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Prim's algorithm for solving minimum spanning tree problem in fuzzy environment

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Abstract. The minimum spanning tree (MST) problem, where the arc costs have fuzzy values, is one of the most studied problems in fuzzy sets and systems area. In this paper, we concentrate on a MST problem on a graph, in which either a trapezoidal fuzzy number or triangular fuzzy number, instead of a real number, is assigned to each arc length. The fuzzy number is able to represent the uncertainty in the arc costs of the fuzzy graph. Two key matters need to be addressed in MST problem with fuzzy numbers. One is how to compare the fuzzy numbers, i.e., the cost of the edges. The other is how to determine the addition of edges to find out the cost of the fuzzy MST. The graded mean integration representation of fuzzy numbers is used here to solve these problems. A famous algorithm to solve the minimum spanning tree problem is Prim's algorithm, where uncertainty is not considered, i.e., specific values of arc lengths are provided. A fuzzy version of classical Prim's algorithm is introduced in this paper to solve the MST problem in fuzzy environment. We use the concept of graded mean integration representation of fuzzy numbers in the proposed algorithm. An illustrative example is also included to demonstrate the proposed algorithm.

2010 AMS Classification: 05C72, 03E72, 08A72, 60A86

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1. Introduction

The minimum spanning problem (MST) is one of the most fundamental and well known combinatorial optimization problems in classical graph theory. It appears in many applications, including transportation, communications, logistics, image processing, cluster analysis, wireless telecommunication networks.

Let $G = (V, E)$ be a connected graph, where V is the set of vertices and E is the set of edges. A tree T is said to be a spanning tree of graph G if T is a sub graph of G and T contains all nodes of G. Since spanning trees are the largest (with maximum number of arcs) tree among all the trees in G, T is also called maximal tree sub graph or maximal T of G. Minimum spanning tree (MST) is the spanning tree in which the sum of lengths of arcs are minimum.

The main MST problems are modelled on a graph. Thus, the vertices of the graph represent the items (points) where the users that demand the facility are, and the arcs reveal the existence of a certain link between the vertices (for instance, roads joining cities). In general, the classical graphs represent deterministic situations, where the items (points) of demand and the relations (links) joining them are exactly known. The classical graph model is not properly represent the problems where uncertainty exists in the description of the items or relations between the items or in both. The fuzzy graph [19, 25, 32, 43, 50, 55, 54] is more precise, flexible and compatible to model those uncertain problems. Fuzzy graph appears in many applications, especially in the field of shortest path problem [47, 48], traffic light control [15], computer networks, pattern recognition, graph coloring [17, 19], decision making [16], automata [35] and expert systems.

Many researchers [6, 20, 26, 37, 51], have studied lots to develop an efficient algorithm for classical MST problem. MSTs also arise in more subtle applications in data storage [38, 42], statistical cluster analysis [7, 22], speech recognition [24] and image processing [49]. Due to the enormous growth of the telecommunication networks, the network applications of the MSTs have attracted a lot of attention during the last decades. For instance, the broadcasting problem, in which the same message must be transmitted to all the nodes within the network (one to all), can be modeled by the MST problem. The MST is also the minimum routing tree for message aggregation (all to one) in distributed environments. Besides, in some of the multicast routing protocols [13], [52], the MST is an effective and reliable ways to multicast the data from a source node to a set of destination nodes. In most of the cases, the arc costs are assumed to be fixed, but such an assumption may be wronged in real life applications and the weights vary with time indeed. For example, the links in the network may be affected due to interferences, congestions, collisions, or some other factors. Consequently, the bandwidths of these links, which are denoted by arc lengths in the graph of the communication network, are nondeterministic, i.e., uncertain. Although in classical graph theory, the costs of the arcs in a MST problem are supposed real numbers, most practical applications, however, have parameters that are not naturally precise (i.e., demands, capacities, costs, time). Uncertainty exists in almost every applications of MST problem. In general, uncertainty is inseparable from the measurement of arc weights which are not known accurately. Some researchers [2, 28, 27, 29, 44], used random variables to handle the uncertainty of the arc weights. This type of MST problem is defined as stochastic minimum spanning tree problem. The major problem with those stochastic minimum spanning tree algorithms is that they are practical when the probability distribution function of the arc weight is assumed to be known. While such an assumption may not be true in realistic applications of MST problem. In these cases to use fuzzy variable for modeling the MST problem is quite appropriate, and the fuzzy minimum spanning tree (FMST) problem appears in a natural way.

The FMST, because it involves ranking and addition operation between fuzzy numbers (particularly trapezoidal fuzzy numbers and triangular fuzzy numbers), is very different from the classical MST, which only involves real numbers. In the FMST problem, the cost of the arcs are fuzzy numbers, and what is hard is to determine an edge being smaller than all the others edges, as the comparison among fuzzy numbers is a method which can be defined in a wide variety of ways. Any case, as it is very well known, it is difficult to determine a fuzzy cost, in a set of them, which is smaller than all the other costs in that set.

Numerous papers have been published on the FMST [5, 11, 30, 31, 56, 57]. The paper by Itoh and Ishii [30] is one of the first paper on this subject and studied a MST problem with fuzzy arc weights as a chance constrained programming based on the necessity measure. Following that, three approaches based on the overall existence ranking index for ranking fuzzy arc weights of spanning trees were presented by Chang and Lee [11]. Almeida *et al.* [5] formulated the MST problem with fuzzy parameters and introduced an exact algorithm to solve this problem. They also proposed a genetic algorithm based on fuzzy set theory and probability theory to determine the MST problem with fuzzy parameters. Janiak and Kasperski [31] employed possibility theory to characterize the optimality of arcs of the graph and to select a MST where the arc weights are specified as fuzzy intervals. Liu [40, 41] introduced the credibility theory including pessimistic value, credibility measure and expected value as fuzzy ranking methods. Based on the credibility theory, Gao and Lu [23] studied the fuzzy quadratic minimum spanning tree problem and formulated it as expected value model, chance-constrained programming and dependent-chance programming according to different decision criteria.

In MST problem in fuzzy environment, ranking of fuzzy numbers is one of the important component. Since fuzzy numbers represent uncertain numeric values, there is an inherent uncertainty in the comparison between fuzzy numbers, i.e., it is very hard to find out whether a fuzzy number is bigger or smaller than others. Many researchers [1, 3, 4, 18, 33, 34, 45, 46, 53] have studied about the ranking of fuzzy numbers. Here, we use canonical representation of operations on triangular fuzzy numbers (TFNs) that are based on the graded mean integration representation method [9] for ranking the fuzzy numbers. In this method, each fuzzy number and the result of addition and multiplication of two fuzzy numbers can be represented as a crisp (real) number. This method is applied in different applications such as portfolio selection [14], evaluation of airline service quality [8], product adoption [39], fuzzy shortest path problem [21, 12] and efficient network selection in heterogeneous wireless networks [10].

In this paper, we propose an algorithm for the MST problem in graphs with fuzzy numbers, i.e., for the FMST problem. The algorithm is a fuzzy version of the classical Prim algorithm. The canonical representation of operations on fuzzy numbers is used in the algorithm for addition and comparison between fuzzy numbers.

The proposed algorithm has various important advantages : (1) Compared with existing algorithms, the proposed algorithm for FMST problem is more efficient due to the fact that the comparing and the addition of fuzzy numbers can be done very easily and straight manner using canonical representation of operations on fuzzy number. (2) In the FMST problem, the FMST and the corresponding cost are the main information for the decision makers to apply the FMST to model the real world problems. Existing algorithms for FMST problem can obtain only the FMST of a graph. It is the purpose of this paper to extend the classical Prim's algorithm that can obtain FMST and the corresponding cost. The computed cost of the FMST is real number not a fuzzy number. Decision makers can easily make their proper decision about the problem based on the cost of the FMST.

The paper is organized as follows. Section II provides some basic concepts and definitions associated with this study. Problem formulation for the solution of the FMST is given in section III. In section IV, we present the fuzzy version of Prim's algorithm for FMST problem. We present a numerical example in section V as a case study to illustrate the algorithm. Finally, we conclude in section VI.

2. Preliminaries

In this section, we have discussed about fuzzy graph, fuzzy number, canonical representation of operations on fuzzy numbers and classical Prim's algorithm to facilitate future discussion.

Definition 2.1. Let V be a finite and non empty set of nodes. A fuzzy graph is a pair of functions $\tilde{G}$=(σ, μ), where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ, i.e., σ: $V \longrightarrow [0,1]$ and μ: $V \times V \longrightarrow [0,1]$ such that

$$\mu\left(u,v\right) \leq min\left(\sigma\left(u\right), \sigma\left(v\right)\right), \forall u,v \in V \tag{2.1}$$

A fuzzy graph is a generalization of crisp graph. In this study, we consider a graph, whose nodes are crisp but lengths/costs of the arcs are represented by fuzzy numbers.

Definition 2.2. The fuzzy number is a fuzzy set with the following conditions:
(i) Convex fuzzy set.
(ii) Normalized fuzzy set.
(iii) The membership function of fuzzy number is piecewise continuous.
(iv) It is defined in the real number.

Many different types of fuzzy numbers are defined in the literature, e.g., TFN, TrFN, L-R fuzzy number and bell shape fuzzy number. Most fuzzy numbers get their names from the sharp of member functions. In this study, we use TFN or TrFN to represent the edge weight of the graph.

Definition 2.3. A triangular fuzzy number (TFN) $\tilde{A}$ is a fuzzy number with a piecewise linear membership function $\mu_{\tilde{A}}$ defined by:

$$\mu_{\tilde{A}}) = \begin{cases} 0 & \text{if } x < \underline{a} \text{ or } x > \overline{a} \\ \frac{(x-\underline{a})}{(a_0-\underline{a})} & \text{if } a_0 \geq x \geq \underline{a} \\ \frac{(\overline{a}-x)}{(\overline{a}-a_0)} & \text{if } \overline{a} \geq x \geq a_0 \end{cases} \tag{2.2}$$

which can be denoted as a triple $\{\underline{a}, a_0, \overline{a}\}$. Here, $\underline{a}, a_0, \overline{a}$ are least possible value, main value and highest possible value receptively. Figure 1 represents the graph of a TFN.

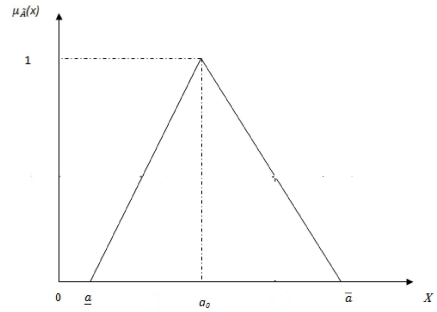

FIGURE 1. A TFN.

Definition 2.4. A trapezoidal fuzzy number (TrFN) $\tilde{B}$ is a fuzzy number with a membership function $\mu_{\tilde{B}}$ defined by:

$$\mu_{\tilde{B}}(x) = \begin{cases} 0 & \text{if } x < \underline{b} \text{ or } x > \overline{b} \\ \frac{(x-b_1)}{(b_1-\underline{b})} & \text{if } b_1 \geq x \geq \underline{b} \\ 1 & \text{if } b_1 \geq x \geq b_2 \\ \frac{(\overline{b}-x)}{(\overline{b}-b_2)} & \text{if } \overline{b} \geq x \geq b_2 \end{cases} \tag{2.3}$$

which can be represented as a quartet $\{\underline{b}, b_1, b_2, \overline{b}\}$. Here, b_1 and b_2 are main values, $\underline{b}$ and $\overline{b}$ are least possible value and highest possible value respectively. Figure 2 represents the graph of a TrFN.

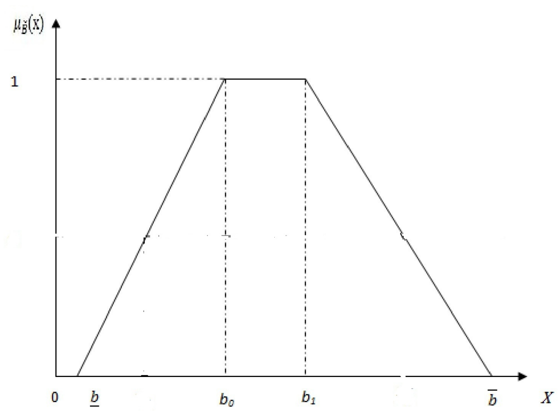

FIGURE 2. A TrFN.

2.1. **Canonical representation of operations on fuzzy numbers.** Defuzzification is an important step in fuzzy optimization and fuzzy decision-making problem. The defuzzification process converts the fuzzy value (fuzzy number) into a crisp value and finding the rank or ordinal positions of n-fuzzy input parameters vector. Many defuzzification methods are present [58] in the literature, but the common defuzzification techniques include first of maximums, centre of area, middle of maximums (MoM) and last of maximums. Different defuzzification methods are used in different types of uncertain problems. In this study, the canonical representation of operation on fuzzy numbers (particularly TFN and TrFN) [9], which are based on the graded mean integration representation method is applied in defuzzification or ranking process to solve the fuzzy minimum spanning tree problem.

Definition 2.5. Let a TFN $\tilde{A} = \{\underline{a}, a_0, \overline{a}\}$, the graded mean integration representation of TFN $\tilde{A}$ is defined as

$$P(\tilde{A}) = \frac{1}{6}(\underline{a} + 4a_0 + \overline{a}). \tag{2.4}$$

Let $\tilde{A} = \{\underline{a}, a_0, \overline{a}\}$ and $\tilde{B} = \left\{\underline{b}, b_0, \overline{b}\right\}$ be two TFNs. By employing, the graded mean integration representation of TFNs $\tilde{A}$ and $\tilde{B}$ can be determined respectively, as follows:

$$P(\tilde{A}) = \frac{1}{6}(\underline{a} + 4a_0 + \overline{a}) \tag{2.5}$$

$$P(\tilde{B}) = \frac{1}{6}(\underline{b} + 4b_0 + \overline{b}) \tag{2.6}$$

The ranking of $\tilde{A}$ and $\tilde{B}$ is defined as follows:
$\tilde{A} \succ \tilde{B}$ if and only if $P(\tilde{A}) > P(\tilde{B})$ or $P(\tilde{A}) = P(\tilde{B})$.
$\tilde{A} \prec \tilde{B}$ if and only if $P(\tilde{A}) < P(\tilde{B})$ or $P(\tilde{A}) = P(\tilde{B})$.
$\tilde{A} \sim \tilde{B}$ if and only if $P(\tilde{A}) = P(\tilde{B})$.
Then, we formulate the order $\succeq$ and $\preceq$:
$\tilde{A} \succeq \tilde{B}$ if and only if $\tilde{A} \succ \tilde{B}$ or $\tilde{A} \sim \tilde{B}$,
$\tilde{A} \preceq \tilde{B}$ if and only if $\tilde{A} \prec \tilde{B}$ or $\tilde{A} \sim \tilde{B}$.
The addition operation $\oplus$ on TFNs $\tilde{A}$ and $\tilde{B}$ can be defined:

$$P(\tilde{A} \oplus \tilde{B}) = \frac{1}{6}(\underline{a} + 4a_0 + \overline{a}) + \frac{1}{6}(\underline{b} + 4b_0 + \overline{b}) \tag{2.7}$$

We know the addition of two TFNs $\tilde{A}$ and $\tilde{B}$

$$\tilde{A} \oplus \tilde{B} = \left(\underline{a} + \underline{b}, a_0 + b_0, \overline{a} + \overline{b}\right), \tag{2.8}$$

$$\begin{aligned} P(\tilde{A} \oplus \tilde{B}) &= \frac{1}{6}\left((\underline{a} + \underline{b}) + 4(a_0 + b_0) + \left(\overline{a} + \overline{b}\right)\right) \\ &= \frac{1}{6}\left(\underline{a} + 4a_0 + \overline{a}\right) + \frac{1}{6}\left(\underline{b} + 4b_0 + \overline{b}\right) \\ &= P(\tilde{A}) + P(\tilde{B}). \end{aligned} \tag{2.9}$$

Definition 2.6. Given a TrFN $\tilde{A} = \{\underline{a}, a_1, a_2, \overline{a}\}$, the graded mean integration representation of TrFN $\tilde{A}$ can be defined as

$$P(\tilde{A}) = \frac{1}{6}(\underline{a} + 2a_1 + 2a_2 + \overline{a}). \tag{2.10}$$

Let $\tilde{A} = \{\underline{a}, a_1, a_2, \overline{a}\}$ and $\tilde{B} = \{\underline{b}, b_1, b_2, \overline{b}\}$ be two TrFNs. The addition operation $\oplus$ of $\tilde{A}$ and $\tilde{B}$ can be defined as follow:

$$P(\tilde{A} \oplus \tilde{B}) = \frac{1}{6}(\underline{a} + 2a_1 + 2a_2 + \overline{a}) + \frac{1}{6}(\underline{b} + 2b_1 + 2b_2 + \overline{b}) \tag{2.11}$$

2.2. **Prim's algorithm for minimum spanning tree problem.** Prim's algorithm is a greedy algorithm that find out a MST of a weighted connected undirected classical graph. This algorithm obtains a subset of the arcs that makes a tree that includes every node, where the total cost of all the arcs in the tree is minimized. This algorithm was introduced by Czech mathematician Vojtech Jarník [36] in 1930, later independently by Robert C. Prim [51] in 1957 and rediscovered by Edsger Dijkstra [20] in 1959. Therefore it is also sometimes called the Jarník algorithm, the DJP algorithm, or the Prim Jarník algorithm.

The algorithm continuously increases the size of a tree, one edge at a time, starting with a tree consisting of a single node, until it spans all nodes.

Step 1. Set V_{new} =$\{x\}$, where x is an arbitrary node (starting point) from V and $E_{new} = \varnothing$.

Step 2. Find an arc (u, v) with minimum weight such that u is in V_{new} and v is not. If there are multiple arcs with the same weight, any of them may be picked.

Step 3. Add v to V_{new} and (u, v) to E_{new}.

Step 4. If $V \setminus V_{new} = \varnothing$, then stop and return V_{new} and E_{new}. V_{new} and E_{new} describe the MST. Otherwise, repeat Step 2 and Step 3.

3. Problem formulation for FMST

Consider a fuzzy graph $\tilde{G}$, consisting of a finite set of n number of nodes $V = \{1, 2, ..., n\}$ and a finite set of m number of arcs $E \subseteq V \times V$. Each arc is represented by e, which is an order pair (i, j), where $i, j \in V$ and $i \neq j$. If the arc e is present in the FMST then $x_e = 1$, otherwise $x_e = 0$. The FMST is formulated as the following linear programming problem.

$$\min \sum_{e \in E} \tilde{c}_e x_e. \tag{3.1}$$

Subject to

$$\sum_{e \in E} x_e = n - 1, \tag{3.2}$$

$$\sum_{e \in \delta(s)} x_e \geq 1 \quad \forall s \subset V, \quad \varnothing \neq s \neq V, \tag{3.3}$$

$$x_e \in \{0, 1\} \quad \forall e \in E. \tag{3.4}$$

Here, $\tilde{c}_e$ is a TrFN that represents the length of the arc e and $\sum$ in (3.1) is the sum of fuzzy numbers using using graded mean integration representation of fuzzy numbers as defined in (2.7) or (2.10). Equation (3.2) ensures that the number of arcs in the FMST is $n - 1$. In (3.3), we use for the cutset of a subset of vertices s, i.e., the arcs that have one node in the set s and the other outside s. So, a spanning tree must have at least one arc in the cutset of any subset of the nodes.

4. Proposed Algorithm for the Fuzzy Minimum spanning tree and its cost

A common algorithm to solve the minimum spanning tree (MST) problem is the Prim's algorithm. In this section, a fuzzy version of Prim's algorithm is proposed to handle MST in a fuzzy environment. In a fuzzy environment, we discuss the MST problem on a graph in which a fuzzy number is assigned to each arc as its arc length. This problem, since it requires comparison and addition between fuzzy numbers, is dissimilar from the conventional MST, which only takes real numbers. The graded mean integration representation of fuzzy numbers is used for comparison and addition between fuzzy numbers. Based on the concept of graded mean integration representation of fuzzy numbers, we introduce a fuzzy version of the classical Prim's algorithm to solve the MST problem with fuzzy numbers.

In this algorithm, we use some variables which are necessary to clarify. An undirected connected fuzzy graph G is defined by an ordered pair $G = (V, E)$, where V is a set of vertices and E is a set of edges. Let $n= |V|$ and $m =|E|$, so we have vertices $V=\{v_1, v_2, ..., v_n\}$ and edges $E=\{e_1, e_2, ..., e_m\}$. V_{new}, E_{new}, $cost_{FN}$ represent the nodes, edges and cost of the corresponding FMST in fuzzy number. The cost of FMST $cost_{FN}$ is converted into crisp number by using graded mean integration representation of fuzzy number and it is stored in $cost_{real}$. The weight of edge e_i is represented by $cost(e_i)$ and $P(cost(e_i))$ denotes the graded mean value of edge e_i. The variable $cost(u, v)$ is the distance between two vertices u and v. An arbitrary

Algorithm 1 Pseudocode of the proposed algorithm.

Input: A nonempty connected fuzzy graph $G = (V, E)$ with fuzzy arcs lengths.
Output: Fuzzy minimum spanning tree $T = (V_{new}, E_{new})$ of G and its cost.

$V_{new} \leftarrow \{x\}$; ▷ x is an starting vertex of T.
$E_{new} \leftarrow \varnothing$; ▷ E_{new} is the arc set of the current tree T
$cost_{FN} \leftarrow$ null; ▷ Corresponding fuzzy number is empty
The P(cost(e_i)) value for each edge e_i, $i =$ 1,2,...,$|E|$ in fuzzy graph G is computed using (2.4);
while $V \setminus V_{new} \neq \varnothing$ **do**
    Find an arc (u, v) with minimum P(cost(u, v)) value such that u is in V_{new} and v is not;
    $cost_{FN} \leftarrow cost_{FN} \oplus \mathrm{cost}(u, v)$; ▷ as described in (2.8).
    $E_{new} \leftarrow E_{new} \cup (u, v)$;
    $V_{new} \leftarrow V_{new} \cup (\{u, v\} \setminus V_{new})$; ▷ add the end vertex in $V \setminus V_{new}$.
end while
$cost_{real} \leftarrow \mathrm{P}(cost_{FN})$ ▷ The cost of FMST is converted in a real number.

node v_1 is chosen from graph. We find the $P(cost(e_i))$ value for each edge e_i, $i =$ 1,2,...,$|E|$ in fuzzy graph G using graded mean integration representation of fuzzy numbers. Start from vertex v_1 and connect it to its nearest neighbor, say v_2. To find the nearest neighbor, first we find all the connecting edge with v_1. Then, among all the connecting edges of v_1 select the edge, i.e., (v_1, v_i) with lowest $P(cost(v_1, v_i))$ value. Similarly, we use same method to find the nearest neighbor for other vertices

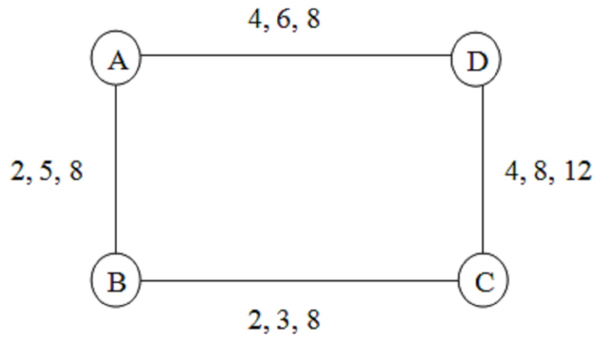

Figure 3. An Example fuzzy graph.

of the graph. Now, consider v_1 and v_2 as one subgraph and connect this subgraph to its closest neighbor. Let, this new vertex be v_k. Next regard the tree with vertices v_1, v_i and v_k as one subgraph and continue the process until all n vertices have connected by $n-1$ edges. The pseudocode of the proposed fuzzy Prim algorithm is shown in Algorithm 1.

In our proposed algorithm, we use an adjacency matrix to represent the fuzzy graph. The linear searching technique is applied to find the minimum weight edge based on the concept of graded mean integration representation of fuzzy number. The addition operation of fuzzy numbers is used to find the cost of the minimum spanning tree. The computational complexity of the proposed algorithm is $O(|V|^2)$.

5. Numerical example

In this section, a numerical example of the FMST problem is used to demonstrate of the proposed algorithm. Consider the fuzzy graph shown in Figure 3, with four nodes and four arcs. The lengths of all arcs of the fuzzy graph are in the form of TFNs and shown in Figure 3.

The staring vertex A is chosen arbitrarily from the vertex set of the graph G. Prim's algorithm starts with the vertex A. Initially, $V_{new} = \{A\}$, E_{new}=$\{\varnothing\}$ and $cost_{FN}$ =$\{0, 0, 0\}$.

Step 1. In this step, we have to find all the edges such that one endpoint is in A and the other is in either B or C or D. The two edges, (A, B) and (A, D), are connected with vertex A. We use the graded mean integration representation of the fuzzy numbers to find the value of $P(cost(A, B))$ and $P(cost(A, D))$. The values of $P(cost(A, B))$ and $P(cost(A, D))$ are 5 and 6 respectively. Among them, the smallest one (A, B) is picked out along with $P(cost(A, B))$ value 5. This cost of (A, B) ,i.e., $cost(A, B)$ value is added with $cost_{FN}$ using the addition operation of fuzzy numbers. Now, $V_{new} = A, B$, E_{new}=(A, B) and $cost_{FN}$ = (2,5,8).

Step 2. In this step, we have to find all the edges such that one endpoint is either in A or B and the other is in C or D. Two edges, (A, D) and (B, C) are found.

Among them, the lightest one (B, C) is chosen out with their value of cost. Now, $V_{new} = A, B, C$, E_{new}=$(A, B), (B, C)$ and $cost_{FN}$ = (4,8,16).

Step 3. Similarly, we add an another edge (A, D) in the spanning tree. Now, $V_{new} = A, B, C, D$, E_{new}=$(A, B), (B, C), (A, D)$ and $cost_{FN}$ = (8,14, 24). Now, the P($cost_{FN}$)=15.

6. Conclusion

In this paper, we introduce the fuzzy version of classical Prim's algorithm to solve the fuzzy minimum spanning tree problem. In a fuzzy minimum spanning tree problem, the fuzzy minimum spanning tree and its corresponding cost are the main information for the decision makers. Our modified Prim's algorithm finds the fuzzy minimum spanning tree and its corresponding cost. We use the graded mean integration representation of fuzzy numbers in our fuzzy Prim's algorithm to solve the fuzzy minimum spanning tree problem. The goal of this study is to build an uncertainty modeling architecture of the MST problem. It handles the uncertainty in arc costs of the fuzzy arc to capture the most available information. In future, we will try to apply our proposed algorithm to the real world problem like transportation systems, logistics management and many other network optimization problems that can be formulated as MST problem and we will also try to improve the complexity of the proposed algorithm using fibonacci heap and adjacency list.

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