$,
(ii) $x \in f(a), \alpha \in \Gamma, r \in S \Rightarrow r\alpha x \in f(a)$ and $x\alpha r \in f(a)$.

Definition 2.29 ([18]). Let S be a $\Gamma-$semiring, E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to [0, 1]^S$, where $[0, 1]^S$ denotes the collection of all fuzzy subsets of S. Then (f, A) is called a fuzzy soft left(right) ideal over S, if for each $a \in A$, the corresponding fuzzy subset $f_a : S \to [0, 1]$ is a fuzzy left(right) ideal of S, i.e.,
(i) $f_a(x + y) \geq \min\{f_a(x), f_a(y)\}$,
(ii) $f_a(x\alpha y) \geq f_a(y)(f_a(x))$ for all $x, y \in S, \alpha \in \Gamma$.

Definition 2.30 ([18]). Let S be a $\Gamma-$semiring, E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to [0, 1]^S$ where $[0, 1]^S$ denotes the collection of all fuzzy subsets of S. Then (f, A) is called a fuzzy soft ideal over S, if for each $a \in A$, the corresponding fuzzy subset $f_a : S \to [0, 1]$ is a fuzzy ideal of S, i.e.,

(i) $f_a(x+y) \geq \min\{f_a(x), f_a(y)\}$,
(ii) $f_a(x\alpha y) \geq \max\{f_a(x), f_a(y)\}$ for all $x, y \in S, \alpha \in \Gamma$.

3. Fuzzy soft $\Gamma-$ semiring homomorphism

In this section, the concept of fuzzy soft $\Gamma-$semiring homomorphism is introduced and studied their properties.

Definition 3.1. Let (f, A) and (g, B) be fuzzy soft sets over $\Gamma-$semirings R and S respectively. Let $\phi : R \to S$ and $\psi : A \to B$ be two functions, where A and B are parameter sets for the crisp sets R and S respectively. Then the pair (ϕ, ψ) is called a fuzzy soft function from R to S.

Definition 3.2. Let (f, A) and (g, B) be fuzzy soft sets over $\Gamma-$ semirings R and S, respectively and (ϕ, ψ) be a fuzzy soft function from R to S. Then (ϕ, ψ) is said to be fuzzy soft $\Gamma-$ semiring homomorphism, if the following conditions hold.
(i) ϕ is a $\Gamma-$ semiring homomorphism from R onto S.
(ii) ψ is a mapping from A onto B.
(iii) $\phi(f_a) = g_{\psi(a)}$ for all $a \in A$.

Definition 3.3. If there exists a fuzzy soft $\Gamma-$semiring homomorphism between (f, A) and (g, B) fuzzy soft semirings, we say that (f, A) is soft homomorphic to (g, B).

Example 3.4. Let M be the additive commutative semigroup of all even natural numbers with 0 and Γ be the additive semigroup of all natural numbers. Then M is a $\Gamma-$semiring, if $a\gamma b$ is defined as usual multiplication of natural numbers a, γ, b, where $a, b \in M, \gamma \in \Gamma$. Let $A = M$ and define

$$f_a(x) = \begin{cases} 0.4, & \text{if } x = 0, \\ \frac{1}{2a}, & \text{if } x \in \{a, 2a, 3a, \cdots\}, \\ 0, & \text{otherwise.} \end{cases}$$

Let N be the additive commutative semigroup of all positive integers with 0 and Γ be the additive semigroup of all natural numbers. Then N is a $\Gamma-$semiring, if $a\gamma b$ is defined as usual multiplication of integers a, γ, b, where $a, b \in N, \gamma \in \Gamma$. Let $B = N$ and define

$$g_b(x) = \begin{cases} 0.4, & \text{if } x = 0, \\ \frac{1}{b}, & \text{if } x \in \{b, 2b, 3b, \cdots\}, \\ 0, & \text{otherwise.} \end{cases}$$

Then (f, A) and (g, B) are fuzzy soft sets. Define $\phi : M \to N$ by $\phi(x) = x$, for all $x \in M$ and $\psi : A \to B$ by $\psi(x) = 2x$, for all $x \in A$. Then fuzzy soft function (ϕ, ψ) is a fuzzy soft $\Gamma-$semiring homomorphism from M onto N and (f, A) is soft homomorphic to (g, B).

Definition 3.5. Let (ϕ, ψ) be a fuzzy soft function from R to S. The pre image of (g, B) under the fuzzy soft function (ϕ, ψ), denoted by $(\phi, \psi)^{-1}(g, B)$, defined by

$$(\phi, \psi)^{-1}(g, B) = (\phi^{-1}(g), \psi^{-1}(B)),$$

is the fuzzy soft set.

Theorem 3.1. *Let* (f, A) *be a fuzzy soft* $\Gamma-$*semiring over a* $\Gamma-$*semiring* S*. If* $\theta : R \to S$ *be onto* $\Gamma-$ *semiring homomophism and for each* $a \in A$*,define* $(\theta f)_a(x) = f_a(\theta(x))$*, for all* $x \in R$*, then* $(\theta f, A)$ *is a fuzzy soft* $\Gamma-$*semiring over* S*.*

Proof. Let $x, y \in R, a \in A$ *and* $\gamma \in \Gamma$. Then

$$\begin{aligned}(\theta f)_a(x+y) =& f_a(\theta(x+y))\\ =& f_a[\theta(x)+\theta(y)]\\ \geq& min\{f_a(\theta(x)), f_a(\theta(y))\}\\ =& min\{(\theta f)_a(x), (\theta f)_a(y)\},\\ \text{and } (\theta f)_a(x\gamma y) =& f_a(\theta(x\gamma y))\\ =& f_a[\theta(x)\gamma\theta(y)]\\ \geq& min\{f_a(\theta(x)), f_a(\theta(y))\}\\ =& min\{(\theta f)_a(x), (\theta f)_a(y)\}.\end{aligned}$$

Thus $(\theta f)_a$ is a fuzzy $\Gamma-$subsemiring of S. So $(\theta f, A)$ is a fuzzy soft $\Gamma-$semiring over S. □

Theorem 3.2. *Let* (α, A) *be a fuzzy soft semiring over* $\Gamma-$*semiring* R*. If* θ *is an endomorphism of* R *and define* $(\alpha\theta)_a = \alpha_a\theta$ *for each* $a \in A$*, then* $(\alpha\theta, A)$ *is a fuzzy soft* $\Gamma-$ *semiring over* $\Gamma-$ *semiring* R*.*

Proof. Let (α, A) be a fuzzy soft semiring over $\Gamma-$semiring $R, x, y \in R, a \in A$ *and* $\gamma \in \Gamma$. Then

$$\begin{aligned}(\alpha\theta)_a(x+y) =& \alpha_a(\theta(x+y))\\ =& \alpha_a[\theta(x)+\theta(y)]\\ \geq& min\{\alpha_a(\theta(x)), \alpha_a(\theta(y))\}\\ =& min\{(\alpha\theta)_a(x), (\alpha\theta)_a(y)\},\\ \text{and } (\alpha\theta)_a(x\gamma y) =& \alpha_a(\theta(x\gamma y))\\ =& \alpha_a[\theta(x)\gamma\theta(y)]\\ \geq& min\{\alpha_a(\theta(x)), \alpha_a(\theta(y))\}\\ =& min\{(\alpha\theta)_a(x), (\alpha\theta)_a(y)\}.\end{aligned}$$

Thus $(\alpha\theta)_a$ is a fuzzy $\Gamma-$subsemiring of R. So $(\alpha\theta, A)$ is a fuzzy soft $\Gamma-$ semiring over R. □

Theorem 3.3. *Let* $\phi : R \to S$ *be an onto homomorphism of* $\Gamma-$*semirings and* (α, A) *be a fuzzy soft left ideal over* $\Gamma-$ *semiring* S*. If for each* $a \in A$*,* $\beta_a = \phi^{-1}(\alpha_a)$*, then* (β, A) *is a fuzzy soft left ideal over* $\Gamma-$ *semiring* R*.*

Proof. Let $a \in A$ *and* $\gamma \in \Gamma$. Then α_a is a fuzzy soft left ideal over $\Gamma-$semiring S. Let $x, y \in R$ *and* $\gamma \in \Gamma$. Then

$$\begin{aligned}\phi^{-1}(\alpha_a)(x+y) =& \alpha_a(\phi(x+y))\\ =& \alpha_a\{\phi(x)+\phi(y)\},\\ \geq& min\{\alpha_a(\phi(x)), \alpha_a(\phi(y))\}\\ =& min\{\phi^{-1}(\alpha_a)(x), \phi^{-1}(\alpha_a)(y)\},\\ \text{and } \phi^{-1}(\alpha_a)(x\gamma y) =& \alpha_a(\phi(x\gamma y))\\ =& \alpha_a\{\phi(x)\gamma\phi(y)\}\\ \geq& \alpha_a(\phi(y))\\ =& \phi^{-1}(\alpha_a)(y).\end{aligned}$$

Thus $\beta_a = \phi^{-1}(\alpha_a)$ is a fuzzy left ideal of $\Gamma-$semiring R. So (β, A) is a fuzzy soft left ideal over $\Gamma-$semiring R. □

Lemma 3.4. *Let R and S be $\Gamma-$ semirings, $\phi : R \to S$ be a $\Gamma-$semiring homomorphism and f be a ϕ invariant fuzzy subset of R. Then $\phi(f)(x) = f(a)$, for each $a \in R$.*

Proof. Let R and S be $\Gamma-$semirings, $\phi : R \to S$ be a $\Gamma-$semiring homomorphism and f be a ϕ invariant fuzzy ideal of R. Suppose $a \in R$ and $\phi(a) = x$. Then $\phi^{-1}(x) = a$. Let $t \in \phi^{-1}(x)$. Then $\phi(t) = x = \phi(a)$. Since f is a ϕ invariant fuzzy subset of R, $f(t) = f(a)$. Thus $\phi(f)(x) = \sup\limits_{t\in\phi^{-1}(x)} f(t) = f(a)$. So $\phi(f)(x) = f(a)$. □

Theorem 3.5. *Let (α, A) be a fuzzy soft left ideal over $\Gamma-$semiring R and ϕ be a homomorphism from R onto S. For each $c \in A, \alpha_c$ is a ϕ invariant fuzzy left ideal of R, if $\beta_c = \phi(\alpha_c), c \in A$, then (β, A) is a fuzzy soft left ideal over $\Gamma-$semiring$S$.*

Proof. Let $x, y \in S$, $c \in A$ *and* $\gamma \in \Gamma$. Then there exist $a, b \in R$ such that $\phi(a) = x, \phi(b) = y, x + y = \phi(a + b), x\gamma y = \phi(a\gamma b)$. Since α_c is ϕ invariant, by Lemma 3.4, we have

$$\begin{aligned}\beta_c(x+y) =& \phi(\alpha_c)(x+y)\\ =& \alpha_c(a+b)\\ \geq& min\{\alpha_c(a), \alpha_c(b)\}\\ =& min\{\phi(\alpha_c)(x), \phi(\alpha_c)(y)\}\\ =& min\{\beta_c(x), \beta_c(y)\},\\ \text{and } \beta_c(x\gamma y) =& \phi(\alpha_c)(x\gamma y)\\ =& \alpha_c(\phi(a\gamma b))\\ =& \alpha_c[\phi(a)\gamma\phi(b)]\\ \geq& \alpha_c(\phi(b))\\ =& \phi(\alpha_c)(y)\\ =& \beta_c.\end{aligned}$$

Thus β_c is a fuzzy left ideal of S. So (β, A) is a fuzzy soft left ideal over S. □

Theorem 3.6. *Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over $\Gamma-$semirings R and S, respectively and (ϕ, ψ) be a fuzzy soft $\Gamma-$semiring homomorphism from (f, A) onto (g, B). Then $(\phi(f), B)$ is a fuzzy soft $\Gamma-$semiring over $\Gamma-$semiring S.*

Proof. Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over $\Gamma-$semirings R and S respectively and (ϕ, ψ) be a fuzzy soft $\Gamma-$semiring homomorphism from (f, A) onto (g, B). By Definition 3.2, ϕ is a $\Gamma-$semiring homomorphism from R onto S and ψ is a mapping from A onto B. For each $b \in B$, there exists $a \in A$ such that $\psi(a) = b$. Define $[\phi(f)]_b = \phi(f_a)$. Let $y_1, y_2 \in S$ *and* $\gamma \in \Gamma$. Then there exist $x_1, x_2 \in R$ such that $\phi(x_1) = y_1, \phi(x_2) = y_2$ and $\phi(x_1 + x_2) = y_1 + y_2$ and $\phi(x_1\gamma x_2) = y_1\gamma y_2$. Now we have

$$\begin{aligned} [\phi(f)]_{\psi(a)}(y_1 + y_2) &= \phi(f_a)(y_1 + y_2) \\ &= f_a[x_1 + x_2] \\ &\geq min\{f_a(x_1), f_a(x_2)\} \\ &= min\{\phi(f_a)(y_1), \phi(f_a)(y_2)\} \\ &= min\{\phi(f)_{\psi(a)}, \phi(f)_{\psi(a)}(y_2)\}, \\ \text{and } [\phi(f)]_{\psi(a)}(y_1\gamma y_2) &= \phi(f_a)(y_1\gamma y_2) = f_a(x_1\gamma x_2) \\ &\geq min\{f_a(x_1), f_a(x_2)\} \\ &= min\{\phi(f_a)(y_1), \phi(f_a)(y_2)\} \\ &= min\{\phi(f)_{\psi(a)}, \phi(f)_{\psi(a)}(y_2)\}. \end{aligned}$$

Thus $\phi(f)_b$ is a fuzzy $\Gamma-$subsemiring of S. So $(\phi(f), B)$ is a fuzzy soft $\Gamma-$semiring over $\Gamma-$semiring S. □

Definition 3.6. Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over a semiring S. Then (f, A) is a fuzzy soft $\Gamma-$subsemiring of (g, B), if it satisfies the following conditions:

(i) $A \subseteq B$,

(ii) f_a is a fuzzy $\Gamma-$subsemiring of g_a, for all $a \in A$.

Theorem 3.7. *Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over S. Then the following statements are true:*

(1) *If $g_b \subset f_b$ for all $b \in B$* and *$B \subset A$, then (g, B) is a fuzzy soft $\Gamma-$subsemiring of (f, A).*

(2) *$(f, A) \cap (g, B)$ is a fuzzy soft $\Gamma-$subsemiring of (f, A) and (g, B), if it is non null.*

Proof. Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over S.

(1) Since $g_b \subset f_b$ for all $b \in B \subset A$, by Definition 3.6, (g, B) is a fuzzy soft $\Gamma-$subsemiring of (f, A).

(2) Let $(f, A) \cap (g, B) = (h, C)$, where $C = A \cap B$. Then (h, C) is a fuzzy soft $\Gamma-$semiring over S. Since $C = A \cap B \subset A$ and $C \subset B$, by Definition 3.6, (h, C) is a fuzzy soft $\Gamma-$subsemiring of (f, A) as well as (g, B). □

Theorem 3.8. *Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over R and (f, A) be fuzzy soft $\Gamma-$subsemiring of (g, B). If $\phi : R \to S$ is a $\Gamma-$semiring homomorphism* *from R onto S, then $(\phi(f), A)$ and $(\phi(g), B)$ are fuzzy soft $\Gamma-$subsemirings over S and $(\phi(f), A)$ is a fuzzy soft $\Gamma-$subsemiring of $(\phi(g), B)$.*

Proof. Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over R and (f, A) be fuzzy soft $\Gamma-$subsemiring of (g, B). Since ϕ is a $\Gamma-$ semiring homomorphism from R onto S, $[\phi(f)]_a = \phi(f_a)$ is a fuzzy $\Gamma-$subsemiring of S for all $a \in A$ and $[\phi(g)]_b = \phi(g_b)$ is a fuzzy $\Gamma-$subsemiring of S for all $b \in B$. Then $(\phi(f), A)$, $(\phi(g), B)$ are fuzzy soft $\Gamma-$semirings over S. Since (f, A) is a fuzzy soft $\Gamma-$subsemiring of (g, B), f_a is fuzzy $\Gamma-$subsemiring of g_a. Thus $\phi(f_a)$ is a fuzzy $\Gamma-$subsemiring of $\phi(g_a)$ for all $a \in A$. So $(\phi(f), A)$ is a fuzzy soft $\Gamma-$ subsemiring of $(\phi(g), B)$. □

Theorem 3.9. *Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over R and S, respectively. If (ϕ, ψ) is a fuzzy soft homomorphism from (f, A) onto (g, B), then the pre-image of (g, B) under fuzzy soft $\Gamma-$semiring homomorphism (ϕ, ψ) is a fuzzy soft $\Gamma-$subsemiring of (f, A) over R.*

Proof. Let (f, A) and (g, B) be fuzzy soft $\Gamma-$semirings over R and S, respectively. Suppose (ϕ, ψ) is a fuzzy soft $\Gamma-$homomorphism from (f, A) onto (g, B). By Definition 3.5, $(\phi, \psi)^{-1}(g, B) = (\phi^{-1}(g), \psi^{-1}(B))$. Define

$$(\phi^{-1}(g))_a(x) = g_{\psi(a)}(\phi(x)), \text{ for all } x \in R \text{ and } a \in \psi^{-1}(B).$$

Let $x_1, x_2 \in R$ and $\gamma \in \Gamma$. Then

$$\begin{aligned}(\phi^{-1}(g))_a(x_1 + x_2) &= g_{\psi(a)}[\phi(x_1 + x_2)] \\ &= g_{\psi(a)}[\phi(x_1) + \phi(x_2)] \\ &\geq min\{g_{\psi(a)}(\phi(x_1)), g_{\psi(a)}(\phi(x_2))\} \\ &= min\{(\phi^{-1}(g))_a(x_1), (\phi^{-1}(g))_a(x_2)\}\end{aligned}$$

and

$$\begin{aligned}(\phi^{-1}(g))_a(x_1\gamma x_2) &= g_{\psi(a)}[\phi(x_1\gamma x_2)] \\ &= g_{\psi(a)}[\phi(x_1)\gamma\phi(x_2)] \\ &\geq min\{g_{\psi(a)}(\phi(x_1)), g_{\psi(a)}(\phi(x_2))\} \\ &= min\{(\phi^{-1}(g))_a(x_1), (\phi^{-1}(g))_a(x_2)\}.\end{aligned}$$

Thus $(\phi^{-1}(g))_a$ is a fuzzy $\Gamma-$subsemiring of R for all $a \in \psi^{-1}(B)$.
So $\big((\phi^{-1}(g)), (\psi^{-1}(B)\big)$ is a fuzzy soft $\Gamma-$subsemiring of (f, A) over R. Hence the proof is complete. □

4. Conclusion

In this paper, we introduced the notion of fuzzy soft $\Gamma-$ semiring homomorphism and studied some properties of homomorphic image of fuzzy soft $\Gamma-$ semiring. In the next paper, we study properties of kernel of fuzzy soft $\Gamma-$ semiring homomorphism, fuzzy soft prime ideals and fuzzy soft filters over $\Gamma-$semirings.

Acknowledgment: Author is thankful to the referee for his valuable suggestions.

References

[1] U. Acar, F. Koyuncu and B. Tanay, Soft sets and soft rings, Comput. Math. Appl. 59 (2010) 3458–3463.
[2] H. Aktaş and N. Çağman, Soft sets and soft groups, Inform. Sci. 177 (2007) 2726–2735.
[3] K. T. Attanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems 20 (1986) 87–96.
[4] *Ö*. Bektaş, N. Bayrak and B. A. Ersoy, Soft $\Gamma-$semirings, arXiv:1202.1496v1 [math.RA] (2012) 1–16.
[5] T. K. Dutta and S. Kar, On regular ternary semirings, Advances in algebra, Proceedings of the ICM Satellite Conference in Algebra and Related Topics, World Scientific (2003) 343–355.
[6] F. Feng, Y. B. Jun and X. Zhao, Soft semirings, Comput. Math. Appl. 56 (2008) 2621–2628.
[7] J. Ghosh, B. Dinda and T. K. Samanta, Fuzzy soft rings and fuzzy soft ideals, Inter. J. of Pure and Appl. Sci. and Techn. 2 (2) (2011) 66–74.
[8] M. Henriksen, Ideals in semirings with commutative addition, Amer. Math. Soc. Notices 5 (1958) 321.
[9] Y. B. Jun and C. Y. Lee, Fuzzy $\Gamma-$Rings, Pusan Kyongnam Math. J. 8 (1992) 163–170.
[10] E. İnan and M. A. *Öztürk* , Fuzzy soft rings and fuzzy soft ideals, Neural Computing and Appl. 21 (Suppl. 1) (2012) 1–8.
[11] H. Lehmer, A ternary analogue of abelian groups, Amer. J. of Math. 54 (2) (1932) 329–338.
[12] W. G. Lister, Ternary rings, Tran. of Amer. Math. Soc. 154 (1971) 37–55.
[13] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, J. Fuzzy Math. 9 (3) (2001) 589–602.
[14] P. K. Maji, R. Biswas and A. R. Roy,On ntuitionistic fuzzy soft sets, J. Fuzzy Math. 9 (3) (2001) 667–692.
[15] P. Majumdar, S. K. Samanth, Generalized fuzzy soft sets, Comp. Math. Appl. 59 (4) (2010) 1425–1432.
[16] D. Molodtsov, Soft set theory-First results, Comput. Math. Appl. 37 (1999) 19–31.
[17] M. Murali Krishna Rao, $\Gamma-$semirings-I, Southeast Asian Bull. Math. 19 (1) (1995) 49–54.
[18] M. Murali Krishna Rao, Fuzzy soft $\Gamma-$semiring and fuzzy soft $k-$ideal over $\Gamma-$semiring, Ann. Fuzzy Math. Inform. 11 (3) (2015) 341–354.
[19] M. Murali Krishna Rao and B.Venkateswarulu, An intuitionistic normal fuzzy soft $k-$ideal over $\Gamma-$semiring, Ann. Fuzzy Math. Inform. 12 (3) (2016) 13–28.
[20] N. Nobusawa, On a generalization of the ring theory, Osaka.J.Math. 1 (1964) 81–89.
[21] M. A. *Öztürk* and E. İnan, Soft Γ-rings and idealistic soft $\Gamma-$rings, Ann. Fuzzy Math. Inform. 1 (1) (2011) 71–80.
[22] M. A. *Öztürk* and E. İnan, Fuzzy Soft Subnear-rings and $(\epsilon, \epsilon \vee q)-$Fuzzy Soft Subnear-rings, Computers and Math. Appli. 63 (3) (2012) 617–628.
[23] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512–517.
[24] M. K. Sen, On $\Gamma-$semigroup, Proc. of International Conference of Algebra and its Application, (1981), Decker Publicaiton, New York 301-308.
[25] T. Shah and S. Medhit, Primary decomposition in a soft ring and a soft module, Iranian J. of Sci. and Tech. 38A3 (Special Issue-Mathematics) (2014) 311–320.
[26] H. S. Vandiver, Note on a simple type of algebra in which cancellation law of addition does not hold, Bull. Amer. Math. Soc. (N.S.) 40 (1934) 914–920.
[27] L. A. Zadeh, Fuzzy sets, Information and control 8 (1965) 338–353.
[28] J. Zhou, Y. Li and T. Yin, Intuitionistic fuzzy soft semigroups, Math. Aeterna 1 (3) (2011) 173 –183.

MARAPUREDDY MURALI KRISHNA RAO (mmarapureddy@gmail.com)
*D*epartment of Mathematics, GIT, GITAM University,
Visakhapatnam-530 045, Andhra Pradesh, India