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A study on lower interval probability function based decision theoretic rough set models

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Abstract. In this paper a new concept of Lower interval probability function based decision theoretic rough set model is introduced and some of its properties are studied. Some comparative discussions are cited between the new concept and some of the previously defined concepts, such as pawlak rough set model, variable precision rough set model, Bayesian decision theoretic rough set model etc. Lastly some example is taken and attribute reduction is done by this new method and various other methods. It is shown that this method gives better result than the previously defined methods. Also some important properties are studied in various other methods.

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1. Introduction

The concept of Rough set was first introduced by Z. Pawlak in 1982 [4]. Later on lots of research has been undertaken in the field of rough set and its generalized form [5, 6, 7]. One of the main application of Rough set is data reduction. Z. Pawlak and other has first introduced a method with the help of discernibility function for reducing the attribute using the rough set introduced by him. Later on W.Ziarko in 1993 introduced Variable precision rough set[17, 18] model and studied attribute reduction for this type of rough set. D. Slezak and W. Ziarko in 2002 introduced Bayesian rough set model [8]. In 2003 they introduced Bayesian version of variable precision rough set model and studied attribute reduction in the respective model. Y. Y. Yao in 2003 introduced probabilistic approach of rough set [11] and in 2007 introduced the concept of Decision theoretic rough set model [10]. The attribute reduction in Decision theoretic rough set model [13, 14] has been studied by Y. Y. Yao and Y. Zhao in 2008. Some more study [16] has been done by them in 2011. In 2012 H. Zhang, J. Zhou, D. Miao, C. Gao [15]has studied some proposition on Bayesian rough set model. In 2003 H. Tanaka, K. Sugihara and Y. Maeda [9]has introduced Interval probability and studied some of its properties.

The aim of this paper is to introduce a new hybrid approach of Lower interval probability function based decision theoretic rough set model. Attribute reduction is done by this method using an arbitrary information table. Using the same information table attribute reduction is done by the other methods defined earlier. It is found that this new method gives more significant attributes after reduction than the other methods. So a comparative study is shown in this paper by the help of some example. Lastly some proposition is studied and the difference with other methods related to this proposition is also shown in this paper. Further researches may be done using original data.

2. Preliminaries

2.1. **Information table([4]).** An information table is the following tuple: $IS = (U, A = At = C \cup D, V, \rho)$, Where U is a finite nonempty set of objects, At is a finite nonempty set of attributes, V is a nonempty set of values of $a \in At$, and ρ is an information function . For classification problems we consider an information table of the form $S = (U, At = C \cup \{D\}, \{V_a\}, \{\rho_a\})$, where C is a set of condition attributes describing the objects, and D is a decision attribute that indicates the classes of objects. Let $\pi_D = \{D_1, D_2, ..., D_m\}$ be a partition of the universe U defined by the decision attribute D. Each equivalence class $D_i \in \pi_D$ is called a decision class. Given another partition $\pi_A = \{A_1, A_2, ..., A_n\}$ of U defined by a condition attribute set $A \subseteq C$ each equivalence class A_j also is defined by $[x]_A = \{y \in U : \forall a \in A(I_a(x) = I_a(y))\}$.

2.2. **Rough set([4]).** Pawlak in 1982 defined the concept of Rough set. According to Pawlak the rough set is defined as below. Typically objective of rough set theory is to form an approximate definition of the target set X $\subseteq$ U in terms of some definable sets especially when the target set is indefinable or vague. The upper and lower approximation of X with respect to equivalence relation A are denoted as $\overline{A}X$ and $\underline{A}X$ respectively and defined as:

$\overline{A}$X = {E: P(X/E)>0, E∈ U/A}, $\underline{A}$X = {E: P(X/E)=1, E∈ U/A}.

2.3. **Variable precision rough set([17]).** In practical applications Pawlaks rough set model cannot deal with data sets which have some noisy data effectively. Lots of information in the boundary region will be abandoned which may provide latest useful knowledge. By applying the parameter the approximate regions can be adjusted and controlled in VPRSM. Given a parameter β, $0 \leq 1 - \beta < P(X) < \beta \leq 1$, three kinds of approximation regions of concepts $X \subseteq U$ with respect to equivalence relation A can be defined as follows:

β Positive region: $POS_A^\beta(X) = \cup\{E : P(X|E) \geq \beta, E \in U|A\}$,

β Negative region: $NEG_A^\beta(X) = \cup\{E : P(X|E) \leq 1 - \beta, E \in U|A\}$,

β Boundary region: $BND_A^\beta(X) = \cup\{E : 1-\beta < P(X|E) < \beta, E \in U|A\}$.
β positive region is the collection of all those elementary sets which can be included in X with the certainty degree not lower than β, β negative region is composed of all those elementary sets which can be included in the complement of X. Since $1-\beta < \beta$, $0.5 < \beta \leq 1$ it indicates that $0 \leq 1-\beta < P(X) < \beta \leq 1$ includes the original 0.5 symmetric variable precision rough set model. When $\beta = 1$ the VPRS will be same as Pawlaks rough set model. In some application however it is not clear how to define the parameters.

2.4. **Decision theoretic rough set([12]).** The decision theoretic rough set model systematically calculates the thresholds by a set of loss functions based on the Bayesian decision procedure. The physical meaning of the loss functions can be interpreted based on more practical notions of costs and risks. By using the thresholds, for a decision class $D_i \in \pi_D$, the probabilistic lower and upper approximations with respect to a partition π_A can be defined based on two thresholds $0 \leq \beta < \alpha \leq 1$ as:

$$\underline{apr}_{(\alpha,\beta)}(D_i|\pi_A) = \{x \in U : p(D_i|[x]_A) \geq \alpha\}$$

and

$$\overline{apr}_{(\alpha,\beta)}(D_i|\pi_A) = \{x \in U : p(D_i|[x]_A) > \beta\}.$$

2.5. **Bayesian rough set model ([8]).** Slezak and Ziarko put forward BRSM in which the prior probability of the event under consideration is chosen as a benchmark value . BRSM is a hybrid product which connects rough set theory and Bayesian reasoning validity and reasonably. It is more appropriate to application problems concerned with achieving any certainty gain during the procedures of prediction or decision making rather than meeting a special certainty goal.
In BRSM three kinds of B approximation regions of concepts $X \subseteq U$ with respect to equivalence relation A can be defined as follows :
B positive region : $POS_A^*(X)$=$\bigcup\{E : P(X/E) > P(X), E \in U/A\}$,
B negative region: $NEG_A^*(X)$=$\bigcup\{E : P(X/E) < P(X), E \in U/A\}$,
B boundary region: $BND_A^*(X)$=$\bigcup\{E : P(X/E) = P(X), E \in U/A\}$.

2.6. **Bayesian rough set depending on coverage ([3]).** The positive, negative and boundary region in Bayesian rough set depending on coverage for an information system $IS = (U, A = C \cup D, V, \rho)$is as follows:
$Pos = POS_{cov(\pi_A)}(D_i) = \{[x]_C : |[x]_C \cap D_i|/|D_i| > P([x]_C)\}$,
Neg=$NEG_{cov(\pi_A)}(D_i) = \{[x]_C : |[x]_C \cap D_i|/|D_i| < P([x]_C)\}$,
$Bnd = BND_{cov(\pi_A)}(D_i) = \{[x]_C : |[x]_C \cap D_i|/|D_i| = P([x]_C)\}$.

2.7. **Bayesian decision theoretic rough set ([1]).** Positive, negative and boundary region are defined as follows :
Positive region: $Pos_{D_i}^*([x]_C) = \{[x]_C : |[x]_C \cap D_i|/|[x]_C| > P(D_i)$,
Negative region: $Neg_{D_i}^*([x]_C) = \{[x]_C : |[x]_C \cap D_i|/|[x]_C| < P(D_i)$,
Boundary region: $Bnd_{D_i}^*([x]_C) = \{[x]_C : |[x]_C \cap D_i|/|[x]_C| = P(D_i)$.

It is actually a Bayesian rough set model depending on decision.Attribute reduction in Bayesian decision theoretic rough set [2] is also studied by the author in 2014.

2.8. **Interval probability function ([9]).** The lower and upper functions conditioned by $B \subseteq X$ are defined as :

$$LB(A|B) = (LB(AB))/(LB(AB) + UB(B - AB))$$

and

$$UB(A|B) = (UB(AB))/(UB(AB) + LB(B - AB)).$$

The lower and upper functions in Bayes' decision problem with IPF are defined as :

$$LB(A|B) = (LB(A, B))/(LB(A, B) + UB(A^c, B))$$

and

$$UB(A|B) = (UB(A, B))/(UB(A, B) + LB(A^c, B)).$$

2.9. **Significance of the reducts ([4]).** Significance of an attribute (reduct) can be evaluated by measuring effect of removing the attribute from an information table on classification defined by the table.

Let C and D be sets of condition and decision attribute respectively and let a be a condition attribute, i.e., $a \in C$. Now the consistency factor $\gamma(C, D) = |POS_C(D)|/|U|$.

If $\gamma(C, D) = 1$,the decision table is consistent and if $\gamma(C, D) \neq 1$,the decision table is inconsistent. But the co-efficient $\gamma(C, D)$ changes when we remove any attribute a. Now we define the significance of any attribute a as-

$$\sigma(C, D)(a) = (\gamma(C, D) - \gamma(C - \{a\}, D))/\gamma(C, D) = 1 - (\gamma(C - a, D)/\gamma(C, D)).$$

And simply denoted by $\sigma(a)$, where $0 \leq \sigma(a) \leq 1$.

In the next section the concept of Lower Interval probability function based decision theoretic rough set model is introduced and some of its properties are studied and by taking some example it has been shown that the new method gives better reduction and also comparative study has been done.

3. Lower interval probability function based decision theoretic rough set model

Here we introduce a new concept of decision theoretic rough set model which is based on Lower Interval probability function. Skowron has proved that reducts are in one-to-one correspondence to the prime implicants of the associated discernibility function in a given decision table. According to this property, discernibility matrices for Lower Interval probability function based decision theoretic rough set model is introduced. Discernibility function for this new model is also introduced.

Definition 3.1. The Lower Interval probability function based decision theoretic positive, boundary and negative regions of $D_i \in \pi_D$ with respect to the equivalence classes can be defined by:

$POS_{(\alpha,\beta)}([x]_C|\pi_D) = \{D_i : LB([x]_C|D_i) \geq \alpha\}$,

$BND_{(\alpha,\beta)}([x]_C|\pi_D) = \{D_i : \beta < LB([x]_C|D_i) < \alpha\}$,

$NEG_{(\alpha,\beta)}([x]_C|\pi_D) = \{D_i : LB([x]_C|D_i) \leq \beta\}$ where $0 \leq \beta < \alpha \leq 1$ and

$LB([x]_C|D_i) = \frac{LB([x]_C, D_i)}{LB([x]_C, D_i) + UB([x]^c_C, D_i)} = \frac{|[x]_C \cap D_i|}{|[x]_C \cap D_i| + |[x]^c_C \cup D_i|}$,
i.e., by $LB([x]_C, D_i)$ we mean the index of intersection of $[x]_C$ and D_i and $UB([x]^c_C, D_i)$ represent the index of union of $[x]^c_C$ and D_i.

Definition 3.2. Discernibility matrix and discernibility function for Lower Interval probability function based decision theoretic rough set model: Constructing a reduct for decision preservation can apply any traditional methods for example, the methods based on the discernibility matrix. Both rows and columns of the matrix correspond to the equivalence classes defined by C. An element of the matrix is the set of all attributes that distinguish the corresponding pair of equivalence classes. Namely the matrix element consists of all attributes on which the corresponding two equivalence classes have distinct values and thus distinct decision making. A discernibility matrix is symmetric. The elements of a positive decision based discernibility matrix $M_{POS_{(\alpha,\beta)}}$ is defined as follows. For any two equivalence classes $[x]_C$ and $[y]_C$,

$$M_{POS_{(\alpha,\beta)}}([x]_C, [y]_C) = \{a \in C : I_a(x) \neq I_a(y) \wedge POS_{(\alpha,\beta)}([x]_C|\pi_D) \neq POS_{(\alpha,\beta)}([y]_C|\pi_D)\}.$$

Skowron and Rauser showed that the set of attribute reducts are in fact the set of prime implicants of the reduced disjunctive form of the discernibility function. Thus a positive decision reduct is a prime implicant of the reduced disjunctive form of the discernibility function.
$f(M_{POS_{(\alpha,\beta)}}) = \wedge\{\vee M_{POS_{(\alpha,\beta)}}([x]_C, [y]_C) : x, y \in U, (M_{POS_{(\alpha,\beta)}}([x]_C, [y]_C) \neq \phi)\}$.
The expression $\vee M_{POS_{(\alpha,\beta)}}([x]_C, [y]_C)$ is the disjunction of all attributes in $M_{POS_{(\alpha,\beta)}}([x]_C, [y]_C)$ indicating that the pair of equivalence classes $[x]_C$ and $[y]_C$ can be distinguished by any attribute in M. The expression $\wedge\{\vee M_{POS_{(\alpha,\beta)}}([x]_C, [y]_C)\}$ is the conjunction of all $\vee M_{POS_{(\alpha,\beta)}}([x]_C, [y]_C)$ indicating that the family of discernible pairs of equivalence classes can be distinguished by a set of attributes satisfying $\wedge\{\vee M_{POS_{(\alpha,\beta)}}([x]_C, [y]_C)\}$. In order to derive the reduced disjunctive form the discernibility function $f(M_{POS_{(\alpha,\beta)}})$ is transformed by using the absorption and distribution laws. Accordingly finding the set of reducts can be modeled based on the manipulation of a Boolean function. Let us now consider an example and show the attribute reduction by the various other previously defined methods and the new method.

Example 3.3. Here in the information table in Table1 we have,
$LB([O_1]_C|M) = LB([O_2]_C|M) = LB([O_6]_C|M)$=2/(2+10)=1/6,
$LB([O_1]_C|Q) = LB([O_2]_C|Q) = LB([O_6]_C|Q)$=1/(1+9)=1/10,
$LB([O_1]_C|F) = LB([O_2]_C|F) = LB([O_6]_C|F)$=0,
$LB([O_3]_C|M) = LB([O_5]_C|M)$=1/(1+11)=1/12,
$LB([O_3]_C|Q) = LB([O_5]_C|Q)$=1/(1+10)=1/11,
$LB([O_3]_C|F) = LB([O_5]_C|F) = 0, LB([O_4]_C|M) = LB([O_7]_C|M) = LB([O_8]_C|M)$ $= LB([O_9]_C|M)$=1/(1+9)=1/10,
$LB([O_4]_C|Q) = LB([O_7]_C|Q) = LB([O_8]_C|Q) = LB([O_9]_C|Q)$=1/(1+8)=1/9,
$LB([O_4]_C|F) = LB([O_7]_C|F) = LB([O_8]_C|F) = LB([O_9]_C|F)$=2/(2+7)=2/9.
Suppose $\alpha = 2/9$ and $\beta = 1/9$. We can obtain the following regions for the equivalence class $[O_1]_C$,

Table 1. An Information Table

	C_1	C_2	C_3	C_4	C_5	C_6	D
O_1	1	0	0	1	0	0	M
O_2	1	0	0	1	0	0	M
O_3	1	0	0	0	0	0	M
O_4	0	0	0	1	1	0	M
O_5	1	0	0	0	0	0	Q
O_6	1	0	0	1	0	0	Q
O_7	0	0	0	1	1	0	Q
O_8	0	0	0	1	1	0	F
O_9	0	0	0	1	1	0	F

Table 2. a reformation of Table1 indicating the decision associated with each equivalence class $[x]_C$

	C_1	C_2	C_3	C_4	C_5	C_6	POS	BND	NEG
O_1	1	0	0	1	0	0	ϕ	{M}	{Q,F}
O_2	1	0	0	1	0	0	ϕ	{M}	{Q,F}
O_3	1	0	0	0	0	0	ϕ	ϕ	{M,Q,F}
O_4	0	0	0	1	1	0	{F}	ϕ	{M,Q}
O_5	1	0	0	0	0	0	ϕ	ϕ	{M,Q,F}
O_6	1	0	0	1	0	0	ϕ	{M}	{Q,F}
O_7	0	0	0	1	1	0	{F}	ϕ	{M,Q}
O_8	0	0	0	1	1	0	{F}	ϕ	{M,Q}
O_9	0	0	0	1	1	0	{F}	ϕ	{M,Q}

$$POS_{(\alpha,\beta)}([O_1]_C|\pi_D) = \{D_i : LB([O_1]_C|D_i) \geq \alpha\} = \phi,$$
$$BND_{(\alpha,\beta)}([O_1]_C|\pi_D) = \{D_i : \beta < LB([O_1]_C|D_i) < \alpha\} = \{M\},$$
$$NEG_{(\alpha,\beta)}([O_1]_C|\pi_D) = \{D_i : LB([O_1]_C|D_i) \leq \beta\} = \{Q, F\}.$$

Similarly we can compute the other regions for all others equivalence classes and we form a reformation of table 1 indicating the decision associated with each equivalence class $[x]_C$ which is shown in Table 2 and the discernibility matrix is shown in Table 3. According to this discernibility matrix in Lower interval probability function based decision theoretic rough set we get the reduction as $\{C_1\}, \{C_5\}$. If we compare the reduction in the new model with other models (Pawlak rough set model, Variable precision rough set model, Bayesian rough set model, Bayesian rough set depending on coverage) we get the comparison study between various model which is shown in Table 4. Hence it is seen that Lower interval probability function based decision theoretic rough set model gives the better reduction.

Example 3.4. Let us consider another information table shown in Table 5, Taking $\alpha = 1/6, \beta = 1/7$, a reformation of Table 5, shown as Table 6, indicates the belonging relationship of all equivalence classes $[x]_C$ to the probabilistic region. and a discernibility matrix is shown in Table 7.

Table 3. Discernibility matrix

	O_1	O_2	O_3	O_4	O_5	O_6	O_7	O_8	O_9
O_1	ϕ								
O_2	ϕ	ϕ							
O_3	ϕ	ϕ	ϕ						
O_4	ϕ	ϕ	ϕ	ϕ					
O_5	ϕ	ϕ	ϕ	{1,4,5}	ϕ				
O_6	ϕ	ϕ	ϕ	{1,5}	ϕ	ϕ			
O_7	{1,5}	{1,5}	{1,4,5}	ϕ	ϕ	ϕ	ϕ		
O_8	{1,5}	{1,5}	{1,4,5}	ϕ	{1,4,5}	{1,5}	ϕ	ϕ	
O_9	{1,5}	{1,5}	{1,4,5}	ϕ	{1,4,5}	{1,5}	ϕ	ϕ	ϕ

Table 4. Comparison study

Model	Attribute Reduction
1. Lower interval probability function based decision theoretic rough set model	$\{C_1\}, \{C_5\}$
2. Pawlak rough set model	Reduction not possible
3. Variable precision rough set model	$\{C_1, C_4\}, \{C_4, C_5\}$
4. Bayesian rough set model	$\{C_1, C_4\}, \{C_4, C_5\}$
5. Bayesian rough set depending on coverage	$\{C_1, C_4\}, \{C_4, C_5\}$

Table 5. An information table

	C_1	C_2	C_3	C_4	C_5	C_6	D
O_1	1	1	1	1	1	1	M
O_2	1	1	0	0	1	1	M
O_3	1	1	1	1	1	1	M
O_4	1	1	0	0	1	1	Q
O_5	1	0	1	0	1	1	Q
O_6	1	0	1	0	1	1	F
O_7	1	1	1	0	0	0	F
O_8	1	1	1	0	0	0	F
O_9	1	0	1	0	1	1	F

After reducing the attribute we get the reduction as $\{C_2, C_4\}$,$\{C_2, C_3, C_5\}$, $\{C_2, C_3, C_6\}$,$\{C_3, C_4, C_5\}$, $\{C_3, C_4, C_6\}$. If we compare the reduction in the new method with other methods (Pawlak rough set model, Variable precision rough set model, Bayesian rough set model, Bayesian rough set depending on coverage) we get the comparison which is shown in below:

(1) Lower interval probability function based decision theoretic rough set model: $\{C_2, C_4\}, \{C_2, C_3, C_5\}, \{C_2, C_3, C_6\}$,

Table 6. A reformation of Table 6

	C_1	C_2	C_3	C_4	C_5	C_6	POS	BND	NEG
O_1	1	1	1	1	1	1	{M}	ϕ	{Q,F}
O_2	1	1	0	0	1	1	ϕ	ϕ	{M,Q,F}
O_3	1	1	1	1	1	1	{M}	ϕ	{Q,F}
O_4	1	1	0	0	1	1	ϕ	ϕ	{M,Q,F}
O_5	1	0	1	0	1	1	{F}	ϕ	{M,Q}
O_6	1	0	1	0	1	1	{F}	ϕ	{M,Q}
O_7	1	1	1	0	0	0	ϕ	ϕ	{M,Q,F}
O_8	1	1	1	0	0	0	ϕ	ϕ	{M,Q,F}
O_9	1	0	1	0	1	1	{F}	ϕ	{M,Q}

Table 7. Discernibility matrix

	O_1	O_2	O_3	O_4	O_5	O_6	O_7	O_8	O_9
O_1	ϕ								
O_2	ϕ	ϕ							
O_3	ϕ	ϕ	ϕ						
O_4	{3,4}	ϕ	{3,4}	ϕ					
O_5	{2,4}	{2,3}	{2,4}	ϕ	ϕ				
O_6	{2,4}	{2,3}	{2,4}	{2,3}	ϕ	ϕ			
O_7	{4,5,6}	ϕ	{4,5,6}	ϕ	{2,5,6}	ϕ	ϕ		
O_8	{4,5,6}	ϕ	{4,5,6}	ϕ	{2,5,6}	ϕ	ϕ	ϕ	
O_9	{2,4}	{2,3}	{2,4}	{2,3}	ϕ	ϕ	ϕ	ϕ	ϕ

$\{C_3, C_4, C_5\}, \{C_3, C_4, C_6\}$

(2) Pawlak rough set model: $\{C_4, C_5\}, \{C_4, C_6\}, \{C_2, C_3, C_4\}, \{C_2, C_3, C_5\}, \{C_2, C_3, C_6\}$

(3) Variable precision rough set model: $\{C_3, C_4\}, \{C_2, C_3, C_5\}, \{C_2, C_3, C_6\}, \{C_2, C_4, C_5\}, \{C_2, C_4, C_6\}$

(4) Bayesian rough set model: $\{C_2, C_3, C_4\}, \{C_2, C_3, C_5\}, \{C_2, C_3, C_6\}, \{C_2, C_4, C_5\}, \{C_2, C_4, C_6\}, \{C_3, C_4, C_5\}, \{C_3, C_4, c_6\}$

(5) Bayesian rough set depending on coverage: $\{C_2, C_3, C_4\}, \{C_2, C_3, C_5\}, \{C_2, C_3, C_6\}, \{C_2, C_4, C_5\}, \{C_2,$
$C_4, C_6\}, \{C_3, C_4, C_5\}, \{C_3, C_4, c_6\}$

In the information table shown in Table 8 the significance of attributes C_1, C_2, C_3, C_5, C_6 are zero but the significance of the attribute C_4 is 1/3. So if we remove attribute C_4 then it will effect the consistent decision rule. Using Lower interval probability function based decision theoretic rough set model we get the

TABLE 8. Information table

Example 3.5.

	C_1	C_2	C_3	C_4	C_5	C_6	D
O_1	1	0	0	1	1	1	M
O_2	1	0	0	1	1	1	M
O_3	1	0	1	0	1	0	M
O_4	1	0	1	0	1	0	M
O_5	1	0	0	0	1	1	Q
O_6	1	0	1	0	1	0	Q
O_7	1	1	1	1	0	1	F
O_8	1	0	0	1	1	1	F
O_9	1	1	1	1	0	1	F

TABLE 9. A reformation of table 8

	C_1	C_2	C_3	C_4	C_5	C_6	POS	BND	NEG
O_1	1	0	0	1	1	1	{M}	ϕ	{Q,F}
O_2	1	0	0	1	1	1	{M}	ϕ	{Q,F}
O_3	1	0	1	0	1	0	{M}	{Q}	{F}
O_4	1	0	1	0	1	0	{M}	{Q}	{F}
O_5	1	0	0	0	1	1	ϕ	ϕ	{M,Q,F}
O_6	1	0	1	0	1	0	{M}	{Q}	{F}
O_7	1	1	1	1	0	1	{F}	ϕ	{M,Q}
O_8	1	0	0	1	1	1	{M}	ϕ	{Q,F}
O_9	1	1	1	1	0	1	{F}	ϕ	{M,Q}

region associated with each equivalence classes $[x]_C$ which is shown in Table 9 and the discernibility matrix is shown in Table 10. Using the discernibility matrix we get the reduction as $\{C_3, C_4\}, \{C_2, C_4, C_6\}, \{C_4, C_5, C_6\}$ Now after finding all the discernibility matrix if we reduce the attribute then we get the following results.

(1) By Lower interval probability function based decision theoretic rough set model the reduction is $\{C_3, C_4\}, \{C_2, C_4, C_6\}, \{C_4, C_5, C_6\}$
(2) By pawlak method the reduction is $\{C_3, C_4\}$, $\{C_4, C_5, C_6\}$,$\{C_2, C_4, C_6\}$
(3) By Variable precision rough set method the reduction is $\{C_3, C_4\}$, $\{C_4, C_5, C_6\}$,$\{C_2, C_4, C_6\}$
(4) By Bayesian decision theoretic rough set method the reduction is $\{C_3, C_4\}$, $\{C_4, C_5, C_6\}$,$\{C_2, C_4, C_6\}$
(5) By Bayesian rough set method depending on coverage the reduction is $\{ C_4\}$, $\{C_3, C_6\}$,$\{C_2, C_3, C_5\}$

Hence we get a comparative study between the various methods of attribute reduction.

Table 10. Discernibility matrix

	O_1	O_2	O_3	O_4	O_5	O_6	O_7	O_8	O_9
O_1	ϕ								
O_2	ϕ	ϕ							
O_3	ϕ	ϕ	ϕ						
O_4	ϕ	ϕ	ϕ	ϕ					
O_5	{4}	{4}	{3,6}	{3,6}	ϕ				
O_6	ϕ	ϕ	ϕ	ϕ	{3,6}	ϕ			
O_7	{2,3,5}	{2,3,5}	{2,4,5}	{2,4,5}	{2,3,4,5}	{2,4,5}	ϕ		
O_8	ϕ	ϕ	ϕ	ϕ	{4}	ϕ	{2,3,5}	ϕ	
O_9	{2,3,5}	{2,3,5}	{2,4,5}	{2,4,5}	{2,3,4,5}	{2,4,5}	ϕ	{2,3,5}	ϕ

Proposition 3.6. *Proposition in Lower Interval probability function based decision theoretic rough set model Suppose an information system* $IS = (U, A = C \cup D, V, \rho), X \subseteq U$, $\forall B \subset D$ *if* $D_i, D_j \in \pi_D (i \neq j)$, $F \in \pi_B$ *and* $F = D_i \cup D_j$, *we have*

(1) *If* $D_i \subseteq POS_{(\alpha,\beta)}([x]_C|\pi_D)$ *and*
$D_j \subseteq POS_{(\alpha,\beta)}([x]_C|\pi_D)$, *then* $F \subseteq POS_{(\alpha,\beta)}([x]_C|\pi_B)$.

(2) *If* $D_i \subseteq NEG_{(\alpha,\beta)}([x]_C|\pi_D)$ *and*
$D_j \subseteq NEG_{(\alpha,\beta)}([x]_C|\pi_D)$, *then* $F \subseteq NEG_{(\alpha,2\beta)}([x]_C|\pi_B)$.

(3) *If* $D_i \subseteq BND_{(\alpha,\beta)}([x]_C|\pi_D)$ *and*
$D_j \subseteq BND_{(\alpha,\beta)}([x]_C|\pi_D)$, *then* $F \subseteq BND_{(2\alpha,\beta)}([x]_C|\pi_B)$.

(4) *If* $D_i \subseteq POS_{(\alpha,\beta)}([x]_C|\pi_D)$ *and*
$D_j \subseteq BND_{(\alpha,\beta)}([x]_C|\pi_D)$, *then* $F \subseteq POS_{(\beta,\beta)}([x]_C|\pi_B)$.

() *If* $D_i \subseteq NEG_{(\alpha,\beta)}([x]_C|\pi_D)$ *and*
$D_j \subseteq BND_{(\alpha,\beta)}([x]_C|\pi_D)$, *then* $F \subseteq NEG_{(\alpha,2\alpha)}([x]_C|\pi_B)$.

Proof. (1) Because $D_i \subseteq POS_{(\alpha,\beta)}([x]_C|\pi_D)$ and
$D_j \subseteq POS_{(\alpha,\beta)}([x]_C|\pi_D)$, $LB([x]_C|D_i) \geq \alpha$ and $LB([x]_C|D_j) \geq \alpha$.
For F,
$LB([x]_C|F)$
$= |[x]_C \cap F|/(|[x]_C \cap F| + |[x]_C^c \cup F|)$
$= |[x]_C \cap (D_i \cup D_j)|/(|[x]_C \cap (D_i \cup D_j)| + |[x]_C^c \cup (D_i \cup D_j)|)$
$= (|[x]_C \cap D_i| + |[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |[x]_C^c| + |D_i| + |D_j|)$
$\geq (|[x]_C \cap D_i| + |[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |[x]_C^c| + |[x]_C^c| + |D_i| + |D_j|)$
$\geq \alpha$.
Consequently, $F \subseteq POS_{(\alpha,\beta)}([x]_C|\pi_B)$.

(2) Since $D_i \subseteq NEG_{(\alpha,\beta)}([x]_C|\pi_D)$ and $D_j \subseteq NEG_{(\alpha,\beta)}([x]_C|\pi_D)$,

$$LB([x]_C|D_i) \leq \beta \text{ and } LB([x]_C|D_j) \leq \beta.$$

That is,

$$(|[x]_C \cap D_i|)/(|[x]_C \cap D_i| + |[x]_C^c \cup D_i|) \leq \beta$$

and

$$(|[x]_C \cap D_j|)/(|[x]_C \cap D_j| + |[x]_C^c \cup D_j|) \leq \beta.$$

Now

$$\begin{aligned}
&(|[x]_C \cap (D_i \cup D_j)|)/(|[x]_C \cap (D_i \cup D_j)| + |[x]_C^c \cup (D_i \cup D_j)|) \\
&= (|[x]_C \cap D_i| + |[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |D_i| + |D_j| + |[x]_C^c|) \\
&= |[x]_C \cap D_i|/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |D_i| + |D_j| + |[x]_C^c|) \\
&\qquad + (|[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |D_i| + |D_j| + |[x]_C^c|) \\
&< |[x]_C \cap D_i|/(|[x]_C \cap D_i| + |D_i| + |[x]_C^c|) + |[x]_C \cap D_j|/(|[x]_C \cap D_j| + |D_j| + |[x]_C^c|) \\
&= LB([x]_C|D_i) + LB([x]_C|D_j) \\
&\leq 2\beta.
\end{aligned}$$

Thus $F = D_i \cup D_j \subseteq NEG_{(\alpha,2\beta)}([x]_C|\pi_B)$.

(3) Since $D_i \subseteq BND_{(\alpha,\beta)}([x]_C|\pi_D)$ and $D_j \subseteq BND_{(\alpha,\beta)}([x]_C|\pi_D)$,

$$\beta < LB([x]_C|D_i) < \alpha \text{ and } \beta < LB([x]_C|D_j) < \alpha.$$

Now

$$\begin{aligned}
&(|[x]_C \cap (D_i \cup D_j)|)/(|[x]_C \cap (D_i \cup D_j)| + |[x]_C^c \cup (D_i \cup D_j)|) \\
&= (|[x]_C \cap D_i| + |[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |D_i| + |D_j| + |[x]_C^c|) \\
&= |[x]_C \cap D_i|/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |D_i| + |D_j| + |[x]_C^c|) \\
&\qquad + (|[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |D_i| + |D_j| + |[x]_C^c|) \\
&< |[x]_C \cap D_i|/(|[x]_C \cap D_i| + |D_i| + |[x]_C^c|) + |[x]_C \cap D_j|/(|[x]_C \cap D_j| + |D_j| + |[x]_C^c|) \\
&= LB([x]_C|D_i) + LB([x]_C|D_j) \\
&< 2\alpha.
\end{aligned}$$

Also

$$\begin{aligned}
&|[x]_C \cap (D_i \cup D_j)|/(|[x]_C \cap (D_i \cup D_j)| + |[x]_C^c \cup (D_i \cup D_j)|) \\
&= (|[x]_C \cap D_i| + |[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |[x]_C^c| + |D_i| + |D_j|) \\
&\geq (|[x]_C \cap D_i| + |[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |[x]_C^c| + |[x]_C^c| + |D_i| + |D_j|) \\
&> \beta.
\end{aligned}$$

So $F = D_i \cup D_j \subseteq BND_{(2\alpha,\beta)}([x]_C|\pi_B)$.

(4) Since $D_i \subseteq POS_{(\alpha,\beta)}([x]_C|\pi_D)$ and $D_j \subseteq BND_{(\alpha,\beta)}([x]_C|\pi_D)$,

$$(|[x]_C \cap D_i|)/(|[x]_C \cap D_i| + |[x]_C^c \cup D_i|) \geq \alpha > \beta$$

and

$$\beta < (|[x]_C \cap D_j|)/(|[x]_C \cap D_j| + |[x]_C^c \cup D_j|) < \alpha.$$

Now,

$$\begin{aligned}
&|[x]_C \cap (D_i \cup D_j)|/(|[x]_C \cap (D_i \cup D_j)| + |[x]_C^c \cup (D_i \cup D_j)|) \\
&= (|[x]_C \cap D_i| + |[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |[x]_C^c| + |D_i| + |D_j|) \\
&\geq (|[x]_C \cap D_i| + |[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |[x]_C^c| + |[x]_C^c| + |D_i| + |D_j|) \\
&> \beta.
\end{aligned}$$

Then $F = D_i \cup D_j \subseteq POS_{(\beta,\beta)}([x]_C|\pi_B)$.

(5) Since $D_i \subseteq NEG_{(\alpha,\beta)}([x]_C|\pi_D)$ and $D_j \subseteq BND_{(\alpha,\beta)}([x]_C|\pi_D)$,

$$(|[x]_C \cap D_i|)/(|[x]_C \cap D_i| + |[x]_C^c \cup D_i|) \leq \beta < \alpha$$

and

$$\beta < (|[x]_C \cap D_j|)/(|[x]_C \cap D_j| + |[x]_C^c \cup D_j|) < \alpha.$$

Now

$(|[x]_C \cap (D_i \cup D_j)|)/(|[x]_C \cap (D_i \cup D_j)| + |[x]_C^c \cup (D_i \cup D_j)|)$
$= (|[x]_C \cap D_i| + |[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |D_i| + |D_j| + |[x]_C^c|)$
$= |[x]_C \cap D_i|/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |D_i| + |D_j| + |[x]_C^c|) + (|[x]_C \cap D_j|)/(|[x]_C \cap D_i| + |[x]_C \cap D_j| + |D_i| + |D_j| + |[x]_C^c|)$
$< |[x]_C \cap D_i|/(|[x]_C \cap D_i| + |D_i| + |[x]_C^c|) + |[x]_C \cap D_j|/(|[x]_C \cap D_j| + |D_j| + |[x]_C^c|)$
$= LB([x]_C|D_i) + LB([x]_C|D_j) < 2\alpha$

Thus, $F \subseteq NEG_{(\alpha,2\alpha)}([x]_C|\pi_B)$. □

Proposition 3.7. *Proposition in Bayesian decision theoretic rough set model Suppose an information system* $IS = (U, A = C \cup D, V, \rho), X \subseteq U, \forall B \subset A$, *if* $[x]_m, [x]_n \in \pi_A (m \neq n)$, *and* $F = [x]_m \cup [x]_n, F \in \pi_B$ *we have*

(1) *If* $[x]_m \subseteq POS^*_{D_i}([x]_C)$ *and* $[x]_n \subseteq POS^*_{D_i}([x]_C)$, *then* $F \subseteq POS^*_{D_i}([x]_{C'})$.
(2) *If* $[x]_m \subseteq NEG^*_{D_i}([x]_C)$ *and* $[x]_n \subseteq NEG^*_{D_i}([x]_C)$, *then* $F \subseteq NEG^*_{D_i}([x]_{C'})$.
(3) *If* $[x]_m \subseteq BND^*_{D_i}([x]_C)$ *and* $[x]_n \subseteq BND^*_{D_i}([x]_C)$, *then* $F \subseteq BND^*_{D_i}([x]_{C'})$.
(4) *If* $[x]_m \subseteq POS^*_{D_i}([x]_C)$ *and* $[x]_n \subseteq BND^*_{D_i}([x]_C)$, *then* $F \subseteq POS^*_{D_i}([x]_{C'})$.
(5) *If* $[x]_m \subseteq NEG^*_{D_i}([x]_C)$ *and* $[x]_n \subseteq BND^*_{D_i}([x]_C)$, *then* $F \subseteq NEG^*_{D_i}([x]_{C'})$.

Here C' *is the condition attribute set in* B.

Proposition 3.8. *Proposition in Bayesian rough set depending on coverage Suppose an information system* $IS = (U, A = C \cup D, V, \rho), X \subseteq U$, $\forall B \subset A$, *if* $[x]_m, [x]_n \in \pi_A (m \neq n)$, *and* $F = [x]_m \cup [x]_n$, $F \in \pi_B$ *we have*

(1) *If* $[x]_m \subseteq POS_{cov(\pi_A)}(D_i)$ *and* $[x]_n \subseteq POS_{cov(\pi_A)}(D_i)$, *then* $F \subseteq POS_{cov(\pi_B)}(D_i)$.
(2) *If* $[x]_m \subseteq NEG_{cov(\pi_A)}(D_i)$ *and* $[x]_n \subseteq NEG_{cov(\pi_A)}(D_i)$, *then* $F \subseteq NEG_{cov(\pi_B)}(D_i)$.
(3) *If* $[x]_m \subseteq BND_{cov(\pi_A)}(D_i)$ *and* $[x]_n \subseteq BND_{cov(\pi_A)}(D_i)$, *then* $F \subseteq BND_{cov(\pi_B)}(D_i)$.
(4) *If* $[x]_m \subseteq POS_{cov(\pi_A)}(D_i)$ *and* $[x]_n \subseteq BND_{cov(\pi_A)}(D_i)$, *then* $F \subseteq POS_{cov(\pi_B)}(D_i)$.
(5) *If* $[x]_m \subseteq NEG_{cov(\pi_A)}(D_i)$ *and* $[x]_n \subseteq BND_{cov(\pi_A)}(D_i)$, *then* $F \subseteq NEG_{cov(\pi_B)}(D_i)$.

Proof. Straight forward. □

Proposition 3.9. *The definition of Bayesian rough set and Bayesian rough set depending on coverage is quite different but in case of positive, negative and boundary region there is some relation between these two model.*

(1) *If an equivalence class* $[x]_C$ *belongs to Bayesian decision theoretic rough set positive region iff its also belongs to Bayesian rough set depending on coverage positive region.*

(2) *If an equivalence class* $[x]_C$ *belongs to Bayesian decision theoretic rough set boundary region iff its also belongs to Bayesian rough set depending on coverage boundary region.*

(3) *If an equivalence class $[x]_C$ belongs to Bayesian decision theoretic rough set negative region iff its also belongs to Bayesian rough set depending on coverage negative region.*

Proof. Only proof of (1) is shown here, other two are similar. Let $[x]_C$ belongs to Bayesian decision theoretic rough set positive region. i.e., for a decision D_i, $|[x]_C \cap D_i|/|[x]_C| > P(D_i)$, i.e.,$|[x]_C \cap D_i| > P(D_i)|[x]_C|$.
Now,$|[x]_C \cap D_i|/|D_i| > P(D_i)|[x]_C|/|D_i| = |[x]_C|/\sum(D_i) = |[x]_C|/\|U| = P([x]_C)$,
i.e., $|[x]_C \cap D_i|/|D_i| > P([x]_C)$.

This implies $[x]_C$ belongs to Bayesian rough set depending on coverage positive region.
Now for the converse part let us assume $[x]_C$ belongs to Bayesian rough set depending on coverage positive region.

i.e., $|[x]_C \cap D_i|/|D_i| > P([x]_C)$,
i.e., $|[x]_C \cap D_i| > P([x]_C)|D_i|$,
i.e., $|[x]_C \cap D_i|/|[x]_C| > P([x]_C)|D_i|/|[x]_C| = |D_i|/|U| = P(D_i)$,
i.e., $|[x]_C$ belongs to Bayesian decision theoretic rough set positive region. □

4. Conclusion

In this paper the concept of attribute reduction using Lower Interval probability function based Decision Theoretic Rough set model is shown. Also considering some example the comparison of Lower Interval probability function based Decision Theoretic Rough Set Model with other methods are shown. It is found that the reduct by this method gives the better reduction. To deal with huge data a Matlab program is to be tested using original data.

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