Annals of **F**uzzy **M**athematics and **I**nformatics
Volume 12, No. 3, (September 2016), pp. 387–395
ISSN: 2093–9310 (print version)
ISSN: 2287–6235 (electronic version)
http://www.afmi.or.kr

Some φ-contrction results using CLRg proprty in probabilistic and fuzzy metric spaces

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Received 8 September 2015; Revised 14 February 2016; Accepted 4 March 2016

Abstract. In this paper our main theorem is in probabilistic metric spaces where we establish a coincidence point result under certain conditions utilizing the weak compatibility and CLRg property. We then make an application of the result in the context of the KM-fuzzy metric space. In another application we obtain a coincidence point theorem in metric spaces. An illustrative example is discussed.

2010 AMS Classification: 47H25, 54H10

Keywords: t-norm, φ- function, Weakly compatible mappings, CLRg property, Common fixed point.

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1. Introduction and mathematical preliminaries

In this paper our aim result is a coincidence point result in probabilistic metric spaces. We also deduce the corresponding results in fuzzy metric spaces as well as in metric spaces. We use two concepts in our theorem, namely, the CLRg property [13, 24] and a control function introduced by Fang [9].

CLRg property, or more elaborately, "common limit in the range of g" property is a concept which has been recently introduced by Sintunavarat et al in 2011 [24]. The concept is given in the following definition.

Definition 1.1 ([24]). Suppose that (X, d) is a metric space and $f, g : X \to X$. The pair of mappings (f, g) is said to satisfy the common limit in the range of g property if there exists a sequences $\{x_n\}$ in X such that

$$\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n) = g(x) \text{ for some } x \in X.$$

The above mentioned property has been extended to coupled mappings in [17]. Further the property has been used in fixed point problems in fuzzy metric spaces by Heirro et al [13]. For a comparison of CLRg property with other properties of pairs of mappings like the compatibility etc, we refer to [1, 17, 24].

A control functions was used in the fixed point studies on probabilistic metric spaces by Choudhury et al in [5] wherein a probabilistic extension of the Sehgal contraction [27] was established. After that several works on fixed point and related studies in probabilistic and fuzzy metric spaces utilized control functions of different types. Some of these works are in [6, 8, 12, 15, 16, 28].

An interesting discussion on the recent development in the uses of control functions in fixed point theory is given by Fang [9]. In the same paper [9], Fang introduced a class of control functions which includes many of such previously known functions. We describe the function in the following definition.

Definition 1.2 ([9])**.** Let Φ denote the class of all functions $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the following condition:

$$\text{for each } t > 0 \text{ there exists } r \geq t \text{ such that } \lim_{n\to\infty} \varphi^n(r) = 0.$$

The class Φ_w is a proper subclass of Φ [see [9]]. Fang utilized the above function in establishing fixed point theorems in both probabilistic and fuzzy metric spaces [9].

Particularly, it was proved in [9], the class Φ in Definition 1.2 includes the class Φ_w introduced by *Ćirić* [7] which is the set of all functions $\eta : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\lim_{n\to\infty} \eta^n(t) = 0$ for all $t > 0$. The class of functions Φ_w was utilized in the fixed point theory of probabilistic metric spaces[7].

We next describe the Menger space which is a particular type of probabilistic metric space on which we work out our main theorem.

Definition 1.3 ([23])**.** A mapping $F : \mathbb{R} \to \mathbb{R}^+$ is called a distribution function if it is non-decreasing and left continuous with $\inf_{t\in\mathbb{R}} F(t) = 0$ and $\sup_{t\in\mathbb{R}} F(t) = 1$, where $\mathbb{R}$ is the set of real numbers and $\mathbb{R}^+$ denotes the set of all non-negative real numbers.

Definition 1.4 ([10, 23])**.** A binary operation $\Delta : [0, 1]^2 \longrightarrow [0, 1]$ is called a t-norm if the following properties are satisfied:
(i) Δ is associative and commutative,
(ii) $\Delta(a, 1) = a$ for all $a \in [0, 1]$,
(iii) $\Delta(a, b) \leq \Delta(c, d)$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0, 1]$.

Generic examples of t-norm are $\Delta_M(a, b) = \min\{a, b\}$, $\Delta_P(a, b) = ab$ etc.

Definition 1.5 ([23])**.** A Menger space is a triplet (X, F, Δ), where X is a non empty set, F is a function defined on $X \times X$ to the set of distribution functions and Δ is a t-norm, such that the following are satisfied:
(i) $F_{x,y}(0) = 0$ for all $x, y \in X$,
(ii) $F_{x,y}(s) = 1$ for all $s > 0$ if and only if $x = y$,
(iii) $F_{x,y}(s) = F_{y,x}(s)$ for all $s > 0$, $x, y \in X$,
(iv) $F_{x,y}(u + v) \geq \Delta\left(F_{x,z}(u), F_{z,y}(v)\right)$ for all $u, v \geq 0$ and $x, y, z \in X$.

Definition 1.6 ([22])**.** Let (X, F, Δ) be a Menger space.

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} F_{x_n,x}(t) = 1$ for all $t > 0$.

(ii) A sequence $\{x_n\}$ in X is called a Cauchy sequence if for each $0 < \varepsilon < 1$

and $t > 0$, there exists a positive integer n_0 such that $F_{x_n,x_m}(t) > 1 - \varepsilon$ for each $n, m \geq n_0$.
(iii) A Menger space in which every Cauchy sequence is convergent is said to be complete.

For a detail description of different concepts of probabilistic metric spaces we refer to the book of Schweizer and Sklar [23]. A comprehension description of the development of fixed point theory in probabilistic metric spaces is given by *Hadžić* and Pap [10]. Some more recent reference are[5, 6, 7, 9, 28].

Fuzzy sets were introduced by Zadeh [29] as an extension of the notion of ordinary sets. Fuzzy concepts made quick headways in different branches of mathematics including functional analysis. Fuzzy fixed point theory is a developed branch of analysis in which some recent references are in [3, 4, 14, 18, 20, 21, 25, 26] One of the early definitions of fuzzy metric space was given by Kramosil and Michalek [19]. This space is called KM-fuzzy metric space. The definition is the following.

Definition 1.7 ([19]). The 3-tuple (X, M, Δ) is called a fuzzy metric space if X is an arbitrary non-empty set, Δ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions for each $x, y, z \in X$ and $t, s > 0$:
(i) $M(x, y, 0) = 0$,
(ii) $M(x, y, t) = 1$ if and only if $x = y$,
(iii) $M(x, y, t) = M(y, x, t)$,
(iv) $\Delta(M(x, y, t), M(y, z, s)) \leq M(x, z, t + s)$,
(v) $M(x, y, .) : [0, \infty) \longrightarrow [0, 1]$ is left continuous,
(vi) $\lim_{t\to\infty} M(x, y, t) = 1$.

Definition 1.8 ([7]). Let Φ_w denote the class of all functions $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying the following condition:
$$\lim_{n\to\infty} \varphi^n(t) = 0 \text{ for all } t > 0.$$

Lemma 1.9 ([9]). *Let $\varphi \in \Phi$, then for each $t > 0$ there exists $r \geq t$ such that $\varphi(r) < t$.*

Lemma 1.10 ([9]). *Let (X, F, Δ) be a Menger space and $x, y \in X$. If there exists a function $\varphi \in \Phi$ such that $F_{x,y}(\varphi(t)) \geq F_{x,y}(t)$ for all $t > 0$, then $x = y$.*

Definition 1.11 ([11]). Let the mappings $f, g : X \to X$ are said to be weakly compatible if $f(g(x)) = g(f(x))$ for all $x \in X$ such that $f(x) = g(x)$.

We use weak compatibility in our theorem. Further we note that a more relaxed condition of occasionally weakly compatible maps has been used in probabilistic metric spaces in [2].

We next define CLRg property in Menger spaces.

Definition 1.12. Let (X, F, Δ) be a Menger space with a continuous t-norm Δ. The two mappings $f, g : X \to X$ are said to have the CLRg property if there exists a sequence $\{x_n\} \in X$ and a point $z \in X$ such that $\lim_{n\to\infty}\{f(x_n)\} = \lim_{n\to\infty}\{g(x_n)\} = g(z)$.

In this paper our primary theorem is in probabilistic metric spaces where we establish a coincidence point theorem under certain conditions utilizing the above mentioned two concepts. We then make an application of the result in the context of the KM-fuzzy metric space. In another application we obtain a coincidence point theorem in metric spaces. An illustrative example is discussed.

2. Major section

We require the result of the following lemma to establish our main theorem.

Lemma 2.1. *Let (X, F, Δ) be a Menger space with a continuous t-norm Δ. Let $f, g : X \to X$ be two mappings with CLRg property, that is, there is a sequence $\{x_n\} \in X$ and $z \in X$ such that $\{f(x_n)\} \to g(z)$ and $\{g(x_n)\} \to g(z)$. Suppose that there exists $\varphi \in \Phi$, such that*

$$F_{f(x),f(y)}(\varphi(t)) \geq F_{g(x),g(y)}(t) \text{ for all } x, y \in X \text{ and } t > 0. \tag{2.1}$$

Then f and g have a coincidence point, that is, $f(z) = g(z)$.

Proof. Since $\varphi \in \Phi$, by Lemma 1.9, for any $t > 0$ there exists $r \geq t$ such that $\varphi(r) < t$.

$$\begin{aligned} F_{f(x_n),f(z)}(t) &\geq F_{f(x_n),f(z)}(\varphi(r)) \\ &\geq F_{g(x_n),g(z)}(r). \end{aligned}$$

Taking limit $n \to \infty$ on both sides of the above inequality, we have

$$\begin{aligned} \lim_{n\to\infty} F_{f(x_n),f(z)}(t) &\geq \lim_{n\to\infty} [F_{g(x_n),g(z)}(r)] \\ &= 1. \end{aligned}$$

Therefore, $\{f(x_n)\} \to f(z)$. Again $\{f(x_n)\} \to g(z)$, so $f(z) = g(z)$, that is, $f$and g have a coincidence point. □

Theorem 2.2. *Let (X, F, Δ) be a Menger space with a continuous t-norm Δ. Let $f, g : X \to X$ be two mappings which satisfy the following conditions:*

(i) *(f, g) is weakly compatible pair,*
(ii) *(f, g) satisfies the CLRg property,*
(iii) $F_{f(x),f(y)}(\varphi(t)) \geq F_{g(x),g(y)}(t)$, (2.2)

for all $x, y \in X$, $t > 0$ where $\varphi \in \Phi$. Then f and g have unique common fixed point, that is, there is a unique $w \in X$ such that $f(w) = g(w) = w$. Further, if $z \in X$ is any coincidence point of f and g, then $f(z) = g(z) = w$.

Proof. Following Lemma 2.1, there exists $z \in X$ such that $f(z) = g(z)$. Let $f(z) = g(z) = w$. We show that w is the only common fixed point of f and g. Since the pair (f, g) is weakly compatible, it follows that

$$f(z) = g(z) \Rightarrow g(f(z)) = f(g(z)) \Rightarrow g(w) = f(w).$$

Now we prove that $f(w) = w$. Putting $x = w$ and $y = z$ in (2.2), for all $t > 0$, we have

$$\begin{aligned} F_{f(w),f(z)}(\varphi(t)) &\geq F_{g(w),g(z)}(t) \\ &= F_{g(w),w}(t) \text{ (since } g(z) = w) \\ &= F_{f(w),w}(t) \text{ (since } g(w) = f(w)). \end{aligned}$$

Since $f(z) = w$, from the above inequality, we have $F_{f(w),w}(\varphi(t)) \geq F_{f(w),w}(t)$. By an application of Lemma 1.10, we have $f(w) = w$.
So, $w = g(w) = f(w)$, that is, w is a common fixed point of f and g.
Let $y \neq w \in X$ be another fixed point of f and g. Therefore, $y = f(y) = g(y)$.
By using (2.2), we have for all $t > 0$,

$$F_{w,y}(\varphi(t)) = F_{f(w),f(y)}(\varphi(t)) \geq F_{g(w),g(y)}(t) = F_{w,y}(t).$$

By an application of the Lemma 1.10, we conclude that $w = y$. Hence f and g have a unique common fixed point. □

3. Results in KM fuzzy metric spaces for φ-contraction

In this section we present a common fixed point result in KM fuzzy metric spaces. This is obtained by an application of the theorem established in the previous section. The result obtained here extends the result of Hierro et al.[13].

Lemma 3.1 ([9]). *Let (X, M, Δ) be a KM-fuzzy metric space with a continuous t-norm Δ at (1,1). Suppose that there exist $x_0, x_1 \in X$ such that $\lim_{t\to\infty} M(x_0, x_1, t) = 1$. Define $Y_0 = \{y \in X : \lim_{t\to\infty} M(x_0, y, t) = 1\}$. Then (Y_0, F, Δ) is a Menger space where F is defined by*

$$F_{x,y}(t) = \begin{cases} M(x, y, t), & \text{if } t \geq 0, \\ 0, & \text{if } t < 0. \end{cases}$$

Theorem 3.2. *Let (X, M, Δ) be a KM-fuzzy metric space with a continuous t-norm Δ. Let $f, g : X \to X$ be two mappings satisfy the following conditions:*
(i) *(f, g) is weakly compatible pair,*
(ii) *(f, g) satisfies the CLRg property,*
(iii) *$M(f(x), f(y), \varphi(t)) \geq M(g(x), g(y), t)$,* *(3.1)*
for all $x, y \in X$, $t > 0$, where $\varphi \in \Phi$. Let there exists $x_0 \in X$ such that

$$\lim_{t\to\infty} M(x_0, f(x_0), t) = 1.$$

Then f and g have unique fixed point in $Y_0 = \{y \in X : \lim_{t\to\infty} M(x_0, y, t) = 1\}$.

Proof. F is defined as in Lemma 3.1. Since (X, M, Δ) is a KM-fuzzy metric space there exists $x_0 \in X$ such that $\lim_{t\to\infty} M(x_0, f(x_0), t) = 1$, then by an application of Lemma 3.1, we have (Y_0, F, Δ) is a Menger space. Since $\varphi \in \Phi$, there exists $r \geq t$ such that $\varphi(r) < t$. By the monotone property of $M(x, y, .)$ and (3.1), we have

$$\begin{aligned} M(f(x), f(y), t) &\geq M(f(x), f(y), \varphi(r)) \\ &\geq M(g(x), g(y), r) \\ &\geq M(g(x), g(y), t) \text{ for all } x, y \in X,\ t > 0. \end{aligned} \tag{3.2}$$

Next we show that f is a mapping from Y_0 into itself.
If $y \in Y_0$, then $\lim_{t\to\infty} M(g(x_0), g(y), \frac{t}{2}) = 1$ and given that $\lim_{t\to\infty} M(x_0, f(x_0), \frac{t}{2}) = 1$. By using (3.2), we have $x, y \in X$, $t > 0$,

$$\begin{aligned} M(x_0, f(y), t) &\geq \Delta\{M(x_0, f(x_0), \tfrac{t}{2}), M(f(x_0), f(y), \tfrac{t}{2})\} \\ &\geq \Delta\{M(x_0, f(x_0), \tfrac{t}{2}), M(g(x_0), g(y), \tfrac{t}{2})\}. \end{aligned}$$

Taking limit $t \to \infty$ on the both sides of the above inequality, we have

$$\begin{aligned} \lim_{t\to\infty} M(x_0, f(y), t) &\geq \lim_{t\to\infty} \Delta\{M(x_0, f(x_0), \tfrac{t}{2}), M(g(x_0), g(y), \tfrac{t}{2})\} \\ &= \Delta\{1, 1\} \\ &= 1, \end{aligned}$$

that is, $f(y) \in Y_0$. This proves f is a mapping from Y_0 into itself.
We have from (3.1) for all $x, y \in Y_0$, $t > 0$,

$$F_{f(x),f(y)}(\varphi(t)) \geq F_{g(x),g(y)}(t),$$

where F is defined as above. This proves that (2.2) is satisfied in (Y_0, F, Δ). Thus we conclude from the Theorem 2.222. that f and g have unique fixed point in $Y_0 = \{y \in X : \lim\limits_{t\to\infty} M(x_0, y, t) = 1\}$. Hence the theorem is proved. □

Remark 3.3. The Theorem 3.2 is an improvement of the Theorem 2.1 of Hierro et al [13] because of (i) the function $\varphi \in \Phi$ more general than the function ϕ which was used in [13], (ii) we use arbitrary continuous t-norm in our theorem.

4. Results in metric spaces for φ-contraction

In this section we present a common fixed point result in metric spaces. This is obtained by an application of the theorem established in the section 2. The result obtained here extends some existing results.

Lemma 4.1 ([9]). *Let (X, d) be a metric space. Define a mapping $F : X \times X \to \mathbb{D}^+$ by*

$$F_{x,y}(t) = \begin{cases} 0, & \text{if } d(x, y) > t > 0, \\ 1, & \text{if } d(x, y) \le t, \end{cases}$$

for all $x, y \in X$. Then (X, F, Δ_M) is a Menger space.

Theorem 4.2. *Let (X, d) be a metric space. Let $f, g : X \to X$ be two mappings satisfy the following conditions:*
(i) *(f, g) is weakly compatible pair,*
(ii) *(f, g) satisfies the CLRg property,*
(iii) $d(f(x), f(y)) \le \varphi(d(g(x), g(y)))$, *(4.1)*
for all $x, y \in X$, where $\varphi \in \Phi$ satisfying $\varphi(t) > 0$ for all $t > 0$,
(iv) *φ is increasing.*
Then f and g have unique fixed point in X.

Proof. We define the mapping $F : X \times X \to \mathbb{D}^+$ by

$$F_{x,y}(t) = \begin{cases} 0, & \text{if } d(x, y) > t > 0, \\ 1, & \text{if } d(x, y) \le t, \end{cases}$$

for all $x, y \in X$. By Lemma 4.1, we have (X, F, Δ_M) is a Menger space. Now we prove (4.1) implies (2.2).

If $F_{g(x),g(y)}(t) = 0$ for $t > 0$, then (2.2) holds. If $F_{g(x),g(y)}(t) = 1$ for $t > 0$,then from the above $d(g(x), g(y)) \le t$. Since φ is increasing, we have that $d(f(x), f(y)) \le \varphi(d(g(x), g(y))) \le \varphi(t)$, which implies that $F_{f(x),f(y)}(\varphi(t)) = 1 = F_{g(x),g(y)}(t)$. Therefore (2.2) holds. So we get the Theorem 4.2 from the Theorem 2.2. □

Note. In the Theorem 4.2 we assume the monotone increasing property of φ which is an additional assumption and is not included in the definition of the class Φ.

5. An illustration

Example 5.1. Let $X = [0, \infty)$. We define the mapping $F : X \times X \to \mathbb{D}^+$ as follows:

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } |x - y| \ge t, \\ 1, & \text{if } |x - y| < t, \end{cases}$$

for all $t > 0$, $x, y \in X$. Let $\Delta(a,b) = \min\{a,b\}$ for all $a, b \in [0,1]$. Then (X, F, Δ) is a Menger space.
Let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be defined by

$$\varphi(t) = \begin{cases} \frac{t}{t+1}, & \text{if } t \in [0,1), \\ -\frac{t}{3} + \frac{4}{3}, & \text{if } t \in [1,2], \\ t + \frac{4}{3}, & \text{otherwise} . \end{cases}$$

It is obvious $\varphi \in \Phi$ but $\varphi \notin \Phi_w$. From the definition of φ, we have $\varphi(t) \geq \frac{t}{t+1}$ for all $t \geq 0$.
Let the mappings $f, g : X \to X$ are defined as follows:

$$f(x) = \begin{cases} \frac{x^2}{x^2+1}, & \text{if } x \in [0,2), \\ 2, & \text{if } t \geq 2, \end{cases}$$

and

$$g(x) = \begin{cases} x^2, & \text{if } x \in [0,2), \\ 1, & \text{if } t \geq 2. \end{cases}$$

It is easy to check that the mappings f and g are weakly compatible at the unique point $x = 0$ and also have the CLRg property.
By the same calculation of [9], we have $|x^2 - y^2| \geq t$.

Now,
$$\begin{aligned} F_{f(x),f(y)}(\varphi(t)) &= \frac{\varphi(t)}{\varphi(t) + |f(x) - f(y)|} \\ &\geq \frac{\dfrac{t}{t+1}}{\dfrac{t}{t+1} + \dfrac{|x^2 - y^2|}{1 + |x^2 - y^2|}} \\ &\geq \frac{t}{t + |x^2 - y^2|} \\ &= F_{g(x),g(y)}(t). \end{aligned}$$

Thus the condition (iii) of the Theorem 2.2 holds. Therefore all conditions of the Theorem 2.2 are satisfied. Here 0 is a unique common fixed point of f and g.

Acknowledgements. The authors thank to the learned referees for suggestions to improve the paper.

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